

**CHAPTER 1**  
**BACKGROUND AND OVERVIEW**  
**OF THE STUDY**

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**1.1 Problem statement and motivation for the study**

The transition from secondary level to tertiary level in mathematics is a complex phenomenon with a whole spectrum of problems and difficult situations to deal with. It is a traumatic experience for students and many students are not prepared for tertiary mathematics (Dreyfus, 1999). Successful entrants already went through a tough matriculation exam, as well as competitive admission applications, and have to adapt to new learning environments, new modes of study, new peers and professors and higher expectations (Luk, 2005:161). They need to deal with these changes, as well as changes in the curriculum and in pedagogy that could be very stressful.

As a result of the implementation of the Revised National Curriculum Statement (RNCS) in the Further Education and Training (FET) band in South Africa from 2006 to 2008, the first year students entering universities in 2009 have been exposed to OBE for their entire school career. There was great concern regarding the preparedness of these students for university studies. Early in 2009 it became apparent from articles in the newspapers that these students were not performing as well as the first years from previous years in mathematics. A diagnostic mathematics test was written by the first year mathematics students of both 2008 and 2009 before starting their mathematics modules at the North-West University. In a comparison of the results of these two groups it was found that there was not a statistically significant difference in the performance of these students. The question can be raised why these students did not perform as expected in their first test at tertiary level? Professor Barry Green from the University of Stellenbosch is of the opinion that the blame

for the students not performing as they should can not solely be put on the school education system that is “insufficient”. He suggests that the different natures of secondary and tertiary education need to be explored by both levels so that each can gain a better understanding of the other (Botha, 2009:9).

## **1.2 Review of literature**

The consequences of this problem are of great concern to mathematics lecturers at tertiary institutions worldwide (Hong *et al.*, 2009:877; Gueudet, 2008:237). Studies in different countries show that there seems to be a widening of the gap between secondary and tertiary mathematics (Brandell *et al.*, 2008, Clark & Lovric, 2008, Gruenwald, *et al.*, 2003). From the lecturers' point of view, students are less competent, with a lack of background knowledge and skills required for entry at university. From the students' point of view, their first year of study has confusing experiences to adapt to, such as the totally new learning environment with new modes of reasoning and thinking. The classes are large and the extent of personal attention, when compared to the secondary school, is drastically lower.

In South Africa there has been a change in the teaching and learning process since 1997 with the implementation of Outcomes Based Education followed by the RNCS in secondary schools from 2006. There was a shift from a teacher centred approach, where the teacher was the person to transfer knowledge, to a learner centred activity-based approach, where the teacher is the facilitator and the learners take responsibility for their own learning (DoE, 2003:2). This shift to an activity-based student centred approach to teaching requires fundamental changes. Whether the teacher is able to make this shift depends on the teachers' system of beliefs and conception of the nature of mathematics.

Research has shown that people's views of mathematics teaching are deeply rooted in their views of mathematics and that they do not discard these views easily (Ernest, 1988). Direct teaching is rooted in the static-formalist view in which mathematics is seen as a fixed and static body of knowledge consisting of a logical and meaningful network of inter-related truths - facts, rules and algorithms (Thompson, 1992:132). The assumption is that one can gradually unfold and discover this body of knowledge in neat chunks and that the lecturer

is able to transfer these chunks of knowledge to the student (Nieuwoudt, 1998:69). In this context understanding of mathematics depends on how well the lecturer explains it. In contrast with this the NCS and the constructivist approach to teaching and learning are based on a relativist–dynamic view of mathematics. It is a dynamic view where mathematics is seen as a continually expanding field in which patterns are generated and then distilled into knowledge that can be used to solve real-life problems (Van de Walle, 2007:13, Nieuwoudt, 1998:71).

A person’s mathematical beliefs include their beliefs about the nature of mathematics, as well as their beliefs about the teaching and the learning of mathematics (Thompson, 1992). Students have certain preferences with regard to the learning of mathematics and lecturers have preferences with regard to their teaching practices. Students and lecturers may have different views about these because they originally stem from different eras. This could create a gap in the transition from secondary to tertiary mathematics.

Mathematics at university is more rigorous and of an abstract nature, in contrast with the more heuristic, concrete approach in secondary school (Luk, 2005:162). The students’ cognitive development should be taken into account when dealing with these. Piaget’s cognitive development theory has influenced how we view the way individuals learn and the processes that people go through while constructing their own knowledge. As students develop through certain stages of cognitive development, they acquire more advanced competencies that assist them in learning mathematics with understanding. Knowledge of different cognitive development theories can assist us to see how a child progresses through certain developmental processes of learning. Teachers cannot push learners through these processes too early and expect of them to learn concepts before they are capable of fully understanding it.

The transition from secondary to tertiary mathematics includes a transition from elementary mathematical thinking to advanced mathematical thinking. Elementary mathematical thinking begins with a focus on concrete objects in order to generalize related concepts and processes, while advanced mathematical thinking refers to the stage when students begin to deal with abstract concepts and deductive proofs (Smith, 2004:1). Edwards *et al.* (2005:

17) define advanced mathematical thinking as: “thinking that requires deductive and rigorous reasoning about mathematical notions that are not entirely accessible to us through our five senses”. In elementary mathematics one describes objects and the description is constructed from experience of the object, but in advanced mathematics objects are defined and the properties constructed from the definition (Semadeni, 2008:7). This indicates that the move from elementary to advanced mathematical thinking requires a reconstruction of thinking: from describing to defining and from convincing to proving (Tall, 1991:20). Due to a lack of exposure to deductive experiences at school level, students tend to have limited advanced reasoning skills and struggle with formal abstractions in their first year (Stewart & Thomas, 2009:960). In addition the formal mathematics language and new laws used by mathematicians on tertiary level alienates students. They have to get accustomed to this new language with new symbols and grammatical rules for reasoning and proving (Gueudet, 2008:242-244). Therefore the transition from elementary to advanced mathematical thinking also needs to include the linking of informal with formal mathematics language.

It is clear that there are diverse factors that are potential causes of insufficient mathematical performance of first year students. The transition from secondary to tertiary mathematics is a major issue that urgently needs a solution that should come from inside the university instead of waiting for decisions imposed from the outside. Lecturers at tertiary institutions put the blame for the under-preparedness of first year students on the new curriculum and the new approach of teaching in secondary schools (Anon., 2009). However, undergraduate lecturers should consider what they can do to address the problem, since the universities cannot influence what happens in schools. Universities should take responsibility for the quality of teaching and learning at university level and cannot remain unchanged, clinging to outdated approaches (Marshall, 2010:69-74). The changes in the school curriculum in South Africa brought challenges for university lecturers with demands for changes in the university curricula. Although changes need time, effort and consensus, it may be worthwhile to unpack these problems in order to search for solutions.

This study explored the transition from secondary to tertiary mathematics against the background of the nature and structure of mathematics. This was achieved through an extensive literature review and empirical investigations regarding the beliefs of students and their lecturers on the nature of mathematics and how mathematics should be learned. The research included an investigation of different cognitive development theories for mathematical development and the cognitive processes through which mathematical understanding develops. The focus is therefore on the nature and structure of mathematics and how the learning of mathematics with understanding takes place.

### **1.3 Research aims**

The aim of the research was to investigate the gap between secondary and tertiary mathematics with respect to three specific domains, namely beliefs on the nature of mathematics, beliefs on the teaching and learning of mathematics and the transition from elementary to advanced thinking.

In an attempt to achieve these aims the following research questions were addressed:

- How the beliefs of the nature and learning of mathematics held by the students compare to those held by their lecturers;
- What the preferences of the students are regarding the learning of mathematics compared to how the lecturers perceive it to be;
- How students master the transition from elementary to advanced thinking by analysis of their reasoning abilities.

Finally, the findings are used to make recommendations on how to facilitate the transition from secondary to tertiary mathematics.

### **1.4 Literature study**

The literature study comprises an in-depth survey of the literature available on beliefs and views of mathematics, the structure of mathematical knowledge and the cognitive development of mathematical thinking. The survey was conducted by means of EBSCOhost and internet search engines.

The following keywords were used: “beliefs”, “worldview”, “paradigm shift”, “modernism”, “post-modernism”, “views of mathematics”, “the nature of mathematics”, “teaching and learning mathematics”, “cognitive development”, “gap between secondary and tertiary mathematics”, “elementary and advanced mathematical thinking”, “learning and teaching styles”, “conceptual and procedural knowledge”, “mathematical under-preparedness”, “transition from secondary to tertiary mathematics”, “representations”, “mathematical reasoning”, “inductive and deductive reasoning”, “cognitive development”.

## **1.5 Empirical study**

### **1.5.1 Research design**

The empirical study consisted of a diagnostic analysis of three domains and therefore a cross-sectional survey design was chosen. The research was done in three parts.

Phase 1: An investigation into the views of mathematics held by the students and the lecturers using a self-constructed questionnaire (see par. 5.3).

Phase 2: An investigation to establish the students’ preferences on how they learn mathematics and how mathematics should be taught, using the Index of Learning Styles (ILS) questionnaire of Felder and Silverman. The results have been compared with the way lecturers want their students to learn and how they themselves prefer to teach (see par. 5.4).

Phase 3: A classification of the questions in the first mathematics test written at tertiary level using a framework developed by Palm *et al.* (2011:229-237) and subsequent analysis of the answers of students to obtain information on the type of reasoning required from students at tertiary level. A framework developed by Palm *et al.* (2011:229-237) was used for the classification and the analyses of the test (see par. 5.5).

## 1.5.2 Study population and sample

The study population for the empirical study consisted of first year university mathematics students. The sample used for this study was the first year students ( $\pm 500$ ) of 2010 enrolled for the module WISN111 at the North-West University (Potchefstroom campus), as well as all the mathematics lecturers of the Faculty of Natural Sciences who were willing to participate in the research.

## 1.5.3 Measuring instruments

Phase 1: The questionnaire was designed to determine the students' and lecturers' conceptions about the nature of mathematics and was based on a questionnaire developed and used by Nieuwoudt (1998). Relevant questions for the present study were extracted from the existing questionnaire.

Phase 2: The Index of Learning Styles (ILS) questionnaire is an instrument designed to assess preferences on the four dimensions of the Felder-Silverman learning style model (Felder & Spurlin, 2005:103). In 1988, they formulated this model to capture the most important learning style differences among engineering students (Felder & Silverman, 1988:674). A version of the ILS that was specifically adapted to be used by lecturers by Visser *et al.* (2006) to analyse lecturers' teaching preferences was used for the purposes of this study.

Phase 3: The first mathematics test written in the first semester of the WISN111 module at the North-West University (Potchefstroom Campus) and the marks that the students obtained for each question.

## 1.5.4 Statistical techniques

Phase 1: An exploratory factor analysis was conducted of the *beliefs questionnaire* determining the Cronbach alpha coefficients. The p-values were used to test if the differences in the means of the students and the lecturers were statistically significant. Cohen's d-

value was used to determine the effect sizes of the differences between the students' choices of each answer and was used to conclude whether these differences were practically significant.

Phase 2: Cronbach alpha coefficients were determined for the *ILS* questionnaire as well. Chi-square tests were used to determine the p-values to evaluate the significant differences between the means of the students and the lecturers. Phi-coefficients were used to determine the effect sizes of the differences between the means of the students and the lecturers. Cohen's d-value was used to determine the effect sizes of the differences between the students' choices of each answer and used to conclude whether these differences were practically significant.

This was done with the help of the Statistical Consultation Services (SCS) at the North-West University (Potchefstroom Campus). The results of the questionnaires on the views and the teaching and learning styles, as well as the results of the tests were analysed and interpreted.

## **1.6 Chapter outline**

Chapter 1: Statement of the problem and motivation for the study.

Chapter 2: The gap between secondary and tertiary level concerning beliefs of the nature and learning of mathematics

This chapter focuses on the different conceptions of what mathematics is and how this will influence the way teaching and learning takes place. The gap between the secondary and tertiary curriculum is discussed, as well as the implications that the intended and the implemented curriculum has on the attained curriculum.

Chapter 3: The gap between secondary and tertiary level concerning beliefs on the structure of mathematics

This chapter focuses on the structure of mathematical knowledge. The processes through which mathematical understanding develops are investigated and discussed with a focus on the



different reasoning processes necessary for mathematical proficiency.

Chapter 4: The gap in the transition from elementary to advanced thinking

In this chapter the cognitive development in a mathematics domain is investigated according to the theories of Piaget, Van Hiele and Tall.

Chapter 5: Empirical study

This chapter provides an outline and description of the empirical study, as well as the results of the different parts of the study.

Chapter 6: Conclusions and recommendations

This chapter offers means to assist lecturers and students in the transition from secondary to tertiary mathematics.

## **1.7 Value of the research**

This study forms part of the project entitled “The transition from secondary to tertiary mathematics” in the Foundations and Ethics subgroup of the Research Focus Area of Business Mathematics and Informatics. This study was undertaken to investigate whether and how the way that incoming first year students process and understand mathematics differ from the nature of tertiary mathematics and how it is presented. As one of the results of this research the researchers could present a profile of first year mathematics students to the university that can assist lecturers to better understand the reasoning skills and cognitive development thinking phases of incoming first year students. Furthermore, this profile can give lecturers an idea of first year mathematics students’ beliefs regarding the nature of mathematics, the way they prefer to learn mathematics, as well as their reasoning proficiency. The research also provides useful information on the cognitive development phases that students go through, as well as how to adapt one’s teaching to lead students to advanced mathematical thinking.