

**ON THE DEVELOPMENT AND
APPLICATIONS OF A
THREE-DIMENSIONAL *AB INITIO*
COSMIC-RAY MODULATION MODEL**

N. E. ENGELBRECHT

On the development and applications of a three-dimensional *ab initio* cosmic-ray modulation model

N. E. Engelbrecht, M.Sc.

Thesis accepted for the degree Doctor Philosophiae in Physics at the Potchefstroom
Campus of the North-West University

Promotor: Prof. R. A. Burger
Co-promotors: Prof. S. E. S. Ferreira
Dr. M. Hitge

November, 2012

*This is my lot; for either still I find
Some imperfection in the chosen theme,
Or see of absolute accomplishment
Much wanting, so much wanting, in myself,
That I recoil and droop, and seek repose
In listlessness from vain perplexity,
Unprofitably travelling toward the grave,
Like a false steward who hath much received
And renders nothing back.*

- Wordsworth

To Remona, Peter, and Herman

Abstract

A proper understanding of the effects of turbulence on the diffusion and drift of cosmic-rays in the heliosphere is imperative for a better understanding of cosmic-ray modulation. This study presents an *ab initio* model for cosmic-ray modulation, incorporating for the first time the results yielded by a two-component turbulence transport model. The latter model is solved for solar minimum heliospheric conditions, utilizing boundary values chosen in such a way that the results of this model are in fair to good agreement with spacecraft observations of turbulence quantities, not only in the ecliptic plane, but also along the out-of-ecliptic trajectory of the *Ulysses* spacecraft. These results are employed as inputs for modelled slab and 2D turbulence energy spectra, which in turn are used as inputs for parallel mean free paths based on those derived from quasi-linear theory, and perpendicular mean free paths from extended nonlinear guiding center theory. The modelled 2D spectrum is chosen based on physical considerations, with a drop-off at the very lowest wavenumbers commencing at the 2D outerscale. There currently exist no models or observations for this quantity, and it is the only free parameter in this study. The use of such a spectrum yields a non-divergent 2D ultrascale, which is used as an input for the reduction terms proposed to model the effects of turbulence on cosmic-ray drifts. The resulting diffusion and drift coefficients are applied to the study of galactic cosmic-ray protons, electrons, antiprotons, and positrons using a three-dimensional, steady-state numerical cosmic-ray modulation code. The magnitude and spatial dependence of the 2D outerscale is demonstrated to have a significant effect on computed cosmic-ray intensities. A form for the 2D outerscale was found that resulted in computed cosmic-ray intensities, for all species considered, in reasonable agreement with multiple spacecraft observations. Computed galactic electron intensities are shown to be particularly sensitive to choices of parameters pertaining to the dissipation range of the slab turbulence spectrum, and certain models for the onset wavenumber of the dissipation range could be eliminated in this study.

Keywords: turbulence, diffusion, drift,
cosmic rays, modulation, heliosphere,
heliospheric current sheet

Opsomming

Insig in die effek wat turbulensie op die diffusie en dryf van kosmiese strale in die heliosfeer het, is noodsaaklik om dieper insig in die modulasie van kosmiese strale te verkry. In hierdie studie word 'n *ab initio* model vir die modulasie van galaktiese kosmiese strale voorgehou, wat vir die eerste keer resultate wat met 'n twee-komponent model vir turbulensie transport verkry is, insluit. Die turbulensie model word opgelos vir 'n tydperk van minimum son aktiwiteit, en die randwaardes wat gebruik word lewer aanvaarbare ooreenstemming met turbulensie data wat deur ruimtetuie verkry is, nie net in die ekliptiese vlak nie, maar ook langs die baan van die *Ulysses* ruimtetuig buite hierdie vlak. Die berekende turbulensie groothede dien as invoer vir die modelle vir die energie spektra vir een- en twee-dimensionele turbulensie. Hierdie turbulensie spektra word dan gebruik om vrye weg lengtes te bereken, ewewydig aan die magneetveld soos beskryf deur kwasi-lineêre teorie, en loodreg op die veld soos beskryf deur uitgebreide nie-lineêre leimiddelpunt teorie. Die model vir twee-dimensionele turbulensie sny af by 'n klein golfgetal, gemotiveer deur fisiese oorwegings, en verseker dat 'n eindige ultraskaal bestaan. Daar bestaan tans geen model of metings vir die golfgetal waar die spektrum moet afsny nie, en dit word die enigste vrye parameter in die studie. Die ultraskaal is van belang vir die turbulente afskaling van dryf. Die dryf koëffisiënt en diffusie koëffisiënte wat bereken is, word dan gebruik om die modulasie van galaktiese protone, antiprotone, elektrone en positrone te bestudeer. Die golfgetal waar die twee-dimensionele turbulensie spektrum afsny, speel 'n belangrike rol in die modulasie proses. 'n Grootte en ruimtelike afhanklikheid van hierdie golfgetal is verkry wat tot aanvaarbare ooreenstemming van model resultate met verskeie kosmiese-straal datastelle lei. Daar word voorts aangetoon dat galaktiese elektrone besonder gevoelig is vir die golfgetal van die een-dimensionele spektrum waar verlies van energie begin, en die spektraal indeks in die verliesgebied. Die modulasie van elektrone kan gebruik word om die struktuur van turbulensie in die verliesgebied te ondersoek, en in hierdie studie kon sekere modelle vir die golfgetal waar die verliesgebied begin, geëlimineer word.

Sleutelwoorde: turbulensie, diffusie, dryf,
kosmiese strale, modulasie, heliosfeer,
heliosferiese neutrale vlak

Abbreviations

Listed below are the acronyms and abbreviations used in the text. For the purposes of clarity, any such usages are written out in full when they first appear.

2D	two-dimensional
3D	three-dimensional
AU	astronomical unit
CIR	corotating interaction region
FLS	fast latitude scan
HMF	heliospheric magnetic field
MHD	magnetohydrodynamic
NLGC	nonlinear guiding center theory
QLT	quasi-linear theory
TPE	transport equation
WKB	Wentzel-Kramers-Brillouin

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