

## CHAPTER 8

### THEORETICAL PREDICTION

Using the values determined from the experimental characterisation in the previous section, the effect of the replacement of the stiff steel connection with soft rubber mounts was predicted using the mathematical models described in section 5.

#### 8.1. Input variables

To predict the difference between the forces in the plates and other elastic elements for the different mounting cases, the mathematical models were programmed in Matlab. They were then run with different input values to describe the differences between the stiff steel-mounted case and the soft rubber-mounted case.

There were certain common input values not dependent on the mounting system. The mass of the system was assumed to remain constant.

From the mass properties from Table 13 were entered into the mass matrix according to Equation (5.17) and is shown in Equation (8.1).

$$[M] = \begin{bmatrix} 22.255 & 0 \\ 0 & 24.007 \end{bmatrix} \quad (8.1)$$

This mass matrix was used in both of the two DOF models – for the stiff steel-mounted and the soft rubber-mounted cases, as the heat exchanger itself was not modified. The other variables, however, were changed to simulate the different mounting conditions that were compared.

##### 8.1.1. Stiff steel-mounted case input variables

For the stiff steel-mounted case, the vibration model was mounted in the same way as the current heat exchanger was mounted to the top of the columns, which was by bolting the top structure directly to the frame.

These stiffness values identified in Table 13 could be combined into the stiffness matrix according to Equation (5.20) to result in the matrix illustrated in Equation (8.2).

$$[K]_R = \begin{bmatrix} 32316997 & -319997 \\ -319997 & 356526 \end{bmatrix} \quad (8.2)$$

The stiffness matrix in Equation (8.2) together with the mass matrix in Equation (8.1), was sufficient to describe the system to be modelled when damping was neglected.

In the model where damping was taken into account, the damping matrix also needed to be specified.

With the values from Table 13, the damping coefficient matrix for the stiff steel-mounted case described in Equation (5.34) was determined, as shown in Equation (8.3).

$$[C]_R = \begin{bmatrix} 43.699 & -43.699 \\ -43.699 & 70.529 \end{bmatrix} \quad (8.3)$$

#### 8.1.2. Soft rubber-mounted case input variables

The other case, to which the stiff steel-mounted case had been compared, was the case where soft rubber mounts were placed between the top structure of the heat exchanger model and the frame. The input variables, therefore, had to be changed to model the new mounting system.

Both the stiffness and the damping input values were affected.

The values obtained from the Table 13 were used to construct the stiffness matrix for the soft rubber-mounted case ( $[K]_M$ ) by using Equation (5.19) resulting in the matrix described in Equation (8.4).

$$[K]_M = \begin{bmatrix} 480834 & -319997 \\ -319997 & 356526 \end{bmatrix} \quad (8.4)$$

From the values indicated in Table 13, the damping coefficient matrix for the soft rubber-mounted case ( $[C]_M$ ) could be determined by using Equation (5.33) and resulting in the matrix shown in Equation (8.5).

$$[C]_M = \begin{bmatrix} 343.979 & -43.699 \\ -43.699 & 70.529 \end{bmatrix} \quad (8.5)$$

The input matrices in Equations (8.1) to (8.5) were entered into the computer programmes for the two DOF model without damping. This is illustrated in Appendix A, while the two DOF model taking damping into account, is illustrated in Appendix B.

## 8.2. Computer programme algorithms

The two mathematical models discussed in sections 5.1 and 5.2 were programmed in Matlab to facilitate the theoretical prediction of the response of the systems with and without taking damping into account.

### 8.2.1. Two DOF model without damping

The Matlab programme used to simulate the mathematical model can be seen in Appendix A.

The process followed by the programme can be illustrated in the form of an algorithm as follows:

- Input variables for the two mounting cases as described in section 8.1 were entered into the programme. The soft rubber-mounted case was identified by the index 1 and the stiff steel-mounted case was identified by the index 2. Only the values of  $k_1$  varied between the two cases ( $k_{1R}$  and  $k_{1M}$ ).
- From the input values, the mass and stiffness matrices were populated, as indicated in the previous section.
- The input force  $F_0$  was described in terms of the forcing frequency, as determined in section 7.6.
- For each of the cases the system was subjected to a number of calculations:

- The *eigenvalues* were calculated to determine the system's natural frequencies.
- For each of the range of forcing frequencies the response of the system was calculated, using a linear solving algorithm for the matrix equation.
  - The solving of the equation resulted in the amplitude of displacement of the top- ( $X_1$ ) and bottom structures ( $X_2$ ).
  - The displacement values were used to calculate the amplitude of the forces in the relative elastic elements ( $k_1$ ,  $k_2$  and  $k_3$ ) for each of the forcing frequencies selected.
- The amplitude of the resulting displacement and forces were plotted over the entire range of frequencies analysed for all the cases.
- The relative magnitudes of the forces calculated for the two different cases were compared and plotted

#### 8.2.2. Two DOF model with damping

The two DOF model taking damping into account could not be programmed to analyse a range of frequencies in one run as the previous model, due to the extremely long calculation time that resulted from the writing and rewriting of ".dat" files between the solver and the running programme.

For this reason, it was decided to design the programme to run on one forcing frequency at a time, with the user changing the parameters between runs. The complete programme can be seen in Appendix B.

The process followed by the programme can be described by the following algorithm:

- The input variables were entered into ".dat" files by the user, named "M.dat", "K.dat", "C.dat" and "w.dat" for the mass matrix, the stiffness matrix, the damping matrix and the frequency of the oscillating force, respectively. The values for the matrices were obtained from Section 8.1.

- As the programme calculated the response in the time domain, the time steps and the total time were determined, using the forcing frequency as guide.
- The calculation was done with the ODE23 function for integration in Matlab for each of the time steps.
- The second half of the time steps (after transient response had faded) was used to calculate resultant forces in the elements. For each of the time steps:
  - The force resulting from the stiffness of the element was calculated;
  - the force resulting from the damping of the element was calculated;
  - the actual displacement of the top and bottom structures were determined; and
  - the top section of Equations (5.41), (5.42), (5.44) and (5.45) were implemented to determine the sum values needed for the calculation of the RMS values.
- The final calculations were done to determine the RMS values of all the forces, the displacements and the total resulting forces in the elements.
- The actual time spectrum values were plotted to ensure that the transient response had died off before the values, used for the RMS calculations, were taken.
- The resulting RMS values were manually recorded for later comparison between different runs.

### 8.3. Results

Using the general input data shown in section 8.1, the simulations were carried out with the different mathematical models to estimate the effects of the design changes.

### 8.3.1. 2 DOF model without damping

The two DOF programme without taking damping into account, provided preliminary results for the effect of a change in design on the response of the system. The complete source code can be seen in Appendix A.

#### 8.3.1.1. *Natural frequencies*

The model predicted the two natural frequencies of the system for both the stiff steel-mounted and the soft rubber-mounted cases. The values of the natural frequencies are shown in Table 14.

**Table 14: Predicted natural frequencies of experimental model**

	Stiff steel-mounted case	Soft rubber-mounted case
First natural frequency [Hz]	19.387	10.038
Second natural frequency [Hz]	603.805	28.683

From the predicted values it can be seen that the addition of the soft rubber mounts and compensators reduced both the natural frequencies.

In the stiff steel-mounted case, the second natural frequency is predicted to be significantly higher than the 50 Hz, which is the highest frequency that the experimental measurements will be done at and would therefore have very little effect on the response in the test range.

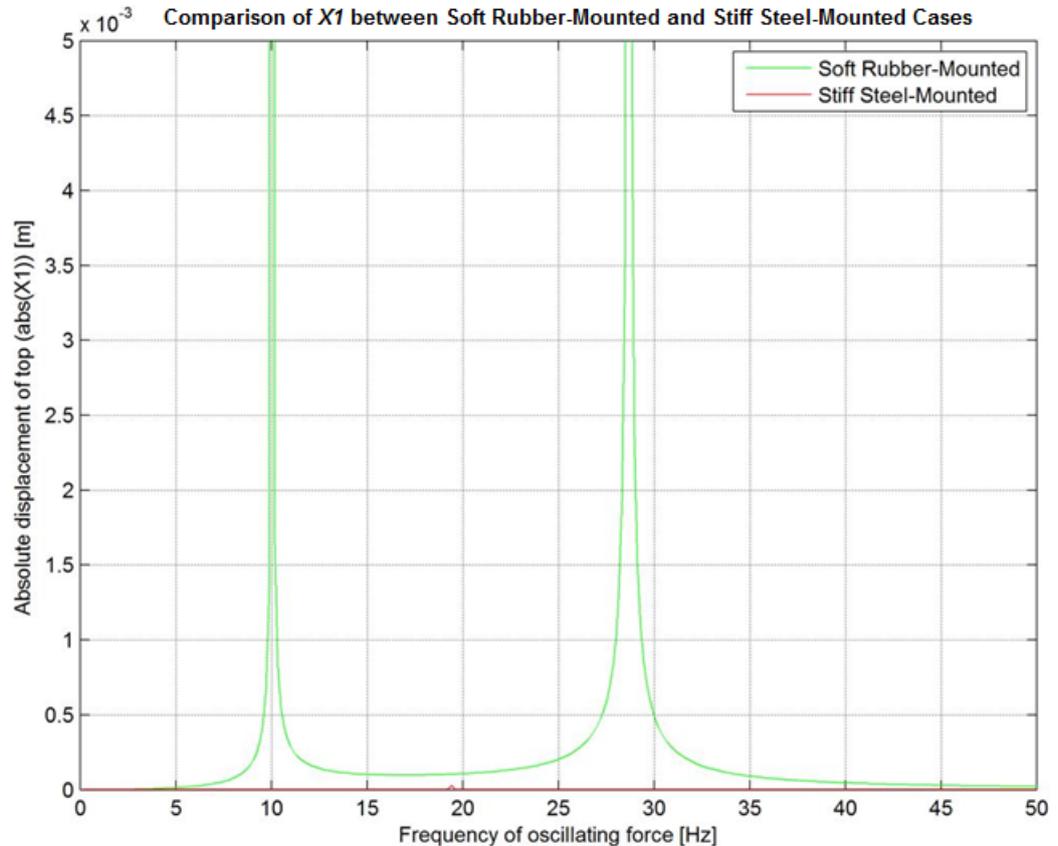
When the soft rubber mounts are added to the system, the second natural frequency is reduced to below the higher range of the experiments. The second natural frequency will therefore affect the response of the system in the range of the experiment.

#### 8.3.1.2. *Displacement*

The reduction of the stiffness of the mounting system and the resulting effects of the second natural frequency would have an impact on the amplitude of the displacement of the system.

With  $x_1$  being the displacement of the top frame of the heat exchanger, the comparison could be made between the case where the heat exchanger was stiff steel-mounted and the case where the heat

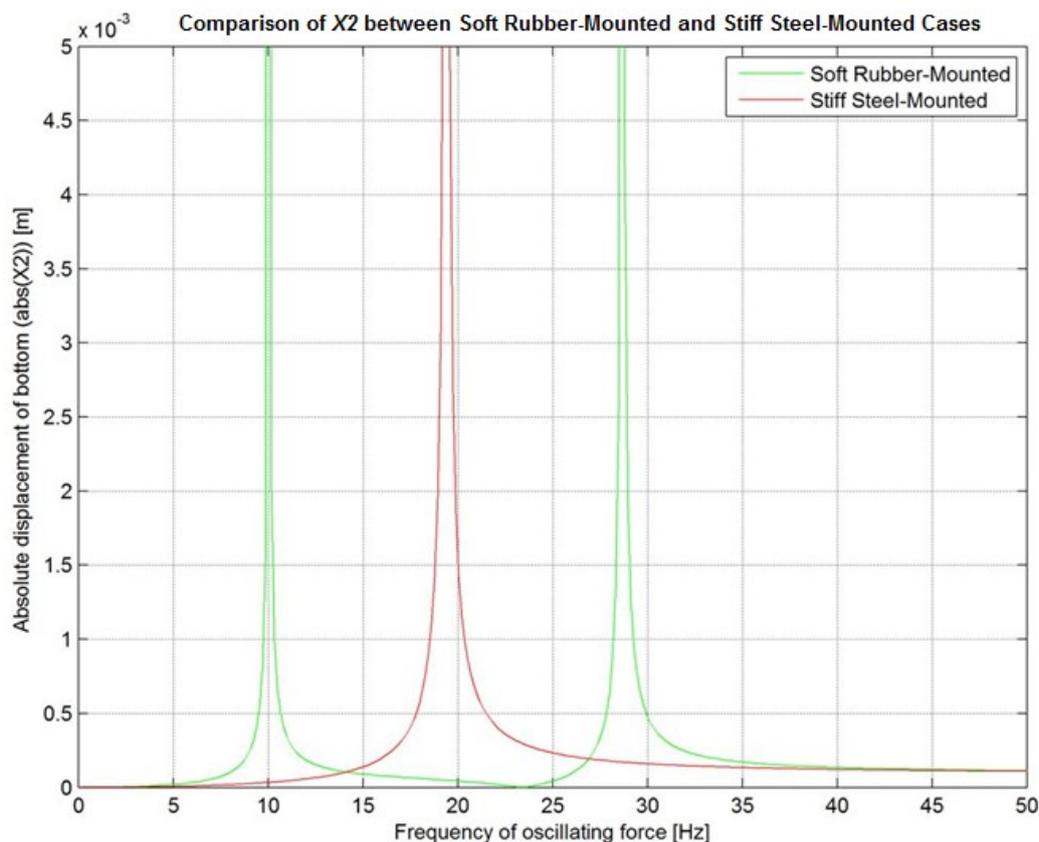
exchanger was mounted on soft rubber mounts, as illustrated in Figure 71.



**Figure 71: Predicted comparison of displacement  $x_1$  for stiff steel-mounted and soft rubber mounted cases**

As was expected, the deflection of the top frame was very small when the model was mounted stiffly (red line) and larger when the soft rubber mounting system was used (green line).

The displacement of the bottom frame of the heat exchanger ( $x_2$ ) could also be compared between the stiff steel-mounted case and the soft rubber-mounted connection case, as illustrated in Figure 72.



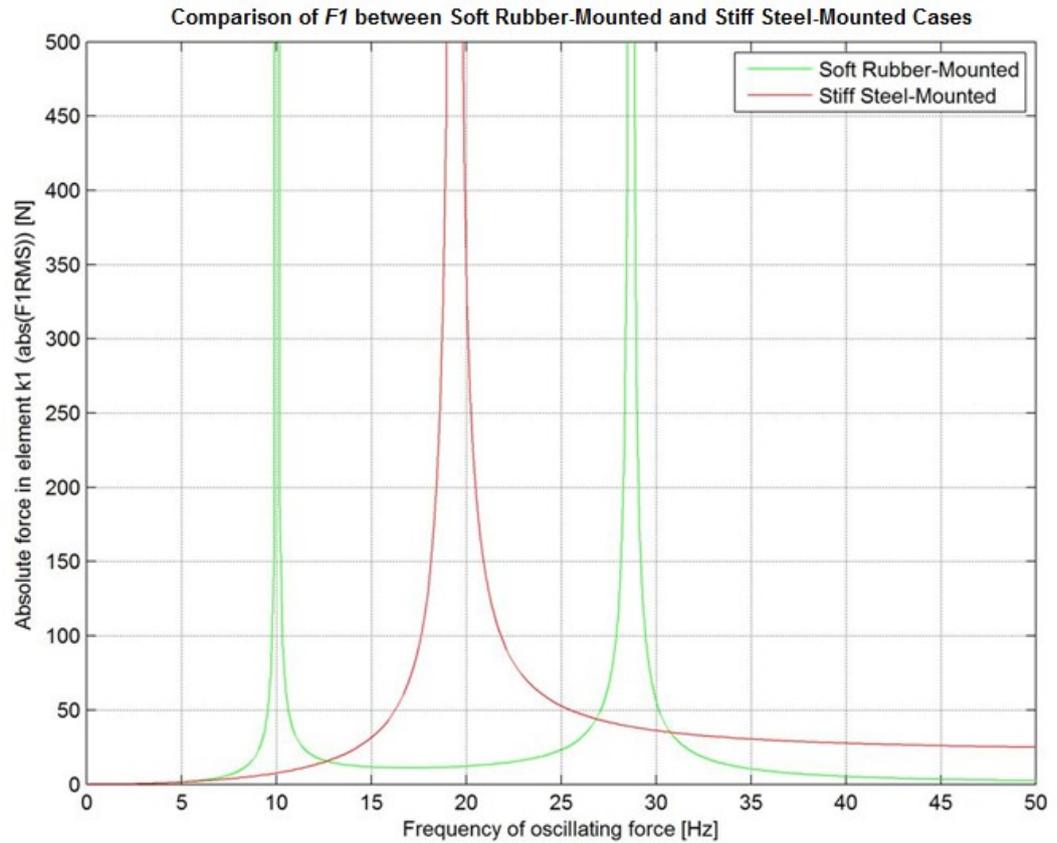
**Figure 72: Predicted comparison of displacement  $x_2$  for stiff steel-mounted and soft rubber mounted cases**

The original stiff steel-mounted system (red) showed a clear natural frequency at 19 Hz. This was the frequency where damage could be caused to the heat exchanger due to the large displacement caused by flow-induced vibration.

The introduction of the mounting system (green) brought another natural frequency into the spectrum and reduced the amplitude of displacement at a range of frequencies. This was especially evident around the previous natural frequency of the stiff steel-mounted system.

#### 8.3.1.3. *Force in the springs*

As there was a direct correlation between the force in a spring and the displacement of the spring according to Equation (5.22), the forces in the spring elements ( $k_1$ ,  $k_2$  and  $k_3$ ) could be compared in the same way as the displacement. Figure 73 illustrates the predicted comparison of the forces in the first spring element ( $k_1$ ).

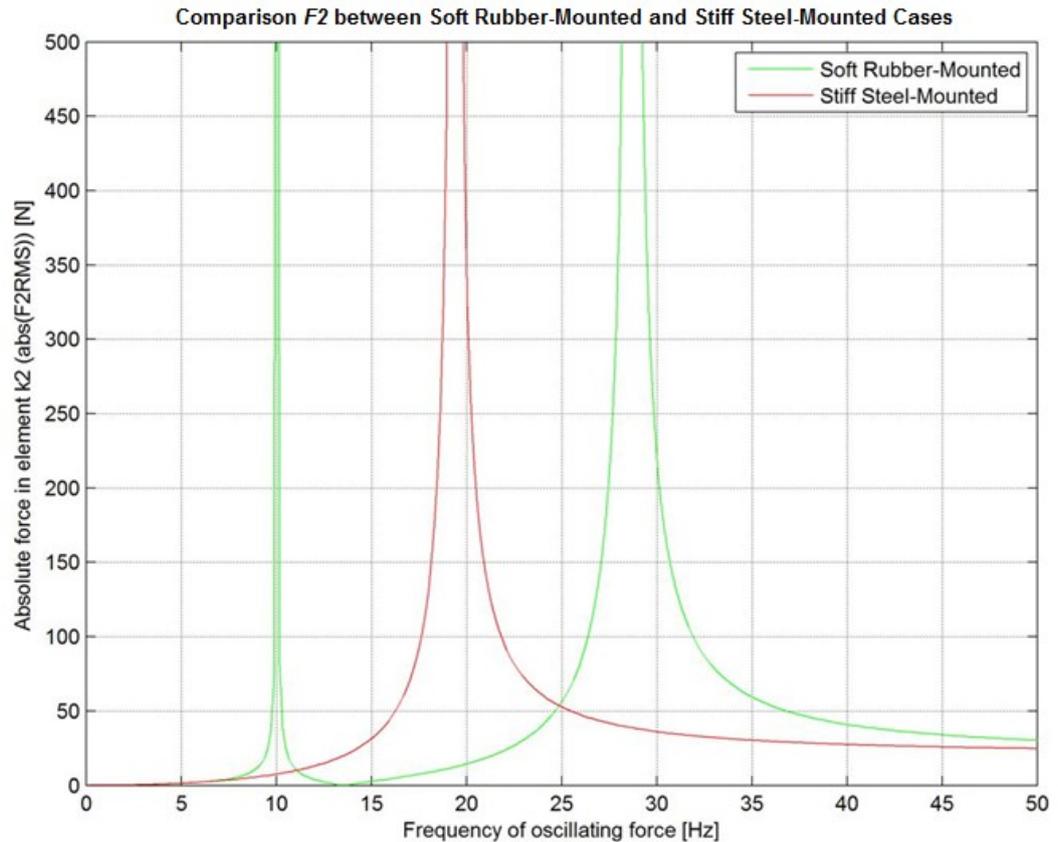


**Figure 73: Predicted comparison of force  $F_{k1}$  for stiff steel-mounted and soft rubber-mounted cases**

It was interesting to note that although the displacement  $x_1$  in the stiff steel-mounted case (red) showed a very small displacement throughout the whole frequency range in Figure 71, the force in the spring element (transferred to the frame) was high and increased considerably when the forcing frequency was close to its natural frequency. This indicated that there were possibly more stresses in the heat exchanger frame when the heat exchanger was bolted tightly (stiff steel-mounted) than was initially thought.

When the structure was placed on the soft rubber mounts (green), the forces were reduced for a range of forcing frequencies, most notably around the original natural frequency of the heat exchanger. The only ranges where the forces appeared to have increased, corresponded with the natural frequencies of the soft rubber-mounted system (with mounts).

The initial failures of the heat exchangers were expected to be caused by the stress and, therefore, the deflection of the plates. The forces in  $k_2$  ( $F_{k_2}$ ), which were the equivalent to the forces in the plates, were very important parameters that had to be reduced. The comparison of the forces in  $k_2$  for the stiff steel-mounted (red) and soft rubber-mounted (green) structures are illustrated in Figure 74.

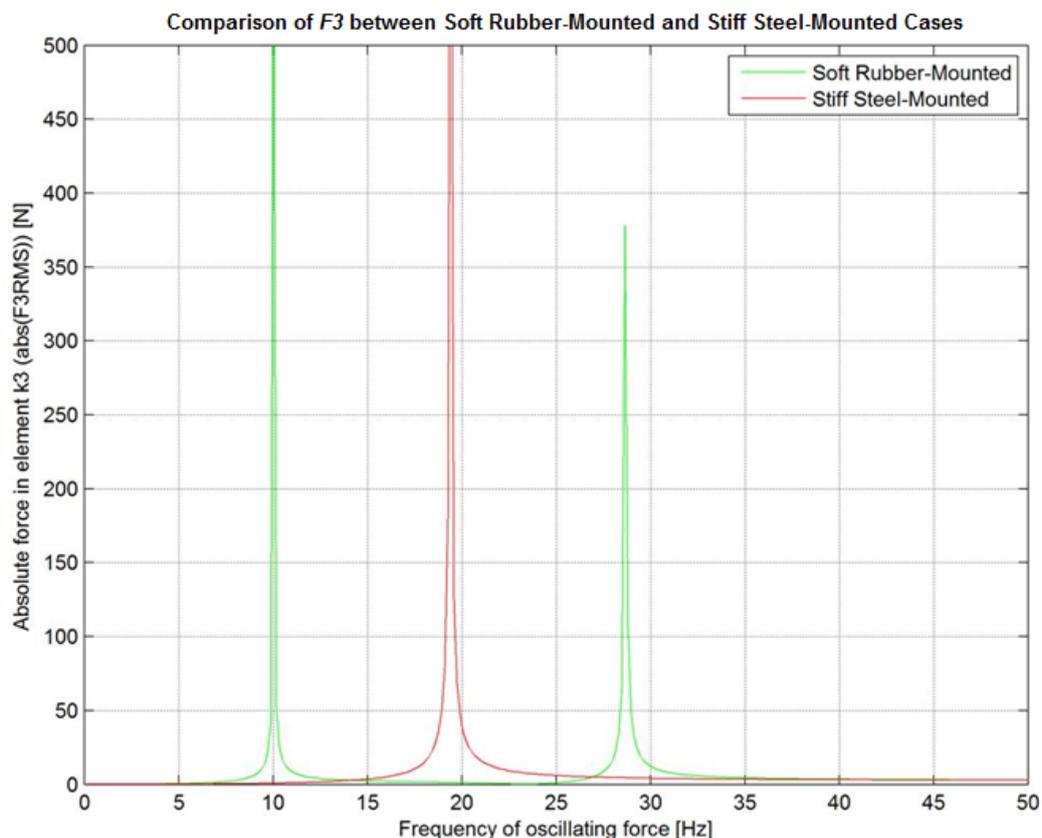


**Figure 74: Predicted comparison of force  $F_{k_2}$  for stiff steel-mounted and soft rubber-mounted cases**

The results showed that for a range of frequencies, again predominantly around the natural frequency of the stiff steel-mounted system, the soft rubber-mounted system (green) had significantly lower forces in the spring elements (plates) than the stiff steel-mounted system (red).

The predicted results, however, indicated that the force in the element (plate pack) would increase around the second natural frequency of the soft rubber-mounted system.

The final spring element ( $k_3$ ), which connected the bottom mass ( $m_2$ ) to the frame could be evaluated for the forces in the element in the same manner. Figure 75 illustrates this comparison between the forces transferred to the frame by the element for the two different mounting cases.



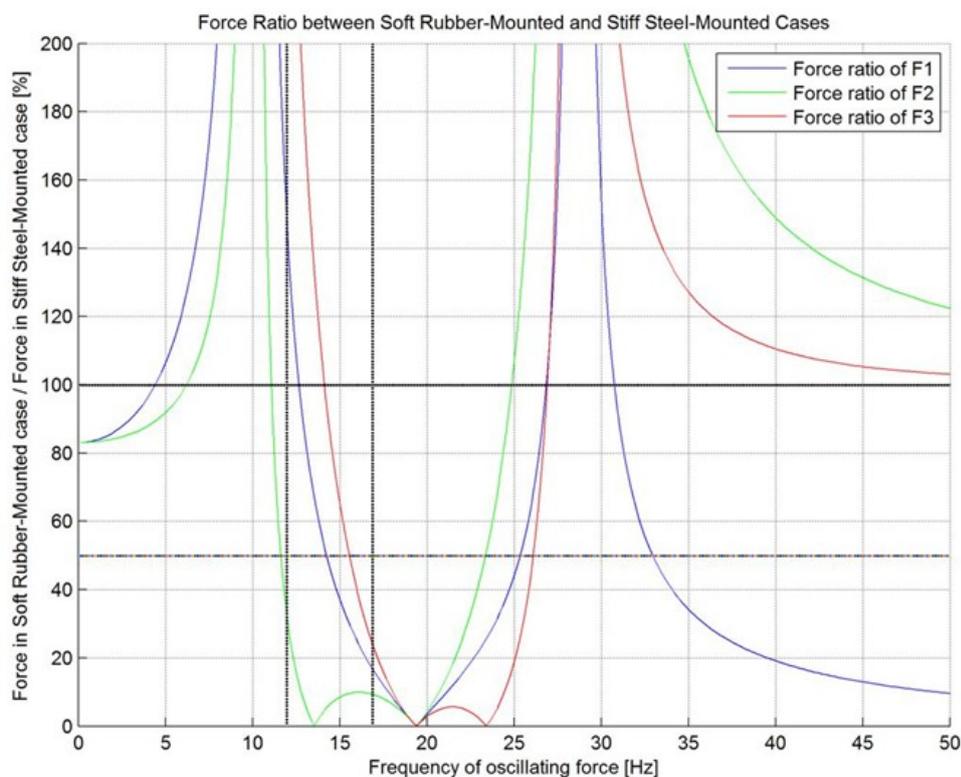
**Figure 75: Predicted comparison of force  $F_{k_3}$  for stiff steel-mounted and soft rubber-mounted cases**

The results of the comparison indicated the same characteristics of the other two elements with the forces being reduced by the addition of the mounting system around the natural frequency of the stiff steel-mounted system and a predicted increase around the natural frequencies of the soft rubber-mounted system.

#### 8.3.1.4. Force ratio between cases

The results of the previous section can be summarised by calculating the ratio of the absolute size of the force in the elements for the soft rubber-mounted case to the absolute size of the force in the elements for the

stiff steel-mounted system. Figure 76 illustrates this value over the entire frequency range, clearly indicating the ranges where the change reduced the forces in the respective elements.



**Figure 76: Predicted comparison of force ratios between the two cases for all the elements**

This illustration again shows the overall trend, namely that the forces in the elements were lower in a range near the natural frequency of the stiff steel-mounted system and increased, as predicted, near the natural frequencies of the soft rubber-mounted system.

#### 8.3.1.5. *Numerical results*

From the predictions of the response over the whole frequency spectrum, shown in Figure 76, two frequencies were selected to prove that the concept that was selected would reduce the dynamic forces in the heat exchanger plates. These two selected frequencies were 12.125 Hz and 17 Hz, which were used as the frequencies where experimental measurements were taken.

As the experimental tests were conducted at only two frequencies, the values of the important parameters are highlighted in Table 15.

Table 15: Predicted dynamic force values of the two DOF model without damping

		75 rad/s ( $\approx$ 12.125 Hz)		106 rad/s ( $\approx$ 17 Hz)	
		mounted	mounted	mounted	mounted
Acceleration of top frame [ $\text{m/s}^2$ ]	$\ddot{X}_1$	0.007	21.347	-0.073	24.431
Acceleration of bottom frame [ $\text{m/s}^2$ ]	$\ddot{X}_2$	7.687	25.593	-78.458	19.006
Displacement of top frame [m]	$X_1$	5.689E-08	-1.705E-04	2.908E-07	-9.770E-05
Displacement of bottom frame [m]	$X_2$	5.693E-05	-1.895E-04	2.909E-04	-7.046E-05
Dynamic force in element $k_1$ [N]	$F_1$	18.205	27.426	93.055	15.714
Dynamic force in element $k_2$ [N]	$F_2$	18.198	6.080	92.982	8.717
Dynamic force in element $k_3$ [N]	$F_3$	2.079	6.923	10.625	2.574
$(F_1 \text{ for soft rubber-mounted}) / (F_1 \text{ for stiff steel-mounted})$		150.65%		16.89%	
$(F_2 \text{ for soft rubber-mounted}) / (F_2 \text{ for stiff steel-mounted})$		33.41%		9.37%	
$(F_3 \text{ for soft rubber-mounted}) / (F_3 \text{ for stiff steel-mounted})$		332.93%		24.22%	

Therefore, the two DOF model without taking damping into account predicted that at a forcing frequency of 75 rad/s ( $\pm 12$  Hz), only the dynamic force in plate pack ( $k_2$ ) would reduce, while the forces in the other elements would increase.

At a forcing frequency of 106 rad/s ( $\pm 17$  Hz), the dynamic forces in all the elements would substantially decrease.

This model, however, did not take any damping into account. To increase the accuracy of the calculation, the two DOF model with damping had to be used.

### 8.3.2. Two DOF model with damping

By including the effects of damping, the results were expected to predict the response of the experimental model more accurately. This added accuracy came at the price of increased computing time, as each forcing frequency had to be calculated separately over a number of time steps.

The Matlab source code and input matrices can be seen in Appendix B.

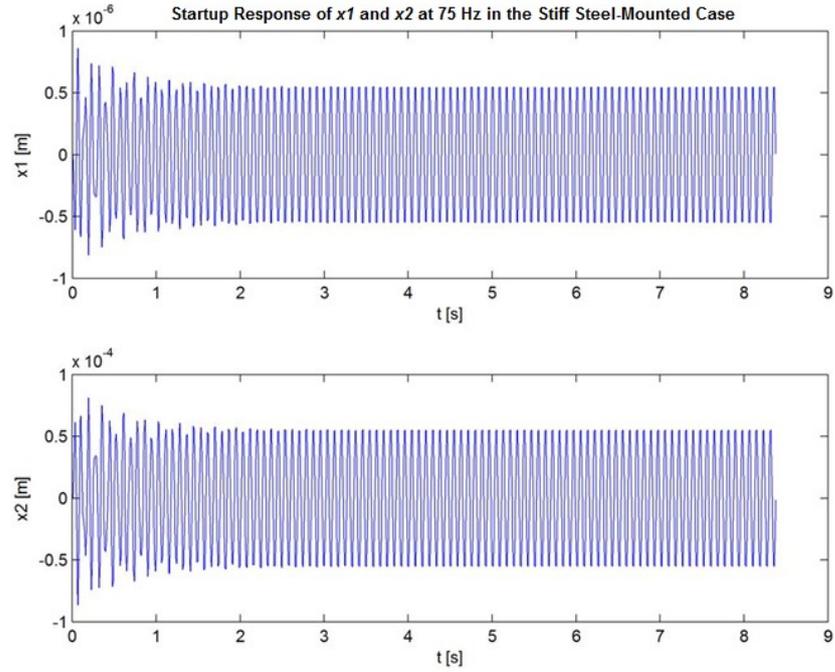
#### 8.3.2.1. *Forcing frequencies*

The two frequencies selected from the previous model (75 rad/s and 106 rad/s) were used as the points where the two DOF model with damping was simulated.

#### 8.3.2.2. *Transient response*

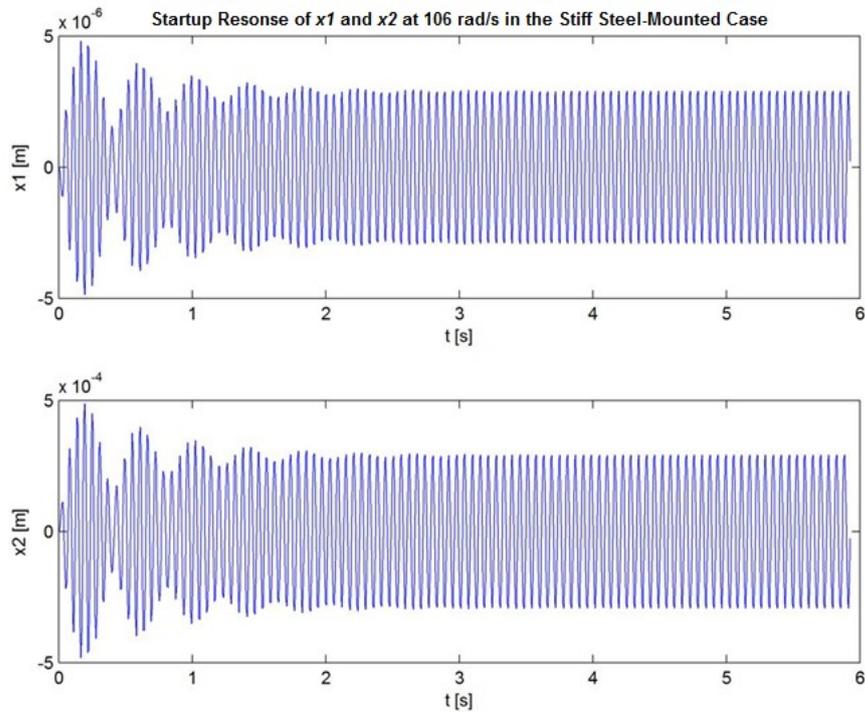
When the system was excited by an oscillating force, a transient start-up response was evident. It was, therefore, important to let this transient response die out before the values were used to compute the RMS values, which were to be compared.

Figure 73 illustrates the start-up response with a forcing frequency of 75 rad/s (11.937 Hz) for the stiff steel-mounted case.



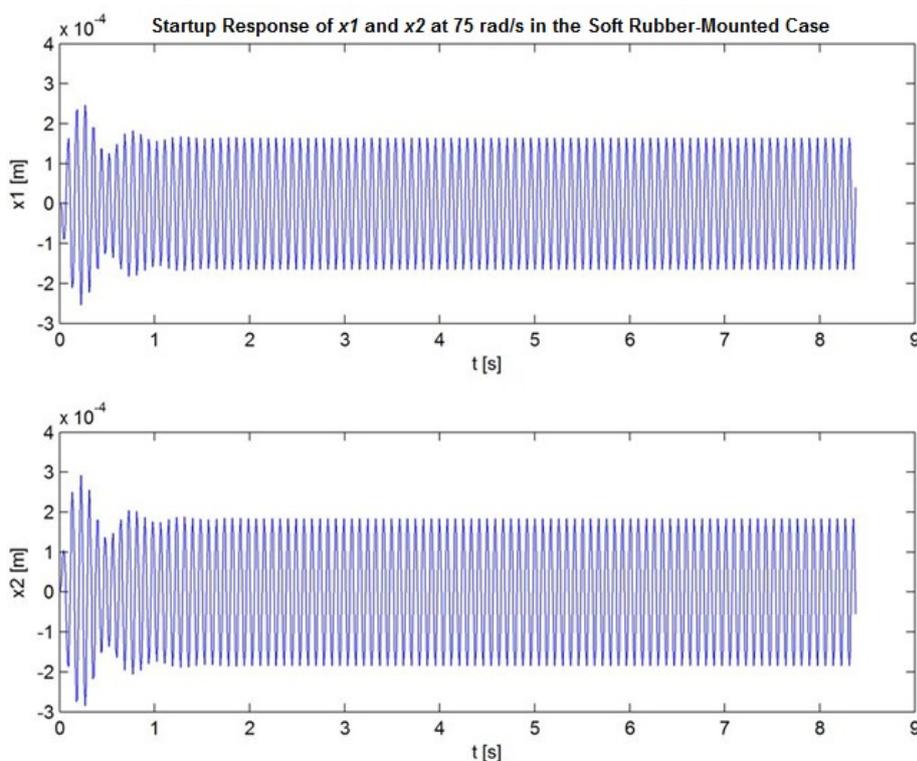
**Figure 77: Predicted transient start-up response of the stiff steel-mounted system at 75 rad/s**

When the stiff steel-mounted model was subjected to a frequency of 106 rad/s (16.870 Hz), the response shown in Figure 78 was predicted.



**Figure 78: Predicted transient start-up response of the stiff steel-mounted system at 106 rad/s**

The soft rubber mounting of the model increased the number of natural frequencies that could be excited during start-up. This complicated the transient response. Figure 79 illustrates the start-up response of the soft rubber-mounted model to a forcing frequency of 75 rad/s (11.937 Hz).

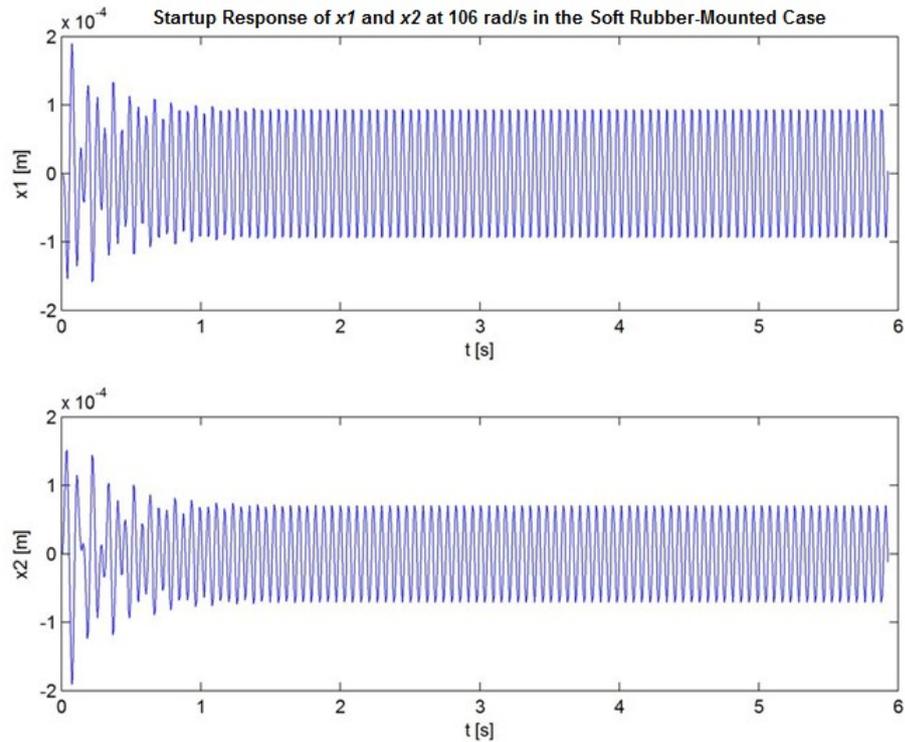


**Figure 79: Predicted transient start-up response of the soft rubber-mounted system at 75 rad/s**

The start-up response when the forcing frequency was increased to 106 rad/s (16.870 Hz), is illustrated by Figure 80.

The computer programme implementing the mathematical model was set up to determine the RMS force value from the second half of the response calculated. In all the cases the transient behaviour subsided in the first half of the measurement.

This indicated that the transient behaviour did not have an effect on the predicted RMS values. It also indicated that the experimental model was not expected to have to contend with transient start-up frequencies once the motor had been running for more than a few seconds.



**Figure 80: Predicted transient start-up response of the soft rubber-mounted system at 106 rad/s**

#### 8.3.2.3. Numerical results

The two most important forcing frequencies were the two values that were going to be verified experimentally. For these values, the displacement and resultant forces were calculated and the results are shown in Table 16.

The two DOF model with damping predicted that there would be an increase in forces at the lowest measured frequency (75 rad/s), but that the dynamic forces in all the elements would reduce significantly at 106 rad/s.

Table 16: Predicted dynamic force values of two DOF model with damping

		75 rad/s ( $\approx 12.125$ Hz)		106 rad/s ( $\approx 17$ Hz)	
		Stiff steel-mounted	Soft rubber-mounted	Stiff steel-mounted	Soft rubber-mounted
Acceleration of top frame [ $m/s^2$ ]	$\ddot{X}_1$	0.007	21.347	0.030	0.865
Acceleration of bottom frame [ $m/s^2$ ]	$\ddot{X}_2$	7.687	25.593	0.439	0.717
RMS force in element $k_1$ [N]	$F_{1kRMS}$	12.875	18.790	65.618	11.078
RMS force in element $k_2$ [N]	$F_{2kRMS}$	12.900	79.017	65.700	38.051
RMS force in element $k_3$ [N]	$F_{3kRMS}$	1.471	4.757	7.493	1.839
RMS force in element $c_1$ [N]	$F_{1cRMS}$	0.000	2.638	0.000	2.197
RMS force in element $c_2$ [N]	$F_{2cRMS}$	0.214	1.310	1.539	0.892
RMS force in element $c_3$ [N]	$F_{3cRMS}$	0.081	0.263	0.585	0.144
RMS combined force in element $k_1$ and $c_1$ [N]	$F_{1RMS}$	12.875	18.975	65.618	11.294
RMS combined force in element $k_2$ and $c_2$ [N]	$F_{2RMS}$	12.901	79.028	65.718	38.061
RMS combined force in element $k_3$ and $c_3$ [N]	$F_{3RMS}$	1.473	4.764	7.515	1.845
$(F_1 \text{ for soft rubber-mounted}) / (F_1 \text{ for stiff steel-mounted})$		147.37%		17.21%	
$(F_2 \text{ for soft rubber-mounted}) / (F_2 \text{ for stiff steel-mounted})$		612.56%		57.92%	
$(F_3 \text{ for soft rubber-mounted}) / (F_3 \text{ for stiff steel-mounted})$		323.38%		24.54%	