

A comparison of the efficient and fractal market hypotheses in developing markets

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Preface

The theoretical work described in this dissertation was carried out whilst in the employ of Aviva Investors (London, UK). Some theoretical and practical work was carried out in collaboration with the Department of Risk Management, School of Economics, North-West University (South Africa) under the supervision of Prof Gary van Vuuren.

These studies represent the original work of the author and have not been submitted in any form to another university. Where use was made of the work of others, this has been duly acknowledged in the text.

Unless otherwise stated, all data were obtained from Bloomberg,TM non-proprietary internet sources, and non-proprietary financial databases of Aviva Investors, London, UK. Discussions with personnel from this institution also provided invaluable insight into current investment trends and challenges faced in the investment risk and portfolio management arena.

The results associated with the work presented in Chapter 3 (Fama-French 3-factor model) has been published in *International Business and Economics Research* (September 2017). The work described in Chapter 4 (Fractal market hypothesis) has been accepted for publication in *Annals of Financial Economics* (August 2018).

The results obtained from these articles and the contributions they make to the existing body of knowledge are summarised in Chapter 5 which also discusses future research opportunities.



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- my girlfriend, Julia Madison, for her selfless patience, love and support throughout this and all my academic and personal ventures, and
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Abstract

The validity and descriptive accuracy of the Capital Asset Pricing Model and the Fama-French Three-Factor Model are tested by describing the variation in excess portfolio returns on the Johannesburg Stock Exchange (JSE). Portfolios of stocks are constructed based on an adapted Fama & French (1993) approach, using a 3×2 annual sorting procedure and based on Size and Book-to-Market metrics, respectively. The sample period spans six years, 2010 to 2015, and includes 46 companies listed on the JSE. The results indicate that both models perform relatively poorly because of inadequate market proxy measures, market liquidity restrictions, unpriced risk factors and volatility inherent in an emerging market environment. The value premium is found to explain a larger proportion of variation in excess returns than the Size Premium and is more pronounced in portfolios with relatively higher book-to-market portfolios.

The Efficient Market Hypothesis (EMH) has been repeatedly demonstrated to be an inferior – or at best incomplete – model of financial market behaviour. The Fractal Market Hypothesis (FMH) has been installed as a viable alternative to the EMH. The FMH asserts that markets are stabilised by matching demand and supply of investors' investment horizons while the EMH assumes the market is at equilibrium. A quantity known as the Hurst exponent determines whether a fractal time series evolves by random walk, a persistent trend or mean reversion. The time-dependence of this quantity is explored for two developed market indices and one emerging market index. Another quantity, intrinsically linked to the Hurst exponent, the fractal dimension of a time series, provides an indicator for the onset of chaos when market participants behave in the same way and breach a given threshold. A causal relationship is found between these quantities: the larger the change in the fractal dimension before breaching, the larger the rally in the price index after the breach. In addition, breaches are found to occur principally during times when the market is trending.

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Chapter 1

Introduction

1.1 Background

The efficient market hypothesis (EMH) asserts that asset prices follow a random walk (Brownian motion) with independent and identically, normally distributed, uncorrelated, relative changes. The EMH has far-reaching implications: investors are rational and homogeneous (all investors use available information in the same way and thus operate on the same investment horizon), financial returns are normally distributed, standard deviations are meaningful risk measures, there is a trade-off between risk and return, and future returns are unpredictable.

The capital asset pricing model (CAPM), an economic model which is founded on the principles of the EMH, employs a single variable – the returns of the local market – to describe and explain market returns. Fama & French (1992) introduced a three-factor model (FF3FM),¹ also based upon the tenets of the EMH, but which includes size and book to market factors (in addition to market index returns) as explanatory variables.

The implications of the EMH have however been widely and consistently rejected in empirical studies. Asset prices do not generally follow random walks, increments are correlated to some extent and are often non-normally distributed. The assertion of homogeneous investment horizons is also demonstrably false. Capital markets comprise investors with considerably different investment horizons, from algorithmic based market-makers (fractions of a second), to noise traders (several minutes), technical traders (days to weeks), fundamental analysts (months) and pension funds (several years). For each of these, market information has a different value and is treated in different ways. Each group also has its own trading rules and strategies which for one group can mean severe losses while for the other it can lead to profitable opportunities. A complex system thus arises which is inadequately described by the oversimplified EMH.

¹ Further adaptations have been proposed, most recently a five-factor model by the same authors (Fama & French, 2015 and Guo, Zhang, Zhang & Zhang, 2017), but while this new model may evolve into a new standard for pricing assets, it does not address prominent questions posed by the three-factor model and raises several new concerns (Xiouros, 2017). Because the five-factor model is 'new' and still relatively untested, focus is on the three-factor model.

1.2 Problem statement

There is no quantitative measure of market efficiency so testing the underlying concepts of the EMH is difficult. Results from suggested tests are also subject to interpretation, particularly so in emerging markets where data are beset with other features such as high volatility and illiquidity. The EMH, nevertheless, remains a popular contemporary framework.

An alternative theory – the FMH – asserts that patterns *are* discernible (and repeatable) in financial markets and describes how participants respond to information by explaining investor behaviour under all market conditions. Establishing recurring market configurations would not eliminate the EMH but would bolster the credibility of the FMH. Little research has been conducted on the FMH using emerging market data.

1.3 Research question

Using the CAPM and FF3FM (as manifestations and consequences of the EMH framework) and emerging market return data, do opponents of the EMH pose valid objections?

Using global data, sourced from both emerging and developed milieus, does the FMH offer a potentially better alternative to the EMH by detecting measurable and repeatable patterns in financial markets?

1.4 Study motivation

The EMH assumes that all information is priced into the market and that this renders the market *efficient* (to varying degrees according to the speed and extent of information dissemination). No quantitative tests exist to establish market efficiency conclusively and unambiguously, so "confirmation" must be obtained via copious, indirect tests such as the CAPM and various incarnations of Fama and French's factor models (Fama & French, 1992, 1993, 1995, 1996, 1998, 2015). Despite prolific research, evidence for market efficiency remains mixed (Thicke, 2017), particularly in emerging markets (Mobarek & Mollah, 2016) which are characterised by high volatility and prone to sustained periods of illiquidity.

To extend the literature on the validity (or not) of the EMH in emerging markets, data from South Africa were used with the CAPM and the three-factor Fama-French model to describe portfolio returns. Positive descriptive results will not necessarily refute or confirm the EMH's relevance in emerging markets. However, such an investigation will provide further information on the applicability of the EMH and extend work undertaken in South Africa to date.

Moving to the second aim of the dissertation, the applicability of the FMH to the South African milieu will be explored.

Emerging market returns are generally higher than developed market returns and considerably more volatile (Harvey, 1995a,b). In addition, emerging markets are less liquid, more prone to political shocks and slower to respond to fiscal stimuli than developed markets (Bekaert, Erb, Harvey, & Viskanta, 1998 and Bekaert, Erb & Harvey, 2016). This makes the emerging market environment a fertile testing ground for the FMH as an alternative to the EMH.

The FMH is based on the most general of the market's characteristics: liquidity (which is completely ignored by the emerging market hypothesis (EMH)). The fractal market hypothesis (FMH) acknowledges that liquidity provides smooth market pricing processes which in turn exerts a stabilising influence on the market. When liquidity ceases, the market's inherent dimensionality alters and becomes *fractal*, the market destabilises, and extreme movements occur. When market participants behave identically, whether by collectively panic-selling or euphoria-buying, they herd and chaos ensues (as measured by the fractal dimension (which $\rightarrow 1$ as herding becomes dominant in the market)). When the fractal dimension is breached, the market rebounds after a herd-induced collapse or collapses after a herd-induced rally (the latter less prevalent). This is how participants react to market information: they behave semi-autonomously at first, then when new information arrives, they herd and – by their collective actions – influence dramatic changes in market returns. These empirical observations are strikingly different from the way "efficient markets" are meant to behave (Joshi, 2014a, b).

The literature concerning the FMH covers the detection of fractality or multifractality of financial assets' price processes in developed markets. The FMH has not, however, been tested extensively in developing markets with respect to its predictions about causes and implications of critical events.

1.5 Dissertation structure

Chapter 2 presents the literature governing the institution of the EMH, the subsequent development of the CAPM and more detailed theories of market behaviour, such as multi-factor models. These frameworks evolved as a natural consequence of market efficiency and are used (along with others – see Figure 2.1) as assessments of its validity. The EMH, however, has been criticised from a variety of opponents. These criticisms are also presented in Chapter

2, along with possible alternatives, such as the AMH and FMH. The development and implementation of these models require other tests and give rise to different consequences for market behaviour such as liquidity evaporation when participants herd and invoke chaos.

Chapter 3 sets out Article 1: *The Capital Asset Pricing Model and Fama French Three-Factor Model in an emerging market environment*. The validity and descriptive accuracy of the Capital Asset Pricing Model and the Fama-French Three-Factor Model are assessed by describing the variation in excess portfolio returns on the Johannesburg Stock Exchange. Portfolios of stocks are assembled based on an adapted Fama & French (1993) approach, using a 3×2 annual sorting procedure and based on size and book-to-market metrics, respectively. Accuracy is determined via the R^2 descriptive statistic. The higher the R^2 , the better the explanatory variables are at explaining market return variability. The sample period spans six years, 2010 to 2015, and includes 46 JSE-listed companies. Both models perform relatively poorly because of inadequate market proxy measures, market liquidity restrictions, unpriced risk factors and volatility inherent in an emerging market environment. The value premium is found to explain a larger proportion of variation in excess returns than the size premium and is more pronounced in portfolios with relatively higher book-to-market portfolios.

Chapter 4 presents Article 2: *Investment implications of the fractal market hypothesis*. The EMH has been repeatedly demonstrated to be an inferior – or at best incomplete – model of financial market behaviour. The FMH was instituted as an alternative to the EMH. The FMH asserts that markets are stabilised by matching demand and supply of investors' investment horizons while the EMH assumes the market is at equilibrium. A quantity known as the Hurst exponent determines whether a fractal time series evolves by random walk, a persistent trend or mean reverts. The time-dependence of this quantity is explored for two developed market indices and one emerging market index. Another metric, the fractal dimension, provides an indicator for the onset of chaos when market participants behave in the same way and breach a given threshold.

A relationship is found between these quantities: the larger the change in the fractal dimension before breaching, the larger the rally in the price index after the breach. In addition, breaches are found to occur principally during times when the market is trending. The existence of such a repeatable phenomenon weakens the argument for the EMH and strengthens the case for the FMH.

Chapter 5 concludes the dissertation by summarising the findings of the entire study and proposing suggestions for future research.

1.6 Specific objectives

Specific objectives of this research are:

1. to ascertain the validity (or otherwise) of the EMH using the CAPM and Fama-French three-factor model in an emerging market milieu;
2. to confirm (or refute) results obtained prior to this work on various global markets;
3. to investigate the application of the FMH – as an alternative to the EMH – on various global markets, especially an emerging market such as the South African JSE;
4. to explore the ramifications of herd behaviour and the onset of chaos if these are detected in markets; and
5. suggest a possible investment strategy which exploits these outcomes.

1.7 Research design

The research design of this dissertation follows in the outline below:

Pose research problem statement and question: The CAPM, FF3FM and emerging market return data, will be used to assess whether the EMH adequately describes market returns. Also, using emerging and developed market financial data, the FMH will be evaluated to determine whether measurable and repeatable patterns arise in market data.

Critical literature review: Critical literature reviews are conducted through Chapters 2 through 4 by consulting existing literature. Adjustments to existing risk management procedures, techniques and methodologies to solve problems are documented and highlighted in the literature studies. The existing literature for this research theme is copious. Where an entirely new approach to risk practices is required, the literature was less obliging, but this was not a constraint in this study, because popular, well-established mathematical techniques

are almost always available for research endeavours and again, abundant literature exists to address these.

Theory building/adapting/testing: Adaptation of existing financial tools and mathematical techniques for practical implementation enjoys rich precedent. The bulk of the results reported in this dissertation were from empirical analyses of historical data derived using known risk metrics with slight innovations for some.

Data collection: Data used were from original sources where possible (e.g. South African Reserve Bank for proprietary data) or third-party, internet-based, electronic databases (e.g. McGregor BFA,² Opendata and Bloomberg™ for historic index prices). Adequate data were available for all the chapters, so sample error was minimised. Data in this study comprised several published, historical time series, available from both proprietary and other non-proprietary sources (e.g. internet databases).

Conceptual development and empirical investigation: This research is intended to provide robust, but practical, solutions for use by investors and traders. As a result, the primary source of analytical work was Microsoft Excel™ since this tool is used by most financial institutions. These spreadsheet-based models use visual basic (a flexible, functional desktop tool available to all quantitative analysts and risk managers) to develop macros to replace onerous and repetitive computing tasks. The empirical study comprises the practical implementation of the research method, using techniques and models developed in Microsoft Excel.™

The variables employed are assembled from various historical time series. All data are available in the public domain. Some pricing data were simulated for illustration.

Illustrate and reason findings: Having analysed the data, obtained meaningful results and displayed these appropriately, the findings were written up into article-style reports for peer review and publication. Chapter 2 has already been published and Chapter 3 has been submitted for publication as detailed in Table 1.2.

Further work: To complement major findings of and ensure the continuation of much needed work not addressed in this dissertation, future work regarding the many consequences of the FMH is proposed for risk theorists and practitioners.

² McGregor BFA was acquired by (and renamed as) IRESS in late 2016.

1.7.1 Literature review

The literature reviews focus on the origin, development, history and applications of the issues identified through problem statements and research questions, in this case the validity (or not) of the EMH. These literature studies explain and clarify the problem of market efficiency and elucidate how previous studies have addressed the problem. An alternative to the EMH – the FMH – is also investigated, and the latter's description of market returns is explored.

1.7.2 Data

Data requirements, frequency and source are shown in Table 1.1 below.

Table 1.1: Data requirements, frequency and source.

#	Topic	Data required	Frequency	Sources
1	The Capital Asset Pricing Model and Fama-French Three Factor Model in an emerging market environment	Accounting (financial statement) data Some time series market data such as risk-free rates for different jurisdictions	Monthly or quarterly	Corporate financial statements
2	Investment implications of the fractal market hypothesis	Index price levels, currency rates, commodity prices	Daily Monthly	Bloomberg

1.7.3 Research output

The research output is shown in Table 1.2 below.

Table 1.2: Research output.

#	Topic	Model	Research methodology
1	Karp, A. and van Vuuren, G. 2017. The CAPM and Fama-French 3 factor model in an emerging market environment. <i>International Business and Economics Research</i> , 16(3): 231 – 256	Linear regression	Empirical investigation using financial statement data
2	Karp, A. and van Vuuren, G. 2018. Investment implications of the fractal market hypothesis. Accepted for publication in <i>Annals of Financial Economics</i>	Rolling regression (H) Simple regression (D)	Rolling regression to establish time dependence for H Empirical analysis using linear regression results to determine breach frequency and change in variables pre- and post breaches

1.8 Conclusion

The conclusion presents a summary of the findings of both topics, providing details of recommendations for possible future research. The next chapter presents a literature survey governing the background information relevant to the dissertation.

Chapter 2

Literature study

2.1 Introduction

The idea of an 'efficient market', namely one in which information regarding the component shares was received and then rapidly processed and adapted was introduced by Fama, *et al.*, (1969). This research was based on (then) empirical observations: stock market prices moved as information became available, sell-offs with bad news and market rallies with good news. The more 'efficient' the market, the faster the processing of the information and the speedier the adjustment of the underlying price. These empirical observations came to be known as the efficient market hypothesis (EMH), but the full theory evolved gradually, with different variants added to its universe as theory and empirical evidence evolved.

2.2 Market efficiency and the EMH

This efficient market concept underwent some refinement and the so-called *weak* form of the EMH was popularised by Malkiel (1973) who suggested that asset prices reflect all past asset price data so technical analysis cannot be used to help with investment decisions.

Jensen (1978) set out the economically realistic idea of what later came to be known as the *semi-strong* version of the EMH, namely that prices do reflect market information, but only to the point where the marginal costs of collecting this outweighed the marginal benefits of acting upon it.

Grossman & Stiglitz (1980) argued that markets exhibit efficiency only when *relevant* information is *rationally* processed. Not all information is available to all market participants, and even if it were, this information is not available *simultaneously* to all participants. Grossman & Stiglitz (1980) thus adjusted the loose concept of market efficiency to embrace the idea that all *available* information is reflected in an efficient market's asset prices. Market information is not costless, a fact which gives rise to the incentivisation of financial gains, but if it were free, prices would rise to their 'fundamental level' (Fama, 1993). Thus originated the strong form of the efficient market hypothesis (EMH) (Grossman & Stiglitz, 1980).

The efficient market hypothesis (EMH) contends that asset prices follow a random walk (Brownian motion). This assertion has profound consequences for the description of these

assets' relative price changes, some of which are that that subsequent price changes represent entirely random departures from previous prices and that they are normally distributed because the data are uncorrelated and independently and identically distributed (Strebel, 1983; Le, 2016 and French, 2017). Standard deviations of relative price changes considered to be meaningful risk measures and there is a trade-off between (this definition of) risk and potential returns. Future returns are entirely unpredictable. There are also deeper consequences: for a true random walk of asset prices, information flow must be unhindered, and share prices must immediately reflect that information. An implicit assumption is that investors are rational and homogeneous (that is, investors all use the available information in the same way and therefore their resulting actions cover the same investment horizon).

The CAPM, an economic model, arose directly from the governing principles of the EMH. Arbitrage pricing theory (APT) and the international CAP model (ICAPM) are also derived from efficient market foundations: both explain returns using linear combinations of market variables (Razzaq, Noveen, Mustafa & Najaf, 2016). Neither of these models are considered here, but see Khurshid (2017) and Tsuji (2017) for recent critiques of APT and ICAPM respectively.

2.3 Asset pricing models and the evolution of the EMH

The CAPM asserts that share returns are adequately described by a single variable – local market returns (Markowitz, 1952a). The CAPM has attracted a sizeable body of literature which is critical of its assumptions and its description and explanation of market returns (a comprehensive review appears in Dayala (2010) and sources therein as well as French, 2017). Still using the tenets of the EMH, Fama & French (1992) introduced a three-factor model (FF3FM),³ which includes size and book to market factors (in addition to the market index's returns) as explanatory variables of share price behaviour. The FF3FM has also attracted criticism (see for example, Silvestri & Veltri, 2011). Opponents of both the CAPM and the FF3FM argue that it is core EMH framework weaknesses that are the root of their problems. Although proponents abound, it is generally believed that the assumptions on which the EMH is based are untenable (Dayala, 2010).

³ Further adaptations have been proposed, most recently a five-factor model by the same authors (Fama & French, 2015 and Guo, Zhang, Zhang & Zhang, 2017). Because the five-factor model is 'new' and untested, focus is here directed at the three-factor model.

Because the EMH generates testable predictions of both asset price movements and asset return movements, considerable research has been conducted to test the empirical informational efficiency of financial markets and thereby establish the validity – or otherwise – of the EMH. Significant empirical evidence is collated and presented in Table 2.1.

Table 2.1: EMH predictions and corroboratory/contradictory empirical evidence.

Prediction	Empirical evidence	Sources
Asset prices move as random walks over time	Approximately true. However: Small positive autocorrelation for short-horizon (daily, weekly and monthly) stock returns Fragile evidence of mean reversion in stock prices at long horizons (3–5 years)	Fama & French (1998) Poterba & Summers (1988) Campbell, Lo & MacKinlay (1997)
New information rapidly incorporated into asset prices	New information incorporated rapidly into asset prices, with some exceptions	Chan, Jegadeesh & Lakonishok (1996) Fama (1998)
Current information cannot be used to predict future excess returns	Short run, shares with high returns continue to produce high returns (<i>momentum effects</i>) Long run, shares with low price-earnings ratios, high book-to-market-value ratios, and other measures of 'value' outperform the market (<i>value effects</i>) FX market: current forward rate helps predict excess returns because it is a biased predictor of future exchange rates	De Bondt & Thaler (1985) Fama & French (1992) Lakonishok, Shleifer, & Vishny (1992) Jegadeesh & Titman (1993) Lakonishok, Shleifer & Vishny (1994) Chan, Jegadeesh & Lakonishok (1996)
Technical analysis should provide no useful information	Technical analysis is in widespread use in financial markets. Mixed evidence about whether it generates excess returns	Levich & Thomas (1993) Osler & Chang (1995) Neely, Weller & Dittmar (1997) Allen & Karjalainen (1999)
Fund managers cannot systematically outperform the market	Approximately true Some evidence that fund managers systematically underperform the market	Lakonishok, Shleifer & Vishny (1992)

		Brown & Goetzmann (1995) Kahn & Rudd (1995)
Asset prices remain at levels consistent with economic fundamentals (ie they are not misaligned)	At times, asset prices appear to be significantly misaligned, for extended periods	Meese & Rogoff (1983) De Long, <i>et al.</i> , (1990) Shleifer & Summers (1990)

Source: Author.

The evidence presented in Table 2.1 provides strong reasons to doubt the assertions of and descriptions provided by the EMH, or at least to question their validity if testing the EMH in a new milieu (Autchariyapanitkul, Chanaim, Sriboonchitta, & Denoeux, 2014; Piamsuwannakit, & Sriboonchitta, 2015).

As a direct result of EMH weaknesses, alternative interpretations of market efficiency have arisen. These include the Fractal Market Hypothesis (FMH) which relaxes some asset price movement constraints and the Adaptive Market Hypothesis (AMH) which employs ideas borrowed from evolutionary theory like fitness assessments and reproductive strategies employed by agents in competition for survival. These concepts, and the tests derived to evaluate them, are shown in Figure 2.1.

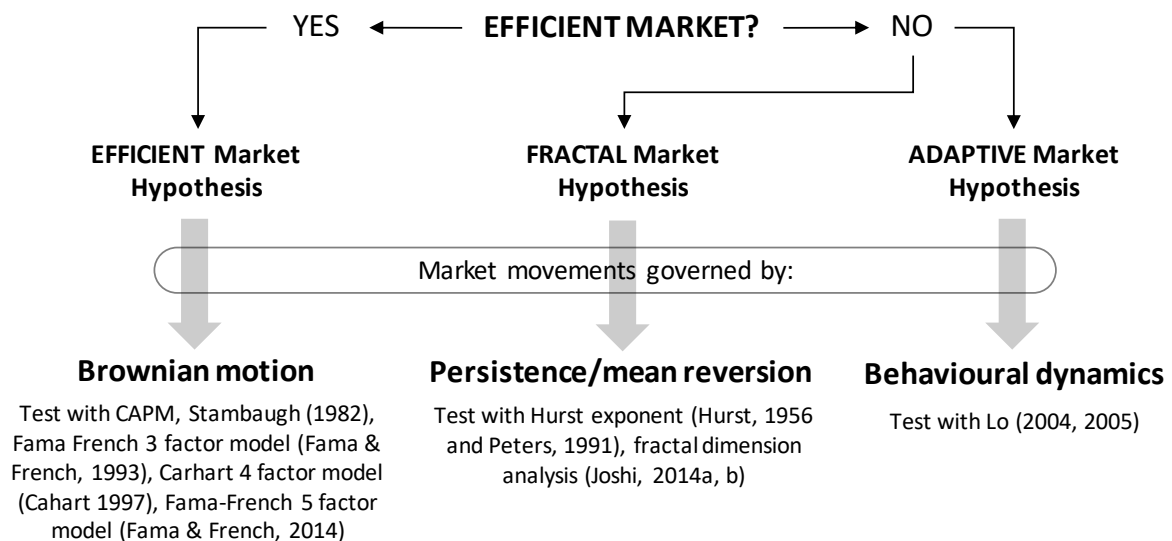


Figure 2.1: Relationship between efficient, fractal and adaptive market hypotheses (Lo, 2012).

Source: Author.

This dissertation explores these links and uses the tests detailed in Figure 2.1 to evaluate the

claims made by competing interpretations of market efficiency. The next section discusses how using the CAPM affirms or contradicts the validity of the EMH.

2.4 EMH validity tests: CAPM

One of the requirements of a functioning economic system is accurate, timeous pricing of the available assets. Early attempts to price assets include the St Petersburg article, published in 1738, which introduced investor utility, risk aversion and premia, and budgeting decisions (Bernoulli, 1954), but it was the emergence of integrated, connected financial markets in the early 20th century that galvanised the endeavour. The need to price assets fairly provided the catalyst for the rapid expansion of fledgling equity and debt markets. The mean-variance framework (Markowitz, 1952a) provided investors with the necessary confidence and encouragement as analysis on optimisation, equilibrium, and investor preference began to be understood, measured and managed. Modern Portfolio Theory (MPT) – on which most subsequent asset pricing models are constructed – further exploited these concepts by assuming that investors are risk averse and that they aim to maximise expected return subject to their risk appetite.

Markowitz's (1952b) work provided the rudimentary foundations of the CAPM, which flourished in the 1960s under joint contributions from Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). The CAPM's great appeal was that it offered powerful, sensible description of risk/return risk relationships (French, 2004; Piamsuwannakit, & Sriboonchitta, 2015; Le, 2016 and French, 2017). Two variants of the CAPM emerged (Sharpe-Lintner (Lintner, 1965 and Sharpe, 1966) and Black (1972)), but they arrived at the same conclusions:

1. the co-variance of asset returns with the market, relative to the risk or variance of the market (β), is both adequate and sufficient in explaining the variation in asset expected returns; and
2. the expected return- β relationship is positive (regression analysis confirms that the relationship between asset returns and β is approximately linear).

The CAPM pioneered asset pricing, but it is burdened with several limitations. By making some unrealistic assumptions, it provides an inadequate representation of financial market behaviour. Roll (1977) argued that it is impossible to observe a strictly diversified market portfolio, and a market index serving as a proxy for such a portfolio would have inherent predictive errors. Estimates of β vary considerably through time (Mullins, 1982). Empirical evidence

showed that asset's expected returns were driven by not only market risk, but a combination of extra risk factors. Basu (1977, 1983) and Banz (1981) for example, first documented what has come to be known as the "size effect" on US stock data. They showed that stocks with high earnings/price ratios, earned significantly higher returns than those with low earnings/price ratios. Moreover, returns for firms with relatively low market value of equity (ME) were found to be significantly higher (return premium of small firms) than firms with large market capitalisations. Small firms, in general, have higher β s than large firms, but differences in observed β s are too small to adequately explain the small-big capitalisation return disparity (Kampman, 2011).

The book-to-market (or value) effect was first explored by Reid, Rosenberg and Lanstein (1985) using US data, and later confirmed by Davis (1994) (also using US data), Lakonishok (1991) (using Japanese data) and Fama & French (1996) using international market data. The effect asserts that a positive relationship exists between a firm's book-to-market ratio (BE/ME) and returns. In addition, a return premium should be added to shares with relatively higher book-to-market ratios.⁴ Research has uncovered other variables which affect the variability of stock returns. These include profitability, liquidity and idiosyncratic volatility – none feature more prominently than the Size and Value effects (Drew, Naughton & Veeraraghavan, 2004). Using these extra variables to test the validity of the EMH (using the FF3FM) is discussed next.

2.5 EMH validity tests: the FF3FM

Although several models have emerged which use more than one factor to explain expected returns, the FF3FM (1993) – which postulates that the cross-sectional variation in the expected asset returns is explained by a combination of three priced factors⁵ (including the market premium) – is by far the most popular.

Fama & French (1993) analysed 25 US-based equity portfolios over 28 years (from July 1963 to December 1991) and found stocks that generally outperformed the market were small-cap and value (high book-to-market ratios)⁶ shares. This prompted the development of the FF3FM

⁴ That is: BE ratio = book value of equity/market value of equity appeared to resonate strongly with expected returns.

⁵ Note: Factors and premiums are used interchangeably throughout this dissertation, as is the book-to-market and value factors.

⁶ Low BE/ME ratio stocks are defined as "growth" stocks and are characterised by increases in capital value rather than high income/profit yielders – they tend to achieve higher growth rates than the market. Value stocks tend to trade at prices which are low relative to its fundamentals and are considered undervalued by the market.

which then formalised the relationship average returns on US stocks could be explained by three factors namely: excess market returns, a book-to-market or value factor, and a size factor. The FF3FM models the size and value effects as risk premia – i.e. as compensation to investors for holding less profitable, more volatile stocks.

Opponents of the FF3FM such as Lakonishok, Shleifer & Vishny (1994) and La Porta (1996), advocate a behavioural explanation for the book-to-market effect: it is merely the result of investors extrapolating past portfolio performance too far forward into the future. This in turn leads to the underpricing of value stocks and overpricing of growth stocks, rather than being as a result of compensation for risk bearing investors (Djajadikerta & Nartea, 2005).

Daniel & Titman (1997) argue that the book-to-market effect is a manifestation of intrinsic investor preferences: they have a higher propensity to hold "growth" stocks than "value" stocks. In response, Fama, French & Davies (2000) applied the FF3FM model to an extended data set (1929-1997) and found that the results of Daniel & Titman's (1997) report were period-specific, leading to spurious conclusions, and inapplicable to other periods.

Griffin (2002), used monthly data from 1981 to 1995, and tested the FF3FM in the United Kingdom, Canada and Japan. Size and value premiums were indeed found to contribute significantly to the explanatory power of the model. Lam (2002), using data from 100 stocks on the Hong Kong Stock Exchange also reported results to support Fama & French's (1996) findings. Australian studies from Faff (2001) and Gaunt (2004) reported that statistical significance and parameter magnitudes were comparable with Fama & French's (1993, 1995) work. Gregory & Michou (2009) applied the three-factor model to the UK stock market and found that size and value factors varied through time. Results were found to be like those attributable to the CAPM, but the FF3FM provided more explanatory power.

Work on emerging markets provide much the same conclusions. Silva (2006) found that the Brazilian market β was statistically significant, and that the explanatory power of the FF3FM model improved with the addition of the Size and BE/ME factors. Pasaribu (2009) found similar results when the model was applied to the Indonesian stock market. Most literature on emerging markets finds that individual stock returns are an increasing function of the book to market ratio and decreasing function of its size (Fama & French, 1998; Drew & Veeraghaven, 2001 and Lockwood, Rodriquez, Goldreyer & Barry, 2002).

Staying with emerging markets, little South African literature regarding the application of the

FF3FM on the JSE exists. Valery (2015) finds that this is justified by a general lack of academic interest in African financial markets, South Africa's status as an emerging, and relatively "immature", market, and a lack of consistent, reliable financial data. Auret & Sinclair (2006), first applied the FF3FM to the JSE using monthly data for shares from all JSE sectors from 1990 to 2000. Return data were adjusted for dividends and capital events and univariate and multivariate regressions were run to test the significance of explanatory variables in estimating excess stock returns. The results confirmed those found by Fama & French (1992): a significant positive relationship was found between the BE/ME factor and expected stock returns.

Basiewicz & Auret (2010) used data on every listed share in the JSE from December 1989 to July 2005.⁷ Firms with missing accounting data, financial statements denominated in foreign currency, and missing market data were omitted from the analysis to reduce potential bias of the results.⁸ The risk-free rate proxy was the three-month T-bill rate: this is the most liquid risk-free South African proxy. Time series regression found the value effect to be highly significant, but the BE/ME factor loses statistical power in describing pricing errors once the size factor is included as an explanatory variable.

The successful implementation of the FF3FM in South Africa is plagued by illiquidity. The FF3FM does not perform well in illiquid markets, estimated returns are biased because of risk parameter mis-measurement (Valery, 2015). Hearn & Piesse (2013) address this issue by adapting the FF3FM model to include a priced liquidity factor in both South Africa and Kenya (Nairobi Stock Exchange) using daily data from 1991 to 2007 (converted to USD to remove volatility effects of currency premiums). Daily stock price returns are divided by daily trading volumes and these coupled with share prices used to assemble liquidity factors. Average liquidity factors were then computed for each stock with *stock illiquidity* defined as the ratio of the absolute value of a share's percentage price change per USD of equity trading volume (Hearn & Piesse, 2013). The inclusion of the liquidity factor significantly improved portfolio return estimation. Although the size factor was found to be as important emerging markets as it is in developed markets; the primary risk in emerging markets is illiquidity (Valery, 2015). Tony-Okeke (2015) confirms this research finding, by showing that a Fama-French liquidity-

⁷ The sample period spanned June 1992 to July 2005 and included 894 companies; previous data were collected to collect prior accounting data which was used to estimate loadings.

⁸ Denominations in foreign currency imply a risk to the real market value of firm operations.

adjusted *four* factor model performs significantly better in explaining expected returns. The value (book value of equity/market value of equity: BE/ME) factor is insignificant on the JSE, but in contradiction to most developed market research, large stocks outperformed small stocks, and liquid stocks outperformed illiquid ones.

Fama & French's (1993) research appears to be country specific: differing market characteristics such as the degree of market sophistication, risk exposures and industry specific market weightings all affect the model's outputs.

Inconclusive – and sometimes contradictory – results obtained from tests conducted on the EMH directed research in different directions, to alternative interpretations of market behaviour. One of these, the FMH, argues that markets are *not* efficient, but *fractal*, i.e. they are not characterised by random walks, but rather, exhibit self-similarity.

2.6 FMH validity tests: the Hurst exponent and fractal dimension

Fractals are geometric shapes, parts of which can be identified and isolated, each of which demonstrates a reduced-scale version of the whole. Mandelbrot (1977) explored and developed fractal geometry mathematically and later applied this research to finance, ultimately using it as a realistic market risk framework. Prices generated from simulated scenarios based on fractal models were found to describe market activity more realistically (Joshi, 2014a and Somalwar, 2016): a description which underlies the FMH.

The FMH asserts that, far from an orderly system of rational, cooperating investors, financial markets behave as nonlinear dynamic systems which teem with interacting agents who rapidly process new information. These interacting agents, or investors, have different investment horizons and hold different market positions for various reasons, so this information is employed in different ways. Considerable price fluctuations may result (which are accurately modelled in calm markets by the FMH and MPT,⁹ and in turbulent trading conditions (*not* predicted by MPT)). FMH and fractal price models can be calibrated to replicate market price accelerations and collapses, key features of heteroscedastic volatility. These price fluctuations are indistinguishable (or 'invariant') at different time scales. This self-similarity implies the

⁹ Modern Portfolio Theory (MPT – which arose from the tenets of EMH) permits the construction of efficient portfolios (those which generate the highest return possible for a given level of risk) while still maintaining the EMH assertion that outperforming the market on a risk-adjusted basis is impossible.

persistence of market prices which would not be observed if returns were indeed independently and identically distributed, as postulated under the EMH. Also, prices deviate from their fundamentals for prolonged periods, and by a greater amount than allowed by the EMH. These empirical observations provide further evidence of market persistence (Carhart, 1997) and encourage a different interpretation of market behaviour other than simple 'efficiency'.

The FMH assumes price changes evolve according to *fractional* Brownian motion, a feature quantified by a quantity known as the Hurst exponent. Hurst (1956) explored the dependencies of long-range time series components (based on the River Nile's flood level observations) and formulated the Hurst exponent, H , which records both the level of autocorrelation of a series and estimates the rate at which these autocorrelations diminish as the time delay between pairs of values increases. Since these key features are also observed in financial time-series, it was postulated that H could be used in the description of market behaviour.

The literature exploring the Hurst exponent in finance and its relationship with the EMH is rich. The range of $H \in [0,1]$ and the EMH is based upon standard Brownian motion processes which assume prices evolve by random walks (which, for such processes, $H = 0.50$). A natural consequence follows from this framework: forecasting future price movements is impossible because price movements are independent and exhibit no autocorrelation, thus technical analysis provides no investor assistance. Deviations from $H = 0.50$ indicate autocorrelation which violates a key tenet of the EMH. Financial time series are also finite, thereby allowing for the possibility that $H \neq 0.50$ (Morales, Di Matteo, Gramatica & Aste, 2012).

Considerable research has focussed on examining H at different times and in different geographies: developed markets are discussed first.

Spanning 10 years (Jan-92 – Dec-02), daily data from both emerging and developed market indices were used to measure $H(t)$, the time-varying H (Cajueiro & Tabak, 2004a, b). Emerging markets had $H > 0.50$, but the long-term trend was towards $H = 0.50$, indicating increasing efficiency over the observation period. Developed markets' H was not statistically different from 0.50. The results for both markets were confirmed by Di Matteo (2007) who used 32 global market indices and Wang, Liu, Gu, Cao & Wang (2010) who used daily data to explore the degree of market efficiency present in the Shanghai stock market.

Grech & Mazur (2004) employed H to forecast market crashes. Three such crashes (1929 and 1987 in the US and 1998 in Hong Kong) were investigated using two years of daily data prior

to the relevant crash in each case. Before each crash, H decreased significantly, as trends dissipated, and volatility soared. During each crash, H increased significantly, as the market exhibited enhanced inefficiency, and investors accelerated the arrival of new information response times. Grech & Pamuła (2008) reached the same conclusions, using daily data from the Polish stock market.

Alvarez-Ramirez, Alvarez, Rodriguez & Fernandez-Anaya (2008) used daily data spanning 60 years from the S&P 500 and Dow Jones indices and found that H displayed erratic dynamic time-dependency. A time-varying evolution of market efficiency was observed with alternating low and high persistent behaviour, i.e. $H > 0.5$ in both cases, but different magnitudes.

The consequences for market efficiency during financial crises were explored by Lim, Brooks & Kim (2008) who found that the 1997 Asian crisis dramatically reduced the efficiency of global stock markets, but within three years efficiency had recovered to pre-crisis levels. The highest level of market efficiency was recorded during post-crisis periods, followed by pre-crisis periods. During crises, markets exhibit high inefficiency.

Vamvakaris, Pantelous & Zuev (2017) examined the persistency of the S&P 500 index using daily data from 1996 to 2010 and found that crises affect investors' behaviour only temporarily ($< six months$). The index also exhibited high anti-persistency (an indication of investor "nervousness", $H < 0.5$) prior to periods of high market instability. Considerable fluctuations of H were observed with a roughly annual frequency and peak to trough amplitude range of 0.2 to 0.4. No prolonged trends of H were recorded.

Work has also been conducted on the behaviour of H in developing markets, such as South Africa. For example, using daily data for 19 months (Jan 01 – Jul 07), Karangwa (2008) found $H \approx 0.50$ on the JSE.¹⁰ Using monthly data for a longer period (i.e. Aug 95 – Aug 07), Karangwa (2008) found $H = 0.58$. Ostaszewicz (2012) used two methods (Higuchi and absolute moments) to measure H using JSE price index data both pre- and post the 2008 crisis period and found $H > 0.50$ predominantly in the pre-2008 crisis period and $H < 0.50$ largely in the post-2008 crisis period. Chimanga & Mlambo (2014) investigated the fractal nature of the JSE and found $H = 0.61$ using daily data from 2000 to 2010. Sarpong, Sibanda & Holden (2016) found $H = 0.46$ for the JSE using daily data from 1995 to 2015 (thereby embracing the full period

¹⁰ Karanaga's (2008) study concluded before the onset of the 2008 credit crisis, so this event and its aftermath were not included in his analysis.

investigated by Chimanga & Mlambo, 2014). Sarpong, *et al.*, (2016) also used the BDS test (Brock, Dechert, Scheinkman & LeBaron, 1996) to verify that JSE price index data exhibit non-random chaotic dynamics rather than pure randomness. These results confirm those obtained by Smith (2008) who, using four joint variance ratio tests, rejected the random walk hypothesis on the JSE.

The mixed results derived from the FMH have directed research into yet other avenues and have fostered enquiries which posit the possibility that market behaviour may be neither efficient nor fractal in nature, but *adaptive*. The interpretation of market performance is known as the AMH (Kima, Shamsuddin & Lim, 2011).

2.7 AMH validity tests: market efficiency and cyclical profitability

The AMH uses concepts borrowed from evolutionary theory. In this framework, investors behave like competing agents who – in their struggle for survival – aim to maximise profits as their *raison d'être*. Assessments of overall fitness suitability, mutation rates, adaptation mechanisms and reproductive strategy success rates have been examined.

Two implications that the AMH would give rise to – were it a true description of market behaviour – are variable market efficiency and cyclical profitability. These characteristics, if found, would confirm the AMH and contradict the EMH. Zhou & Lee (2013) used prices from the US real estate investment trust (REIT) market and confirmed both implications using the automatic variance ratio test of Choi (1999) and the automatic portmanteau test of Escanciano & Lobato (2009).

Using data from the Brazilian (Sao Paulo) stock exchange from Jan 1995 to Dec 2012, Dourado & Tabak (2014) found strong evidence in favour of variable, adaptive market behaviour. Hiremath & Narayanb (2016) used both linear and nonlinear methods to evaluate the AMH empirically in the Indian stock market. Cyclical profitability was found using linear methods, while nonlinear tests exhibited evidence of periods of alternating efficiency and inefficiency. Similar results were confirmed for the Japanese stock market using time-varying auto-regressive models (Noda, 2016).

Kim, Li & Perry (2017) found evidence of market adaptability: upward price drifts between announcement and effective dates almost disappeared in the years from 2010 to 2013. No evidence was found of positive price drifts between announcement dates and effective dates

and much of newly added stock price impact occurred before the relevant market opened on the day just prior to the announcement.

The jury is still out on which of the three interpretations of market behaviour (EMH, FMH, AMH) is correct. Each hypothesis has its critics, and each makes assumptions – often unrealistic. The EMH has a long pedigree, so it has attracted considerably more research and literature than the FMH and AMH. The latter two frameworks, while still relatively new, explain aspects of market behaviour which the EMH has proved incapable of doing, but the evidence for these successes has been principally assembled in large, liquid, developed markets. More research needs to be conducted on developing markets, such as South Africa. The next two chapters tackle precisely these issues: Chapter 3 evaluates the FF3FM and contrasts the results with those obtained from the CAPM to assess the validity of the EMH in an emerging market environment. Chapter 4 then explores the FMH in a global context (using developed and developing markets for comparison) and examines some interesting consequences for investors if the FMH is indeed an accurate description of market behaviour.

Chapter 3

The Capital Asset Pricing Model and Fama-French Three Factor Model in an Emerging Market Environment

Adam Karp¹¹ and Gary van Vuuren¹²

Abstract

This article tests the validity and descriptive accuracy of the Capital Asset Pricing Model and the Fama-French Three-Factor Model, by assessing the variation in excess portfolio returns on the Johannesburg Stock Exchange and determining which model fared better at explaining share return variability. Portfolios of stocks were constructed based on an adapted Fama & French (1993) approach, using a 3×2 annual sorting procedure, based on Size and Book-to-Market metrics respectively. The sample period spans six years, 2010 to 2015, and includes 46 companies listed on the JSE. Results show that both models perform relatively poorly – with low R^2 values – because of inadequate market proxy measures, market liquidity restrictions, unpriced risk factors and volatility inherent in emerging markets. The value premium explains a larger proportion of excess return variation than the size premium and is more pronounced in portfolios with higher book-to-market portfolios.

Keywords: Capital asset pricing model, value, three-factor model, liquidity

3.1 Introduction

The notions of risk and return form the body of fundamental first principles of rational investing. Since the advent of modern financial systems, and the emergence of sophisticated markets, the question of how and what return premiums risk bearing assets should bear, in the presence of such risk, has been one which financiers and economists alike, have long been concerned. If the relationship between risk and return can be understood, and subsequently measured with suitable descriptive accuracy then the implications of such estimation are far-reaching.

From a corporate investing perspective, asset-pricing models can generate evaluations of the cost of firm equity,¹³ a key component in the appraisal of capital budgeting as well as capital structure decisions. For individual investors, they serve as asset differentiation mechanisms;

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¹³ The rate of return paid to equity investors as risk compensation.

comparative tools which can be used to assess and decide on the composition of portfolio holdings, depending on investor preference.

One of the earliest asset pricing models of Sharpe (1964) and Lintner (1965) developed using the foundational groundwork of Markowitz's (1952a) mean-variance portfolio framework, led to the Capital Asset Pricing Model (henceforth, CAPM). The CAPM describes how the expected return on an asset or portfolio of assets is a linear function of the markets systematic risk component or market risk. Subsequent models, such as Arbitrage Pricing Theory introduced by Ross (1976) and later augmented by Chen, Roll & Ross (1986), introduced the notion of multivariate asset pricing models which estimated asset returns, in a manner which did not distinguish between the causality of macro and micro return predictors.

Fama & French (1993) extended the CAPM by showing that returns could be predicted by three factors, namely: market, size and value, the outcome of which resulted in the formulation of the Fama-French Three Factor Model (henceforth, FF3FM). This finding has since been tested extensively with congruent findings occurring in many markets. While extensive studies have been applied to developed markets, specifically the US and Western Europe, the literature regarding the application of such models in emerging markets is sparse. This article undertakes an empirical evaluation with the following objectives:

- to test the ability and validity of the CAPM and the FF3FM as descriptive models in explaining excess stock returns on the Johannesburg Stock Exchange (Henceforth, JSE).
- to compare the performance of the CAPM relative to the FF3FM, to ascertain which model outperforms the other, with respect to explanatory power.
- if indeed there are significant size and value factors which affect stock returns, to determine which factor explains the larger proportion of the variation in stock returns.

This work adapts that used in Fama & French (1993) to accommodate South African data.

South Africa is a middle-income, emerging financial market.¹⁴ In 2013, South Africa was ranked the 19th largest stock exchange in the world by market capitalisation¹⁵ and the largest exchange in Africa (≈ 400 listed companies (JSE, 2013)). While the JSE may be a relatively well-

¹⁴ One which has a low to middle per capita income.

¹⁵ Around \$1 007bn at the start of 2014.

established exchange, there is a scarcity of literature with respect to asset pricing model's applications and those involving the application of the FFTFM (1993, 1995). This work aims to add to the available literature.

The remainder of this article is structured as follows: Section 3.2 covers some preliminaries which are necessary for the discussion which follows. This is followed by Section 3.3 which covers the literature surrounding general asset pricing, with specific focus on the CAPM and FF3FM. Section 3.4 examines the data and methodology employed. Section 3.5 discusses the analysis and results, while Section 3.6 concludes and provides recommendations for further studies.

3.2 Preliminaries

3.2.1 The CAPM

The CAPM, as presented in the works of Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966), relies on a series of stringent assumptions. A fundamental notion is that investors hold well-diversified portfolios, implying that idiosyncratic risk can be diversified away and the only risk for which investors are compensated is attributable to a systematic, non-diversifiable risk component (represented by the market).¹⁶ Other assumptions underlying the model are that investors:

1. aim to maximise economic utilities (asset quantities are given and fixed),
2. are rational and risk-averse,
3. are broadly diversified across a range of investments,
4. are price takers, i.e., they cannot influence prices,
5. can lend and borrow unlimited amounts under the risk-free rate of interest,
6. trade without transaction or taxation costs,
7. deal with securities that are highly divisible (all assets are perfectly divisible and liquid),
8. have homogeneous expectations, and
9. assume all information is available at the same time to all investors (Bodie, Kane & Marcus, 2008).

¹⁶ Idiosyncratic risk is the specific risk associated with a company or asset, while systematic risk refers to risk attributable to the market and its movements, which cannot be diversified away.

3.2.2 A brief note on β

Systematic risk is measured by the β of a portfolio, defined as:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \quad (3.1)$$

where: $Cov(R_i, R_m)$ = The covariance of asset/portfolio relative to the market, $Var(R_m)$ = the variance of the market and $\beta_i = \beta$ of portfolio i . The expected return according to the CAPM, is then given as a linear function of the sum of the market risk-free rate of interest and the product of the β and excess return, such that

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \quad (3.2)$$

where: $E(R_i)$ = The expected return on asset/portfolio i , R_f = The risk-free rate of interest, β_i = The β value of asset/ portfolio i and $E(R_m)$ = the expected return on the market. (3.2) may be re-written:

$$E(R_i - R_f) = \alpha + \beta_i[E(R_m) - R_f] + \epsilon_i \quad (3.3)$$

where all the elements defined in (2) are the same in (3), $E(R_i - R_f)$ = the expected excess returns on portfolio i , α = intercept of the estimated regression line, $\beta_i[E(R_m) - R_f]$ = the excess return on the market premium and ϵ_i = random error component.

3.2.3 The FF3FM

The FF3FM served as a tool to address the shortfalls and complications associated with the CAPM. Fama & French (1993, 1995, 1996) found that approximating returns using two other factors (size and value) in conjunction with the original market factor as presented by the CAPM, could significantly improve stock return estimation. The size of a firm is defined as the market capitalisation (henceforth, ME):

$$ME = (\text{Share price}) \times (\text{number of outstanding shares in issue}) \quad (3.4)$$

The value premium of a firm – which is best represented by the Book-to-Market ratio (henceforth, BE/ME), reflects the firm's fundamental accounting value relative to current market value given by:

$$\frac{BE}{ME} = \frac{(\text{Book Value of Equity})_{t-1}}{(\text{Market Value of equity})_t} \quad (3.5)$$

Both the size and value premiums are captured in the model by engineering two portfolios

called "small minus big" (SMB) and "high minus low" (HML), in line with the methods used in Fama & French (1993, 1996) – and will be dealt with in further sections. The relationship between these three factors and the expected return on asset i can be approximated as follows:

$$E(R_i) = \alpha + b_i \cdot [E(R_m)] + s_i \cdot E(SMB) + h_i \cdot E(HML) \quad (3.6)$$

where $E(R_i)$ = expected return on asset/portfolio i , R_f = risk-free rate, $E(R_m)$ = expected return on the market, $E(SMB)$ = expected return of the size factor, $E(HML)$ = expected return on the value factor, b, s, h = factor coefficients and α = regression intercept. (3.6) can be re-written to yield (3.7) which is used to run the multiple linear regressions for the FF3FM:

$$E(R_i) - R_f = \alpha + b_i \cdot [E(R_m) - R_f] + s_i \cdot E(SMB) + h_i \cdot E(HML) + \epsilon_i \quad (3.7)$$

where all elements described previously are equivalent, α is the regression intercept and ϵ_i is the random error regression component.

3.3 Literature review

Asset pricing has long been an area of considerable interest, with initial contributions dating back as early as the 18th century (Bernoulli, 1738).¹⁷ The emergence of an integrated global community and the development and sophistication of financial markets have been the catalysts for its ever-increasing prominence. The uses of asset pricing models are vast: they serve as tools for management in the undertaking of capital budgeting decisions, pricing equity, as well as determining the cost of capital. These are all elements intrinsic to the operations of firms and investors.

The 20th century saw the emergence of works which have underpinned the fundamental ideas regarding mean-variance optimisation, equilibrium analysis, and investor preference. Most notable was the "mean-variance" model contribution of Markowitz (1952a, b), forming the basis of Modern Portfolio Theory (MPT) – on which most subsequent asset pricing models are currently built. The model is a single period model, which assumes investors are risk adverse. Portfolio selection is undertaken at time $t - 1$ and stochastic returns are determined at time t . The aim of the investor is to maximise expected return subject to their risk appetite.

¹⁷ Specifically, the St Petersburg's article, *published* in 1738 which detailed and formed the basis of economic theory regarding risk aversion, risk premium and utility.

3.3.1 The development of the CAPM

Markowitz's (1952a) work presents a direct and rudimentary foundation to the CAPM, developed during the 1960s, with collective contributions from Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). The allure of the CAPM is that is described as offering powerful and intuitively pleasing predictions with respect to expected return-risk relationships, in a rational equilibrium market (French, 2004).

Tests conducted on both versions of the model namely: The Sharpe-Lintner (Sharpe, 1964 and Lintner, 1965) and Black (1972) both arrive at the same conclusions. These inferences are two-fold. First, is the proposition that the co-variance of asset returns with the market, relative to the risk or variance of the market (β), is both adequate and sufficient in explaining the variation in asset expected returns; and secondly that the expected return- β relationship is positive, and symbiotic in nature. Early cross-sectional tests and time-series regressions applied to both forms of the model, suggested that the relationship between asset returns and β were found to be approximately linear. The addition of other explanatory variables also led to no significant explanatory improvement (perhaps because of the immature nature of financial markets at the time), resulting in a premature conclusion that the market proxy portfolio was indicative of a "stand-alone "indicator of risk.

3.3.2 Limitations of the CAPM

The CAPM pioneered the way in which assets are priced, however it is encumbered with several limitations. Firstly, the model makes a series of unrealistic assumptions and may be an inadequate representation of the behaviour of financial markets. Secondly, historical estimates of β s are problematic as they have been found to vary considerably through time (Mullins, 1982). Roll (1977) criticised the CAPM by suggesting that it is impossible to observe a strictly diversified market portfolio, and a market index serving as a proxy for such a portfolio would inherently have predictive errors.

3.3.3 The size effect

The refutations with respect to the CAPM model stem primarily from empirical evidence. Expected returns of assets were found not merely to be driven because of market risk, but instead a combination of additional risk factors. Basu (1977, 1983) and Banz (1981) for example, first documented what has come to be known as the "size effect" on US stock data. They

showed that stocks with high earnings/price ratios, earned significantly higher returns than those with low earnings/price ratios. Moreover, returns for firms with relatively low ME were found to be significantly higher (return premium attached to small firms) than firms with relatively large market capitalisations.

The response in defence of this finding points to the idea that small firms, in general, have higher β s than large firms. The resultant β differences, however, were not significant enough to adequately explain the small-big capitalisation return disparity (Kampman, 2011).

3.3.4 Book-to-market/value effect

The book-to-market effect or value effect was first documented by Reid, Rosenberg and Lanstein (1985) using US data, and later confirmed by Davis (1994) (also using US data), Lakonishok (1991) (Japanese data) and Fama & French (1996) using international market data. The effect postulates that there exists a positive relationship between a firm's book-to-market ratio (BE/ME) and returns, and a return premium should be given to stocks with relatively higher book-to-market ratios.¹⁸ Fama & French (1992), showed that a cross-sectional regression of book-to-market ratios on realised returns yielded a positive coefficient that was approximately six standard deviations different from zero.

While the literature has uncovered various other variables which affect the variability of stock returns (such as profitability, liquidity and idiosyncratic volatility) none features more prominently in the literature than the size and value effects (Drew, Naughton & Veeraraghavan, 2004). Treatment of other variables will not be dealt with explicitly here, as they fall outside the scope of this article.

3.3.5 The FF3FM

The above-mentioned considerations suggest that the single factor CAPM is not entirely suitable for explaining the relationship between risk and return. While many models have emerged because of the persistent issues associated with CAPM, none are perhaps used as extensively as the FF3FM (1993). The model postulates that the cross-sectional variation in the expected returns of an asset is a function not only of the market premium, but instead, a combination of three priced factors.¹⁹

¹⁸ That is: BE ratio = Book Value of Equity/Market Value of Equity appeared to resonate strongly with expected returns.

¹⁹ Note: factors and premiums are used interchangeably throughout the article, as are book-to-market and value factors.

Fama & French (1993) conducted a study which analysed a total of 25 US based equity portfolios (spanning a period from July 1963 through to December 1991) and found that the cross-section of average return on US stocks could be explained overwhelmingly by three factors namely: excess market returns, a book-to-market or value factor, and a size factor.

Fama & French (1993) found that two classes of stocks tended to outperform the market. The first being small-capped stocks and the second being "value" stocks (stocks with high book-to-market ratios).²⁰ As a result, the FF3FM, models the size and value effects as risk premia – i.e. as compensation to the investor for holding less profitable, more volatile stocks.

This logic has not been without contention, however, opponents such as Lakonishok, Shleifer & Vishny (1994) and La Porta (1996), advocate a behavioural explanation in the belief that the book-to-market effect is a result of investors extrapolating past portfolio performance too far forward into the future. This would in turn lead to the underpricing of value stocks and overpricing of growth stocks, rather than being as a result of compensation for risk bearing investors (Djajadikerta & Nartea, 2005).

Daniel & Titman (1997) provide a characteristic explanation in which they argue that the book-to-market effect is a manifestation of characteristics of firms, which are intrinsic to investor preferences. Investors were found to have a higher propensity to hold "growth" stocks versus "value" stocks. In response to this critique Fama, French & Davies (2000) applied the FF3FM model to an extended data set (1929-1997) and argued that the results of Daniel & Titman's (1997) report were period-specific, leading to spurious conclusions, and not applicable to other time periods.

Another concern surrounding The FF3FM was the notion of 'data mining'. Fama & French (1996) respond to this by applying the Three Factor Model to other sets of data, spanning different periods, and show that the same relation between the variables is observed.

It is important to note that the FF3FM (1993) was developed and tested primarily on US data, most of stocks on which are comprised of industrially intensive industries, and thus, the resultant conclusions may only be relevant to markets with a set of characteristics.

²⁰ Low BE/ME ratio stocks are defined as "growth" stocks and are characterised by increases in capital value rather than high income/profit yielders – they tend to achieve higher growth rates than the market. Value stocks tend to trade at prices which are low relative to its fundamentals and are considered undervalued by the market.

3.3.6 Evidence from developed markets

Griffin (2002), using monthly data from 1981 to 1995, tested the FF3FM in the United Kingdom, Canada and Japan and reported that the size and value premiums do indeed contribute significantly to the explanatory power of the model. Lam (2002), using data for 100 Stock Exchange of Hong Kong (SEHK) listed stocks also reported results to support Fama & French's (1993) findings. Australian studies attributed to Faff (2001) and Gaunt (2004) report that the statistical significance, and parameter magnitudes, are comparable with Fama & French (1993, 1995) to a partial degree – noting a significant size effect with little evidence to suggest a significant book-to-market effect. This contrasts with Kassimatis's (2008) findings, which concluded that the FF3FM did not provide convincing evidence. More recently, Gregory & Michou (2009) applied the three-factor model on the UK stock market, in which the size and value factors were found to vary through time, and overall results were found to be similar, yet more explanatory compared with those of the CAPM.

3.3.7 Evidence from emerging markets

Silva (2006) found that the Brazilian market β was found to be statistically significant, and the explanatory power of the model improved with the addition of the Size and BE/ME factors. Pasaribu (2009) found similar results when the model was applied to the Indonesian stock market. Overwhelmingly, most of the literature on emerging markets point to the idea that returns on individual stocks tend to be an increasing function of the book to market ratio and decreasing function of its size (Fama & French, 1998; Drew & Veeraghaven, 2001 and Lockwood, Rodriquez, Goldreyer & Barry, 2002).

3.3.8 Evidence from South Africa

There exists a sparse set of South African literature with respect to the application of the FF3FM on the JSE. Valery (2015) mentions that since South Africa is an emerging, and relatively "immature" market, the lack of academic interest in the general African financial markets; and more precisely the lack of consistent and reliable data could be reasons to justify this. Auret & Sinclair (2006), were among the first to apply the Three-Factor Model to the JSE and in their study, monthly data for stocks from all sectors of the JSE were assembled from 1990 to 2000. Return data were obtained, adjusted for dividends and capital events and a thin trading filter was used to ensure that the trading volume of each share exceeded at least

one per period. Univariate and multivariate regressions were then undertaken to test the significance of the explanatory variables with respect to estimating excess stock returns. As per Fama & French (1992), a significant positive relationship was found between the BE/ME factor and expected stock returns. In addition, when the value factor was applied to the model of van Rensburg & Robertson (2003), it almost entirely subsumed the size factor (as evidenced in terms of explanatory power).

Basiewicz & Auret (2010) used data on every listed share in the JSE from December 1989 to July 2005.²¹ Firms with missing accounting data, financial statements denominated in foreign currency, and missing market data were omitted from the analysis – to reduce potential bias of the results.²² The proxy used for the risk-free rate was the three-month T-bill rate. This contrasts with one-month treasury bills available in the US and other developed markets – however, the three-month instrument is the most liquid risk-free proxy South Africa has. Utilising time series regressions, it was found that the FF3FM was able to account, significantly, for the value effect. However, the BE/ME factor loses statistical power in describing pricing errors once the size factor is included as an explanatory variable.

A recurring issue for the successful application of the FF3FM in South Africa, is liquidity. Evidence suggests that the FF3FM does not perform well in illiquid markets as this may result in biases in estimated returns through the mis-measurement of risk parameters (Valery, 2015). Since the largest capped company in South Africa is a small cap firm in the US, this raises concern for potential modelling issues. Hearn & Piesse (2013) addressed the liquidity issue by augmenting the FF3FM model to include a priced liquidity factor in both South Africa and Kenya (Nairobi Stock Exchange). Monthly data were collected from 1991 to 2007 and converted into USD to remove volatility effects of currency premiums when calculating excess returns. Stock price returns were computed daily and then divided by daily trading volumes. Daily trading volumes and share price levels were used to construct liquidity factors. An average liquidity factor was computed monthly for each stock. Stock illiquidity was measured and defined as the ratio of the absolute value of the percentage price change of a stock per US\$ of equity trading volume (Hearn & Piesse, 2013). The liquidity factor was found to significantly improve portfolio return estimation. The study showed that illiquidity was both a consistent

²¹ The sample period spanned June 1992 to July 2005 and included 894 companies; previous data were collected to collect prior accounting data which was used to estimate loadings.

²² Denominations in foreign currency imply a risk to the real market value of firm operations.

and a priced characteristic in South Africa and Kenya. While the size factor was found to be as important a component in emerging markets as it is in developed markets, the primary risk in emerging markets is illiquidity (Valery, 2015).

Tony-Okeke (2015) supports this finding, by showing that a Fama-French liquidity adjusted-four factor model performs significantly better in explaining expected returns. Moreover, Tony-Okeke (2015) concludes that the value (BE/ME) factor is insignificant on the JSE, and that in contrast to popular findings, large stocks outperformed small stocks, with liquid stocks outperforming illiquid ones.

The literature indicates that some results were consistent with the findings of Fama & French (1993, 1996), and others not. The resultant findings of Fama & French (1993) are very much country specific. Differing market characteristics, the extent of market sophistication, risk exposures and industry specific market weightings are all issues which may affect the model's outcomes. The most persistent problem with previous studies on the JSE are associated with illiquidity. To address this and to avoid liquidity adjustments, this work explored the top 50 companies (by market capitalisation and trading volume) on the JSE. The data have been adapted to rid them of potential bias in line with suggestions of Basiewicz & Auret (2010) and Valery (2015). The time period of the model encapsulates an expansionary phase of the South African business cycle (2010 – 2015) to avoid robustness issues.

3.4 Data and methodology

The period under consideration extended from January 2010 to January 2016.²³ The reason for this choice was firstly to address the issue of potential estimate bias. *Ceteris paribus*, the longer the time horizon of estimation, the higher the probability of the β values of the factors changing over the period (Bartholdy and Peare, 2005). Secondly, the Fama & French (1993) model has been shown to perform unsatisfactorily in periods of downturn or economic contraction. This article assesses the FF3FM in a period of positive average economic growth in South Africa. Thirdly, since the FF3FM has been shown to lack robustness during downturns in developed markets, an emerging market environment (South Africa) which tends to exhibit greater volatility than developed markets would exacerbate the issue.

²³ Fama & French (1993) used 28 years, quarterly sampling (112 data points) and addressed only 25 companies. Griffin (2002) used 14 years of quarterly data (56 data points), Faff (2001) 6 years of monthly data (72 points), Kassimatis (2008) used 5 years of monthly data (60 points) and Auret & Sinclair (2006) only used 10 years of monthly South African data (120 points).

Initially, all relevant data were collected on 53 of the most liquid JSE shares (rebalanced annually), over a period of six years (40 of which constituted the components of the JSE Top 40 index). This list included a variety of industries which would ensure that the portfolios would be well diversified.

3.4.1 The market index

Motivating the choice of a market index proved difficult. The FTSE/JSE All-Share index which represents 99% of the JSE's ME and is constructed using 164 companies was used as a proxy for the market index (JSE, 2013). The FTSE/JSE Top 40 Index thus provides a more appropriate proxy for the data pool. After data adjustments were made, the total number of firms used in the study stood at 46. Thus, most of the firms in the sample were members of the Top 40 in each of the respective years. Moreover, the JSE board use a liquidity screening process in which companies are filtered from the index, should they be too illiquid. Although the index is comprised of 40 companies, this composition represents over 85% of the total ME of JSE listed companies (Johannesburg Stock Exchange, 2013).

3.4.2 The risk-free rate

Conventional studies in developed markets such as the US and Europe make use of one-month T-bills or government bond equivalents as the risk-free approximation. For South Africa, the shortest term risk-free instrument is the highly liquid, 3-month treasury-bill, and is the rate used in the study (SARB, 2016). Annualised rates were retrieved from the South African Reserve Bank (SARB) after which average monthly rates were calculated using $R_{f'} = \sqrt[12]{1 + R_f} - 1$ where $R_{f'}$ = the monthly percentage rate of interest (%) and R_f = the annual interest rate.

3.4.3 Value of book and market equity

An important component for computing risk factors for the FF3FM involves BE/ME ratios.

To address the problem of liquidity, the study used the ALSI top 40 Index as a proxy for the market, in line with Valery (2015). Basiewicz & Auret (2010), applied the three-factor model to 200 companies in the JSE, involved setting restrictions on price and liquidity. In that case,

stocks which had share turnovers below 0.001²⁴ were excluded.

3.4.4 Data adjustment prior to portfolio construction

The original data sample comprised 53 companies. Return data were adjusted for dividends and capital events and univariate and multivariate regressions were run to test the significance of explanatory variables in estimating excess stock returns. Companies associated with missing or incomplete data were removed from the sample to prevent any potential bias when the portfolios were formed, and the respective regressions run. Any companies falling on a median, upper or lower quartile split, when partitioning companies into ME sizes, were excluded. Companies which issued financial statements denominated in foreign currency were also excluded (Basiewicz & Auret, 2010). Listed companies on the JSE are quoted in rands (ZAR), as is the calculated yield on the market index and risk-free rate. An exchange rate conversion to address this would be both difficult – constant rebalancing would need to occur to reflect daily movements – and inaccurate; this would cause a loss of real value for companies. After adjustments, the final database comprised 46 stocks.

3.4.5 Portfolio construction

To create portfolios which track the size and value factors, the sample of firms were sorted annually by the ME and BE/ME ratio like Fama & French (1993). For the size consideration, firms were stratified each year by a median value and are classified as either "big" (henceforth, B), for companies lying above the median, or "small" (henceforth, S), for companies lying below the median. Companies which fall on the median are excluded from the analysis to avoid bias. Similarly, firms are sorted annually into three other sub-groups based on book-to-market ratios. The low group (L) contains firms with the lowest 30% BE/ME ratios, the medium ratio group with the middle 40% (M), and a large group with the highest 30% (H) – see Table 3.1.

Table 3.1. Summary of the general partitioning procedure of portfolios based on percentile split and on size and book-to-market ratios.

Size	Book to market		
	Above 70%	Between 30% and 70%	Below 30%

²⁴ Share turnover as a proxy for share liquidity is computed by dividing the aggregate shares traded over a set period by the average number shares outstanding for the specified period. The higher the value of the ratio, the more liquid is the share of the company or portfolio.

Above 50%	Big/High (B/H)	Big/Medium (B/M)	Big/Low (B/L)
Below 50%	Small/High (S/H)	Small/Medium (S/M)	Small/Low (S/L)

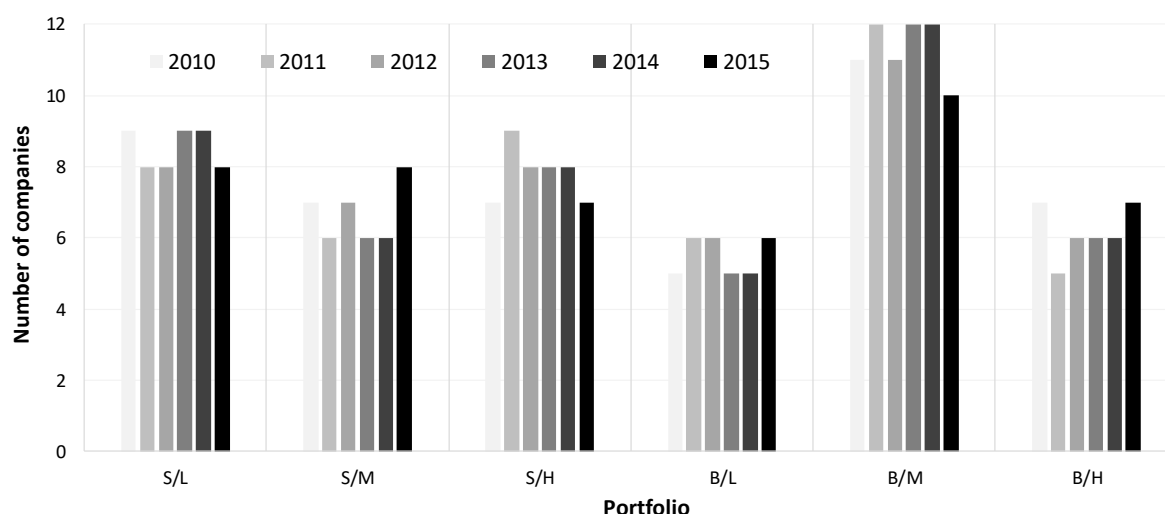
Source: Author.

High ratio firms are often referred to as *value firms* as they appear to provide an investor with good value, since they sell at low multiples of their respective book values (Bodie, Kane & Marcus, 2008). The portfolio intersections of the two sizes and three book-to-market categories are then found, for each respective year, such that six portfolios are formed: S/L, S/M, S/H, B/L, B/M, and B/H²⁵ and their monthly returns computed. Table 3.2 depicts the partitioning procedure on a value determined basis, while Figure 3.1 shows the number of companies in each portfolio group from 2010 to 2015.

Table 3.2. Value specific partitioning based on the sample data used in the study, indicating threshold category values.

Partitioning cutoffs	Cutoff proportion	2010	2011	2012	2013	2014	2015	Period average
Market cap (ZAR bn)	50%	42 115	40 372	58 961	61 874	75 542	84 073	60 490
BE/ME	30%	0.260	0.327	0.284	0.255	0.261	0.250	0.273
BE/ME	70%	0.580	0.710	0.622	0.651	0.675	0.812	0.675

Figure 3.1. Distribution of the number of companies in each portfolio from 2010 to 2015, re-balanced annually.



²⁵ As a matter of clarity, S/L corresponds to the portfolio of stocks which is classified as both small and having a low book-to-market ratio. Similarly, B/H corresponds to the portfolio which is big in size and has a high book-to-market ratio etc.

Source: Author calculations.

Explanatory Variables

SMB Factor

Following the formation of the intersection portfolios, the size premium, SMB, was constructed and is defined as the resultant difference of returns between small and large firms. More precisely, that is the difference in monthly returns between the sum of an equally weighted long position in the small sized portfolios and the sum of a short, equally weighted position in the low big sized groups (Bodie, Kane & Marcus, 2008). Mathematically this can be expressed as:

$$SMB = \frac{1}{3} \left(\frac{S}{L} + \frac{S}{M} + \frac{S}{H} \right) - \frac{1}{3} \left(\frac{B}{L} + \frac{B}{M} + \frac{B}{H} \right)$$

where: $S/L, S/M, S/H, B/L, B/M$, and B/H = the intersection of each respective portfolio formed on size and book-to-market values.

HML Factor

The book-to-market effect was captured by calculating the difference in the monthly returns between firms with relatively high BE/ME ratios, and firms with relatively low BE/ME ratios. Medium portfolios were excluded from the calculation, as Fama & French (1993) note that HML variable performs best when defined in the fashion employed. The high minus low (HML) factor can be understood as the difference in monthly returns between an equally weighted long position in high BE/ME ratio portfolios coupled with an equally weighted short position in low BE/ME portfolios.

$$HML = \frac{1}{2} \left(\frac{S}{H} + \frac{B}{H} \right) - \frac{1}{2} \left(\frac{S}{L} + \frac{B}{L} \right)$$

where: HML = the high – low or "value" factor, S/H = the monthly excess return on the small-high portfolio; B/H = the big-high portfolio and S/L = the small-low portfolio; B/L = the big-low portfolio.

3.4.6 Explanatory regression variables

Excess returns ($R_i - R_f$) on each of the six portfolios are used as dependent variables in the subsequent regression. Fama & French (1993) used more portfolios (25) due to their higher

number of classification fields. Such partitioning was deemed inappropriate because the sample size was far smaller and thus avoided having small numbers of stocks in each portfolio.

3.4.7 Statistical techniques

Descriptive Statistics

The first four moments of each excess return portfolio are considered (monthly and annually) to provide a general characteristics profile for the different stock portfolios.

Moment Scaling

When comparing dissimilar metrics, different scaling procedure applications are required to address the time-varying dynamics of return distributions. Correct scaling is imperative to ensure that metrics are compared on an equal footing (Gabrielsen, Kirchner, Liu & Zagaglia, 2012). Table 3.3 summarises the scaling requirement from a monthly to n -period value, applied to the data.

Table 3.3. Monthly to an n -period value scaling formulae summary, employed for the first four moments of the return distributions.²⁶

MOMENT SCALING	Mean	Standard deviation	Skewness	Kurtosis
Monthly value	$\mu_m(\%)$	$\sigma_m(\%)$	S_m	K_m
n -period scaling	$(1 + \mu_m)^n - 1$	$\sigma_m \times \sqrt{n}$	$\frac{S_m}{\sqrt{n}}$	$\frac{K_m + 3 \cdot (n - 1)}{n}$

Source: Gabrielsen, Zagaglia, Kirchner & Liu (2012).

Time-Series Regression and Backward Elimination

Time-series regressions were applied to the various portfolios.

Paired Sample t tests and the p -Value Approach

Partial t - tests were conducted on each portfolio and their respective factors coefficients to test for significance at the 5% level.

Durbin Watson Test Statistic

Auto-correlation amongst residuals could underestimate the true variance of the regression model and may lead to the rejection of the null hypotheses when it is in fact true (type 1

²⁶ μ = the mean, σ = the standard deviation, S = skewness, K = kurtosis.

error). To address the potential issue of auto-correlation amongst residuals, the Durbin Watson test statistic is used. The following hypothesis is tested using a critical value comparison approach:

$H_0: \rho = 0$ (there exists no auto-correlation present amongst the residuals)

$H_1: \rho \neq 0$ (auto-correlation amongst the residuals exists)

The test statistic was computed for each portfolio using:

$$d = \frac{\sum_{t=2}^T (e_t - e_{t-1})^2}{\sum_{t=1}^N e_t^2}$$

where: $e_t = y_t - \hat{y}_t$; and y_t and \hat{y}_t are the observed and predicted values of the response variable at time t respectively and d = The Durbin Watson test statistic.

Augmented Dickey Fuller Test (ADF) Result

The Augmented Dickey Fuller test was also run to test for stationarity. The ADF test, tests the null hypothesis of whether a unit root exists in the data. If a unit root is present, appropriate transformations of the data are needed (log normal transformations are useful). The ADF tests the following hypothesis:

H_0 = the data exhibit non-stationarity and

H_1 = the data exhibit stationarity.

Multicollinearity

In the FF3FM, it is necessary to test whether high correlation exists among the explanatory variables. If high correlations exist difficulties in distinguishing between individual variable effects are encountered as both variables may explain the same thing. Multicollinearity leads to spurious regressions, inflated R^2 values and inaccurate significance levels.

Factor Analysis

The first two moments of the SMB and HML factors are computed over the 2010-2015 period, as well as the 2010-2014 period. The omission of 2015 was carried through as a comparative procedure to illustrate the effects of market volatility on each factor.

3.4.8 Portfolio performance²⁷

Jenson's Alpha (α): Jenson's α (Jensen, 1967), is a performance evaluation procedure which tests whether the intercept (α) of a regression is statistically significant. Positive α s imply outperformance of the market benchmark (Top 40 index), whilst negative α s imply under-performance. For each individual portfolio the hypothesis tested at the 5% level is:

H_0 : The intercept (α) of the regression model for the CAPM/FF3FM is not significant and

H_1 : The intercept (α) of the regression model for the CAPM/FF3FM is significant.

Significant α s indicate that the regression model omits other factors, which should be priced.

Sharpe Ratio: The Sharpe ratio is a simple risk-adjusted return measure (Sharpe, 1964). For each of the six portfolios the Sharpe ratio measure is calculated and compared and indicates the level of excess return obtained for a specific volatility tolerance. The ratio is:

$$S_i = \frac{(R_i - R_f)}{\sigma_i}$$

where: S_i = the Sharpe ratio for portfolio i , R_i = average return on portfolio i (annual), R_f = average risk-free rate (annual) and σ_i = average standard deviation of portfolio i (annual).

The Sharpe ratio is considered under the caveat that potential abnormalities such as kurtosis and skewness cause severe problems with the ratio (Brown, 2016).

Other measures: Jensen's alpha provides a convenient (and well-known) portfolio performance measure – i.e. the portfolio returns even when the market return is 0%. It informs the investor immediately of the portfolio's outperformance over and above the market in which the portfolio operates.

The Sharpe ratio is also a standard, well-known and well-used performance measure – convenient for comparing risk free investments.

Although there are issues with both the above measures, this has not prevented copious use of both. For this reason, and because both are standard fare in evaluating portfolio performance for other work (see Hearn & Piesse, 2013; Tony-Okeke, 2015; Basiewicz & Auret 2010 and Valery, 2015).

²⁷ The principal factor of Fama & French (1993) factor models is portfolio performance evaluation (ex ante/ex post).

The Sortino ratio was not used because this would require knowledge of the actual downside returns to calculate a downside volatility. The Omega ratio was also not considered for the same reason and the Treynor ratio requires knowledge of a market beta, which would involve many more assumptions.

3.5 Results and discussion

Durbin Watson test result

To ensure ordinary least squares (OLS) regression was appropriate, auto-correlation amongst the portfolio residuals²⁸ were tested, with results indicated in Table 3.4, under the hypotheses (at the 5% level):

$H_0: \rho = 0$ (no auto-correlation present amongst the residuals)

$H_1: \rho \neq 0$ (auto-correlation amongst the residuals exists).

In each case the Durbin Watson statistic exceeds the critical value (for $n = 80, k = 3$), the null hypotheses cannot be rejected, and it is concluded that no auto-correlation among residuals is present at the 5% level (DW stat > 1.72).

Table 3.4. Summary output of the Durbin Watson (DW) test results for sample size $n = 71$.

DW upper critical 5% for $n = 80\%$	DW statistic	Portfolio	Reject/Accept H_0	n
1.72	1.983	S/L	Do not reject	71
	2.096	S/M		
	2.117	S/H		
	2.092	B/L		
	2.192	B/M		
	1.981	B/H		

Source: Author calculations.

The stationarity of the time series data was tested using the Augmented Dickey Fuller test (Table 3.5). In each case, the null and alternate hypotheses tested are:

H_0 = the data exhibit non-stationarity and

H_1 = the data exhibit stationarity.

²⁸ The tests are only conducted on excess returns – see Fama & French's (1993) original work, Griffin (2002), Faff (2001), Kassimatis (2008), Bartholdy and Peare (2005) and Auret & Sinclair (2006). All used (as stipulated by Fama & French, 1993) excess returns only – this was a key consideration. None used portfolio returns.

All portfolios except S/H were significant at the 5% level, implying that the data for the other eight portfolios are significant. On closer inspection of the excess returns on the S/H portfolio, they appeared to be two large values (in absolute terms) in the latter stages of 2015. These outliers were removed (mean replacement was introduced) and the test was re-run to yield a p -value of 3.29%, implying that all nine portfolios became significant ($\alpha = 0.05$) after the adjustment process.

Table 3.5. Dickey Fuller test results for all portfolios. The lag order is assumed to be zero.

Portfolio	DF test statistic	p -value	Accept/Reject H_0	Lag order	n
S/L	-3.58	4.23%	Reject	0	71
S/M	-3.78	2.53%			
S/H	-3.69	3.29%			
B/L	-4.51	1.00%			
B/M	-4.47	1.00%			
B/H	-4.44	1.00%			
$R_m - R_f$ factor	-4.38	1.00%			
SMB factor	-3.69	3.27%			
HML factor	-3.50	4.82%			

Source: Author calculations.

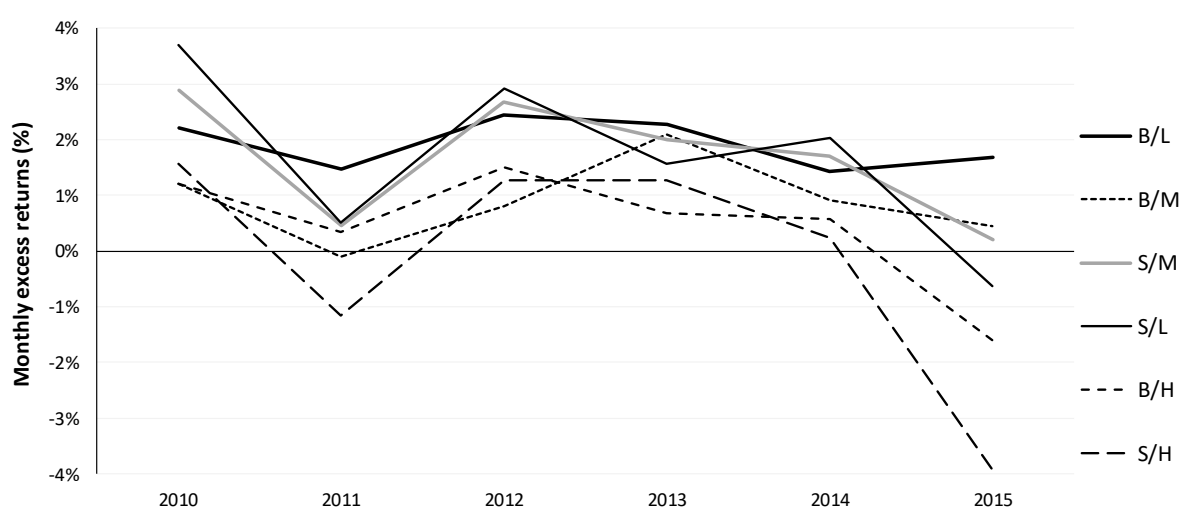
3.5.1 Descriptive statistics

Statistical metrics and portfolio returns: Table 3.6 summarises the first four moments of the data, spanning the entire sample period (2010-2015). The average excess return across the six portfolios during the sample period was 1.06% (13.5% annually). This value is expected considering that the analysis was conducted over an expansionary phase of the South African macro-economic business cycle.²⁹ On a portfolio-specific level, the B/L portfolio obtained the highest average monthly return of 1.91%, followed by S/L (1.63%), S/M (1.62%), B/M (0.90%), B/H (0.42%) and the lowest value attributable to S/H (-0.14%) respectively. This finding again, stands in contrast to Fama & French (1993, 1996) and the general historical observation, that on average, small and value stocks yield higher returns, relative to stocks which are classified as being both big and growth stocks.

²⁹ Figure 3.4 shows that South Africa was in a clear expansionary phase over much of the period, although losing some steam towards the very end.

Cumulative excess return data present in Table 3.6 show that if excess returns for the pair of three portfolio intersections are aggregated separately over the sample period, then the small stock portfolios outperform the large group portfolios by a small margin (128% versus 125%). the ranking order of returns based on the size and value premiums (as evident in much of the literature in developed markets) does not hold true. Figure 3.2 shows the first moment of the six portfolios over the sample period.

Figure 3.2. Average monthly portfolio excess returns (2010-2015). 2014 shows a decline in excess returns across most portfolios. This is likely due to the large degree of volatility present in the market at the time.

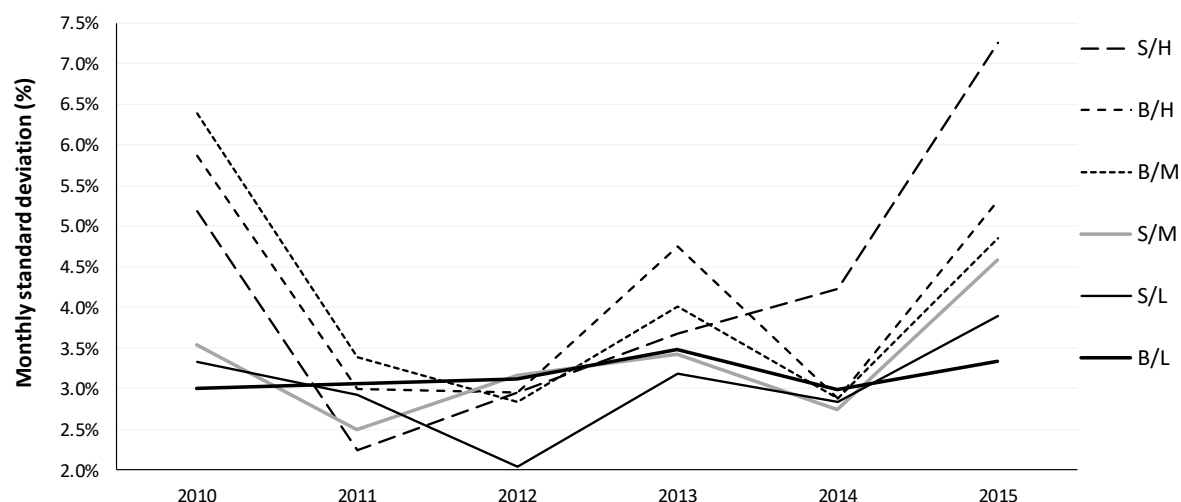


Source: Author calculations.

Standard Deviation: Higher standard deviations of the portfolios are associated with higher average excess returns. In general, the higher standard deviations of the portfolios are also characterised by higher a range of returns (i.e. the difference between the highest monthly excess return value and lowest). The exception to the above two observations is the S/H portfolio – a mining and minerals predominate portfolio, which returned on average, the highest monthly standard deviation (4.73%) of all the portfolios over the sample period (Figure 3.3). This could be explained by the volatility associated with the mining sector, not only from the typical market sense but also from an investment sense (Statistics South Africa, 2015). Foreign investment exhibits a strong positive relationship with economic growth, which in turn tends to increase general return levels and decrease volatility (Rahman, 2015). South Africa harbours some of the best mineral deposits in the world, yet it increasingly lacks an environment conducive to foreign investment. The Investment Attractiveness Index (IAI), computed by the Frasier Institute (2014), which measures mineral potential in conjunction with policy

perceptions, ranked South Africa 66th, lower than the Democratic Republic of Congo and Ghana (Seccombe, 2016).

Figure 3.3. Average monthly portfolio standard deviations (2010-2015), as in the case with the previous graph the latter half of 2014 is associated with high volatility, evident on the graph with spikes in portfolio standard deviation.



Source: Author calculations.

Skewness and Kurtosis: The returns are approximately normally distributed. Normal distributions exhibit skewness values close to zero (perfectly symmetrical) and a kurtosis of around 3. In both cases the data exhibit favourable results, with an average annual skewness = -0.06 and an average annual kurtosis = 2.74 .

Table 3.6. Descriptive portfolio statistics (2010-2015).

Descriptive metrics	S/L	S/M	S/H	B/L	B/M	B/H	Average
Mean	1.63%	1.63%	-0.14%	1.91%	0.90%	0.43%	1.06%
Standard deviation	3.77%	3.50%	4.73%	3.33%	4.12%	4.16%	3.94%
Skewness	-0.15	0.010	-0.17	0.003	-0.001	-0.05	-0.06
Kurtosis	2.79	2.71	2.79	2.77	2.68	2.77	2.75
High %	9.39%	8.87%	9.03%	10.08%	8.60%	10.42%	9.40%
Low %	-8.87%	-6.56%	-14.15%	-6.96%	-7.84%	-10.78%	-9.19%
Cumulative returns	200.8%	201.1%	-16.8%	269.6%	78.5%	27.2%	126.7%

Source: Author calculations.

3.5.2 Explanatory variables: risk factors

Arithmetic means over 2010-2015 for SMB and HML factors are -0.04% and -1.63% respectively. Although both negative, the SMB is considerably lower than the HML. The HML's large negative value over 2010-2015 is a result of low BE/ME portfolios outperforming high

BE/ME portfolios by a larger differential. This contradicts the Fama & French (1993) observation that value stocks (high BE/ME ratios) outperform growth stocks (low BE/ME ratios).

To illustrate the effect of volatility on factor performance, 2015 (a turbulent year in South Africa, particularly the latter half) has been omitted from the computation. Once 2015 data are removed, both the SMB (0.28%) and HML (−1.29%) increase. The HML becomes less negative and both factor standard deviations decrease by $\approx 11\%$. The SMB becomes positive, implying that small firms outperform relatively larger ones, as expected.

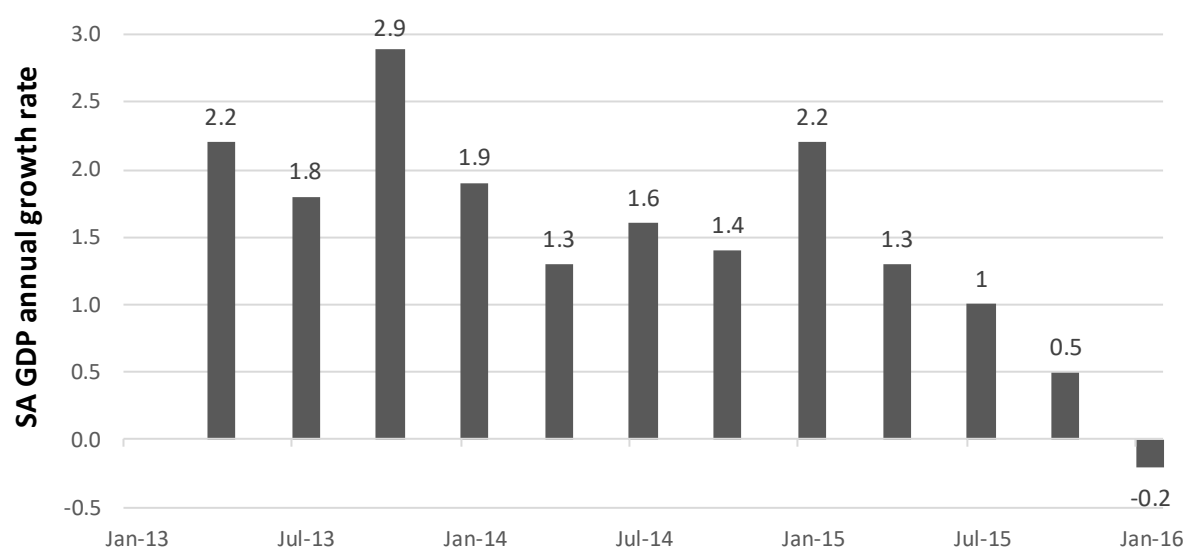
The effect on the market factor is less pronounced (Table 3.7). The mean value increases from 0.46% for the 2010-2015 periods to 0.57% when 2015 is omitted (2010-2014). This is indicative of higher macroeconomic growth rates prior to 2015 which translates to higher market returns (proxied by the top 40 market index), while the standard deviation was static. Figure 3.4 provides the growth rates of the South African economy from 2013 to 2016.

Table 3.7. Factor summaries for both 2010-2015 and 2010-2014.

		Market Premium	SMB	HML
2010-2015	Mean	0.47%	-0.04%	-1.63%
	Standard Deviation	3.85%	2.34%	3.49%
2010-2014	Mean	0.57%	0.29%	-1.30%
	Standard Deviation	3.86%	2.08%	3.13%

Source: Author calculations.

Figure 3.4. South African growth rate, measured by GDP from Jul-13 to Jan-16.



Source: TradingEconomics, 2016.

3.5.3 Factor correlation interpretation

For the 2010-2015 period (Table 3.8), it is observable that a positive, yet weak correlation exists between the HML (0.065) and SMB (0.24) factor portfolios, and the excess market portfolio ($R_m - R_f$) respectively. A possible reason for these low correlations may stem from the fact that most of the companies listed in the sample are Top 40 Index constituents and represent multinational companies with well-diversified operations world-wide. Steinhoff International Holdings Limited, for example, is a parent company which manufactures and distributes household commodities and furniture-based products throughout Sub-Saharan Africa and Europe (Steinhoff, 2015). Imperial Holdings Limited, a vehicle related import distribution retail, and rental company, as another example, operates in over 1 200 locations and 31 countries, with operations spanning five continents (Imperial, 2016).

Many of the shares used are listed in other stock markets such as the London Stock Exchange (LSE) and the New York Stock Exchange (NYSE). Share returns are affected by market movements in these foreign markets, in addition to domestic movements.

Current (2016) South African regulations permit foreign domiciled companies to be treated as domestic based listings and the last decade has seen foreign exchange rules and tight equity holding regulations (for domestic investors) relaxed (Valery, 2015). While this poses an important regulatory exchange shift in terms of enhancing the JSE as a more attractive listing destination, it hampers traditional asset pricing models in explaining returns. This is observed when pricing the market factor as a market proxy, which becomes harder to capture.

The positive, yet weak, correlation between HML and SMB (0.083) is expected, as most of the stocks in the sample represent the largest MEs stocks on the JSE, and many of companies across the portfolio intersections operate within in the same industry and are thus subject to similar forces. Moreover, because portfolios are rebalanced annually according to the relative benchmarks of BE/ME and ME, movements of stocks into different portfolios could serve as part of the explanation. For example: Old Mutual began in the B/M portfolio in 2010 after which it transitioned to the B/H portfolio. Initially, it would form part of the SMB calculation, yet after the transition it was included in both the SMB and HML calculations.

Evidence of this weak correlation serves to rule out any potential issue of multicollinearity. Were the data found to be highly correlated, small changes in the data could result in erratic

changes in coefficient estimates and impair the descriptive power of models used in the regression (Table 3.8).

Table 3.8. Summary of correlation matrix amongst factors 2010-2015 and 2010-2014.

		$R_m - R_f$	SMB	HML
2010 – 2015	$R_m - R_f$	1		
	SMB	0.243	1	
	HML	0.065	0.083	1
2010 – 2014	$R_m - R_f$	1		
	SMB	0.247	1	
	HML	0.099	-0.241	1

Source: Author calculations.

3.5.4 CAPM regression results

2015 was a tumultuous year for the South African market. Table 3.9 represents the CAPM regression of each portfolio using the market premium as the sole risk factor. The results suggest that the CAPM performs poorly in describing the stock return variation as evidenced by low adjusted R^2 values which range from 0.031 to 0.062. All the small grouped stocks, namely S/L, S/M and S/H's market premiums are statistically insignificant at the 5% level, suggesting that while the CAPM performs unfavourably overall, it performs better for big group stocks relative to small group stocks.

The negative β s indicate the lack of co-movement between portfolios and market. This is not surprising: many of the company constituents are well diversified and multinational and are subject to international market movements: the top 40 Index, as a market proxy, fails to explain stock returns.

When 2015 is removed from the analysis such that the regression spans the period from 2010-2014, all adjusted R^2 values increase (Table 3.10). Although these increases are slight (0.0093 to 0.0252), it renders the S/L portfolio statistically significant at the 5% level, where previously it was not. This increased performance is potentially because of the omission of the 2015 market volatility within South Africa, as well as shortening the descriptive period, which in turn reduces the possibility of unbiased β estimates.

These results support the findings of Fama & French (1992), which argue that the CAPM fails to capture a significant portion of stock return variation.

Table 3.9. CAPM regressions 2010-2015.

2010 – 2015	β	Adjusted R^2	Standard error	p -value B
S/L	-0.206	0.031	3.736	0.077
S/M	-0.042	-0.012	3.544	0.698
S/H	-0.063	-0.012	4.794	0.673
B/L	-0.238	0.062	3.251	0.020
B/M	-0.253	0.042	4.059	0.047
B/H	-0.265	0.046	4.095	0.040

Source: Author calculations.

Table 3.10. CAPM regressions 2010-2014.

2010 – 2014	β	Adjusted R^2	Standard error	p -value B
S/L	-0.245	0.056	3.450	0.040
S/M	-0.094	-0.005	3.287	0.400
S/H	-0.058	-0.014	3.764	0.647
B/L	-0.248	0.068	3.213	0.026
B/M	-0.272	0.052	3.967	0.046
B/H	-0.275	0.060	3.763	0.034

Source: Author calculations.

3.5.5 FF3FM regression results

Initial regression results (2010-2015)

The result summary (Table 3.11) indicates that the FF3FM performs relatively weakly. The adjusted R^2 values are poor, ranging from 0.113 to 0.500 across the portfolios. Considering the market environment, this is not surprising.

For the coefficient estimates on the market factor, four of the six loadings are insignificant at the 5% level, with only S/L and B/L as exceptions. Again, the plausible explanation for this stems from the multi-national listings and company diversification discussed in the previous section. In view of the HML (value) factor, four of the six portfolios are deemed significant at the 5% level, except for the S/L ($p = 43.1\%$) and S/M ($p = 97.6\%$) portfolio. The portfolio constituents of the S/L and S/M portfolios respectively show that over the sampling period an average of 61% (ranging from 50% to 69%) of the companies were outside the JSE Top 40.

The implication of this is twofold. First, it suggests, potentially, that using a book-to-market ratio and ME measure as risk proxies for firms below certain value "thresholds" is not appropriate. Portfolio constituents of the low and medium groups in studies conducted on developed markets are often superior in both sheer size and BE/ME ratio values to those of emerging markets.

Secondly, the risk characteristic profiles of small grouped companies are likely to differ from that of large grouped companies, which could explain the weak descriptive power of the HML in this case. Some examples, among others could include: operating constraints, level of foreign market exposure, cost profiles and production constraints (economies of scale in production industries).

Controlling for size, it also found that the loadings on HML increases monotonically from the low to high BE/ME portfolios in both small and big groups and is consistent with Fama & French (1995).

The results for the SMB (Size) factor indicate consistent significance of the coefficients at the 5% level ($\alpha = 0.05$) among all six portfolios, with $0.11\% < p < 2.49\%$. All three big sized portfolios load negatively on SMB, while all three small sized portfolios have positive loadings. This is consistent with Fama & French's (1995) "small firm effect" that smaller firms tend to outperform large ones, under the caveat that HML and the market premium is controlled for.

The adjusted R^2 values have increased, with increases in BE/ME and ME directly, confirming that the model performs better on larger based value stocks (high BE/ME ratios) – see Table 3.11.

Table 3.11. Initial regression results 2010-2015.

2010 – 2015	β	S	H	Adjusted R^2	p-value B	p-value S	p-value H
S/L	-0.275	0.504	-0.097	0.103	0.019	0.009	0.431
S/M	-0.130	0.593	0.003	0.113	0.221	0.001	0.976
S/H	-0.199	0.576	0.878	0.500	0.068	0.002	0.000
B/L	-0.167	-0.358	-0.304	0.213	0.082	0.025	0.004
B/M	-0.195	-0.538	0.369	0.194	0.105	0.007	0.005
B/H	-0.243	-0.430	0.720	0.433	0.018	0.011	0.000

Source: Author calculations.

Removal of insignificant variables and disjoint tests

After the initial regression tests, backward elimination conducted at the 5% level ($\alpha = 0.05$) was carried out and the regressions were re-run (Table 3.12). The adjusted R^2 values fell slightly, from 0.113 to 0.482, with the highest values attributable to the portfolios with high BE/ME ratios (S/H and B/H respectively). The issue of the market premium insignificance from four of the six portfolios is clearly indicative of the lack of an efficient market proxy, and possible solutions for this are dealt with in Section 3.6.

Table 3.12. Regression results after the removal of insignificant variables 2010-2015.

2010 – 2015	β	S	H	Adjusted R^2	p-value B	p-value S	p-value H
S/L	-0.279	0.494	-	0.108	0.017	0.010	-
S/M	-	0.541	-	0.119	-	0.002	-
S/H	-	0.498	0.868	0.481	-	0.006	0.000
B/L	-	-0.424	-0.313	0.189	-	0.008	0.003
B/M	-	-0.615	0.359	0.173	-	0.002	0.007
B/H	-0.243	-0.430	0.720	0.433	0.018	0.011	0.000

Source: Author calculations.

The loadings and significance of the remaining factors in each portfolio are dissimilar to the previous regression, indicating that although the model is weak in descriptive prowess, it is stable. The SMB factor is a significant variable in all six of the portfolios, while the HML is significant in only four of the six.

To determine which of these factors contributes more in explaining overall return variations, two additional regressions were run (results in Tables 3.13 and 3.14) in disjoint tests. The HML and market premium were considered together and the SMB and market premium. Adjusted R^2 values indicate that the HML factor ($0.026 < \text{adjusted } R^2 < 0.428$) explains a much larger portion of the variation in expected stock returns than the SMB ($0.071 < \text{adjusted } R^2 < 0.126$). Moreover, the HML factor appears to be a more valuable variable for firms with high BE/ME ratios, with adjusted $R^2 = 0.428$ for the S/H and 0.385 for the B/H portfolios respectively. The results indicate that the BE/ME ratio is a more powerful descriptive variable than firm size, an important finding consistent with Fama & French (1992).

Table 3.13. Regressions using SMB and the market premium (2010-2015).

$R_m - R_f$ SMB	β	SMB	Adjusted R^2	p value B	p value S
S/L	-0.279	0.494	0.108	0.0017	0.010
S/M	-0.130	0.593	0.126	0.217	0.001
S/H	-0.162	0.670	0.080	0.270	0.007
B/L	-0.180	-0.390	0.121	0.076	0.020
B/M	-0.179	-0.499	0.106	0.155	0.018
B/H	-0.212	-0.353	0.071	0.103	0.099

Source: Author calculations.

Table 3.14. Regressions using HML and the market premium (2010-2015).

$R_m - R_f$ SMB	β	HML	Adjusted R^2	p value B	p value H
S/L	-0.202	-0.074	0.021	0.086	0.563
S/M	-0.044	0.030	-0.026	0.689	0.804
S/H	-0.115	0.904	0.428	0.303	0.000
B/L	-0.219	-0.321	0.164	0.024	0.003
B/M	-0.273	0.344	0.115	0.027	0.012
B/H	-0.305	0.700	0.385	0.004	0.000

Source: Author calculations.

Omission of 2015 in the regression (2010-2014)

To illustrate the effect of model robustness in terms of explanatory power during periods of volatility time series regressions were conducted throughout the sample period with the omission of 2015 (i.e. 2010-2014). Five of the six portfolios report adjusted R^2 that were higher, when 2015 was excluded, except for the S/H portfolio (23% decrease). The S/H portfolio is found to be comprised of two thirds mining companies and one third logistics established companies. 2014 proved to be a worse year for the mining sector (irrespective of the other macro-economic volatility) than 2015. 2014 saw a 1.4% reduction in overall mining production and a growth rate of 3.5% was achieved in 2015 (Statistics South Africa, 2015). Mining strikes and diminished investor confidence further served to amplify volatility, which affected the descriptive power of the factors on the S/H portfolio.

3.5.6 Comparison between CAPM and FF3FM

The FF3FM outperforms the CAPM with respect to capturing variation in responses. The adjusted R^2 values for the initial regressions are significantly higher, amounting to differences as high as 49% within the individual portfolios.

3.5.7 Portfolio performance evaluation

Jensen's α

H_0 : The intercept (α) of the regression model for the CAPM/FF3FM is not significant and

H_1 : The intercept (α) of the regression model for the CAPM/FF3FM is significant.

All six α values for the FF3FM portfolios are positive and statistically significant at the 5% level, implying that the expected return is underestimated and the alternative hypothesis (H_1) can be accepted. This underpins the fact that the model fails to capture a portion of systematic risk that should be proxied for by other factors (e.g. liquidity). In the case of the CAPM, five of six of the portfolios are significant at the 5% level, S/H being highly insignificant. These results should be treated with caution, however, as the α s should be considered in conjunction with R^2 model values (Table 3.15).

Table 3.15. α (regression intercept) summary for CAPM and FF3FM over 2010-2015.

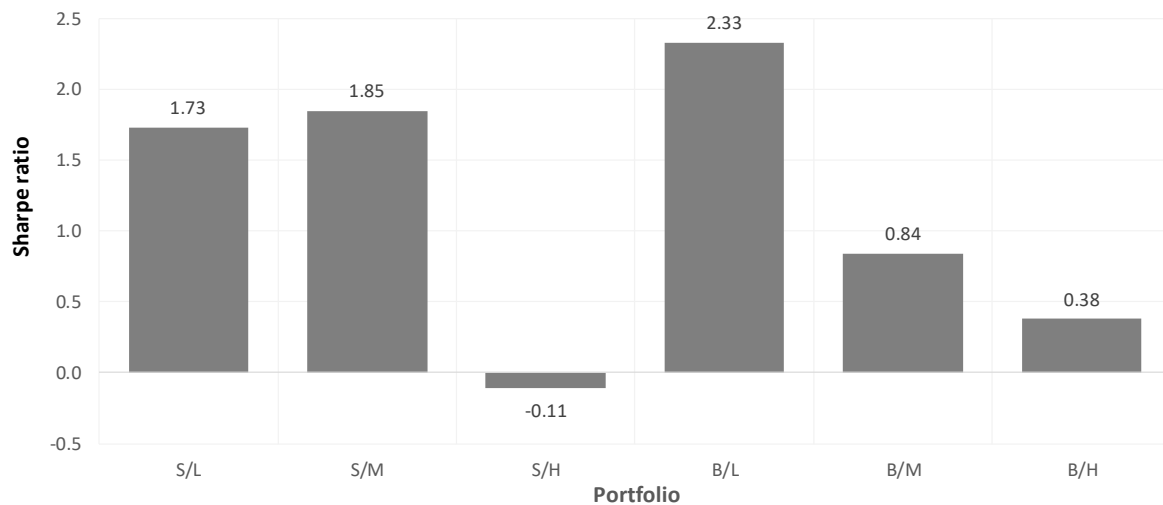
Portfolio	FF3FM α	p -value	CAPM α	p -value
S/L	0.016	0.11%	0.017	0.024%
S/M	0.017	0.02%	0.016	0.023%
S/H	0.014	0.24%	-0.001	18.958%
B/L	0.015	0.04%	0.020	0.000%
B/M	0.016	0.21%	0.010	3.892%
B/H	0.017	0.01%	0.006	4.862%

Source: Author calculations.

Sharpe Ratio

From a risk-adjusted performance basis, the big sized portfolio group slightly outperformed the small sized group (1.18 vs. 1.16) (Figure 3.5). B/L performed the best, whilst S/H performed the worst. In the big portfolios the Sharpe ratio decreased monotonically as the book to market value became higher. This may be an important consideration for future studies.

Figure 3.5. The Sharpe Ratio for each portfolio over the sample period (2010-2015).



Source: Author calculations.

3.6 Conclusion

Using an adapted approach of Fama & French (1993, 1995) a final sample of 46 JSE stocks, spanning a period from 2010 to 2015, were sorted and stratified, by size and book-to-market values - into a 3×2 assortment of portfolios. The portfolios were re-balanced annually on a relative value basis and used in the evaluation of the multiple regressions on both the CAPM and the FF3FM. Results indicate that both models are limited in their explanatory ability, however, the FF3FM ($0.113 < \text{adjusted } R^2 < 0.500$) clearly outperforms the CAPM ($0.031 < \text{adjusted } R^2 < 0.064$) in all portfolios. A common problem amongst both models, points to the lack of a sufficient market proxy, and is one of the key reasons for the low performance. Other reasons for the poor performance include the liquidity issue and inherent volatility present in South Africa during the sample period.

On a risk-adjusted performance basis, the portfolios are non-monotonic in nature, with B/L and S/H performing the best and worst respectively. This stands in contrast to Fama & French (1993), who observe a monotonic relationship of small firms outperforming big firms. The results in this respect are inconclusive, as the data sample limitations in both size and period cannot be discounted. Larger studies should be applied to ascertain this.

Two disjoint-paired tests of both the SMB and market factors and HML and market factors, indicated that although both factors were significant, the HML factor explained a much larger proportion of the variation in excess stock returns, in contrast to the SMB factor. This implies

that it is a stronger descriptive variable when applied to the JSE,³⁰ supporting Fama & French's (1995) small firm effect.

3.7 Recommendations for further study

Time-series regression: time horizon and sample size

Often one of the most salient limitations associated with time-series regression is the quality of data sample. The period may be too short to fully capture the relationships between variables, and the sample may not be fully representative and/or non-synchronous trading may be evident (Basiewicz & Auret, 2010). The explanatory variables span only a six-year period (2010-2015), whereas Fama & French (1993;1996) include several decades of data. The implication of this is that the possibility of noise and bias to estimates, because of these ideas, cannot easily be refuted. A possible solution would be to include a sample spanning a longer time horizon, as well as including more companies.

Inclusion of unpriced factors

Perhaps the most obvious procedure would be to look at other risk factors which may better capture risk characteristics of asset behaviour in the South African market. A key issue identified in the context of South Africa was liquidity. Thus, a priced liquidity factor could be a significant additive to the regression model. Another finding among emerging markets is the historical volatility of loading estimations, due to time-varying components. Adjusting the model for the time-varying β s could help.

The extension of the FF3FM to the FF5FM (Fama & French, 2014), which includes profitability and investment, or momentum (Carhart, 1997) may better proxy systematic risk for emerging markets (see also Jegadeesh & Titman, 2001).

Portfolio partitioning and factor computation

A key component influencing the portfolios descriptive metrics is the constituents in the portfolio. While the method employed in this study sought to partition based on overall ME and BE/ME value, an interesting adjustment to further study could involve the partitioning of portfolios based instead on industry sectors. This may result in better explain the cross-section variation in returns but must be considered in light of the models "threshold effect". SMB and

³⁰ This was the case in four of the six portfolios – except for S/L and S/M.

HML factors could also be constructed on an industry basis, which would also then highlight the effects across industries better (see, for example, Derwall, Guenster, Bauer, & Koedijk, 2005).

Market proxy

Previous sections make extensive mention of the lack of explanatory power provided for the market proxy. A way around this issue could be to use industry-based indices as the market benchmark for respective share/portfolio descriptors. This would likely add significant power in the explained variation in the response, for example: A mining index could be used in the description of mineral-based companies or a financial index could be used in the description of financial asset returns. This is important, as different sectors are likely proxied by different risks – especially in emerging markets where volatility is higher than developed markets. A more general approach could involve a market index to proxy for small, medium and large market caps, when estimating different sized portfolios.

Reduction in bias and estimation

Following Dimson (1979) and Basiewicz (2010) – the latter of which appears to be the most comprehensive FF3FM study on the JSE – note that the largest problem with thin trading, is the bias in computed β s, and thus, regression results. A lag period for factor β s could be introduced.

Dummy variables and interaction

Different market sectors exhibit differing degrees of volatility. A potential model extension could also include a series of indicator variables (such that the recording process accounts for each binary description level represented by each JSE sector), which may improve the asset pricing model's statistical descriptive accuracy. Further studies could also test for statistical interactions between factors.

Alternative techniques

One of the underlying assumptions underpinning multiple linear regression is that the model follows a linear-relationship. Do current models indeed follow a linear relationship? Following the work of Koenker (2005) and Allen, Kumar-Singh & Powell, (2009), further tests for factor linearity, and to tackle some of the issues discussed associated with time-series, with respect

to the response, could be tested using the method of quantile regression.³¹ If non-linearity is found, data transformation could be applied to the series. Other helpful methods include principal component and factor analysis.

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³¹ An advantage of quantile regression, relative to the ordinary least squares regression, is that the quantile regression estimates are more robust against outliers in the response measurements.

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Chapter 4

Investment implications of the fractal market hypothesis

Adam Karp³² and Gary van Vuuren³³

Abstract

The Efficient Market Hypothesis (EMH) has been repeatedly demonstrated to be an inferior – or at best incomplete – model of financial market behaviour. The Fractal Market Hypothesis (FMH) has been installed as a viable alternative to the EMH. The FMH asserts that markets are stabilised by matching demand and supply of investors' investment horizons while the EMH assumes the market is at equilibrium. A quantity known as the Hurst exponent determines whether a fractal time series evolves by random walk, a persistent trend or mean reverts. The time-dependence of this quantity is explored for two developed market indices and one emerging market index. Another quantity, the fractal dimension of a time series, provides an indicator for the onset of chaos when market participants behave in the same way and breach a given threshold. A relationship is found between these quantities: the larger the change in the fractal dimension before breaching, the larger the rally in the price index after the breach. In addition, breaches are found to occur principally during times when the market is trending.

Key words

Efficient market hypothesis, Fractal market hypothesis, Hurst exponent, fractal dimension

JEL classification

C52, G11

4.1 Introduction

A central tenet of modern portfolio theory (MPT) is the concept of diversification: an assembly of several different assets can achieve a higher rate of return and a lower risk level than any asset in isolation (Markowitz, 1952). MPT has enjoyed remarkable success – it is still in wide use today (2018) – but it has also attracted a large and growing critical literature (e.g. Michaud, 1989; Elton & Gruber, 1997 and Mehdi & Hawley, 2013 and references therein). An example of these criticisms is that MPT relies on the statistical independence of underlying asset price changes. This renders predictions of future market movements impossible. Sources of instability and market risk are also assumed to be exogenous under MPT. Were this true, the economic system would converge to a steady-state path, entirely determined

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by fundamentals and with no associated opportunities for consistent speculative profits in the absence of external price shocks. Empirical evidence, however, shows that prices are not only governed by fundamentals, but also by non-linear market forces and factor interactions which give rise to endogenous fluctuations.

Asset returns are also assumed to be normally distributed, but this omits (or assigns very low probabilities to) large return outliers. This is not an attribute of financial markets: they are characterised by long periods of stasis, punctuated by bursts of activity when volatility escalates – often rapidly and without warning. A consequence of the normal distribution assumption, then, is that these large market changes occur too infrequently to be of concern. Classical financial models, such as the efficient market hypothesis, embrace the precepts of MPT, so these abrupt market events are omitted from their frameworks.

The efficient market hypothesis, with its three varieties (weak, semi-strong and strong) evolved from the MPT (Fama, 1965). Strong form efficiency is considered impossible in the real world (Grossman & Stiglitz, 1980) so only the weak and semi-strong forms of the EMH are empirically viable: both take for granted what Samuelson (1965) proved: that future asset price movements are determined entirely by information not contained in the price series; they *must* follow a random walk (Wilson & Marashdeh, 2007). The literature is, however, replete with evidence that weak and semi-strong forms of efficiency are inaccurate descriptions of financial markets (for example, Jensen, 1978; Schwert, 2003; Zunino, *et al.*, 2008; Piamsuwannakit, & Sriboonchitta, 2015; Le, 2016 and French, 2017), so alternative descriptions must be sought.

Two alternatives to *efficient* markets have evolved: the Adaptive (AMH) and Fractal (FMH) market hypotheses. The former offers a biological assessment of financial markets – specifically an evolutionary framework in which markets (and market agents: assets and investors) adapt and evolve dynamically through time. This evolution is fashioned by simple economic principles which, like natural selection, punish the unfit (through extinction) and reward the fit (through survival) as agents compete and adapt – not always optimally (Farmer & Lo, 1999; Farmer, 2002; Lo, 2002; 2004; 2005). Survival is paramount, even if that requires temporarily abandoning profit and utility maximisation. Unlike the EMH, the AMH allows for an unstable, dynamic risk/reward relationship in which arbitrage opportunities arise and close depending

on prevailing macro and microeconomic conditions which in turn affect the success of investment strategies.

The FMH relaxes the EMH's random walk requirement of asset prices using a concept designed by Hurst (1951, 1956). Whilst exploring the annual dependence of water levels on the river Nile, Hurst (1951, 1956) noted that the ebbs and flows were not random (as expected), but rather displayed persistence and mean-reversion. High levels one year tended to be followed by high levels the next (and vice versa). In other periods, sharp reversions toward the mean were recorded. Hurst's (1956) observations led to the formulation of the Hurst exponent, H , which effectively measures the degree of persistence prevalent in a time series: higher values suggest directional similarity (persistence) and lower values imply directional heterogeneity (reversion to the long-run mean: the further away from the mean, the stronger the tendency to return to it).

The relationship between these competing hypotheses and some of the tests used to determine their validity is summarised in Figure 4.1.

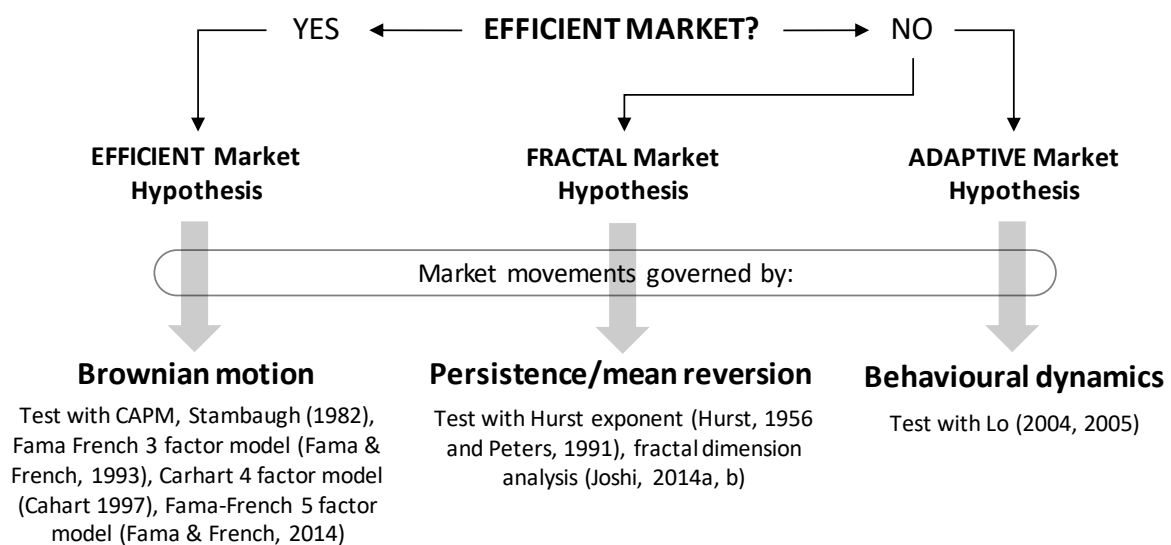


Figure 4.1: Relationship between efficient, fractal and adaptive market hypotheses (Lo, 2012).

Source: Author.

The remainder of this article proceeds as follows. The literature study in Section 4.2 provides a brief overview of salient features of the EMH. The EMH and the less-explored FMH, which addresses some of the former's shortcomings, are also discussed and compared here. Section 4.3 presents the data used to explore the FMH approach. If market movements are indeed

described by fractal geometry, the implications for financial markets are profound. A diminishing fractal dimension, for example, indicates herding behaviour until critical values are breached, leading to chaos. This section introduces the theoretical constructs of fractal geometry prevalent in financial time series. The results of the investigation on some global markets are presented in Section 4.4 as well as an empirical discussion on the implications of these results. Section 4.5 concludes.

4.2 Literature survey

The phrase 'efficient market', introduced by Fama, *et al.*, (1969), originally defined a market which received, processed and adapted to new information quickly. A more contemporary definition, which considers rational processing of relevant information, asserts that all available information is reflected in an efficient market's asset prices (Fama, 1991). If the relevant information *were* free, prices would rise to their 'fundamental level', but financial incentives arise if procurement costs are not zero. This is the strong form of the efficient market hypothesis (EMH) (Grossman & Stiglitz, 1980). The economically realistic, semi-strong version of the EMH, argues that prices reflect information, but only to the point where the marginal costs of collecting the information outweigh the marginal benefits of acting upon it (through expected profits) (Jensen 1978). The weak form of the EMH suggests that asset prices reflect all past asset price data so technical analysis is of no help in forming investment decisions.

The EMH generates several testable predictions regarding the behaviour of asset prices and returns, so much empirical research is devoted to gathering important evidence about the informational efficiency of financial markets and establishing the validity – or otherwise – of the EMH. Some of the more important evaluations are presented in Table 4.1.

Table 4.1: EMH predictions and empirical evidence.

Prediction	Empirical evidence	Sources
Asset prices move as random walks over time	Approximately true. However: Small positive autocorrelation for short-horizon (daily, weekly and monthly) stock returns Fragile evidence of mean reversion in stock prices at long horizons (3–5 years)	Poterba & Summers (1998); Fama & French (1992); Campbell, Lo & MacKinlay (1997);

New information rapidly incorporated into asset prices	New information usually incorporated rapidly into asset prices, with some exceptions	Chan, Jegadeesh & Lakonishok (1996); Fama & French (1998);
Current information cannot be used to predict future excess returns	Short run, shares with high returns continue to produce high returns (<i>momentum effects</i>) Long run, shares with low price-earnings ratios, high book-to-market-value ratios, and other measures of 'value' outperform the market (<i>value effects</i>) FX market: current forward rate predicts excess returns (it is a biased predictor of future exchange rates)	De Bondt & Thaler (1985); Fama & French (1992); Jegadeesh & Titman (1993); Lakonishok, Shleifer & Vishny, (1994); Goodhart (1988)
Technical analysis should provide no useful information	Although technical analysis is in widespread use in financial markets, there is contradictory evidence about whether it can generate excess returns	Levich & Thomas (1993); Osler & Chang (1995); Neely, Weller & Dittmar (1997); Allen & Karjalainen (1999)
Fund managers cannot systematically outperform the market	Approximately true Some evidence that fund managers <i>can</i> systematically underperform market	Lakonishok, Shleifer & Vishny (1992); Brown & Goetzmann (1995); Kahn & Rudd (1995)
Asset prices remain at levels consistent with economic fundamentals (i.e. they are not misaligned)	Asset prices appear to be significantly misaligned for extended periods at times	Meese & Rogoff (1983); De Long, Shleifer, Summers & Waldman (1990)

Source: Author.

Modern Portfolio Theory (MPT) allows for the construction of efficient portfolios (those which generate the highest return possible for a given level of risk) while still maintaining the EMH assertion that *outperforming* the market on a risk-adjusted basis is impossible (Elton & Gruber, 1997).

Far from an orderly system of rational, cooperating investors, financial markets are instead characterised by nonlinear dynamic systems of interacting agents who rapidly process new information. Investors with different investment horizons and holding different market positions employ this information in different ways. Considerable price fluctuations are observed, and these are indistinguishable or 'invariant' on different time scales, as illustrated in Figure 4.2 which demonstrates this phenomenon for crude oil prices using 70 daily, weekly, monthly and quarterly prices. It is impossible to say which of these is which with the axes (deliberately, in this case) unlabelled.

This self-similarity implies market price persistence which would not be observed if returns were indeed independently and identically distributed, as postulated under the EMH. Further evidence of market persistence is shown by prices which deviate from their fundamentals for prolonged periods, and by a greater amount than allowed by the EMH (Carhart, 1997)

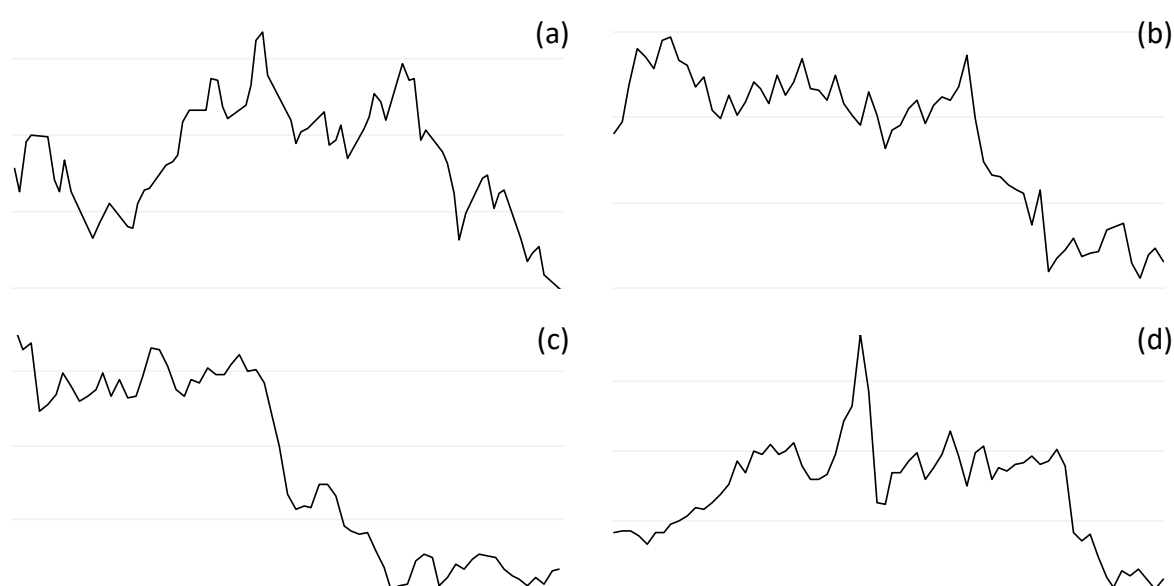


Figure 4.2: (a) Daily, (b) weekly, (c) monthly and (d) quarterly crude oil prices measured over 70 periods in each case. Without time-axis labels, these series trace a geometric pattern which appears indistinguishable across different timescales.

Source: Author calculations.

These empirical facts have created the need for a more realistic description of market movements than that described by the EMH – a need which was first satisfied by Mandelbrot (1977) who argued that *fractals* (geometric shapes, parts of which can be identified and isolated, each of which demonstrates a reduced-scale version of the whole) provided such a realistic

market risk framework. Prices generated from simulated scenarios based on these fractal models reflect more realistic market activity (Joshi, 2014a and Somalwar, 2016).

The quantification of self-similar structures is non-trivial: an analogy usually invoked in the literature is that of the changing length of a coastline, depending on the ruler used to measure it (Feder, 1988 and Cajueiro & Tabak, 2004a). Differences in estimation arise when line segments (as characterised by a ruler) are used to measure lengths of nested, self-similar, structures (Anderson & Noss, 2013). The fractal nature of financial markets has led to the formulation of the FMH which replicates patterns evident in calm markets (predicted by MPT) as well as highly turbulent trading conditions (not predicted by MPT). The FMH and fractal price models may also be calibrated to replicate market price accelerations and collapses, key features of heteroscedastic volatility.

The principal differences between the EMH and the FMH are summarised in Table 4.2 below. Note that all the assumptions in the EMH column are false, whilst those in the FMH column are true.

Table 4.2: Summary of differences between the EMH and the FMH.

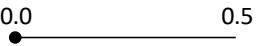
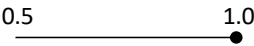
EMH	FMH
Return distribution is Normal (Gaussian)	Return distribution is non-Normal (non-Gaussian)
Stationary process (distribution mean does not change)	Non-stationary process (distribution mean changes)
Returns have no memory (no trends)	Returns have memory (trends)
No repeating patterns at any scale	Multiple repeating patterns at all scales
Continuously stable at all scales	Possible instabilities at any scale

Source: Author.

The FMH assumes price changes evolve according to *fractional* Brownian motion, a feature quantified by the Hurst exponent. Hurst (1956) explored long-range time series component dependences and formulated the Hurst exponent, H , which records both the level of autocorrelation of a series and estimates the rate at which these autocorrelations diminish as the time delay between pairs of values increases. The range of $H \in [0,1]$. The EMH is based upon standard Brownian motion processes which assume prices evolve by random walks i.e., $H = 0.5$. A natural consequence follows from this framework: forecasting future price movements

is impossible because price movements are independent and exhibit no autocorrelation, thus technical analysis provides no assistance to investors. Deviations from $H = 0.5$ indicate autocorrelation which violates a key principle of the EMH. The finite nature of financial time series allows for $H \neq 0.5$, so this possibility must be accounted for (Morales, Di Matteo, Gramatica & Aste, 2012). Table 4.3 records the differences in time series depending on subranges of H : Figure 4.3 shows different time series for three sub-regions of H .

Table 4.3: Characteristics of time series dependency on H .

Range	$H \in [0, 0.5)$	$H \approx 0.5$	$H \in (0.5, 1]$
	0.0  0.5	$< 0.5 >$	0.5  1.0
Auto-co-variance	$< 0 \forall \text{ lags}$	$= 0 \forall \text{ lags}$	$> 0 \forall \text{ lags}$
Behav- iour	Anti-persistent	Brownian	Persistent
Statisti- cal inter- pretation	Decrements (increments) more likely to be proceeded by increments (decrements)	Decrements/increments equally likely	Increments (decrements) more likely to be proceeded by increments (decrements)
Charac- ter	Reverts to the mean more frequently than a random one	Random motion	Exhibit long-memory and "trends" and "cycles" of varying length
Sources	Barkoulas, <i>et al.</i> , (2000), Kristoufek (2010)	Osborne (1959)	Mandelbrot & van Ness (1968)

Source: Author calculations.



Figure 4.3: S&P 500 price series for 18-month periods in which (a) $0 < H < 0.5$ (mean-reverting), (b) $H \approx 0.5$ (Brownian motion) and (c) $0.5 < H < 1.0$ (trending).

Source: Author calculations.

The literature exploring the Hurst exponent in finance and its relationship with the EMH is rich. Using daily data from both emerging and developed market indices spanning 11 years (Jan-92 – Dec-02), Cajueiro & Tabak (2004a, b) calculated $H(t)$, the time-varying H . For the emerging markets $H > 0.5$ but the long-term trend was towards $H = 0.5$, indicating increasing efficiency over the observation period. Developed markets' H was not statistically different from 0.5. The results for both markets were confirmed by Di Matteo (2007) who used 32 global market indices and Wang, Liu, Gu, Cao & Wang (2010) who used daily data to explore the efficiency of Shanghai stock market.

Grech & Mazur (2004) employed H to forecast market crashes. Three such crashes (1929 and 1987 in the US and 1998 in Hong Kong) were investigated using two years of daily data prior to the relevant crash in each case. *Before* each crash, H decreased sharply, an indication of vanishing trends and increasing volatility while *during* each crash, H *increased* significantly, a sign of enhanced inefficiency. Using daily data from the Polish stock market, Grech & Pamuła (2008) reached the same conclusions.

Alvarez-Ramirez, *et al.*, (2008) used daily data spanning 60 years from the S&P 500 and Dow Jones indices and found that H displayed erratic dynamic time-dependency. A time-varying evolution of market efficiency was observed with alternating low and high persistent behaviour, i.e. $H > 0.5$ in both cases, with different magnitudes.

The consequences for market efficiency of financial crises were explored by Lim, Brooks & Kim (2008) who found that the 1997 Asian crisis dramatically reduced the efficiency of global stock markets. Within three years, however, efficiency had recovered to pre-crisis levels. The highest level of market efficiency was recorded during post-crisis periods, followed by pre-crisis periods. During crises, markets exhibit high inefficiency.

Using daily data from Jan 01 to Jul 07, Karangwa (2008) found $H \approx 0.5$ for the JSE. Note that Karanaga's (2008) study concluded before the onset of the 2008 credit crisis, so this event and its aftermath were not included in the analysis. Using monthly data for a longer period (i.e. Aug 95 – Aug 07), Karangwa (2008) found $H = 0.58$. In a more recent study, Ostaszewicz (2012) used two methods (Higuchi and absolute moments) to measure H using JSE price index data both pre- and post the 2008 crisis period and found $H > 0.5$ predominantly in the pre-2008 crisis period and $H < 0.5$ predominantly in the post-2008 crisis period. Chimanga &

Mlambo (2014) investigated the fractal nature of the JSE and found $H = 0.61$ using daily data from 2000 to 2010. By sector, the values for the JSE were as shown in Figure 4.4.

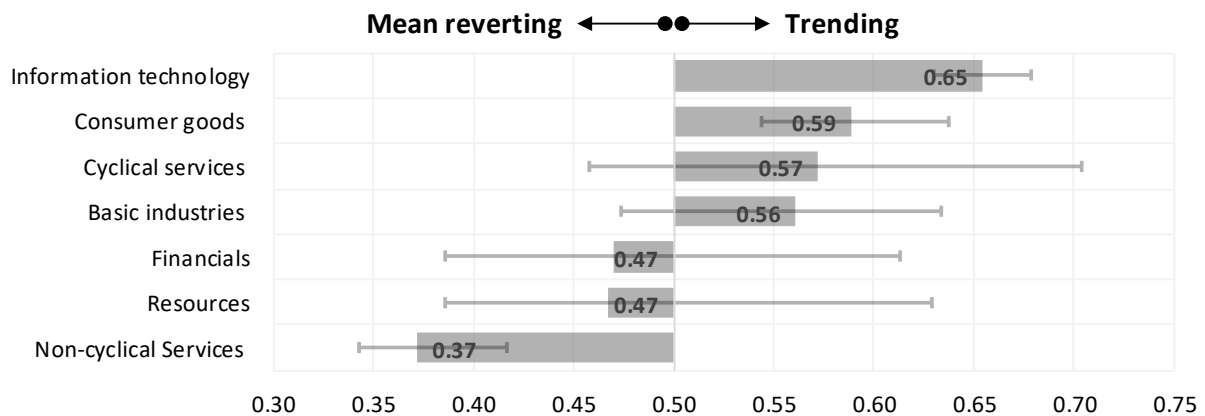


Figure 4.4: Average H s measured on various JSE sectors over the period 2000 – 2010. Error bars indicate maximum and minimum values obtained from individual shares within the relevant sector.

Source: Author calculations.

Sarpong, Sibanda & Holden (2016) found $H = 0.46$ for the JSE using daily data from 1995 to 2015 (thereby embracing the full period investigated by Chimanga & Mlambo, 2014). In addition, Sarpong, *et al.*, (2016) used the BDS test (Brock, Dechert, Scheinkman & LeBaron, 1996) to verify that JSE price index data exhibit non-random chaotic dynamics rather than pure randomness. These results confirm those obtained by Smith (2008) who, using four joint variance ratio tests, rejected the random walk hypothesis on the JSE.

Vamvakaris, Pantelous & Zuev (2017) examined the persistency of the S&P 500 index using daily data from 1996 to 2010 and found that crises affect investors' behaviour only temporarily ($< \text{six months}$). In addition, the index exhibited high anti-persistency (an indication of investor "nervousness", $H < 0.5$) prior to periods of high market instability. Considerable fluctuations of H were observed with a roughly annual frequency and amplitude (from peak to trough) of 0.2 to 0.4. No prolonged trends of H were recorded.

4.3 Data and methodology

4.3.1 Data

The data used to calibrate the FMH (via the estimation of the Hurst exponent) comprise 22.5 years (Jul 95 to Dec 17) of daily market index prices³⁴ for developed (S&P 500, FTSE 100) and

³⁴ This technique was also tested on individual assets, foreign exchange rates and commodity prices and all were found to be equally accurate.

emerging market stock exchanges (the JSE). Three years (36 months) of daily index prices were used to determine H^{36} . The data sample was then rolled forward by one month and the next realisation of the Hurst exponent calculated, i.e. H^{37} . This was repeated until the latest Hurst exponent in the data sample was calculated, i.e. for end Dec 17, using the three years of data from Jan 15 to Dec 17.

This sample size was selected to include at least three full South African business cycles. This has been shown to be \approx seven years (Botha, 2004 and Thomson & van Vuuren, 2016). In addition, these data embrace a period of non-volatile growth (2003 – 2008), and considerable turbulence (1998 – 2000 [the Asian crisis and the dotcom crash] and 2008 – 2011 [the credit crisis]).

The same indices were used for the fractal dimension, D , analysis to establish whether breaching of a given D led to herding behaviour (and a resulting collapse or rally in price). The fractal dimensions of gold and oil prices were investigated over the same period for calibration purposes and to confirm earlier work undertaken by Joshi (2014a, b).

4.3.2 Methodology

Standard Brownian motion describes the trajectory of a financial asset price, S_t , through time by integrating the differential equation (Areerak, 2014):

$$dS_t = S_t(\mu dt + \sigma dW_t), \quad (4.1)$$

where S_t is a financial asset price at time t , dS_t is the infinitesimal change in the asset's price over time dt , μ is the expected rate of return that the asset will earn over dt and σ the expected volatility. dW_t is a Weiner process described by $\varepsilon\sqrt{t}$ where ε is a random number drawn from a standard normal distribution. The solution of (4.1) is:

$$S_t = S_0 \exp\left(\mu t - \frac{\sigma^2}{2} t + \sigma W_t\right), \quad (4.2)$$

where S_0 is the initial asset price. In principle, S_t describes the asset's price trajectory through time, but in practice many features of financial assets are not captured by this formulation. Cont (2001) assembled a group of stylised statistical facts which describe several financial assets. While not exhaustive, this list includes empirical evidence that financial asset returns are characterised by:

1. insignificant linear autocorrelations (Cont, 2001),
2. heavy tails and conditional heavy tails (even after adapting returns for volatility clustering) of unconditional return distributions which can be described by power-laws or Pareto-like tails with finite tail indices (Horak & Smid, 2009),
3. asymmetric gains and losses – larger drawdowns than upward movements (Horak & Smid, 2009),
4. different distributions at different timescales. Known as "aggregational Gaussianity" the return distribution approaches a normal distribution as $t \rightarrow \infty$ (Cont, 2001),
5. a high degree of return variability at all timescales (Di Matteo, Aste & Dacorogna, 2005),
6. homoscedasticity, or volatility clustering: the clustering of high-volatility events and low volatility events in time (Cont, 2001)
7. long-range dependence of return data, characterised by the slow decay (as a function of time) of the autocorrelation of absolute returns, often as a power law with exponent $0.2 \leq \beta \leq 0.4$ (Cont, 2001),
8. negative correlation of the asset's volatility and its returns (Chordia, Roll & Subrahmanyam, 2008),
9. higher-than-expected correlation between trading volume and volatility (Blume, Easley & O'Hara, 1994) and
10. time scale asymmetry: fine-scale volatility is better predicted than coarse-grained measures rather than the other way around (Di Matteo, Aste & Dacorogna, 2005).

These features are generally *not* captured by standard Brownian motion, which has led to the development of *fractional* Brownian motion. In this formulation, (4.1) becomes:

$$dS_t = S_t(\mu dt + \sigma dZ_t), \quad (4.3)$$

where $dZ_t = \varepsilon \sqrt{t^{2H}}$ and H ($0 \leq H \leq 1$) is the Hurst parameter. The respective Wiener processes (dW_t in (4.1) and dZ_t in (4.3)) have many features in common, but also exhibit strikingly different properties. The Wiener process dZ_t is self-similar in time, while dW_t is self-affine (Mandelbrot, 1977 and Feder, 1988). Fractional Brownian motion, for example, captures dependence among returns. A generalised solution for (4.3) is:

$$S_t = S_0 \exp \left(\mu t - \frac{\sigma^2}{2} t^{2H} + \sigma Z_t \right). \quad (4.4)$$

If $0 \leq H < 0.5$ changes in S_t are negatively correlated and if $0.5 \leq H < 1$ they are positively correlated. Correlation also increases with H (Shevchenko, 2014).

4.3.3 Hurst exponent, H

A variety of methods for estimating H are discussed in the literature, each with associated advantages and drawbacks. Approaches include rescaled-range analysis (proposed by Hurst (1951) himself), wavelet transformations (Simonsen & Hansen, 1998), neural networks (Qian & Rasheed, 2004) and the visibility-graph approach (Lacasa, *et al.*, 2009). The most commonly used methodology is rescaled-range analysis, and this will be adopted here as it is also the technique used to determine the fractal dimension, D , also known as the Hausdorff-Besicovitch dimension (Hausdorff, 1919 and Manstavičius, 2007).

Hurst (1951) asserted that the variation of fractal time series is related to the horizon over which the time series are assessed by a power law relationship. Starting with a de-measured time series (to ensure stationarity), define Y_k as the sum of k increments of this series, extending to n increments. The adjusted range (the 'distance' travelled over n time increments) is defined as the difference between the maximum and the minimum of the series:

$$\{Y_1, Y_2, \dots, Y_n\} \text{ or } R_n = \max(Y_k) - \min(Y_k), 1 < k < n.$$

If Y is a time series characterised by Gaussian increments (i.e. a random walk) then this range increases with the product of the series' standard deviation and \sqrt{n} . Hurst (1951) generalised this relationship to:

$$\left(\frac{R}{\sigma}\right)_n = cn^H, \quad (4.5)$$

where σ is the standard deviation of the stationary time series' n observations and H is the Hurst exponent. Rescaling the series by determining the quotient of the range and σ measures time series that do not exhibit finite variance (or fractals). This method makes no assumption regarding the underlying distribution of increments; only how they scale with time, as measured by H . The theoretical value of the positive constant, c , is:

$$c = \sqrt{\frac{\left(2H \cdot \Gamma\left(\frac{3}{2} - H\right)\right)}{\Gamma\left(\frac{1}{2} + H\right) \cdot \Gamma(2 - 2H)}}, \quad (4.6)$$

where $\Gamma(\cdot)$ is the Gamma function.

The H exponent captures the degree of persistence in a time series, irrespective of the time scale over which it is measured. For a time series with an observed $H > 0.5$ implies that a large value of the series in one period is likely to be followed by a larger value in a later period (the reverse applies if $H < 0.5$ so such a series is mean-reverting). H may be calculated using ordinary least squares regression after taking the logarithm of (4.5):

$$\ln\left(\frac{R}{\sigma}\right)_n = \ln(c) + H \cdot \ln(n).$$

Using many different increments, n , and regressing $\ln\left(\frac{R}{\sigma}\right)$ on $\ln(n)$ gives a straight line with $c = \exp(y - \text{intercept})$ – see (6) and $H = \text{regression line slope}$.

Peters (1991) provides the following process for determining H .

Using a time series of $N + 1$ prices $\{P_t\}$, calculate the time series of N returns, $\{X_t\}$ such that $X_t = \ln(P_t/P_{t-1})$. Divide the return time series (length N) into A contiguous subperiods, each of length n (so $A \cdot n = N$). Label each subperiod l_a with $a = 1, 2, 3, \dots, A$. Label each element in l_a as N_k where $k = 1, 2, 3, \dots, n$. For each subperiod, calculate the mean: $e_a = \frac{1}{n} \sum_{k=1}^n N_{k,a}$ as shown in Figure 4.5.

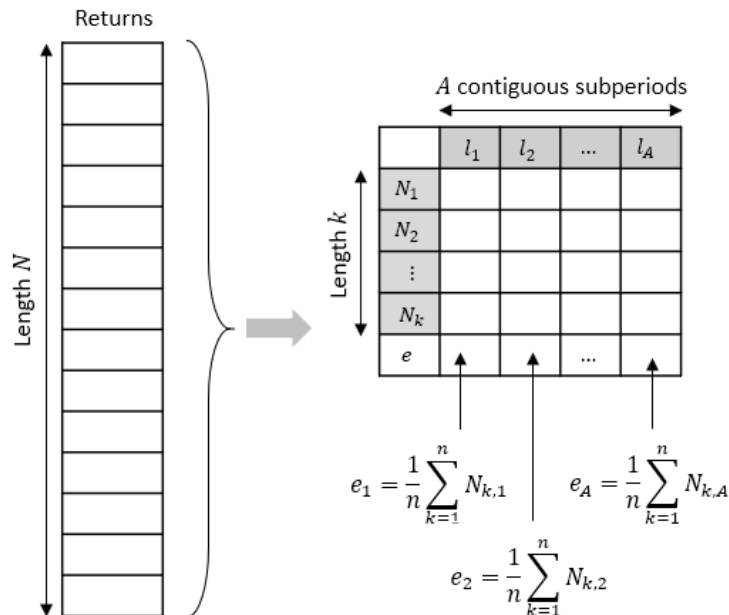


Figure 4.5: Applying Peters (1991) procedure for measuring e_a s.

Source: Author calculations.

The time series of cumulative departures from the mean, for each subperiod l_a , are then

$$X_{k,a} = \sum_{i=1}^k (N_{i,a} - e_a) \forall k = 1, 2, 3, \dots, n.$$

Define the range as the difference between the maximum and minimum value of $X_{k,a}$ within each subperiod l_a : $R_{l_a} = \max(X_{k,a}) - \min(X_{k,a})$ where $1 < k < n$. The sample standard deviation, σ , for each subperiod l_a is:

$$\sigma_{l_a} = \sqrt{\frac{1}{n} \sum_{k=1}^n (X_{k,a} - e_a)^2}.$$

A rescaled range, R_{l_a}/σ_{l_a} for each subperiod, l_a , is then determined, the average of which is:

$$\left(\frac{R}{\sigma}\right)_n = \frac{1}{A} \sum_{a=1}^n \frac{R_{l_a}}{\sigma_{l_a}}.$$

The length n is then increased until there are only two subperiods $\left(= \frac{N}{2}\right)$. A least squares regression is performed, with $\ln(n)$ as the independent variable and $\ln\left(\frac{R}{\sigma}\right)_n$ as the dependent variable. The slope of the regression is H and the y-intercept, c , as shown in Figure 4.6 for a single three-year period, as an example. In the subsequent month, this process is followed again using three years of data prior to *that* month, and the next H and c are calculated.

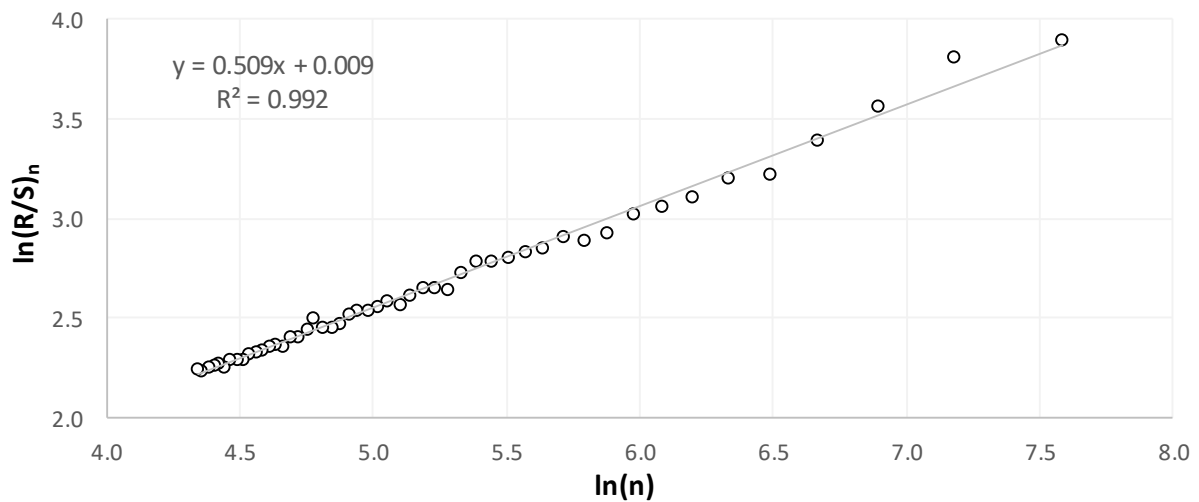


Figure 4.6: Regression results, Mar 06 – Mar 09. $H = 0.509$ and $c = \exp(0.009) = 1.009$.

Source: Author calculations.

The evolution of H was examined using this technique over the two-decade period spanning Jan 98 to Jan 18. This reveals the characteristic nature of markets over this period: persistence, random walks or mean reversion. The fractal dimension, D , discussed in the next section, and H are related (4.8) although a different technique (4.7) is used to measure D in this case as it provides more granular (daily) estimates than (4.8). When D approaches and breaches a given threshold, the market tends to become chaotic, and given that the market exhibits a level of predictability after the onset of chaos (and the threshold breach), this tendency that may be exploited by investors.

4.3.4 Fractal dimension, D

Joshi (2014a, b) described the fractal structure of a financial market using the definition of the fractal dimension, D and the rescaled range. The estimation of the time series' fractal dimension rests on the assertion that stock markets are complex adaptive systems – and thus embedded within them is an endogenous tipping point of instability (i.e. no explicit exogenous trigger is required).

Market stability rests on balancing supply and demand (liquidity) and the fractal structure of financial markets optimises this liquidity. When different investors, with many different investment horizons are all active in the market, the market is characterised by a rich fractal structure. Investors with different investment periods focus on different buy and sell signals: traders on technical data and momentum (short horizons) and pension funds on structural fundamentals and valuation (long horizons) for example. Sharp one day sell-offs are interpreted by traders as a sell signal while pension funds interpret this as a buying opportunity. There is ample market liquidity: large price moves are not inevitable (Joshi, 2014a).

If the trader's horizon becomes dominant, however, and liquidity evaporates when sell orders far outweigh the number of buy orders, the fractal structure of the market collapses and violent price corrections become manifest. This is the endogenous tipping point and by monitoring the fractal dimension, discussed below, such thresholds may be monitored and employed as early indicators of market corrections. The lower the fractal dimension, the more unstable the market it measures.

Breaching a fractal dimension threshold of 1.25 triggers market corrections. This empirical limit appears identical across asset classes, geographies and time periods – it is not theoretically derived. It is impossible, however, to ascertain the *magnitude* of the subsequent adjustment or its direction, i.e. the ensuing correction may be > 0 or < 0 (Joshi, 2014b, 2017).

The measurement of D , the fractal dimension, is described by Joshi (2014a, b). If an asset's price is P_i on day i , its one-day log return, r_i , on day i is:

$$r_i = \ln\left(\frac{P_i}{P_{i-1}}\right).$$

The scaling factor, n , is used to determine the n -day log return, $R_{i,n}$, on day i :

$$R_{i,n} = \ln\left(\frac{P_i}{P_{i-n}}\right),$$

as well as the scaled return, $N_{i,n}$, on day i :

$$N_{i,n} = \frac{\sum_{i-n}^i \text{abs}(r_i)}{\text{abs}\left(\frac{R_{i,n}}{n}\right)} = \frac{\sum_{i-n}^i \text{abs}\left(\ln\left(\frac{P_i}{P_{i-1}}\right)\right)}{\text{abs}\left(\frac{\ln\left(\frac{P_i}{P_{i-n}}\right)}{n}\right)},$$

and the scaled fractal dimension, $D_{i,n}$, on day i :

$$D_{i,n} = \frac{\ln(N_{i,n})}{\ln(n)} = \frac{\ln\left[\frac{\sum_{i-n}^i \frac{\text{abs}\left(\ln\left(\frac{P_i}{P_{i-1}}\right)\right)}{\text{abs}\left(\frac{\ln\left(\frac{P_i}{P_{i-n}}\right)}{n}\right)}\right]}{\ln(n)}, \quad (4.7)$$

The theoretical relationship between H and D is given by (Schepers, van Beek & Basingthwaite, 2002):

$$D = H - 2, \quad (4.8)$$

but (4.7) provides a much more granular (daily) estimate of D than (4.8) since H (in 4.8) is a *monthly* value, determined using (4.5).

4.4 Results and discussion

4.4.1 Hurst exponent, H

How H changes over time is useful to market participants: economists to ascertain the nature of the prevailing markets (persistent or mean-reverting), government strategists to establish the economy's current position in the business cycle, long-term investors to exploit market rallies and busts and short-term investors to exploit mean reversion conditions.

The rolling H was explored for three market indices: two in developed markets (US and UK) and one in an emerging market (South Africa). Figure 4.7(a) shows the results for the S&P 500: Cajueiro & Tabak (2004a, b) found similar results for developed markets ($H \approx 0.5$). Grech & Mazur (2004) found that H decreased sharply before market crashes showing a rapid decrease in trend. This is clearly shown for the September 2001 and September 2008 events – particularly for the latter. After this event, H increases steadily (over three years) from a market dominated by mean-reverting to one characterised by random walk prices.

Figure 4.7(b) shows the rolling constant, $c(t)$, for the S&P500 measured over the same period. Because $c(t)$ depends so heavily on $H(t)$, Figures 4.7(a) and (b) have similar profiles. However, the magnitude of $c(t)$ and how far it strays from $c = 1$ may provide deeper insights at a later stage.

The rolling H for the FTSE 100 is shown in Figure 8 on the same vertical and timescale as Figure 4.7(a). Again, in line with the findings of Cajueiro & Tabak (2004a, b), $H \approx 0.5$. Unlike the results obtained by Grech & Mazur (2004) no sharp decrease of H was observed for the crisis which affected the S&P 500. The events of September 2001 occurred on US soil and so were more damaging to the US economy than the UK economy. The financial crisis of 2008, however, was global in impact and of considerable severity, yet the UK market appears to have been unaffected.

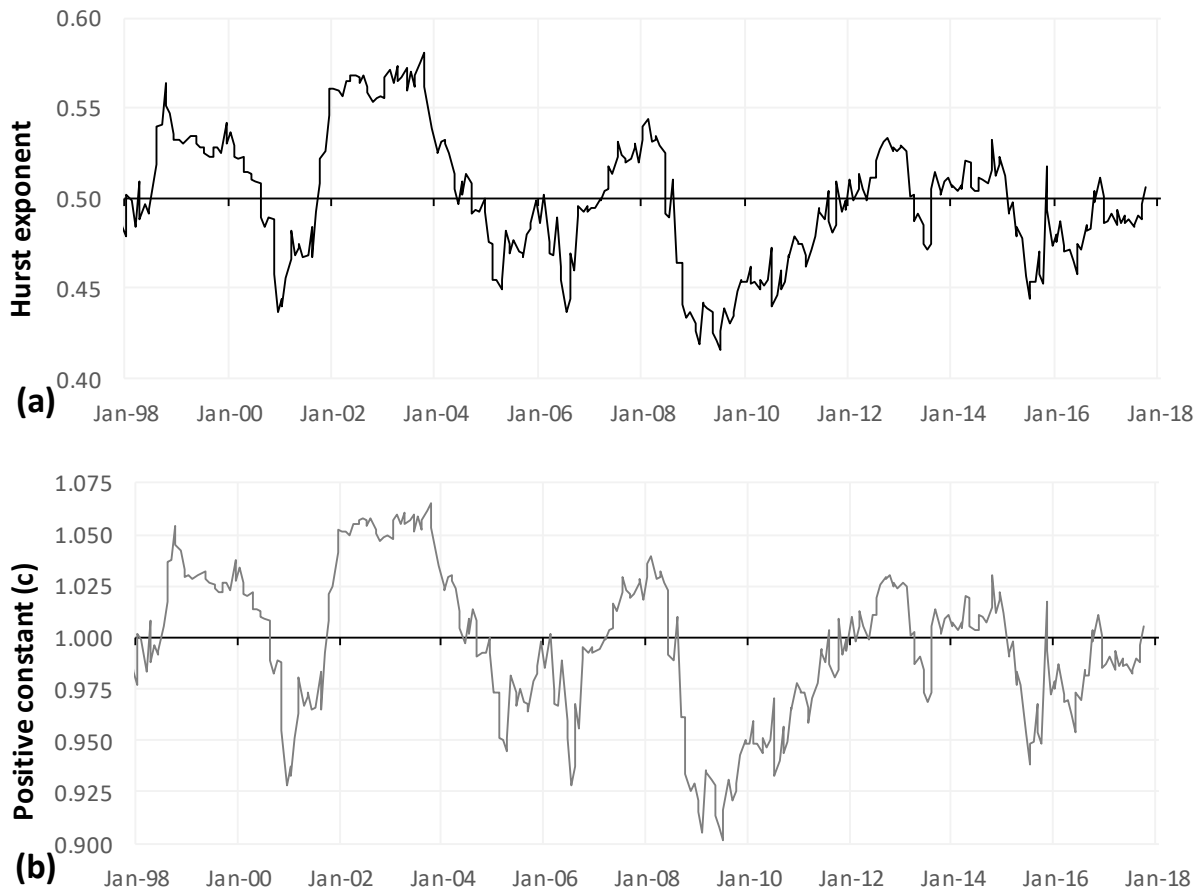


Figure 4.7: Rolling (a) $H(t)$ and (b) $c(t)$ for the S&P 500 from Jan 98 – Dec 17.

Source: Author calculations.

The FTSE 100 exhibits slight persistence ($H > 0.5$) between the time of the onset of the 2008 crisis and early 2012 when the sovereign crisis (which affected several European countries, including the UK albeit not as dramatically) began (Gärtner, Griesbach & Jung, 2011) – see Figure 4.8. At this point the market changes gradually to become slightly mean-reverting and has since followed a random walk since 2014. From 2012, the behaviour of H for the FTSE 100 closely resembles that of the S&P 500 over the same period. These developed market results reinforce results obtained previously (e.g. Alvarez-Ramirez, *et al.*, 2008).



Figure 4.8: Rolling $H(t)$ for the FTSE 100 from Jan 98 - Dec 17 (note the same vertical scale as used for the S&P 500 for comparison).

Source: Author calculations.

The JSE All Share index displays behaviour significantly different from that of developed market indices (Figure 4.9). Until 2006, the JSE trends strongly, unaffected by the 'dotcom' crisis in 00 or the events of Sep 01. These results confirm and update those found by Karangwa (2008) and Chimanga & Mlambo (2014).

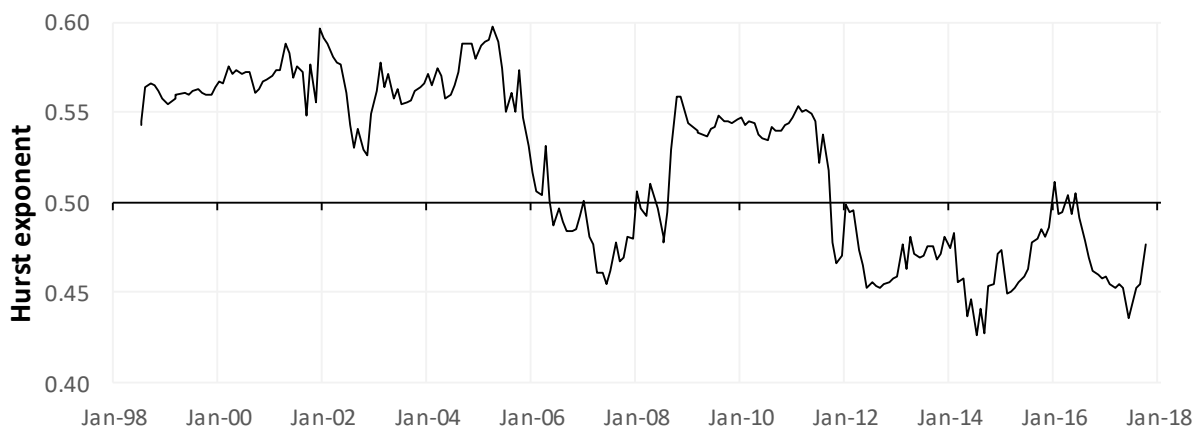


Figure 4.9: Rolling $H(t)$ for the JSE All Share from Jan 98 - Dec 17.

Source: Author calculations.

Between 2006 and the start of the 2008 financial crisis, market prices on the JSE evolve by random walk, but changes to a trending market rapidly at the onset of the crisis – the opposite of what is observed in developed markets. This could be because developing markets – in particular South Africa – largely escaped the consequences of the crisis because it occurred in a period of sustained growth for the country and strong fundamentals (Zini, 2008). South African financial institutions were also relatively robust and did not issue credit as freely and loosely as their global counterparts (Mnyande, 2010). In a trend similar to global markets

though, JSE prices have become slightly mean-reverting or become random walks since 2012. Smith (2008) also found statistically significant results that $H < 0.5$, but over a shorter horizon and using daily (rather than monthly) data. Sarpong, *et al.*, (2016), using daily JSE index data spanning 20 years from 1995 to 2015 also found $H < 0.5$ prior to 2012 and $H \approx 0$ after 2012. The South African market was also found to be more "sectorised" or heterogenous with respect to H ; different market sectors are characterised by different values of H and these values tend to persist over time.

4.4.2 Fractal dimension, D

Analysis of D for the JSE All Share generated interesting results, previously unexplored. The majority (95%) of threshold breaches occur when $H > 0.5$. Only 5% of breaches occur during periods when the market exhibits periods of random walk or mean reversion behaviour. This fact alone provides valuable information to market participants, but the percentage change in D – i.e. the rate of change or "speed" of the change of D also provides information about subsequent market movements.

A breach is classified as an event in which $D \rightarrow 1.25$ from 'above', i.e. $D > 1.25$. There is no theoretical explanation for why this threshold value is significant. It does appear to be empirically consistent across markets, eras, geographies and asset types. When D breaches 1.25 from 'below' (when preceding fractal dimension is < 1.25) this is *not* deemed to be a breach of interest. When threshold breaches were first identified these occurred primarily during times when the South African market was trending, i.e. between 1998 and 2006 (the same results were obtained for the two developed market indices). Four such prominent breaches are shown in Figure 4.10a. The behaviour of the market index over the same period is shown in Figure 4.10b, illustrating the impact of breaches. The shaded area links the timescales on Figures 4.10a and b during the four breaches observed during this period.

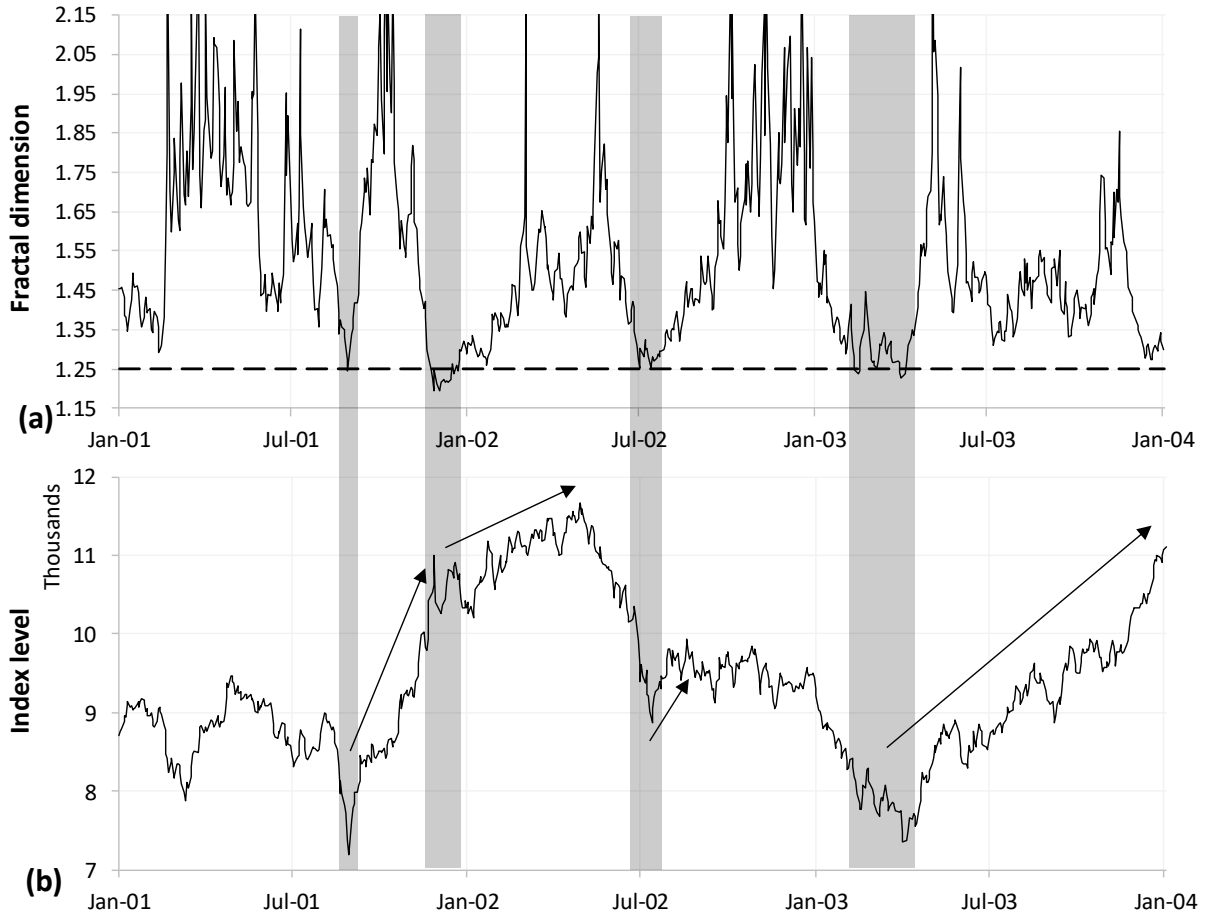


Figure 4.10: (a) Fractal dimension, D over the three-year period between Jan 01 and Jan 04 showing several breaches (shaded) i.e. when $D \leq 1.25$ and (b) the JSE All Share index over the same period showing the behaviour of index prices post breaching.

Source: Author calculations.

Next, the rate of change of D was determined over one trading week (5 days) prior to the breach. (over which time D decreases considerably and rapidly, but not instantaneously). One day is too short a time to capture this time and over two weeks, D has often recovered to pre-breach levels, so one week appears to be an appropriate time to capture a significant, persistent decrease:

$$\frac{D_{t_0} - D_{t-5}}{D_{t-5}},$$

where t_0 is the time D first breaches $D = 1.25$. After t_0 price changes tend to be significant (generally $> 5\%$), sustained and positive. One trading month, (22 days) was selected over which to measure index price changes, i.e.

$$\frac{P_{t+22} - P_{t_0}}{P_{t_0}}.$$

Of course, price changes could be measured over shorter or longer periods than one month, and changes in D could be ascertained over shorter or longer periods than one week, but this approach provides a convenient, simple framework to analyse the effect of breaches on asset prices. The results are shown in Figure 4.11.

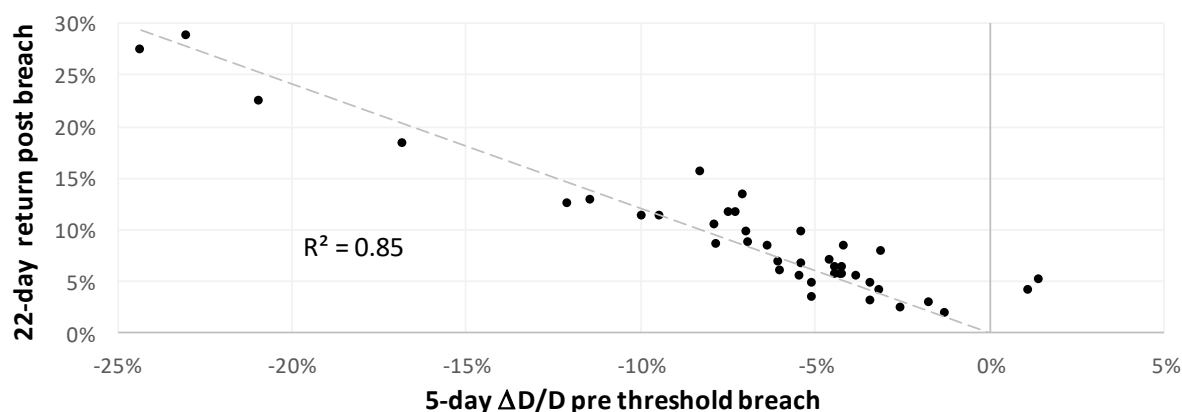


Figure 4.11: Simple regression of one-month index return *post*-breath ($D \leq 1.25$) against a five-day pre-breath percentage change in fractal dimension ($\Delta D/D$). The period analysed was Jul 95 to Dec 17, i.e. the full data sample.

Source: Author calculations.

Regression analysis indicates that the larger $\Delta D/D$ over the week prior to a breach, the larger the positive change – over a month – of the index price. $R^2 = 0.85$ indicating a statistically significant result. Similar results were obtained for the developed market indices. The slope of the line is -1.2 , so for a 1.0% five-day pre-breath drop in $\Delta D/D$, *ceterus paribus* leads to a 1.2% increase in the post-breath, one-month price series. These results could have significant consequences for investors, and could serve as a complementary tool to support, rationalise and justify investment decisions.

Fractal dimension analysis (Figure 4.10) measures market fractality and provides an indication of the level of chaos in the market. When market participants herd and this behaviour begins to dominate the market, the fractal dimension $D \rightarrow 1$ and this provides an early warning about the subsequent magnitude of the market "correction". When the breach is reached ($D < 1.25$) the market rallies – and the rally magnitude again depends on the relative size of the pre-breath change in D , i.e. the magnitude of $\Delta D/D$.

4.5 Conclusions and suggestions

This article examined the fractal properties of developed and developing market indices and examined the evolution of these fractal properties over a two-decade period. The FMH, using

empirical evidence, posits that financial time series are self-similar, a feature which arises because of the interaction of investors with different investment horizons and liquidity constraints. The FMH presents a quantitative description of the way financial time series change; so after the testing of observed, empirical properties of financial market prices, forecasts may be formalised and implemented. Under the FMH paradigm, liquidity and the heterogeneity of investment horizons are key determinants of market stability, so the FMH embraces potential explanations for the dynamic operation of financial markets, their interaction and inherent instability. During 'normal' market conditions, different investor objectives ensure liquidity and orderly price movements, but under stressed conditions, herding behaviour dries up liquidity and destabilises the market through panic selling.

This work also established a relationship between the change in a time series' fractal dimension (before breaching a threshold) and both the magnitude and direction of the subsequent change in the time series. This relationship was found to be prevalent during times of strong price persistence – a feature detectable by elevated Hurst exponents. These results suggest potential investment strategies.

Additional extensions could include more detailed calibration – perhaps by OLS – of the optimal pre-breach period for $\Delta D/D$ and optimal post-breach period for $\Delta P/P$. A comprehensive application of these results to other market indices and asset classes is also needed. Whether the relationship above holds for all asset classes (and, if so, whether the requirement that $H > 0.5$ is a necessary or sufficient prerequisite), also needs to be ascertained.

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Chapter 5

Conclusions and suggestions for future research

5.1. Summary and conclusions

The EMH providing a robust analytical framework for understanding the time evolution of asset prices. Evidence gathered over the past 50 years (1968 – 2018) suggests that the EMH provides a description of market behaviour to a reasonable approximation. Short horizon asset price movements are robustly modelled using the random walk mechanics: information becomes available and is swiftly absorbed, integrated and reflected in asset prices. Although fund managers occasionally report stellar stock market outperformance, this is relatively rare, short-lived and inconsistent. So, although the EMH enjoyed some success early after its institution in the late 1960s, copious research has subsequently cast doubts on the descriptive accuracy of market returns. It turns out that the EMH does go *some* way in explaining market behaviour, but too many anomalies have been identified – which are not adequately explained – to ignore its shortcomings. The EMH is far less successful at describing several other, observed features of market behaviour such as post-earnings-announcement drift (a robust market phenomenon which seems not to depend on the sample period employed) and the forward exchange rate bias (which is considerably more biased as a spot exchange rate predictor than economic fundamentals).³⁵ These anomalies cause long-run prices to misalign – this contradicts the fundamentals of the EMH. The EMH, therefore, although a good starting point to formulate ideas about asset price evolution and describe market returns, still does not adequately explain many important, empirical market behaviour characteristics.

The FMH and the AMH, which provide alternatives to the EMH, explain market behaviour better than the EMH, but they are *complementary* concepts rather than *superseding* concepts. A universal, comprehensive explanation of market behaviour continues to elude research: a panacea is yet to be identified. It remains the ambit of competent analysts to explore forecasts made by each of these hypotheses, test them scrupulously and reject the hypothesis which explains market behaviour with the least descriptive accuracy.

5.1.1. Article 1: *The Capital Asset Pricing Model and Fama-French Three Factor Model in an Emerging Market Environment*

³⁵ Cases in point are the anomalous behaviour of the US dollar in the 1980s and the Japanese Yen in the 1990s.

Fama & French's (1993, 1995) three-factor model was applied to portfolios which comprised combinations of 46 JSE stocks, spanning a six-year period from 2010 to 2015. These stocks were sorted by size and book-to-market values into a 3×2 assortment of portfolios, which were then re-balanced annually on a relative value basis and used for multiple regression evaluations using both the CAPM and the FF3FM. Both models were limited in describing stock performance, although the FF3FM ($11.3\% \leq R^2 \leq 50.0\%$) clearly outperforms the CAPM ($3.1\% \leq R^2 \leq 6.3\%$) for all portfolios. A common problem which affects both models is the absence of a suitable market proxy – this is a key factor for the low performance. Other factors include liquidity problems and the high market volatility experienced in South Africa during the sample period.

The portfolios are non-monotonic (on a risk-adjusted performance basis) with B/L performing the best and S/H the worst, respectively, in direct contrast to Fama & French (1993), who observed a monotonic relationship of small firms outperforming big firms. Although this study used roughly the same amount of data as most other international studies, sample size and period limitations cannot be ignored. The results are inconclusive, but larger and more comprehensive studies should be applied to rectify this.

Two disjoint-paired tests of both the SMB [and market factors] and HML [and market factors], indicated that although both factors were significant, the HML factor explained a larger proportion of stock return excess variation than the SMB factor. This implies that it is a stronger predictor when applied to the JSE,³⁶ supporting Fama & French's (1995) small firm effect.

The CAPM and the FF3FM are tests for the EMH (see Figure 2.1), and neither – therefore – tests for herding behaviour or the onset of market chaos since these characteristics govern *inefficient* markets. The results obtained from these models, then, indicates that the EMH cannot generate the requisite descriptive accuracy (at least not on developing market data, such as South Africa). Alternative ideas such as the AMH and FMH should be considered.

5.1.2. Article 2: Investment implications of the fractal market hypothesis

The fractal properties which characterise developed and developing market indices were examined, and the evolution of these properties over two-decades interrogated. The FMH posits that financial time series are *self-similar*, a feature which arises because of the interaction

³⁶ This was the case in four of the six portfolios – except for S/L and S/M.

of investors with different investment horizons and various liquidity constraints. It presents a quantitative explanation of how financial data grow through time. If accurate, the FMH asserts that forecasts of financial market prices may be formalised.

Under the FMH paradigm, liquidity and the heterogeneity of investment horizons are critical components of market stability. The FMH attempts to explain the dynamic functioning of financial markets, their interaction with market constituents and their implicit instability. The variety of different investor objectives during 'normal' market conditions ensure that liquidity is abundant and price movements evolve in an orderly way. Under stressed conditions, however, market participants' herding behaviour (empirically observed) disrupts markets through fire sales, pressure selling and depleted liquidity.

The relevance of the FMH to global markets was examined. Determining the Hurst exponent is a crucial first step of the FMH. Using developed and developing market index data, the evolution of H through time was explored to establish the behaviour of H under various market conditions. Results obtained largely agreed with – and extended – prior research.

A data series' H and its fractal dimension, D , are linked so the latter could also be investigated. D is an important component of the FMH as it tends to 1 when herding behaviour dominates a market place. After choosing a suitable threshold to indicate the onset of herding behaviour, a relationship was established between the change in a time series' fractal dimension (before breaching a threshold) and both the magnitude and direction of the subsequent change in the time series. This was found to be prevalent during times of strong price persistence – a feature detectable by elevated Hurst exponents. These results encourage further research and advocate potential investment strategies.

5.2. Suggestions for future research

5.2.1. Article 1: *The Capital Asset Pricing Model and Fama-French Three Factor Model in an Emerging Market Environment*

Time horizon and sample size. Time-series regression is limited by the quality of the underlying sample data. The period may be too short (so the sample may not be sufficiently representative) to capture the true relationships between variables. Non-synchronous trading may also be evident (Basiewicz & Auret, 2010). The explanatory variables in this study span only a six-year period, whereas Fama & French (1993; 1996) used several decades of data. It is, therefore, possible that noise may have biased the estimates. Also, a downturn/contraction

period should also be investigated since this sample mostly covered an expansionary period. A possible solution would be to include a sample spanning a longer time horizon and including more companies.

Inclusion of unpriced factors: Additional risk factors, which may better capture risk characteristics of asset behaviour in the South African market, should also be sought. A priced liquidity factor could be a significant additive to the regression model. The historical volatility of loading estimations, due to time-varying components is also a feature of developing markets such as South Africa. The model could be adjusted to consider time-varying β s.

Extending the FF3FM to the FF5FM (Fama & French, 2014), which includes profitability and investment (or momentum) could provide a better systematic risk proxy for emerging markets.

Portfolio partitioning and factor computation: The method used in this study sought to partition the data according to overall ME and BE/ME values. A possible adjustment could involve the partitioning of portfolios based, instead, on industry sectors. This may result in a better explanation of the cross-section variation in returns but must be considered in light of the models "threshold effect". SMB and HML factors could also be constructed on an industry basis, which would also then highlight the effects across industries better.

Market Proxy: The lack of explanatory power was an issue in this research. A solution may be to use industry-based indices as a market benchmark for respective share/portfolio predictions. This could add significant power to the explained response variation. A mining index could be used in the prediction of mineral-based companies or a financial index could be used in the prediction of financial assets, for example. Because different sectors are proxied by different risks – especially in emerging markets where volatility is higher than developed markets – this approach would be sensible. A more general approach when estimating different sized portfolios could involve a market index to proxy for small, medium and large market caps.

Reduction in bias and estimation: Dimson (1979) and Basiewicz (2010) noted that the largest problem with thin trading, was the computed β bias (and, by association, regression results). A lag period for factor β s could be introduced.

Dummy variables and interaction: Different market sectors exhibit differing degrees of volatility so a model extension could include indicator variables (such that the recording process accounts for each binary prediction level represented by each JSE sector). This may improve the statistical descriptive accuracy of the pricing model. Tests for statistical factor interactions could also be included.

Alternative techniques: An underlying assumption of multiple linear regression is that the model follows a linear-relationship, but it is by no means clear that the relationships are indeed linear. Tests for factor linearity could be included in future studies. Other issues associated with time-series such as response times, could be tested using quantile regression.³⁷ If non-linearity were established, the series could be transformed using principal component and factor analysis.

5.2.2. Article 2: *Investment implications of the fractal market hypothesis*

In the current study, the choice of these values was somewhat arbitrary. An extension to this work could therefore include a more detailed calibration – perhaps by OLS – of the optimal pre-breach period for $\Delta D/D$ and optimal post-breach period for $\Delta P/P$.

A comprehensive application of these results to other market indices and asset classes is also needed. Whether the relationship above holds for all asset classes (and, if so, whether the requirement that $H > 0.5$ is a necessary or sufficient prerequisite), also needs to be ascertained.

This analysis interrogated only overall market indices, but prior research has found that H varies by market sector, so sectoral analysis could be included in future research. In addition, there is every reason to presume that different results would arise from individual share price (or commodity price or exchange rate) studies.

Whether relationships exist between $\Delta D/D$ and $\Delta P/P$ and different market conditions, or different economies (developed or developing) could also be attempted.

³⁷ An advantage of quantile regression, relative to the ordinary least squares regression, is that the quantile regression estimates are more robust against outliers in the response measurements.

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