A multi-instrument ionospheric Faraday rotation analysis

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Contents

1 Introduction 5
  1.1 Radio Interferometry and the Van Cittert-Zernike Theorem . . . . . . . . . . . . 5
    1.1.1 Radio Frequency Interference . . . . . . . . . . . . . . . . . . . . . . . . 7
    1.1.2 Calibration . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 7
  1.2 Polarization . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8
    1.2.1 Faraday Rotation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 9
    1.2.2 Stokes Parameters . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 12
  1.3 Faraday Rotation Extraction From Radio Data . . . . . . . . . . . . . . . . . . . 12
    1.3.1 RM Models . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
  1.4 Outlier Detection . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 17
    1.4.1 Kernel Density Estimation . . . . . . . . . . . . . . . . . . . . . . . . . . 17
    1.4.2 Percentile . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 19
    1.4.3 Mean Absolute Percentage Error (MAPE) . . . . . . . . . . . . . . . . . . 19
  1.5 Problem Statement . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 19
  1.6 Existing Work That Models Ionospheric Faraday Rotation From TEC . . . . . . 19
  1.7 Research Goal . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 24
    1.7.1 Aims . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25
    1.7.2 Objectives . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25
  1.8 Chapter Layout . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25

2 Data Selection and Processing 26
  2.1 KAT-7 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 26
  2.2 MeerKAT . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
  2.3 TrigNet . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
  2.4 Sample Data . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29
  2.5 TEC Data . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 32
2.6 Data Processing ......................................................... 32
   2.6.1 Exploring the data ........................................ 32
   2.6.2 Flagging ....................................................... 33
   2.6.3 Delay Calibration ............................................ 33
   2.6.4 Bandpass Calibration ........................................ 33
   2.6.5 Gain Calibration ............................................. 33
   2.6.6 Setting the flux scale ...................................... 34
   2.6.7 Polarization Calibration .................................... 34
   2.6.8 Applying the Calibrations and Imaging ..................... 42
2.7 Self Calibration ..................................................... 43
2.8 Extracting the Polarized Signal .................................. 43
2.9 Rotation Measure Modelling ..................................... 49
2.10 Model-fit Evaluation ............................................. 49
2.11 Temporal Analysis ............................................... 50

3 Results and Discussion 51
   3.1 Rotation measure modelling ................................... 51
   3.2 Temporal Analysis ............................................. 66

4 Summary, Conclusion and Future Work 75
Declaration of Authorship

I, Michael Roger Saharini, know the meaning of plagiarism and declare that all of the work in the thesis, save for that which is properly acknowledged, is my own.

Signed: [Signature]

Date: 17/11/2017
Abstract

Radio interferometers are used for polarimetric imaging. This is one of the types of studies that will be done at the Southern African MeerKAT telescope and in turn, the Square Kilometre Array (SKA) telescope. The polarization of the radiation coming to the interferometer from an astronomical source can be altered by any magneto-ionic plasma along the line-of-site, whether the plasma is associated with the source itself or it is separate from the source. However, the polarization of the radiation that is measured is altered by the ionosphere due to Faraday rotation. We therefore need to remove the effects of this ionospheric Faraday rotation. Unfortunately, this is made difficult by the variability of the ionosphere due to space weather. Therefore, the relationship between the total electron content (TEC) of the ionosphere and the ionospheric Faraday rotation needs to be determined.

O’Sullivan et al. [2012] showed that modelling the polarization angle and the degree of polarization dependences with wavelength squared is vital in measuring the true Faraday depth structure of extragalactic radio sources. This project aimed to extend the methods used by O’Sullivan et al., and potentially other methods, to extract Faraday rotation parameters from existing KAT-7 and MeerKAT data and to make progress towards linking these parameters to the change in TEC of the ionosphere over the SKA site in the Karoo.

Three KAT-7 observations and one MeerKAT commissioning observation were flagged and calibrated, during which the calibration procedures and results were studied in detail, including polarization calibration. The Stokes Q and U parameters, which describe the polarization properties, were extracted. Three different outlier detection methods were compared and used to remove the outliers in the Q and U data. Different polarization models were then fitted to the Q and U data, to extract the rotation measure (RM) properties of the sources.

The first KAT-7 observation showed that 3C286 was best described by a three RM-component model. The other two KAT-7 observations and the MeerKAT observation all showed that 3C138 was also best described by a three RM-component model.

The time-variabilities of the polarization properties of these sources were analysed and compared to total electron content (TEC) data from a nearby TrigNet station, as well as the change in TEC (dTEC). We could not come to an exact conclusion about the relationship between the ionosphere properties and the rotation measure since these observations were not carried out within the same time window or the data from surrounding TrigNet stations were missing. We showed that there is scope for such a multi-instrument analysis and this can be coordinated and carried out in the future with the SKA pathfinder, MeerKAT.

Keywords: Faraday rotation, ionosphere, total electron content, rotation measure, polarization, KAT-7, MeerKAT, SKA, radio astronomy, calibration, polarimetry, outlier detection, TrigNet.
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## List of Figures

1.1 Schematic diagram of a basic interferometer ........................................ 6
1.2 Known radio frequency interference bands from geostationary satellites in the KAT-7 band. Colours: Yellow - GSM 900 Mobile and Thuraya; Purple - Aeronautical Navigation; Grey - Meteosat, Fengyun and Iridium; Blue - Inmarsat; Red - Afristar; Cyan - Express AM1 and AM44; Green - Galileo, Beidou, GPS (L1, L2 and L3) and GLONASS .................................................. 7
1.3 A schematic diagram of an electromagnetic plane wave .............................. 9
1.4 Rotation measure spread function from O’Sullivan et al. (2012) .................. 15
1.5 Polarization data for PKS B1903-802 and single RM component model from O’Sullivan et al. (2012) ............................................................... 16
1.6 A schematic diagram of an ionospheric pierce point ................................ 20
1.7 ionFR-modelled ionospheric Faraday depths from Sotomayor-Beltran et al. (2013) 21
1.8 Polarization angle after different stages of CASA calibration .................... 23
1.9 Polarization angle after three calibration stages ..................................... 24
2.1 MeerKAT technical specifications (from www.ska.ac.za) .......................... 27
2.2 TrigNet station distribution ...................................................................... 28
2.3 Flow chart showing the steps of the procedure used to process the data ...... 36
2.4 KAT_DAT1 observation header ............................................................... 37
2.5 Section of the KAT_DAT1 observing schedule ........................................ 37
2.6 Other KAT_DAT1 information from listobs ............................................ 37
2.7 Unflagged and uncalibrated PKS B1934-638 data from KAT_DAT1 .......... 38
2.8 Flagged but uncalibrated PKS B1934-638 data from KAT_DAT1 ............... 38
2.9 Delay calibration for KAT_DAT1 ............................................................. 39
2.10 Bandpass amplitudes for KAT_DAT1 ..................................................... 40
2.11 Unflagged cross-hand phase solutions of 3C286 .................................... 41
2.12 Flagged cross-hand phase solutions of 3C286 ....................................... 41
2.13 Gain phase solutions for KAT_DAT1 .................................................... 45
2.14 Image of 3C286 ................................................................. 46
2.15 Stokes U vs Stokes Q for the first 6 scans of 3C286 in KAT_DAT1 .............. 47
2.16 Stokes U vs Stokes Q for the last 5 scans of 3C286 in KAT_DAT1 .............. 47
2.17 Stokes V vs Stokes U for the first 6 scans of 3C286 in KAT_DAT1 .............. 48
2.18 Stokes V vs Stokes U for the last 5 scans of 3C286 in KAT_DAT1 .............. 48
3.1 Polarization data for 3C286 from KAT_DAT1 with single RM model ............ 52
3.2 Polarization data for 3C286 from KAT_DAT1 and single RM model with external Faraday dispersion ................................................................. 52
3.3 Polarization data for 3C286 from KAT_DAT1 and two RM model ............... 53
3.4 Polarization data for 3C286 from KAT_DAT1 and three RM model ............... 53
3.5 Polarization data for 3C286 from KAT_DAT1 and three RM model ............... 54
3.6 Polarization data for 3C138 from KAT_DAT1 with single RM model ............. 55
3.7 Polarization data for 3C138 from KAT_DAT2 and single RM model with external Faraday dispersion ................................................................. 56
3.8 Polarization data for 3C138 from KAT_DAT2 and two RM model ............... 56
3.9 Polarization data for 3C138 from KAT_DAT2 and three RM model ............... 57
3.10 Polarization data for 3C138 from KAT_DAT2 and three RM model ............... 57
3.11 Polarization data for 3C138 from KAT_DAT3 with single RM model .......... 58
3.12 Polarization data for 3C138 from KAT_DAT3 and single RM model with external Faraday dispersion ................................................................. 58
3.13 Polarization data for 3C138 from KAT_DAT3 and two RM model ............... 59
3.14 Polarization data for 3C138 from KAT_DAT3 and three RM model ............... 59
3.15 Polarization data for 3C138 from KAT_DAT3 and three RM model ............... 60
3.16 Polarization data for 3C138 from MKAT_DAT1 with single RM model ........ 61
3.17 Polarization data for 3C138 from observation (4) and single RM model with external Faraday dispersion ................................................................. 62
3.18 Polarization data for 3C138 from MKAT_DAT1 and two RM model ............... 62
3.19 Polarization data for 3C138 from MKAT_DAT1 and three RM model ............... 63
3.20 Polarization data for 3C138 from MKAT_DAT1 and three RM model ............... 63
3.21 Polarization properties over time for 3C286 in KAT_DAT1 ..................... 67
3.22 Average polarization properties for each scan of 3C286 in KAT_DAT1 .......... 67
3.23 Gaussian Process Regression of polarization angle for 3C286 in KAT_DAT1 ... 68
3.24 TEC over Sutherland 28-07-2013 - 30-07-2013 ................................. 68
3.25 TEC data compared with change in polarization angle of 3C286 in KAT_DAT1 . 69
3.26 Polarization properties over time for 3C138 in KAT_DAT2
3.27 Average polarization properties for each scan of 3C138 in KAT_DAT2
3.28 Gaussian Process Regression of polarization angle for 3C138 in KAT_DAT2
3.29 TEC over Sutherland 15-11-2013 - 18-11-2013
3.30 TEC data compared with change in polarization angle of 3C138 in KAT_DAT2
3.31 Polarization properties over time for 3C138 in KAT_DAT3
3.32 Average polarization properties for each scan of 3C138 in KAT_DAT3
3.33 Gaussian Process Regression of polarization angle for 3C138 in KAT_DAT3
3.34 TEC over Sutherland 17-11-2013 - 20-11-2013
3.35 TEC data compared with change in polarization angle of 3C138 in KAT_DAT3
4.1 Ionospheric Faraday depths on the 29th of July from ionFR program
4.2 Ionospheric Faraday depths on the 16th and 17th of November, 2013, from ionFR program
4.3 Ionospheric Faraday depths on the 18th and 19th of November, 2013, from ionFR program
List of Tables

2.1 Basic KAT-7 specifications .......................................................... 26
2.2 Dataset descriptions ........................................................................ 30
2.3 Polarization properties of 3C138 and 3C286 ............................... 31
3.1 Rotation measure modelling results ............................................... 65
Chapter 1

Introduction

The ionosphere is defined as the layer of the Earth’s atmosphere that is ionized by the sun’s radiation. This radiation causes electrons to be removed from gas particles in the atmosphere, creating a ionized plasma consisting of loose electrons and positively charged molecules. Studying the ionosphere is important because this ionized plasma causes fluctuations in the amplitude and phase of incoming radio waves from celestial objects and satellites. Therefore, the ionosphere affects technologies that involve passing radio signals through the atmosphere or bouncing signals off it. These include various navigational and communicational technologies, as well as radio astronomy. This effect is known as ionospheric scintillation and the extent of the fluctuations depends on the density of the ionosphere, which is constantly changing owing to the variable radiation coming from the sun. Different types of equipment and methods can be used to monitor the properties of the ionosphere. A useful property of the ionosphere is the total electron content (TEC), which is the number density of electrons integrated along the line of sight. Another effect that the ionosphere has on electromagnetic waves, which has particular implications for radio astronomy, is Faraday rotation. In this chapter, radio interferometry will be introduced and some aspects of electromagnetic polarization and Faraday rotation will be described and derived. The aims of this thesis will then be discussed and finally, the thesis layout is presented.

1.1 Radio Interferometry and the Van Cittert-Zernike Theorem

Due to the diffraction of light through a circular aperture, the angular resolution of a telescope is proportional to $\lambda/D$, where $\lambda$ is the wavelength of the radiation being observed and $D$ is the diameter of the dish. However the largest fully steerable radio dishes are on the scale of about 100m in diameter. At radio wavelengths ($\sim$1mm to 100000m), this limits us to angular resolutions of a few arcseconds. However, multiple radio antennas can be interconnected together to form an equivalent big telescope called a radio interferometer.

A basic interferometer consists of one pair of antennas, as shown in Figure 1.1. A radio wave from an extraterrestrial source is received by both antennas but because there is a slight difference in path lengths, there is a time delay between the two measurements. The correlator multiplies and time-averages the two voltages which yields an output response. Like the double slit exper-
Figure 1.1: A basic interferometer setup with two antennas labelled 1 and 2. $\mathbf{b}$ is the baseline vector, which is the vector between the two antennas, $\tau_g$ is the delay between the two signals due to the different path lengths, $V_1$ and $V_2$ are the voltages produced by the signal at each antenna and $R$ is the response function. (Image from www.cv.nrao.edu/course/astr534/Interferometers1.html)

The output of an interferometer is an interference pattern but because the antennas are directional, the response is this interference pattern multiplied by the power pattern, which we call the primary beam, of the individual antennas. We refer to the output of the interferometer as the visibilities. When an interferometer is being used to observe an extended, monochromatic and incoherent intensity distribution $I_\nu$, the complex visibility function measured by a co-planar baseline $pq$ is given by the following 2-dimensional Fourier Transform:

$$V_{pq}(u, v, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_\nu e^{-2\pi i (ul + vm)} dldm,$$

(1.1)

where $(u, v, 0)$, known as the $uv$-plane, is the coordinate system given to the visibility space, and $l$ and $m$ are coordinates in the directional cosine coordinate system, which is used to describe the intensity distribution (Foster [2016]). This is a theorem that van Cittert and Zernike developed independently (van Cittert [1934] and Zernike [1938]) and is therefore known as the van Cittert-Zernike Theorem.

The phase of the visibilities tells us about the position of the source, whereas the amplitude tells us about the intensity and shape of the source. The angular resolution now becomes proportional to $\lambda/B$, where $B$ is the distance vector between the two antennas, which we call the baseline. This baseline can easily be made bigger than the biggest single dishes and we can therefore reach much higher resolutions with interferometers.

To improve the sensitivity of the interferometer, as well as the imaging fidelity, more antennas can be added. $B$ is then the longest baseline.
1.1.1 Radio Frequency Interference

![Figure 1.2: Known radio frequency interference bands from geostationary satellites in the KAT-7 band. Colours: Yellow - GSM 900 Mobile and Thuraya; Purple - Aeronautical Navigation; Grey - Meteosat, Fengyun and Iridium; Blue - Inmarsat; Red - Afristar; Cyan - Express AM1 and AM44; Green - Galileo, Beidou, GPS (L1, L2 and L3) and GLONASS](image)

While observing a radio source, there are often unwanted signals that are picked up by the telescope. These signals are radiated by a variety of natural sources, intentional radiators and unintentional radiators. These include lightning, power lines, microwaves, consumer electronics, mobile services, wireless devices, two-way radios, etc. Figure 1.2 shows some frequency ranges that are known to often contain radio frequency interference (RFI) due to satellites. RFI can also originate within the antennas themselves. Because RFI sources are all much closer than the target sources, the unwanted signals are usually much stronger than the signal from the target. Luckily, the interference is usually only at some of the frequencies within the telescope’s range and/or for small time intervals. In the early days of radio astronomy, RFI was removed by inspecting the data and manually removing outliers. Due to the huge amount of data generated by new radio telescopes, this process is becoming more complex and tedious. Automated methods are therefore being developed and tested (Offringa et al. [2012] and Mosiane [2017]). This process is called flagging.

1.1.2 Calibration

Once the data has been cleaned from RFI, the next step is to calibrate the signal. There are several factors in radio interferometry, both internal and external, that can cause errors in the measurements. Some of these errors can be corrected using known information or electronics, whilst others require the observations to be calibrated. First generation calibration involves correcting the observed signal using observations of sources with known parameters. Observations of the calibration sources are interspersed with the target field observations in order to account for the changes in the observational parameters.

- A very bright, invariant source, that is point-like or well-modelled and has a known flux, is used to calibrate the absolute flux density of the targets.
• Bandpass calibration is required to correct the errors along the frequency axis of the observation. This requires a very bright, invariant source, that is point-like or well-modelled and has a known spectrum. The same calibrator can be used to correct a phase delay error which manifests as a linear ramp in the bandpass.

• The complex valued gains of an observation may be altered by local effects. Therefore, a bright source that is close to the target in the sky (within 2° for high frequency observations and within 20° for low frequency observations), is needed to perform gain calibration, because the gain calibrator needs to be observed through the same atmospheric conditions as the target source. If no bright calibrator exists close to the target, a weaker but still close source should be used rather than looking for a bright source further away.

• To perform accurate polarization calibration, a bright, polarized source is observed over a range of parallactic angles spanning at least 60 degrees.

The absolute flux density, delay and bandpass calibration can be performed with the same calibrator. If this calibrator is sufficiently close to the target, it can also be used as the gain calibrator. However, this is rarely the case, and a second source is usually used for gain calibration.

1.2 Polarization

As a light wave travels through space, it may pass through magnetic media that alter some of its properties. We can study these changes to find out something about the regions in space that this light has travelled through. To understand the details of the changes, one needs to understand polarization of light.

The electric field of every electromagnetic (EM) wave can be expressed as the superposition of two orthogonal independent waves:

\[ E_x = E_1 \cos(kz - \omega t + \delta_1) \]  
\[ E_y = E_2 \cos(kz - \omega t + \delta_2). \]

The phase difference between these two waves determines the polarization of the EM wave. If the phase difference is a multiple of \( \pi \), namely if \( \delta = \delta_1 - \delta_2 = m\pi \), where \( m = 0, \pm 1, \pm 2, \ldots \), then the EM wave is said to be linearly polarized. On the other hand, if the phase difference is \( \delta = \frac{1}{2}(1 + m)\pi \)

where \( m \) has the same values as previously, then the wave is said to be circularly polarized. We can also express any elliptically polarized wave as the superposition of two orthogonal circularly polarized waves. Depending on the orientation of the plane of polarization, circularly polarized waves are either left-handed or right-handed.
When we measure radiation from an astronomical radio source, the radiation has a wide frequency range. In any bandwidth within this range, the radiation is made up of many independent waves which have a variety of polarizations. Depending on the emission mechanism of the source, the polarization of these waves are usually not completely random and therefore, the resulting radiation is usually polarized.

1.2.1 Faraday Rotation

The signals that are measured by radio interferometers are the superposition of one right-handed circularly polarized wave and one left-handed circularly polarized wave. As the signal travels through a medium, the two circularly polarized waves have different indices of refraction and therefore propagate through the medium at different speeds. This produces a relative phase shift and this rotates the plane of polarization of the original signal. This rotating effect is known as Faraday rotation.

Faraday rotation is the change that was mentioned at the beginning of the introduction. As a signal travels from an extragalactic source, it passes through various media before reaching our equipment. Polarimetric imaging is the study in which the changes in the signal’s polarization along its path are modelled to determine properties of the media that the signal has passed through, and ultimately this information is used to map regions in space.

The trouble with this, is that the Earth’s ionosphere also contributes a significant amount of Faraday rotation. Therefore, to study the polarization of the signal, we need to remove the effect of the ionospheric Faraday rotation.

Figure 1.3: An electromagnetic plane wave propagating in the z-direction. (Image from https://www.researchgate.net/post/Picture_of_the_very_initial_portion_of_an_em_wave-any_thoughts)

Let us look at the mathematics behind Faraday rotation. The force exerted on an electron by the electric and magnetic fields of an electromagnetic wave is usually negligible. However, if there is
a background magnetic field, the force may become significant and Faraday rotation may occur. Let us define the z-axis to be the direction of propagation, as seen in Figure 1.3 and let this also be the direction of the background magnetic field. This restricts the electric field vector $\mathbf{E}$ and the displacement vector $\mathbf{s}$ of the dipole oscillation to the $(x,y)$ plane. We also assume that each electron has a natural frequency $\omega_0$ and therefore a restoring force of $f = \omega_0^2 m_e$, where $m_e$ is the mass of the electron. The equation of motion of the electron is then

$$m_e \frac{d^2 \mathbf{s}}{dt^2} + f \mathbf{s} = -e \left( \mathbf{E} + \frac{1}{c} \frac{d \mathbf{s}}{dt} \times \mathbf{B} \right),$$

where $\mathbf{B}$ is the magnetic field vector. If the electromagnetic wave has a frequency $\omega$, all quantities will have an $e^{-i\omega t}$ time dependence when in equilibrium. The equation of motion then reduces to

$$(\omega_0^2 - \omega^2) s_x - i \Omega \omega s_y = -\frac{e}{m_e} E_x$$  \hspace{1cm} (1.7)$$

$$(\omega_0^2 - \omega^2) s_y - i \Omega \omega s_x = -\frac{e}{m_e} E_y$$  \hspace{1cm} (1.8)$$

where $\Omega = \frac{eB}{m_e c}$ is the cyclotron frequency. We now use the circularly polarized expressions of the electric field and displacement vectors:

$$E_\pm \equiv E_x \pm i E_y \text{ and } s_\pm \equiv s_x \pm i s_y$$  \hspace{1cm} (1.9)$$

Substituting (1.9) into (1.7) and (1.8) and adding and subtracting the equations from each other, we get

$$(\omega_0^2 - \omega^2 - \omega \Omega) s_+ = -\frac{e}{m_e} E_+$$  \hspace{1cm} (1.10)$$

$$(\omega_0^2 - \omega^2 + \omega \Omega) s_- = -\frac{e}{m_e} E_-$$  \hspace{1cm} (1.11)$$

We then use the induced dipole moments

$$P_\pm = \frac{n_e e^2 E_\pm}{m_e (\omega_0^2 - \omega^2 \mp \omega \Omega)}$$  \hspace{1cm} (1.12)$$

to find the polarizability

$$\chi_\pm \equiv 4\pi \frac{P_\pm}{E_\pm}$$  \hspace{1cm} (1.13)$$

and from this we can extract the refractive index for left-handed and right-handed circularly polarized waves:

$$n_\pm = \sqrt{1 + \frac{4\pi n_e e^2}{m_e (\omega_0^2 - \omega^2 \mp \omega \Omega)}}$$  \hspace{1cm} (1.14)$$
where $n_e$ is the number density of electrons. We can see that $n_+ > n_-$ and therefore the right-handed wave will propagate with a slightly higher phase velocity than the left-handed wave. The right-handed circularly polarized wave will have an electric field of

$$E^R_x(z, t) = E_0 \cos \left( \frac{w}{c} [n_- z - ct] \right)$$
$$E^R_y(z, t) = -E_0 \sin \left( \frac{w}{c} [n_- z - ct] \right)$$

(1.15)

and similarly, the left-handed circularly polarized wave will have an electric field of

$$E^L_x(z, t) = E_0 \cos \left( \frac{w}{c} [n_+ z - ct] \right)$$
$$E^L_y(z, t) = -E_0 \sin \left( \frac{w}{c} [n_+ z - ct] \right)$$

(1.16)

We add these fields together to get the electric field of a linearly polarized wave propagating through a magnetized medium:

$$E_x(z, t) = 2E_0 \cos \left( \frac{w}{c} [nz - ct] \right) \cos \left( \frac{w}{2c} [n_+ - n_-] \right)$$
$$E_y(z, t) = 2E_0 \cos \left( \frac{w}{c} [nz - ct] \right) \sin \left( \frac{w}{2c} [n_+ - n_-] \right)$$

(1.17)

where $n \equiv (1/2)(n_+ + n_-)$ is the average index of refraction. In most cases the frequencies of electromagnetic radiation and the natural oscillation of atomic dipoles are much larger than the cyclotron frequency, i.e. $\omega, \omega_0 >\!> \Omega$. We then find

$$n \approx 1 + \frac{4\pi n_e e^2}{m_e (\omega_0^2 - \omega^2)}$$

(1.18)

and therefore

$$n_+ - n_- \approx \frac{4\pi n_e e^2 \omega \Omega}{m_e (\omega_0^2 - \omega^2)^2}.$$  

(1.19)

The polarization angle $\chi$ relative to the $x$-axis is

$$\chi = \arctan \left( \frac{E_y}{E_x} \right) = \frac{\omega}{2c} (n_+ - n_-) z.$$ 

(1.20)

Therefore, the change in polarization angle with propagation distance is

$$\frac{d\chi}{dz} = \frac{2\pi n_e e^2 \omega^2 \Omega}{m_e c (\omega_0^2 - \omega^2)^2}$$

(1.21)
We can further approximate this by assuming $\omega \gg \omega_0$, which is usually true. We then find

$$\Delta \chi = \frac{e^3 \lambda^2}{2\pi (m_e c^2)^2} \int_{\text{LOS}} n_e(z) B_{\parallel}(z) dz \quad (1.22)$$

where $\lambda = 2\pi c / \omega$ is the wavelength, $B_{\parallel}$ is the component of the magnetic field along the line of sight and the integral is calculated along the line of sight (LOS) from the source to the observer. For convenience we define the Rotation Measure (RM) as

$$\text{RM} = 2.64 \times 10^{-17} \text{Gauss}^{-1} \int_{\text{LOS}} n_e(z) B_{\parallel}(z) dz \quad (1.23)$$

so that

$$\Delta \chi = \text{RM} \lambda^2 \quad (1.24)$$

### 1.2.2 Stokes Parameters

A radio interferometer measures a correlation function of the field. Therefore, for convenience, we represent the polarization properties of a signal using the Stokes parameters, which, for a linear feed, are defined as follows:

$$I = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle \quad (1.25)$$

$$Q = \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle \quad (1.26)$$

$$U = 2\text{Re}(E_x E_y^*) \quad (1.27)$$

$$V = -2\text{Im}(E_x E_y^*) \quad (1.28)$$

From the Stokes parameters (Myserlis [2017]), we can calculate the polarization fraction and polarization angle of the EM wave:

$$p = \frac{\sqrt{Q^2 + U^2 + V^2}}{I^2} \quad (1.30)$$

and

$$\Psi = \arctan \left( \frac{U}{Q} \right) \quad (1.31)$$

### 1.3 Faraday Rotation Extraction From Radio Data

O’Sullivan [2012] studied the Faraday depth structure of four strongly polarized, unresolved, radio-loud quasars using the Australia Telescope Compact Array (ATCA) with 2GHz of instantaneous bandwidth from 1.1 to 3.1 GHz. They spectrally resolved the polarization structure of spatially unresolved radio sources and fitted various Faraday rotation models to the data. They show that two of the sources require a more complex description than a simple rotation measure.
(RM) component modified by depolarization from a foreground Faraday screen. They suggest that additional RM components come from polarized structure in the compact inner regions of the radio sources themselves and not from polarized emission from Galactic or intergalactic foreground regions.

Radio-loud Active Galactic Nuclei (AGN) produce powerful jets of relativistic plasma and the polarized radiation from these jets can be used as a probe to study the strength and structure of magnetic fields in our galaxy. RM surveys such as the Polarization Sky Survey of the Universe’s Magnetism (POSSUM) (Gaensler et al. [2010]) will measure the RM’s of about 3 million of these extragalactic background sources. Therefore, the algorithms that extract the polarization and RM properties from the datasets need to be extensively tested and optimized.

We will use the same notation as O’Sullivan et al. and define the complex linear polarization as

\[ P = Q + iU = p e^{2\pi i} \psi \]  

where \( I, Q \) and \( U \) are the measured stokes parameters and \( \psi \) is the observed polarization angle. They also define \( q = Q/I \) and \( u = U/I \) and the measured magnitude of the degree of linear polarization becomes

\[ p = \sqrt{q^2 + u^2} \]  

and the polarization angle is

\[ \psi = \frac{1}{2} \arctan \frac{u}{q} \]

The fractional values are used because it decouples depolarization effects from simple spectral index effects when analysing the dependence of polarization with wavelength, as well as minimising errors in the estimate of the RM using RM synthesis.

If there are different regions of polarized emission sampled within each resolution element, then each of these regions may experience different amounts of Faraday rotation caused by the magneto-ionic materials between the source of the emission and the observer. Therefore, the Faraday depth, \( \phi \), is used to describe the Faraday rotation of a particular region of polarized emission (Burn [1966]):

\[ \phi = \int_{\text{telescope}}^{\text{emission}} n B \cdot dl \text{ rad m}^{-2} \]  

where \( n \) is the free electron density (in cm\(^{-3}\)), \( B \) is the magnetic field (in \( \mu \text{G} \)) and \( l \) is the distance along the line of sight (in pc).

If there is a background source of emission and only pure rotation due to a foreground magneto-ionic medium, then we have the trivial case where the Faraday depth is equal to the RM and we get

\[ \psi = \psi_0 + RM \lambda^2 \]

Rather than only modelling depolarization as a single RM component, O’Sullivan et al. also consider the case of multiple RM components that are either along the line of sight or intrinsic to the source itself. These multiple RM components can cause both increases and decreases
in the degree of polarization with $\lambda^2$, as well as sometimes deviations from a linear $\Psi(\lambda^2)$ behaviour. Following on from existing work, O’Sullivan et al. conclusively demonstrate the effect of multiple RM components in two cases by considering many different Faraday rotation models to simultaneously describe both $\Psi(\lambda^2)$ and $p(\lambda^2)$.

They first created Uniformly-weighted Stokes I, Q and U images for each source in 10MHz intervals and then deconvolved using the Högbom CLEAN algorithm (Hogbom [1974]). Each image was smoothed to the resolution at the lowest frequency in order to remove any resolution dependent effects. The emission at the positions of the sources in each Stokes I image was taken and a table of $q$ and $u$ as a function of $\lambda^2$ was created, at intervals corresponding to 10MHz. Errors in each channel measurement were assigned by taking the rms noise from a small region around the source position in the clean-residual images.

The RM synthesis technique (Brentjens and de Bruyn [2005]) was then used to extract the polarized signal over a wide range of possible Faraday depths. A spectrum of complex polarization versus Faraday depth was created following the equation

$$F(\phi) = \frac{\sum_{j=1}^{N} w_j P_j e^{-2i\phi(\lambda_j^2 - \lambda_0^2)}}{\sum_{j=1}^{N} w_j}$$

where $N$ is the number of input maps, $P_j$ is the complex polarization at channel $j$ and $w_j$ are the weights, which is the inverse square of the rms noise. The reference wavelength ($\lambda_0$) was defined as

$$\lambda_0^2 = \frac{\sum_{j=1}^{N} w_j \lambda_j^2}{\sum_{j=1}^{N} w_j}.$$  

Figure 1.4 shows the Rotation Measure Spread Function (RMSF) for the observations, which is the normalised response function in Faraday depth space to the incomplete $\lambda^2$ sampling, due to radio interference flagging. With a perfect $\lambda^2$ coverage, the RMSF would be a delta function.

RMCLEAN (Heald et al. [2009]) was used to deconvolve the ‘dirty’ RM spectrum in an attempt to recover information lost due to the incomplete $\lambda^2$ coverage.

### 1.3.1 RM Models

The simplest model of a polarized signal modified by Faraday rotation is

$$P = Q + iU = p_0 e^{2i(\Psi_0 + RM\lambda^2)}$$

where $p_0$ is the original degree of polarization of the synchrotron emission, $\Psi_0$ is the original angle of polarization at emission and RM is the rotation measure which describes the Faraday rotation due to the magneto-ionic material through which the polarized signal is travelling. The change of degree of polarization with wavelength, as can be seen in the data, can be caused by mixing of the emitting and rotating media, or the finite spatial resolution of the observations. The three commonly listed mechanisms of this depolarization that O’Sullivan [2012] consider are:
1. **Differential Faraday rotation (DFR):** if the emitting and rotating regions are co-spatial and are in the presence of a regular magnetic field, then the plane of polarization of the emission at the far side of the region is rotated by a different amount to that of the light that is emitted at the near side, which causes depolarization when summed over the whole region. If we consider a uniform slab, then we have

\[ P = p_0 \frac{\sin R \lambda^2}{R \lambda^2} e^{2i \Psi_0 + \frac{1}{2} R \lambda^2} \]  

(1.40)

where \( R \) is the Faraday depth through the region.

2. **Internal Faraday dispersion (IFD):** if the emitting and rotating regions contain a turbulent magnetic field, then the plane of polarization performs a random walk through the region, which causes depolarization. For identical distributions of all the ingredients of the magneto-ionic material along the line of sight, we can model the modified signal as

\[ P = p_0 e^{2i \Psi_0} \left( 1 - e^{2i R \lambda^2 - \frac{2 \varsigma^2_{RM} \lambda^4}{2 R \lambda^4}} \right) \]  

(1.41)

where \( \varsigma_{RM} \) is the internal Faraday dispersion of the random field. Here, \( \Psi_0 = \pi/2 \) for a purely random anisotropic magnetic field.

3. **External Faraday dispersion/beam polarization:** this is caused by a purely external, non-emitting Faraday screen. There are two possible cases. The first case is turbulent magnetic fields, in which many turbulent cells are within the synthesised beam, causing depolarization. The second case is a regular magnetic field, where any change in the strength or direction of the field within the observing beam will result in depolarization. In either case, the effects can be described by

\[ P = p_0 e^{-2 \sigma^2_{RM} \lambda^4} e^{2i \Psi_0 + R \lambda^2} \]  

(1.42)

where \( \sigma_{RM} \) is the dispersion about the mean RM across the source on the sky.
Another possible mechanism for depolarization is multiple interfering RM components along the line of sight or even on the plane of the sky on smaller scales than the spatial resolution of the observation.

An example of the modelling results from O’Sullivan [2012] can be seen in Figure 1.5. This figure shows the polarization data for one of their target sources, PKS B1903-802, and the corresponding best-fit single RM-component model. PKSB 1903-802 is a flat spectrum quasar with a spectral index of -0.04, where spectral index (α) is defined as $S \propto \nu^{-\alpha}$ (Healey [2007]). It is a known calibrator from the ATCA calibrator catalogue. O’Sullivan [2012] also found that the source is strongly polarized across the entire 2 GHz band of the ATCA. The simple RM model in Figure 1.5 provides a decent description of the polarization angle but the degree of polarization is clearly not constant. They found that both an external Faraday dispersion model and a two RM-component model provide an excellent fit to the data. This supports the conclusion that in order to properly study Faraday rotation, both the polarization amplitude and angle need to be modelled.

**Figure 1.5:** An example of the modelling results from O’Sullivan [2012]. Polarization data for PKS B1903-802, and the corresponding best-fit single RM-component model. Top left: $q$ (open circles) and $u$ (full circles) data vs. $\lambda^2$, fitted with the model $q$ (dot-dashed line) and $u$ (dashed line). Top right: $p$ vs. $\lambda^2$ data over-plotted by the model (solid line). Bottom left: $\Psi$ vs. $\lambda^2$ data over-plotted by the model (solid line). Bottom right: $u$ vs. $q$ data over-plotted by the model (solid).
1.4 Outlier Detection

Even though most of the RFI is removed during the flagging stage, as discussed in Section 1.1.1, once the polarization data has been extracted, some outliers maybe still be present and these need to be flagged in order not to bias our analysis. The following methods of outlier detection were considered for this study:

1.4.1 Kernel Density Estimation

Outlier detection algorithms were traditionally based on assumptions involving statistical distribution. But in the era of data mining, it is becoming increasingly difficult to properly statistically model the complex data. Hence the need for efficient but flexible outlier detection that doesn’t require modelling the data. Schubert [2014] present a local density-based outlier detection for low-dimensional data. They attempt to advance the state of art by keeping a clean connection to the statistical roots while increasing the flexibility of the earlier algorithms.

Distance-based detection involves determining the number of objects within a certain distance from the object in question and if this is a smaller fraction of the database than a threshold fraction, then the object is considered an outlier. Some distance-based methods are based on the distances to the $k$ nearest neighbours. Both of these methods are simple density estimates. Local density methods measure the ratios of the local density around an object and the local density around its neighbouring objects. An example of such a method is the Local Outlier Factor (LOF), which determines the local density of each object $o$ in a database $D$ and compares it to the average density estimates for the $k$ nearest neighbours of $o$. There are many modifications and extensions of this method. The method presented in this paper uses kernel density estimation (KDE) to improve the quality of density-based outlier detection.

KDE for Outlier Detection

Like other existing local density-based methods, this algorithm performs density estimation and then compares the densities within local neighbourhoods. But Schubert et al. propose the use of classic kernel density estimation directly, rather than experimenting with non-standard kernels with no good reason.

Density Estimation Step

The kernel function that is best to use in this method depends on the situation but they suggest to use either the Gaussian or Epanechnikov kernels of bandwidth $h$ and dimensionality $d$:

$$ K_{\text{gauss}, h}(u) := \frac{1}{(2\pi)^{d/2}h^d} e^{-\frac{1}{2} \frac{u^2}{h^2}} $$

(1.43)
These radially symmetric versions only require one bandwidth to be estimated, instead of needing full bandwidth matrices, which is a difficult problem. The balloon estimator is:

$$K_{\text{epanechnikov}, h}(u) := \frac{3}{4h^d} \left(1 - \frac{u^2}{h^2}\right)$$ \hspace{1cm} (1.44)

A classic approach to estimating the local kernel bandwidth $h(o)$ is to use the nearest-neighbour distances, i.e. $h(o) = k - \text{dist}(o)$. If the kernel function $K(o - p)$ is negligible beyond the $k$-nearest neighbour, we can ignore these for density estimation, giving:

$$n \cdot \text{KDE}_{kNN}(o) := \sum_{p \in kNN(o)} K_{h(o)}(o - p)$$ \hspace{1cm} (1.45)

where $kNN(o)$ are the $k$-nearest neighbours of $o$. The parameter $k$ can be hard to choose so Schubert et al. suggest extending the method to use a range of $k = k_{\text{min}} ... k_{\text{max}}$ to produce a density estimate for each $k$.

**Density Comparison Step**

For density comparison, they assume that not only do the local densities vary, but the variability itself is also sensitive to locality. Therefore, to standardize the deviation from normal density, the z-score transformation is applied. The z-score of $x \in X$ is defined as:

$$z(x, X) := \frac{x - \mu_X}{\sigma_X}$$ \hspace{1cm} (1.47)

where $\mu_X$ is the mean of the set $X$ and $\sigma_X$ is the standard deviation (if $\sigma_X = 0$, then $z(x, X) := 0$). When using multiple $k$-values, an average z-score is used:

$$s(o) := \text{mean}_{k_{\text{min}} ... k_{\text{max}}} z\left(\text{KDE}(o), \{\text{KDE}(p)\}_{p \in kNN(o)}\right)$$ \hspace{1cm} (1.48)

**Score Normalization Step**

Assuming the resulting scores are approximately normally distributed, the normal cumulative density function $\Phi$ can be used to normalize the scores to the range $[0;1]$ and then the rescaling

$$\text{norm}(p, \phi) := \frac{\phi \cdot (1 - p)}{\phi + p}$$ \hspace{1cm} (1.49)

can be applied to obtain the proposed outlier score:

$$\text{KDEOS}(o, \phi) := \text{norm}(1 - \Phi(s(o)), \phi),$$ \hspace{1cm} (1.50)

where $\phi$ is the expected rate of outliers, which can be intuitively interpreted as a significance threshold.
1.4.2 Percentile

A simple outlier detection method involving the 25th and 75th percentiles (\(Q_1\) and \(Q_3\) respectively) and the interquartile range (IQR) was tested. Any point that lay below \(Q_1 - 1.5IQR\) or above \(Q_3 + 1.5IQR\) was considered an outlier, where \(IQR = Q_1 - Q_3\).

1.4.3 Mean Absolute Percentage Error (MAPE)

The mean absolute percentage error (MAPE) method was tested. In this method, the unbiased median absolute deviation from the median (UMdAD) is calculated and any point that lies more than 3UMdAD above or below the median is considered an outlier. The median absolute deviation (MdAD) is the median of the absolute deviation from the median:

\[
\text{MdAD} = \text{median}(|X - \text{median}(X)|) \quad (1.51)
\]

and \(\text{UMdAD} = \text{MdAD}/0.6745\) (Levenbach [2015]).

1.5 Problem Statement

As discussed previously, radio interferometers are used for polarimetric imaging. This is one of the types of studies that will be done at the Southern African MeerKAT telescope and in turn, the Square Kilometre Array (SKA) telescope. However, the polarization of the radiation that is measured is altered by the ionosphere due to Faraday rotation, as explained in Section 1.2.1.

We therefore need to remove the effects of this ionospheric Faraday rotation. Unfortunately, this is made difficult by the variability of the ionosphere due to space weather. Therefore, the relationship between the TEC and the ionospheric Faraday rotation needs to be determined.

O’Sullivan [2012] show that modelling the polarization angle and the degree of polarization dependences with wavelength squared is vital in measuring the true Faraday depth structure of extragalactic radio sources. This project aims to extend the methods used by O’Sullivan et al., and potentially other methods, to extract Faraday rotation parameters from existing KAT-7 and MeerKAT data and to make progress towards linking these parameters to the change in TEC of the ionosphere over the SKA site in the Karoo.

In the following section, we will review some of the work that has been carried out that addresses the problem.

1.6 Existing Work That Models Ionospheric Faraday Rotation From TEC

Sotomayor-Beltran [2013] present a code called \textit{ionFR} which takes GPS-derived total electron content maps and the most recent release of the International Geomagnetic Reference Field and
models ionospheric Faraday rotation for a specific epoch, geographic location, and line-of-sight. They define Faraday depth as

\[ \phi(l) = 0.81 \int_{\text{source}}^{\text{observer}} n_e \mathbf{B} \cdot d\mathbf{l} \, \text{rad/m}^2, \]  

(1.52)

where \( n_e \), the electron density \((\text{cm}^{-3})\), and \( \mathbf{B} \), the magnetic field, are integrated along the line-of-sight (LOS) and \( d\mathbf{l} \) is the infinitesimal path length in pc.

Figure 1.6: A schematic diagram of a signal piercing the ionosphere, which is approximated to be a thin shell (red line).

If we assume that the ionosphere is a thin spherical shell, as shown in Figure 1.6, the Faraday depth of the ionosphere is then calculated at the ionospheric pierce point (IPP). Eqn. 1.52 then becomes

\[ \phi_{\text{ion}} = 2.6 \times 10^{-17} \text{TEC}_{\text{LOS}} B_{\text{LOS}} \, \text{rad/m}^2, \]  

(1.53)

where TEC_{LOS} is the total electron content at the geographic coordinates of the IPP and B_{LOS} is the geomagnetic field intensity in gauss at the IPP.

In this work by Sotomayer-Beltran et al., the TEC data files were taken from the Centre for Orbit Determination in Europe (CODE), which has spatial resolutions of \( \Delta_{\text{lon.}} = 5^\circ \) and \( \Delta_{\text{lat.}} = 2.5^\circ \) and a time resolution of 2 hours, and the Royal Observatory of Belgium (ROB) which has spatial resolutions of \( \Delta_{\text{lon.}} = 0.5^\circ \) and \( \Delta_{\text{lat.}} = 0.5^\circ \) and a time resolution of 15 minutes.

Sotomayor-Beltran et al. compare results from the ionFR code with Faraday depths extracted from measurements from the Low Frequency Array (LOFAR), using RM-synthesis (Brentjens and de Bruyn [2005]). Figure 1.7 shows some of their results.

LOFAR is a radio interferometer that uses a novel phased-array design and covers a less explored low-frequency range of 10-240MHz (van Haarlem [2013]). The model output of ionFR shows good...
agreement with the measured Faraday depths for all of the LOFAR observing campaigns used in the investigations. However, with the TEC data from ROB, Faraday depth measurements are limited to precisions of about 0.05 rad/m². So Sotomayor-Beltran et al. suggest that, to take full advantage of LOFAR’s low observing frequency and large fractional bandwidth, more sophisticated and improved calibration methods will need to be developed. For higher frequency observations and larger bandwidths, such as those that will be used with SKA, the uncertainty associated with ionospheric Faraday depth may be large enough to be comparable with $\phi_{\text{ion}}$ during solar maximum. Additionally, the ionospheric equatorial anomaly will sometimes be directly over the SKA site, making accurate calibration even more important.

The *ionFR* code presents an alternative to using GPS receivers co-located with the radio telescope, but to improve the precision, more precise and better geographically resolved TEC maps are needed. This leads to the need for accurate ionospheric conditions to be extracted directly from MeerKAT and SKA data, if one one wants to push for the best quality data and images.

---

**Figure 1.7:** Some results from Sotomayor-Beltran [2013]. Observed Faraday depths and ionFR-modelled ionospheric Faraday depths toward B0834+06 as a function of time during midday (observations: blue circles, left axis labels; model: red triangles, right axis labels). Upper panels: eleven LOFAR HBA observations using the international station near Onsala, Sweden. Middle panels: eleven LOFAR Superterp HBA observations. Lower panels: eleven LOFAR HBA observations using the international station near Nançay, France. Panel a) shows the ionFR model using CODE TEC data and IGRF11; panel b) shows the ionFR model using ROB TEC data and IGRF11.
Daniel Hayden (Hayden [2013]), an intern at SKA South Africa in 2013, explored two methods of using a radio interferometer to determine ionospheric properties. The first one had already been successfully used on the Very Large Array (VLA) (Helmboldt [2012]) and is only useful for measuring the difference in TEC between two different lines-of-sight. This method therefore determines TEC gradients but not the actual TEC values, and was not explored much further. The second method, inspired by O’Sullivan [2012], has the potential to extract absolute TEC values but relies on calibrating data to remove all distortions except for the effects of the Faraday Rotation caused by the ionosphere.

Hayden then investigated the Common Astronomy Software Applications (CASA) (McMullin et al. [2007]) package’s calibration procedure and found that it does not directly solve for the effects of ionospheric Faraday Rotation. However, taking a look at the mathematics behind the calibration, he found that these effects may be addressed within other parts of the calibration. The measurement equation can be stated as (Cotton, 1999):

\[ v = (J_i \otimes J_k^*)Ss, \]  

where \( J_i \) is the product of all Jones matrices for antenna \( i \), \( \otimes \) is the Kronecker product, \( s \) is the true Stokes visibility vector and \( S \) is the matrix that transforms this vector into the four correlations. \( J_i \) can be written as:

\[ J_i = G_i D_i R_i P_i \]  

where

\[ G_i = \begin{bmatrix} g_{ip} & 0 \\ 0 & g_{iq} \end{bmatrix} ; D_i = \begin{bmatrix} 1 & d_{ip} \\ -d_{iq} & 1 \end{bmatrix} ; R_i = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} ; P_i = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \]  

and \( p \) and \( q \) represent the two feeds. \( G_i \), the gain matrix, represents uncorrected distortions due to the atmosphere and electronics. \( D_i \) represents imperfections in the feed polarization response. In other words it models how much of the signal from each feed leaks into the other. \( R_i \) models the Faraday Rotation of the electric vector due to the ionosphere, over an angle \( x \). \( P_i \) represents the rotation of an altitude-azimuth mounted antenna as seen by the source while the antenna tracks the source.

The fact that the diagonal elements in \( G_i \) and the off diagonal elements in \( D_i \) need not be identical, means that the effects of \( R_i \) may be absorbed into the other Jones matrices. To see if this may be happening in the CASA calibration, Hayden then computed plots of the polarization angle as a function of time after the different stages of the calibration, as shown in Figure 1.8.

From Figure 1.8, we can see that after every stage, except maybe the second cross-hand delay correction, there is a change in the behaviour of the polarization angle. Therefore, the effect of ionospheric Faraday Rotation may be absorbed into each of these stages. However, the method by which the parallel and cross-hand relay corrections are calculated should not be affected by the presence of ionospheric Faraday Rotation and it is therefore believed that these effects are not absorbed into these stages. These two calibration stages were applied successively as well as a gain correction with the diagonal terms forced to be equal and Figure 1.9 shows the polarization angle after each correction.

Hayden concludes that CASA cannot be used to calibrate the measured visibilities without removing the ionospheric Faraday Rotation effects, or to isolate these effects. He then attempted
to solve for $R_i$ explicitly. To simplify this process, it was assumed that $J_i = J_k$ in Equation 1 and only the real parts of this equation were considered, in order to minimize the number of unknowns. $J$ is then expressed as

$$J = GDRP$$

(1.57)

and the Kronecker product becomes

$$J \otimes J = (GDR \otimes GDR)(P \otimes P),$$

(1.58)

where

$$GDR = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix} \begin{bmatrix} 1 & l \\ -l & 1 \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}.$$  

(1.59)

This gives four unknowns, namely $g_1, g_2, l$ and $x$. To reduce the range of possible solutions for $g_1$ and $g_2$, the calibration stages which should not absorb ionospheric Faraday Rotation were applied to the measured visibilities. Hayden then wrote a program that calculates all the possible combinations of $(GDR \otimes GDR)$ for a single time and single frequency, and then finds the values for $(g_1, g_2, l, x)$ that minimize the chi-square:

$$\chi^2 = \sum_{\text{correlations}} [v - (P \otimes P)Ss]^2.$$  

(1.60)

Multiple degeneracies were found for $x$ and $l$, indicating that the system is ill-conditioned. Hayden suggests attempting to improve the ill-conditioning by adding information to the system. He also suggests that two possible points of such information may be:
1. The ionospheric RM should remain constant over the frequency range, and $x$ is related to the RM through the square of the frequency. Therefore, for a particular RM, the chi-square can be calculated as:

$$\chi^2 = \sum_{\text{correlations}} \sum_{\text{frequencies}} [v - (P \otimes P)Ss]^2.$$  \hspace{1cm} (1.61)

2. $l$ should remain constant over time, while the other unknowns can be allowed to change with time.

### 1.7 Research Goal

For polarimetric imaging, the instrumental polarization should be determined through observations of calibrator sources spread over a wide range of parallactic angle. A phase calibrator can be chosen to double as a polarization calibrator. If a bright unpolarized source is known, it can be used for correcting for the polarization leakage terms. Calibrating instrumental polarization for a linear feed is somewhat more complicated than for circular feeds because polarization effects appear in all the correlations at first or zeroth order. Ionospheric Faraday rotation is always notable at 20cm (1.4GHz) and the maximum rotation measure under quiet solar conditions is 1 or 2 radians/m$^2$. This will induce rotation of the plane of polarization of about 5° at 20cm. Under active conditions, this can become more severe. Having reviewed what has already been
done with KAT-7 data and other telescopes, and with the upcoming MeerKAT telescope, there is a need to derive a methodology to extract and understand the change in TEC over the SKA SA site. The aims and objectives are further summarised below:

1.7.1 Aims

- To study and understand polarimetric imaging using radio interferometers
- To study and understand ionospheric Faraday rotation
- To study the existing calibration process and how it addresses, if at all, Faraday rotation
- To determine the relationship between the TEC and ionospheric Faraday rotation in order to remove these rotation effects from polarimetric data.

1.7.2 Objectives

- To investigate the CASA calibration procedure used for KAT-7 and MeerKAT
- To extend the methods used by O’Sullivan et al. (2012), and other potential methods, to model ionospheric Faraday Rotation
- To extract Faraday rotation parameters from existing KAT-7 and MeerKAT data and to link these parameters to the change in TEC of the ionosphere over the SKA site in the Karoo.

1.8 Chapter Layout

This thesis contains four more chapters, the contents of which are summarized below:

- Chapter 2 - Data Selection and Analysis: This chapter presents the sample data and gives a brief description of each of the instruments that produced these datasets, ie. the KAT-7 and MeerKAT telescopes and TrigNet. The complete procedure used to analyse the data is presented. This includes the flagging and calibration procedures that were performed on the radio telescope data, the modelling procedure used to extract rotation measures from the polarization data and the temporal analysis of the polarization data.

- Chapter 3 - Results and Discussion: Here, the final results from the analysis procedure are presented, as well as some discussion of the results.

- Chapter 4 - Summary, Conclusion and Future Work: This chapter summarizes the aims and results of the thesis, as well as the main conclusions drawn from the study. Ideas for future continuation of the study will also be discussed.
Chapter 2

Data Selection and Processing

This chapter presents a brief overview of the four datasets that were selected for this study, as well as the instruments that produced these datasets. We then describe the analysis procedure as applied to the first dataset, KAT_DAT1. The same procedure was then applied to the other datasets, KAT_DAT2, KAT_DAT3 and MKAT_DAT1.

2.1 KAT-7

The Karoo Array Telescope (KAT-7) (Foley [2016]) is a 7-element radio interferometer with a maximum baseline of 185m and a minimum of 26m. It was built as an engineering test-bed for the 64-dish MeerKAT array, which is the South African pathfinder for the Square Kilometre Array (SKA). KAT-7 is situated near the SKA core site in the Karoo Desert, in the Northern Cape province of South Africa. Table 2.1 shows some basic information about KAT-7.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>30.7148° S, 21.388° E, altitude 1054m</td>
</tr>
<tr>
<td>Number of antennas</td>
<td>7</td>
</tr>
<tr>
<td>Dish diameter (m)</td>
<td>12</td>
</tr>
<tr>
<td>Baselines (m)</td>
<td>26 - 185</td>
</tr>
<tr>
<td>Frequency Range (MHz)</td>
<td>1200 - 1950</td>
</tr>
<tr>
<td>Instantaneous Bandwidth (MHz)</td>
<td>256</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear non-rotating (Horizontal + Vertical) Feed</td>
</tr>
<tr>
<td>$T_{sys}$ (K)</td>
<td>&lt;35 across the entire frequency band</td>
</tr>
<tr>
<td>Antenna efficiency at L-band (%)</td>
<td>≈ 30 for all elevations &gt; 30°</td>
</tr>
<tr>
<td>Primary beam FWHM at 1.8GHz (°)</td>
<td>66</td>
</tr>
<tr>
<td>Angular resolution at 1.8GHz (arcmin)</td>
<td>1.0</td>
</tr>
<tr>
<td>Continuum Sensitivity</td>
<td>3</td>
</tr>
<tr>
<td>Angular scales (arcmin)</td>
<td>1.5mJy in 1 minute (256MHz bandwidth, $1\sigma$)</td>
</tr>
<tr>
<td></td>
<td>3 - 22</td>
</tr>
</tbody>
</table>

Table 2.1: Some basic information about the Karoo Array Telescope (KAT-7) (Foley [2016])
The Digital Back-End (DBE) of the KAT-7 system is a Field Programmed Gate Array (FPGA)-based, flexible packetised correlator which uses the Reconfigurable Open Architecture Computing Hardware (ROACH). When using linearly-polarized feeds, total-power measurements can easily be corrupted by linear polarization and polarization calibration is therefore key in achieving the highest quality total-power imaging. For this reason, the KAT-7 correlator always computes all four complex polarization products (XX, YY, XY, YX) for all baselines.

2.2 MeerKAT

MeerKAT, on the other hand, is a 64-dish radio interferometer that is being built as the precursor to the SKA telescope, also situated near the SKA core site in the Karoo Desert, in the Northern Cape province of South Africa. Although it is still in progress, MeerKAT has already been making observations and produced its First Light image in 2016, using 16 dishes. Some technical specifications of MeerKAT are shown in Figure 2.1.

![MeerKAT Technical Specifications](from www.ska.ac.za)

The MeerKAT telescope has a much bigger bandwidth (∼856 MHz in the continuum correlator
model) than that of the KAT-7 telescope, as well as observing at a slightly lower frequency range. This makes MeerKAT a better instrument for an ionospheric study such as this, because when the rotation models, which are a function of wavelength, are fitted to the data, there are more data points and the resulting model parameters should be more accurate.

2.3 TrigNet

TrigNet is a distribution of 67 Global Navigation Satellite System (GNSS) base stations that are spread across South Africa. Each station contains a continuously operating dual-frequency reference receiver which records Global Positioning System (GPS) and GLONASS observables on the L1 (1575.42 MHz) and L2 (1227.60 MHz) frequencies every second. The data is streamed directly to the TrigNet control centre in Cape Town, where the data is processed and then published on the TrigNet website. A map of the TrigNet station distribution is shown in Figure 2.2. Note that there is now a station at the KAT-7/MeerKAT site, shown by the red arrow in the figure. However, this station only began recording data in April 2016.

The ionosphere causes dispersion in electromagnetic waves, i.e. signals with different frequencies travel through the ionosphere at different speeds. Therefore, when two frequencies are used, as is the case with dual-frequency receivers, the time-delay between the two signals can be used to easily determine the TEC (El-Naggar [2011]):

\[
\text{TEC} = \frac{c}{40.3} \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \cdot \delta \tau_{f_1 f_2}
\]

where \(c\) is the speed of light in a vacuum and \(\delta \tau_{f_1 f_2}\) is the travel time difference between the two signals. This gives you the slant TEC (sTEC) which is the electron density integrated over the path from the GPS satellite to the base station. To remove the dependence on elevation angle of the signal path, the sTEC is converted to the vertical TEC (vTEC) using a mapping function.
For this study, the TEC data was extracted from the TrigNet GPS data using the GPS-TEC application (Available at http://seemala.blogspot.co.za/), as used by the South African National Space Agency.

2.4 Sample Data

Three KAT-7 observations (labelled as KAT_DAT1, KAT_DAT2 and KAT_DAT3 respectively) and one MeerKAT commissioning observation (labelled as MKAT_DAT1) were selected for the investigation because they involved observations of the popular polarization calibrators 3C138 and 3C286, whose polarization properties are well-known (Perley and Butler [2013]). Table 2.2 shows some basic information about the different observations and Table 2.3, taken from Perley and Butler [2013], shows the basic polarization properties of the two quasars 3C138 and 3C286, which are of interest in this work.
<table>
<thead>
<tr>
<th>Dataset Label</th>
<th>Telescope</th>
<th>Date</th>
<th>Central Frequency (MHz)</th>
<th>Bandwidth (MHz)</th>
<th>Frequency Resolution (kHz)</th>
<th>Fields</th>
<th>Field Use</th>
<th>Integration Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KAT_DAT1</td>
<td>KAT-7</td>
<td>29 July 2013</td>
<td>1328.1953</td>
<td>256</td>
<td>390.625</td>
<td>PKS B1934-638</td>
<td>Flux Calibrator, Phase Calibrator, Target</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PKS 1313-333, M83</td>
<td></td>
<td>5280</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3C286</td>
<td></td>
<td>27500</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1870</td>
</tr>
<tr>
<td>KAT_DAT2</td>
<td>KAT-7</td>
<td>16 and 17 November 2013</td>
<td>1328.1953</td>
<td>256</td>
<td>390.625</td>
<td>PKS B1934-638</td>
<td>Flux Calibrator, Target, Phase Calibrator and Polarization Calibrator</td>
<td>1140</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PKS 1313-333, MACSJ0553</td>
<td></td>
<td>23150</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3C138</td>
<td></td>
<td>4380</td>
</tr>
<tr>
<td>KAT_DAT3</td>
<td>KAT-7</td>
<td>18 and 19 November 2013</td>
<td>1328.1953</td>
<td>256</td>
<td>390.625</td>
<td>PKS B1934-638</td>
<td>Flux Calibrator, Target, Phase Calibrator and Polarization Calibrator</td>
<td>690</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PKS 1313-333, MACSJ0553</td>
<td></td>
<td>16020</td>
</tr>
<tr>
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<td>3C138</td>
<td></td>
<td>3230</td>
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<tr>
<td>MKAT_DAT1</td>
<td>MeerKAT</td>
<td>16 April 2017</td>
<td>1283.8955</td>
<td>856</td>
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<td>PKS B1934-638</td>
<td>Flux Calibrator, Target, Phase Calibrator and Polarization Calibrator</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PKS 0153-410, NGC641</td>
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<td>476</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3C138</td>
<td></td>
<td>24</td>
</tr>
</tbody>
</table>

Table 2.2: Basic details of the datasets used. Columns: (1) Dataset label; (2) Telescope used for observation; (3) Date of observation; (4) Central frequency of original dataset; (5) Bandwidth of original dataset; (6) Frequency resolution of observation; (7) Names of sources observed; (8) Use of field; (9) Integration time of source in observation.
Table 2.3: Polarization properties of 3C138 and 3C286. Columns: (1) Frequency; (2) The degree of polarization of 3C138; (3) The angle of polarization of 3C138; (4) The degree of polarization of 3C286; (5) The angle of polarization of 3C286 (Perley and Butler [2013])

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>3C138 p (%)</th>
<th>3C138 χ (°)</th>
<th>3C286 p (%)</th>
<th>3C286 χ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050</td>
<td>5.6</td>
<td>-14</td>
<td>8.6</td>
<td>33</td>
</tr>
<tr>
<td>1450</td>
<td>7.5</td>
<td>-11</td>
<td>9.5</td>
<td>33</td>
</tr>
<tr>
<td>1650</td>
<td>8.4</td>
<td>-10</td>
<td>9.9</td>
<td>33</td>
</tr>
<tr>
<td>1950</td>
<td>9.0</td>
<td>-10</td>
<td>10.1</td>
<td>33</td>
</tr>
<tr>
<td>2450</td>
<td>10.4</td>
<td>-9</td>
<td>10.5</td>
<td>33</td>
</tr>
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<td>2950</td>
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<td>-10</td>
<td>10.8</td>
<td>33</td>
</tr>
<tr>
<td>3250</td>
<td>10.0</td>
<td>-10</td>
<td>10.9</td>
<td>33</td>
</tr>
<tr>
<td>3750</td>
<td>-</td>
<td>-</td>
<td>11.1</td>
<td>33</td>
</tr>
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<td>4500</td>
<td>10.0</td>
<td>-11</td>
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<td>33</td>
</tr>
<tr>
<td>5000</td>
<td>10.4</td>
<td>-11</td>
<td>11.4</td>
<td>33</td>
</tr>
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<td>6500</td>
<td>9.8</td>
<td>-12</td>
<td>11.6</td>
<td>33</td>
</tr>
<tr>
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<td>-8</td>
<td>11.9</td>
<td>33</td>
</tr>
<tr>
<td>12800</td>
<td>8.4</td>
<td>-7</td>
<td>11.9</td>
<td>33</td>
</tr>
<tr>
<td>13700</td>
<td>7.9</td>
<td>-7</td>
<td>11.9</td>
<td>34</td>
</tr>
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<tr>
<td>36500</td>
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<td>-24</td>
<td>13.1</td>
<td>36</td>
</tr>
<tr>
<td>43500</td>
<td>6.5</td>
<td>-27</td>
<td>13.2</td>
<td>36</td>
</tr>
</tbody>
</table>
2.5 TEC Data

For some reason, no TrigNet data were available from the stations closest to the KAT-7 site (PSKA and CALV) for the dates of the observations described in Table 2.2. The closest station that had the required data was the Sutherland station which is shown by the blue arrow in Figure 2.2 on page 28. We shall, however, use these data and see if there is any fluctuation that can imply a change in the ionosphere for that period.

2.6 Data Processing

As previously mentioned in Section 1.1, in synthesis imaging, one accumulates a large volume of visibilities from which we produce an estimate of the sky brightness. We are interested in getting the true sky brightness distribution. However, there are many impediments to this question and the data need to be processed. Data processing is a crucial step in radio astronomy and needs to be carried out with extreme care since the final results will depend on each of the steps. In the following sections, each step used in the processing of the data will be described. A flow chart showing the steps of the data processing procedure can be seen in Figure 2.3.

The calibration setup, flagging and calibration procedures were done following Riseley [2014]. CASA version 4.6.0 was used for the calibration and some of the tasks mentioned below are stored in a python file called kat7_pothelpers.py, which had to be executed before the pre-calibration steps.

2.6.1 Exploring the data

The visibilities from the KAT-7 and MeerKAT telescopes are originally stored in Hierachical Data Format (HDF5) and some initial flagging is done using an in-house flagging tool, to remove effects such as known satellite RFI and edge truncation due to the bandpass. Some elevation cut-off (> 20° above the horizon) is also applied, to avoid ground pick ups. The HDF5 is then converted to a Measurement Set file (.ms) using an in-house package (h5toms.py). Such a MS file contains a main table, which holds the visibility data, and sub-tables which contain additional information regarding the observation. To view the tables, one can list the contents of the MS directory itself or use the browsetable task in CASA. One may also retrieve some basic information about the observation by using the listobs task. Figures 2.4 to 2.6 shows some sections of the output of listobs for KAT_DAT1. The plotms task was used to visualize the unflagged and uncalibrated data. An example of raw data is shown in Figure 2.7.
2.6.2 Flagging

The data were then flagged using CASA’s flagdata task, set on rflag mode which detects outliers based on the Rflag Algorithm developed by Eric Greisen in AIPS in 2011. In RFlag, the data is iterated-through in chunks of time, statistics are accumulated across time-chunks, thresholds are calculated at the end, and applied during a second pass through the dataset. The time deviation threshold and frequency deviation threshold were both set to 3.0. The data was then viewed using the plotms task and if necessary, more flagging was done by viewing each scan as a function of frequency/channel and outliers were manually removed. Figure 2.8 shows the same data as Figure 2.7 but after flagging was performed.

2.6.3 Delay Calibration

As mentioned in Section 1.1 and shown in Figure 1.1, there is a delay between the signals being read by the different antennas due to the pass difference. To account for this, an equivalent delay is added to the system using a correlator model, which takes antenna position and timing into account. Small errors in this correlator model create a time-constant linear phase slope as a function of frequency in the correlated data. Delay calibration requires a strong and discrete source and in this dataset, the delay was calibrated with PKS B1934-638, using the gaincal task with the gaintype parameter set to ‘K’. Figure 2.9 shows the delays found for KAT_DAT1. For the KAT-7 telescopes, the delays should always be less than about 5.12ns. Figure 2.9 shows that all the delays in this example are much less than 5ns, and are therefore acceptable.

2.6.4 Bandpass Calibration

There may be residual errors in the amplitude and phase response as a function of frequency are also occur. These errors are a property of the passband and the removal of these errors also require a strong, discrete source, and is known as bandpass calibration. As is often the case, the same calibrator was used for both phase and bandpass calibration in this dataset, and the extracted bandpass amplitudes for each antenna can be seen in Figure 2.10.

2.6.5 Gain Calibration

While the above-mentioned effects are generally from properties that are assumed to be largely constant over the observation period, there are other errors that arise from conditions within the instrument and atmospheric conditions, which are variable over time. These cause errors in both the amplitude and phase of the incoming radiation. The time-variability of these conditions require a gain calibrator to be observed regularly, in between the scans of the target source. The changes can then be interpolated over the target scans, which can then be combined into a single image. In this observation, PKS 1313-333 was used as the gain calibrator.
2.6.6 Setting the flux scale

To calibrate the flux density scale of the observation, a few scans of PKS B1934-638, a source whose flux model we have (Reynolds), were included in the observation. The scans of this flux calibrator, along with its known flux density model, were used to scale the flux density of the complex gain calibrator, which was thereafter transferred to the other sources. The setjy task was used to do this and the flux calibrator’s scale was set to 14.795 Jy at 1.4503 GHz.

2.6.7 Polarization Calibration

Once the preliminary calibration carried out, the next step is to perform the polarization calibration. The polarization calibration procedure on CASA was based on the Atacama Large Millimeter/submillimeter Array (ALMA) since it also has a linear feed. Due to ALMA having different sign conventions to KAT-7 and MeerKAT, the feed angles had to be rotated using a task called rotatefeed which takes the RECEPTOR ANGLE parameter from the FEED table in the MS and changes it from what it is originally, usually [0,0], to [\pi/2, \pi/2].

KAT-7 and MeerKAT have a linear feed basis, which makes the calibration of the instrumental polarization more complex because the polarization effects are present at first or zeroth order for all four correlations when dealing with linear feeds. For the circular feed basis, the effects only appear in the parallel hand correlations at second order. Therefore, some iteration is required to isolate the gain calibration if the polarization of the source is not previously known.

The polarization properties of the polarization calibrator (3C286) were estimated using the qufromgain task (with paoffset=90.0) on the gain table that was derived from the previous calibration steps. The initial gain table values were derived with the assumption that all sources are unpolarized, which is not true because there will be at least some instrumental polarization and the polarization calibrator will have an intrinsic polarization. qufromgain prints the fractional Stokes model and polarization position angle (\chi) for each source present in the gain table, which, for the polarization calibrator, was \( q = -0.0267459446398, \ u = 0.0226907881362, \ p = 0.0350744553902 \) and \( \Psi = 69.8446567887 \).

The next step was to derive the cross-hand delay terms in the standard manner. A polarization model for the source had been produced but at this stage, it’s better to use a non-zero Stokes model, i.e. smodel=[1.0, 0.0, 1.0, 0.0].

Next, the cross-hand phase and source polarization were derived, again using a non-zero Stokes model. This step gave the following output:

Spw = 0 (ich=300/601):

X-Y phase = -54.2536 deg.
Fractional Poln: \( Q = 0.0615013, \ U = 0.0686826; \ P = 0.0921938, \ X = 24.0787\)deg.
Net (over baselines) instrumental polarization: -0.000493329

The cross-hand phase solutions were then plotted, as shown in Figure 2.11. There were some

\footnote{Setting paoffset to 90$^\circ$ is essential to ensure the RECEPTOR ANGLE change is registered by qufromgain.}
channels where the phase jumped significantly and these were removed so that the XY phase solutions were consistent across all frequency channels, as seen in Figure 2.12.
Figure 2.3: Flow chart showing the steps of the procedure used to process the data
Figure 2.4: The header section of the information about KAT\_DAT1 given by the listobs task, showing the observation date, time and duration. Note that this is an 11 hour observation, with good parallactic coverage.

![MeasurementSet](image)

Figure 2.5: A section of the KAT\_DAT1 observing schedule, given by the listobs task. The observation consists mostly of a number of similar scheduling blocks. Note that PKS B1934-638, which is the flux, delay and bandpass calibrator, is only observed once in the block, while the gain and polarization calibrators, PKS 1313-333 and 3C286 are observed between each scan of the target source M83.

Figure 2.6: Other useful information about KAT\_DAT1 given by the listobs task, including frequency information, list of fields and antenna locations used for this observing run.
Figure 2.7: Uncalibrated PKS B1934-638 data (XX and YY) from observation KAT_DAT1 as a function of channel number, before flagging.

Figure 2.8: Uncalibrated PKS B1934-638 data (XX and YY) from observation KAT_DAT1 as a function of channel number, after flagging.
Figure 2.9: The delay calibration for KAT_DAT1. The horizontal axis represents antenna index and the vertical axis is the time delay (in nano seconds) relative to the reference antenna, which was antenna 4 (index 3) in this observation. The blue and green points represent the X and Y antenna gains respectively.
Figure 2.10: The bandpass amplitude as a function of channel number for KAT_DAT1. Each plot corresponds to an antenna and the blue and green points represent the X and Y gains respectively. The gap at around channels 450 to 500 shows were data were removed from flagging.
Figure 2.11: The cross-hand phase solutions of 3C286 before flagging. Note the presence of outliers that need to be flagged.

Figure 2.12: The cross-hand phase solutions of 3C286 after flagging.
The ambiguity in phase sign of the polarization model was corrected using the `xyamb` task. The resulting full-Stokes model was compared with reference values. The model was reasonable but could be better. Therefore, for the remainder of the procedure, a manually-derived full-Stokes model was used, using the values from Perley and Butler [2013] in Table 2.3 for `polfrac` and `pangle`:

CASA <39>: q0=polfrac*cos(2.*pangle*(pi/180.))
CASA <40>: u0=polfrac*sin(2.*pangle*(pi/180.))
CASA <41>: smodel=[1.,q0,u0,0.]

The gain calibration was then revised using the full-Stokes model. The gain phase solutions are shown in Figure 2.13.

The instrumental leakage terms were then derived. The `refant` parameter was set to ‘’ here because defining a reference antenna would assume that it was ideal and did not experience any leakage. The derived leakages were then generalised to the cross-hands using the `Dgen` task. To automatically scale the Stokes Q, U and V flux densities of the polarization calibrator alongside the Stokes I flux density, the `fluxscale` task was used:

CASA <68>: myflux=fluxscale(vis=invis,caltable='G2',fluxtable='F2',reference=[fcal],transfer=[pcal,ical])

This gave fluxes for PKS 1313-333 and 3C286 of 1.0997±0.0111 Jy and 14.8352±0.0257 Jy, respectively.

### 2.6.8 Applying the Calibrations and Imaging

With the calibration solutions obtained, the calibrations were applied and the calibrator sources were imaged for verification. The imaging was done using the Hög bom CLEAN algorithm (Hogbom [1974]) with 2500 iterations and a gain threshold of 0.1 Jy. Figure 2.14 shows the image of 3C286 that was produced. An integrated flux of 14.468 Jy and a rms noise of 0.013 Jy were found. According to Perley and Butler [2013], the spectral flux density of 3C286, at a frequency \( \nu \) between 1 GHz and 50 GHz, is given by:

\[
\log(S_E) = 1.252 - 0.461 \log(\nu) - 0.172 \log(\nu)^2 + 0.034 \log(\nu)^3. \tag{2.2}
\]

Therefore, the expected flux of 3C286 (Figure 2.14), at a frequency of 1.333 GHz, should be around 15.55 Jy. Our resulting flux deviates from the theoretical flux by 6.9% which is within an acceptable range. The theoretical noise, given by:

\[
\sigma = \frac{\text{SEFD}}{\sqrt{N(N-1)\Delta t \Delta \nu}}, \tag{2.3}
\]

where SEFD is the system equivalent flux density (Jy), defined as the flux density of a radio source that doubles the system temperature (for KAT-7, SEFD \( \approx \) 1100Jy), \( N \) is the number of antennas used, \( \Delta t \) is the total on-source integration time in seconds and \( \Delta \nu \) is the channel width.
in Hz. Therefore, for this image, where 3C286 had a total integration time of 1870s and the channel width was 390.625 kHz, the theoretical noise is 0.00628. Using standard calibration and imaging procedures we do expect the rms noise to be on average about 5 times the theoretical noise, as seen in previous published papers from KAT-7 (Scaife [2015]).

Finally, a measurement set containing only the polarization calibrator data, was created using the split CASA task. The remainder of the procedure was performed on this measurement set. Because the polarization calibration was done using 3C286 as the calibrator source, the resulting polarization angles found for 3C286 throughout the remainder of the study, may be incorrect as one cannot calibrate the polarization of a source using the source itself. However, the change in polarization angle will be correct and therefore the rotation measures that were extracted are unaffected by this. For example, the polarization properties of 3C286 that were extracted from the above process were $p = 8.4 \pm 0.6\%$ and $\chi = 31.8 \pm 0.7^\circ$, which is in reasonable agreement with the values found by Perley and Butler [2013], as shown in Table 2.3.

### 2.7 Self Calibration

Self Calibration (Pearson and Readhead [1984]) is a method of calibration that involves using a model of the source as a calibrator. The source is usually imaged and this image is used as the initial model. The process is repeated, each time using the new image as the model, until the improvement is negligible.

Self calibration was attempted but did not improve the signal-to-noise ratio and was therefore found not to be useful for this study.

### 2.8 Extracting the Polarized Signal

A cube of channel images of 3C286 was created using the CASA task `clean` in `channel` mode. In order to determine how many channels to average over, Eqn. 2.3 was used. The required integration time or channel width depends on the desired noise level. The dynamic range of an image is given by

$$\text{DR} = \frac{\text{Source Flux}}{\sigma}. \quad (2.4)$$

For KAT_DAT2, where the source flux was about 14 Jy, to get a dynamic range of 10, we require a noise level of 1.4Jy. Equation 2.3 then says that the required channel width is only 7.86 Hz. The channel width of the data is 390.625 kHz. Therefore, even for a DR of 1000, which would require a channel width of 78.6 kHz, no channel averaging should be required. However, in practice, it was found that at least an averaging over 5 channels was required. With 601 channels in total originally, this would give a cube of 120 images.

The CASA task `imstat` was used in a loop to extract the Stokes parameters from each image, as well as the rms noise from a small region around the source position in the residual images, which was used as the errors for each channel measurement. This provides a range of $Q$, $U$ and $V$ values as a function of $\lambda^2$. 

43
Most astrophysical sources only show linear polarization, which means we expect the V values to be close to zero. However, a phase difference between the X and Y channels will lead to a rotation of U in to V. According to Pizzo [2011], this phase difference is calculated as follows:

\[ \delta_{X-Y} = \arctan \left( \frac{V}{U} \right) + n\pi, \] (2.5)

where \( n = 0, 1 \). There are two solutions to Equation 2.5, one for each value of \( n \). The incorrect solution will flip the sign of the RM of the source. Therefore, if the sign is known beforehand, it is easy to know which is the correct solution. During the polarization calibration procedure, the X-Y phase difference is minimized, bringing the V values down to close to zero. Figures 2.17 and 2.18 show the V values plotted against the U values of 3C286 in KAT_DAT1 after polarization calibration, where each of the data points is the average U and V value over 5 channels. In these figures, the X-Y phase difference was determined by fitting a straight line to the UV data and it can be seen that the phase difference in each scan has been brought down to a small angle and that the V values are close to zero. The average X-Y phase difference over all the scans is \( \delta_{X-Y} = 0.7^\circ \). Because the sources that are being dealt with are known to be linearly polarized (V=0), only the Q and U stokes values were used henceforth. In addition, the Q and U values were divided by the stokes I values to give the partial stokes parameters \( q \) and \( u \).

The above reduction methodologies were also carried out on KAT_DAT2, KAT_DAT3 and MKAT_DAT1, producing \( q(\lambda) \) and \( u(\lambda) \) data for each. Rotation measure models were then fitted to this data, as described in the following section.
Figure 2.13: The gain phase solutions as a function of time for KAT_DAT1. Each plot corresponds to an antenna and the blue and green points represent the X and Y gains respectively. These plots show that the phase gains in all the antennas are relatively constant over time, except for some jumps in antennas 2 and 7, which are due to phase wrap and not a problem. Therefore, antennas are stable in phase. Antenna 4 shows zero phase gains because it is the reference antenna.
Figure 2.14: Image of 3C286 produced after the calibration of KAT_DAT1. The integrated flux of the source is 14.468 Jy, the rms noise is 0.013 Jy and the synthesized beam has major and minor axes of 485.8 arcsec and 204.5 arcsec, respectively.
Figure 2.15: The Stokes Q and Stokes U data plotted for the first 6 scans of 3C286 in KAT_DAT1. Each point represents the average Q and U values for a bin of 5 channels, with Q on the x-axis and U on the y-axis.

Figure 2.16: The Stokes Q and Stokes U data plotted for the last 5 scans of 3C286 in KAT_DAT1. Each point represents the average Q and U values for a bin of 5 channels, with Q on the x-axis and U on the y-axis. The points were grouped into 5 frequency ranges which are represented by the different symbols.
Figure 2.17: The Stokes U and Stokes V data plotted for the first 6 scans of 3C286 in KAT_DAT1. Each point represents the average U and V values for a bin of 5 channels, with U on the x-axis and V on the y-axis. The points were grouped into 5 frequency ranges which are represented by the different symbols.

Figure 2.18: The Stokes U and Stokes V data plotted for the last 5 scans of 3C286 in KAT_DAT1. Each point represents the average U and V values for a bin of 5 channels, with U on the x-axis and V on the y-axis. The points were grouped into 5 frequency ranges which are represented by the different symbols.
2.9 Rotation Measure Modelling

Once we were convinced of our calibration and imaging, we looked at the RM modelling. The different models mentioned in the introduction on page 14 (Eqns 1.39 - 1.42) were fitted to both the $q(\lambda^2)$ and $u(\lambda^2)$ data simultaneously. Firstly, the simple single RM component model (Eqn 1.39) was fitted, which does not describe any change in the degree of polarization $p(\lambda^2)$. To account for changes in $p(\lambda^2)$, a single RM component plus a depolarization model were fitted. Multiple RM component models were then also tried. O'Sullivan et al. believe that the most physically reasonable models are either solely Faraday thin components (e.g. emission from the radio galaxy only) or one Faraday thick component (e.g galactic slab or mixed emitting and rotating region in the source) plus Faraday thin component(s). Therefore, only these combinations were considered. Differing spectral indices of individual components were not considered, and hence, multiple component models were constructed simply as $P = P_1 + P_2 + \ldots + P_N$.

2.10 Model-fit Evaluation

The method of maximum likelihood was used to fit the different models to the $q$ and $u$ data and find the best fit model parameters. The likelihood is the probability of obtaining the data, $d$, given a model of the source and some characterisation of the noise. In our case, the data are the $q(\lambda^2)$ and $u(\lambda^2)$ values. The prior likelihood of a specific RM value for an observation of a single channel $i$, assuming Gaussian noise, is

$$P_i(d_i|RM) = \frac{1}{\pi \sigma_q \sigma_u} \exp \left( -\frac{(q_i - q_{\text{model},i})^2}{2 \sigma_q^2} - \frac{(u_i - u_{\text{model},i})^2}{2 \sigma_u^2} \right)$$

(2.6)

where $\sigma_q, \sigma_u$ is the single channel rms. Then, for a total of $N$ channels, the prior likelihood is

$$P(d|RM) = \prod_{i=1}^{N} P_i(d_i|RM)$$

(2.7)

The optimize package was used in Python to maximize the above equation to find the maximum likelihood, $L \equiv \max(P(d|RM))$.

The Bayesian Information Criterion was then used to distinguish between the goodness-of-fit of models with different degrees of freedom (Schwarz [1978], Trotta [2008]):

$$\text{BIC} \equiv -2 \log P(d|k) \approx -2 \log L + k \log N$$

(2.8)

where $k$ is the number of free parameters in the model. $\text{BIC}_{\text{model}_1} - \text{BIC}_{\text{model}_2} > 100$ is required to significantly favour model 2. The reduced chi-square ($\chi^2_\nu$) goodness-of-fit values were also calculated to quantitatively evaluate the fit of individual models to the data:

$$\chi^2_\nu = \frac{1}{k} \left[ \left( \sum \frac{q_i - q_{\text{model},i}}{\sigma_q^2} \right) + \left( \sum \frac{u_i - u_{\text{model},i}}{\sigma_u^2} \right) \right]$$

(2.9)
2.11 Temporal Analysis

The *Pyrap* package was used in Python to extract the different correlation products from the measurement sets and calculate the Stokes parameters as a function of time. This could be done for any given channel. The Stokes parameters were then used to calculate the polarization properties \( p \) and \( \Psi \) (Eqns 1.30 and 1.31). The simple percentile method was used to remove outliers (see Section 1.4 on page 17). The measurement set consists of multiple scans of each of the sources and each scan spans several time stamps. The results were averaged over all the time stamps for each scan. Gaussian Process Regression, a supervised learning regression method, was then used to predict values for the times where there was no data. This data was then compared to the TEC data from TrigNet, and to the change in TEC data (\( \Delta \text{TEC} \)), where the \( i \)th point of \( \Delta \text{TEC} \) is calculated as 
\[
\Delta \text{TEC}[i] = \text{TEC}[i] - \text{TEC}[i - 1].
\]

**Conclusion:**

In this chapter, we described the datasets used in this study and the instruments used to retrieve the data. We also looked at the procedure used to process the data and the imaging of the calibrator sources. We found that the calibration results were acceptable and as expected. The imaged calibrator showed the expected flux and noise levels. We then applied model fitting to the polarization data to extract various parameters for further analysis in Chapter 3.
Chapter 3

Results and Discussion

In this chapter, the results obtained from the procedure explained in Chapter 2 are fitted with the various rotation measure models described in Section 1.3.1, to extract the required parameters such as rotation measures, polarization angles and polarization fractions.

3.1 Rotation measure modelling

For all four observations analysed (KAT_DAT1, KAT_DAT2, KAT_DAT3 and MKAT_DAT1), the model fitting procedure showed that the single RM model, the single RM-component with a foreground screen model, and the two RM model all produced poor fits which shows that these simpler models were insufficient to account for the complex polarization structures of 3C138 and 3C286. Therefore, in all the datasets, the sources were found to be best described by a three RM-component model, either three Faraday-thin components or two Faraday-thin components with one Faraday-thick component.

The resulting models are plotted against the data in Figures 3.1 to 3.20. Each figure corresponds to the results of one model being fitted to a particular observation. The data and resulting model are plotted for fractional Stokes $q$ and $u$ as a function of frequency squared, polarization fraction ($p$) as a function of frequency squared, polarization angle ($\Psi$) as a function of frequency squared, and fractional Stokes $u$ vs $q$.

- In KAT_DAT1, both of the three RM-component models had nearly an equally good fit, with the three Faraday-thin component model being slightly better. This best-fit model is shown in Figure 3.15. The RMs of each component were found to be 5.422 rad/m$^2$, 2.208 rad/m$^2$ and 9.042 rad/m$^2$, in order of highest polarization percentage to lowest.
Figure 3.1: Polarization data for 3C286 from KAT_DAT1. The solid red lines show the best-fit single RM model (Eqn. 1.39). Top: Fractional Stokes $q$ (blue) and $u$ (green) data as a function of frequency squared ($\lambda^2$) fitted with the corresponding model. Middle: Polarization fraction ($p$) data as a function of frequency squared ($\lambda^2$) plotted with the model. Bottom: Polarization angle ($\Psi$) data as a function of frequency squared ($\lambda^2$) plotted with the model. The Bayesian Information Criterion for this fit is 3.28.

Figure 3.2: Polarization data for 3C286 from KAT_DAT1 as plotted in Figure 3.1, but now modelled (solid red line) by a single RM model with depolarization from external Faraday dispersion (Eqn. 1.42). The Bayesian Information Criterion for this fit is 7.78.
Figure 3.3: Polarization data for 3C286 from KAT_DAT1 as plotted in Figure 3.1, but now modelled (solid red line) by a two RM-component model. The Bayesian Information Criterion for this fit is 16.8.

Figure 3.4: Polarization data for 3C286 from KAT_DAT1 as plotted in Figure 3.1, but now modelled (solid red line) by a three RM-component model (two Faraday thin and one Faraday thick (Eqn. 1.40)). The Bayesian Information Criterion for this fit is 30.28.
Figure 3.5: Polarization data for 3C286 from KAT_DAT1 as plotted in Figure 3.1, but now modelled (solid red line) by a three RM-component model. The Bayesian Information Criterion for this fit is 30.28.
In KAT_DAT2 and KAT_DAT3, the procedure shows that 3C138 is also best described by three RM components with no foreground screen. However, the RMs found differ quite drastically between the two datasets. KAT_DAT2 produced RMs of 7.11 rad/m$^2$, -0.656 rad/m$^2$ and 10.6 rad/m$^2$, in order of highest polarization percentage to lowest. KAT_DAT3, on the other hand, produced RMs of 7.33 rad/m$^2$, 13.2 rad/m$^2$ and -2.92 rad/m$^2$, again in order of highest polarization percentage to lowest. These models are shown in Figures 3.10 and 3.15 respectively.

Figure 3.6: Polarization data for 3C138 from KAT_DAT2. The solid red lines show the best-fit single RM model (Eqn. 1.39). Top: Fractional Stokes $q$ (blue) and $u$ (green) data as a function of frequency squared ($\lambda^2$) fitted with the corresponding model. Middle: Polarization fraction ($p$) data as a function of frequency squared ($\lambda^2$) plotted with the model. Bottom: Polarization angle ($\Psi$) data as a function of frequency squared ($\lambda^2$) plotted with the model. The Bayesian Information Criterion for this fit is 3.26.
Figure 3.7: Polarization data for 3C138 from KAT_DAT2 as plotted in Figure 3.6, but now modelled (solid red line) by a single RM model with depolarization from external Faraday dispersion (Eqn. 1.42). The Bayesian Information Criterion for this fit is 7.74.

Figure 3.8: Polarization data for 3C138 from KAT_DAT2 as plotted in Figure 3.6, but now modelled (solid red line) by a two RM-component model. The Bayesian Information Criterion for this fit is 16.7.
Figure 3.9: Polarization data for 3C138 from KAT_DAT2 as plotted in Figure 3.6, but now modelled (solid red line) by a three RM-component model (two Faraday thin and one Faraday thick (Eqn. 1.40)). The Bayesian Information Criterion for this fit is 30.1.

Figure 3.10: Polarization data for 3C138 from KAT_DAT2 as plotted in Figure 3.6, but now modelled (solid red line) by a three RM-component model. The Bayesian Information Criterion for this fit is 30.1.
Figure 3.11: Polarization data for 3C138 from KAT_DAT3. The solid red lines show the best-fit single RM model (Eqn. 1.39). Top: Fractional Stokes $q$ (blue) and $u$ (green) data as a function of frequency squared ($\lambda^2$) fitted with the corresponding model. Middle: Polarization fraction ($p$) data as a function of frequency squared ($\lambda^2$) plotted with the model. Bottom: Polarization angle ($\Psi$) data as a function of frequency squared ($\lambda^2$) plotted with the model. The Bayesian Information Criterion for this fit is 3.33.

Figure 3.12: Polarization data for 3C138 from KAT_DAT3 as plotted in Figure 3.11, but now modelled (solid red line) by a single RM model with depolarization from external Faraday dispersion (Eqn. 1.42). The Bayesian Information Criterion for this fit is 7.87.
Figure 3.13: Polarization data for 3C138 from KAT_DAT3 as plotted in Figure 3.11, but now modelled (solid red line) by a two RM-component model. The Bayesian Information Criterion for this fit is 16.96.

Figure 3.14: Polarization data for 3C138 from KAT_DAT3 as plotted in Figure 3.11, but now modelled (solid red line) by a three RM-component model (two Faraday thin and one Faraday thick (Eqn. 1.40)). The Bayesian Information Criterion for this fit is 30.59.
Figure 3.15: Polarization data for 3C138 from KAT_DAT3 as plotted in Figure 3.11, but now modelled (solid red line) by a three RM-component model. The Bayesian Information Criterion for this fit is 30.59.
Finally, as mentioned before, because MKAT_DAT1 comes from the commissioning phase of the MeerKAT telescope, there are still many parameters that have not yet been characterised for the telescope. Nevertheless, we tried preliminary analysis on the data and extracted the polarization parameters of the known calibrator, 3C138. However, with a wider bandwidth and a much lower frequency range as compared to KAT-7, the RM modelling was more tedious.

Additionally, in this particular observation, even after outliers were removed, the $q$ and $u$ values show a much more variable and complicated behaviour over the wavelength range than in the other observations. However, MKAT_DAT1 also showed 3C138 to be best described by the same three RM-component model, with resulting RMs of $-0.577 \text{ rad/m}^2$, $8.89 \text{ rad/m}^2$ and $13.0 \text{ rad/m}^2$, in the same order. This model is shown in Figure 3.20. Literature values for multiple-RM-component models were not found for 3C138 and 3C286. However, Perley and Butler [2013] present a single-component RM value of $0 \text{ rad/m}^2$ for both 3C138 and 3C286.

Figure 3.16: Polarization data for 3C138 from MKAT_DAT1. The solid red lines show the best-fit single RM model (Eqn. 1.39). Top: Fractional Stokes $q$ (blue) and $u$ (green) data as a function of frequency squared ($\lambda^2$) fitted with the corresponding model. Middle: Polarization fraction ($p$) data as a function of frequency squared ($\lambda^2$) plotted with the model. Bottom: Polarization angle ($\Psi$) data as a function of frequency squared ($\lambda^2$) plotted with the model. The Bayesian Information Criterion for this fit is 5.06.
Figure 3.17: Polarization data for 3C138 from observation (4) as plotted in Figure 3.16, but now modelled (solid red line) by a single RM model with depolarization from external Faraday dispersion (Eqn. 1.42). The Bayesian Information Criterion for this fit is 11.34.

Figure 3.18: Polarization data for 3C138 from MKAT_DAT1 as plotted in Figure 3.16, but now modelled (solid red line) by a two RM-component model. The Bayesian Information Criterion for this fit is 23.89.
Figure 3.19: Polarization data for 3C138 from MKAT_DAT1 as plotted in Figure 3.16, but now modelled (solid red line) by a three RM-component model (two Faraday thin and one Faraday thick (Eqn. 1.40)). The Bayesian Information Criterion for this fit is 42.72.

Figure 3.20: Polarization data for 3C138 from MKAT_DAT1 as plotted in Figure 3.16, but now modelled (solid red line) by a three RM-component model. The Bayesian Information Criterion for this fit is 42.72.
Table 3.1 shows the best-fit parameters from the maximum likelihood modelling of the rotation measure components for each model, for each dataset. Table 3.1 also presents the $\chi^2_{\nu}$ and Bayesian information criterion values for each model-fit, to show which model best describes each dataset.
<table>
<thead>
<tr>
<th>Dataset</th>
<th>Source</th>
<th>Model</th>
<th>RM1</th>
<th>(p_0)</th>
<th>(\Psi_0)</th>
<th>RM2</th>
<th>(p_0)</th>
<th>(\Psi_0)</th>
<th>RM3</th>
<th>(p_0)</th>
<th>(\Psi_0)</th>
<th>(\sigma_{RM})</th>
<th>(\chi^2)</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3C286</td>
<td>Single RM, no screen</td>
<td>0.585</td>
<td>8.91</td>
<td>36.42</td>
<td>0.591</td>
<td>10.13</td>
<td>36.40</td>
<td>-0.426</td>
<td>8.33</td>
<td>28.68</td>
<td>2.208</td>
<td>12.18</td>
<td>32.13</td>
</tr>
<tr>
<td>2</td>
<td>3C138</td>
<td>Single RM, no screen</td>
<td>-7.26</td>
<td>-2.00</td>
<td>120.50</td>
<td>-7.26</td>
<td>-1.99</td>
<td>120.50</td>
<td>-6.99</td>
<td>59.25</td>
<td>72.11</td>
<td>7.36</td>
<td>-26.75</td>
<td>52.63</td>
</tr>
<tr>
<td>3</td>
<td>3C138</td>
<td>Single RM, no screen</td>
<td>1.06</td>
<td>7.83</td>
<td>-1.39</td>
<td>3.08</td>
<td>-9.43</td>
<td>74.59</td>
<td>1.59</td>
<td>-7.04</td>
<td>70.70</td>
<td>-2.92</td>
<td>-4.94</td>
<td>88.85</td>
</tr>
<tr>
<td>4</td>
<td>3C138</td>
<td>Single RM, no screen</td>
<td>0.151</td>
<td>-6.45</td>
<td>90.21</td>
<td>0.137</td>
<td>-7.51</td>
<td>90.26</td>
<td>4.88</td>
<td>-14.5</td>
<td>65.02</td>
<td>5.02</td>
<td>-13.8</td>
<td>99.71</td>
</tr>
</tbody>
</table>

Table 3.1: Rotation measure modelling results. Columns: (1) Dataset number referring to Table 2.2; (2) Name of the source; (3) Description of model fitted; (4) Rotation measure of first component; (5) Polarization fraction of first component; (6) Intrinsic polarization angle of first component; (7) Rotation measure of second component; (8) Polarization fraction of second component; (9) Intrinsic polarization angle of second component; (10) Rotation measure of third component; (11) Polarization fraction of third component; (12) Intrinsic polarization angle of third component; (13) Dispersion about mean RM; (14) Rms noise level in fractional polarization; (15) Reduced chi-square goodness-of-fit value; (16) Bayesian information criterion for model comparison.
3.2 Temporal Analysis

Since we have a data cube with the various polarization values, we extracted a time series for each observation. The polarization angle variation with time can be a proxy for the TEC variation. Figure 3.21 shows the polarization properties, \( q, u, p \) and \( \Psi \), of 3C286 as a function of time, from KAT_DAT1. It can be seen in this figure that the data points are separated into the time ranges of the different scans of the source. The average value for each scan is shown in Figure 3.22, where each error bar is given by the standard deviation of the data in that scan.

Figure 3.23 shows the prediction found by the Gaussian processes (GP) regression applied to the polarization angle data. GP was used as it is a non-parametric and therefore finds a distribution over all possible functions that fit the data, without previously specifying any parameters. It uses the prior distribution and updates as it is fed the data, and thus produces the posterior distribution over the functions.

Figure 3.24 shows the vTEC measurements from the Sutherland TrigNet station for the day of KAT_DAT1, as well as for the day before and the day after. Each colour represents the measurements from a particular GPS satellite.

During daytime, the radiation from the sun is obviously increased, causing more molecules in the ionosphere to be ionized and therefore more individual electrons are floating around. At night, the electrons then lose energy and bond onto molecules again. This explains the increase in TEC during the daytime, which is seen in Figure 3.24. The top panel in Figure 3.25 shows the same vTEC measurements but averaged over the different satellites for each time stamp. The difference between each adjacent vTEC measurement is shown in the middle panel of Figure 3.25, where some outliers were removed manually. The bottom panel of the figure is the same as Figure 3.23, put there to compare the change in polarization angle to the TEC and \( \Delta \text{TEC} \) data.

Figures 3.26 to 3.35 show the same plots for KAT_DAT2 and KAT_DAT3. These plots do not show any significant correlation between the change in polarization angle and the dTEC. It is difficult to make any other conclusions without taking the geomagnetic field into account, which is beyond the scope of this research work shown here.
Figure 3.21: Polarization properties over time for 3C286 in KAT_DAT1, showing every data point in each scan. Top: Fractional Stokes parameters $q$ (blue) and $u$ (green). Middle: Polarization fraction $p$. Bottom: Polarization angle $\Psi$.

Figure 3.22: Polarization properties over time for 3C286 in KAT_DAT1, showing the average over all the points in each scan. Error bars given by the standard deviation. Top: Fractional Stokes parameters $q$ (blue) and $u$ (green). Middle: Polarization fraction $p$. Bottom: Polarization angle $\Psi$. 

67
Figure 3.23: Gaussian Process Regression prediction and 95% confidence interval for change in polarization angle over time for 3C286 in KAT_DAT1

Figure 3.24: Vertical total electron content (vTEC) data calculated from GPS data from the Sutherland TrigNet Station over three days starting on the 28th of July, 2013. The different colours show the different GPS satellites used to obtain the data.
Figure 3.25: The same total electron content (TEC) data from Figure 3.24 and the results from the Gaussian Process Regression for change in polarization angle over time for 3C286 in KAT1_DAT1 as shown in Figure 3.23. Top: Vertical total electron content (vTEC) averaged over the different satellites for each timestamp. Middle: Change in vTEC (ΔTEC), calculated from the averaged data shown on top. Bottom: Gaussian Processes Regression results from Figure 3.23.

Figure 3.26: Polarization properties over time for 3C138 in KAT1_DAT2, showing every data point in each scan. Top: Fractional Stokes parameters q (blue) and u (green). Middle: Polarization fraction p. Bottom: Polarization angle $\Psi$. 
Figure 3.27: Polarization properties over time for 3C138 KAT_DAT2, showing the average over all the points in each scan. Error bars given by the standard deviation. Top: Fractional Stokes parameters $q$ (blue) and $u$ (green). Middle: Polarization fraction $p$. Bottom: Polarization angle $\Psi$.

Figure 3.28: Gaussian Process Regression prediction and 95% confidence interval for change in polarization angle over time for 3C138 in KAT_DAT2.
Figure 3.29: Vertical total electron content (vTEC) data calculated from GPS data from the Sutherland TrigNet Station over four days starting on the 15th of November, 2013. The different colours show the different GPS satellites used to obtain the data.

Figure 3.30: The same total electron content (TEC) data from Figure 3.29 and the results from the Gaussian Process Regression for change in polarization angle over time for 3C138 in KAT_DAT2 as shown in Figure 3.28. Top: Vertical total electron content (vTEC) averaged over the different satellites for each timestamp. Middle: Change in vTEC (ΔTEC), calculated from the averaged data shown on top. Bottom: Gaussian Processes Regression results from Figure 3.28.
Figure 3.31: Polarization properties over time for 3C138 in KAT-DAT3, showing every data point in each scan. Top: Fractional Stokes parameters $q$ (blue) and $u$ (green). Middle: Polarization fraction $p$. Bottom: Polarization angle $\Psi$.

Figure 3.32: Polarization properties over time for 3C138 in KAT-DAT3, showing the average over all the points in each scan. Error bars given by the standard deviation. Top: Fractional Stokes parameters $q$ (blue) and $u$ (green). Middle: Polarization fraction $p$. Bottom: Polarization angle $\Psi$. 
Figure 3.33: Gaussian Process Regression prediction and 95% confidence interval for change in polarization angle over time for 3C138 in KAT_DAT3

Figure 3.34: Vertical total electron content (vTEC) data calculated from GPS data from the Sutherland TrigNet Station over four days starting on the 17th of November, 2013. The different colours show the different GPS satellites used to obtain the data.
Figure 3.35: The same total electron content (TEC) data from Figure 3.34 and the results from the Gaussian Process Regression for change in polarization angle over time for 3C138 in KAT_DAT3 as shown in Figure 3.33. Top: Vertical total electron content (vTEC) averaged over the different satellites for each timestamp. Middle: Change in vTEC (ΔTEC), calculated from the averaged data shown on top. Bottom: Gaussian Processes Regression results from Figure 3.33.
Chapter 4

Summary, Conclusion and Future Work

Main Objectives

The main objectives of this research were to investigate the CASA calibration procedure used for KAT-7 and MeerKAT, including polarization calibration, and to extend the methods used by O’Sullivan et al. (2012), and potentially other methods, to extract Faraday rotation parameters from existing KAT-7 data and to make progress towards linking these parameters to the change in TEC of the ionosphere over the SKA site in the Karoo.

Three KAT-7 observations and one MeerKAT commissioning observation were flagged and calibrated, during which the calibration procedures and results were studied in detail, including polarization calibration. Therefore, the first objective was reached.

The Stokes Q and U parameters, which describe the polarization properties, were extracted. Three different outlier detection methods were compared and used to remove the outliers in the Q and U data. Different polarization models, as discussed on page 14 (Eqns 1.39 - 1.42), were then fitted to the Q and U data, to extract the rotation measure (RM) properties of the sources. The data and fitted models are plotted in Figures 3.1 to 3.15. The resulting best fit parameters from this process are summarised in Table 3.1. The first KAT-7 observation showed that 3C286 was best described by a three RM-component model. The other two KAT-7 observations and the MeerKAT observation all showed that 3C138 was also best described by a three RM-component model.

The time-variabilities of the polarization properties of these sources were then plotted and compared to total electron content (TEC) data from a nearby TrigNet station, as well as the change in TEC (dTEC). These can be seen in Figures 3.21 to 3.35. We could not come to an exact conclusion about the relationship between the ionosphere properties and the rotation measure since these observations were not carried out within the same time window or the data were missing. Therefore, this objective was not quite reached. We have showed that there is scope for such a multi-instrument analysis and this can be coordinated and carried out in the future.
Future Work

The different RM components extracted by the modelling procedure could be investigated further, to see if one of the components is much more time-variable than the others. This component would correspond to the ionosphere, as the ionosphere’s properties vary on a time scale of minutes to hours, whereas the properties of the media which are responsible for the other RM components, would vary on a much, much longer time scale and could be considered to be constant throughout the observations.

A tool that would be worth while exploring further is RM synthesis (Brentjens and de Bruyn [2005]). As explained previously, RM synthesis is a method of extracting rotation measures from radio data. Sotomayor-Beltran [2013] use RM synthesis to compare RM’s extracted from LOFAR data to the RM’s modelled by their ionFR program. In addition, Sotomayor-Beltran [2013] make the comparison without performing any polarization calibration on the radio data. It would therefore be potentially useful to perform a similar comparison using KAT-7 or MeerKAT data, which have bigger bandwidths than LOFAR, and to first perform polarization calibration as this may affect the results. We used ionFR to model the ionospheric Faraday depths for the line of sight of KAT-7 observing 3C286 on the date of KAT_DAT1, and 3C138 on the dates of KAT_DAT2 and KAT_DAT3. These can be seen in Figures 4.1 to 4.3. The gaps are due to either missing TEC data or missing geomagnetic field data. Unfortunately, data was not available to do so for the MKAT_DAT1 observation. We plan to apply the RM synthesis technique to extract the RM’s from the radio data, as a function of time, and compare them to these models.

Figure 4.1: Ionospheric Faraday depths for the line of sight of KAT-7 observing 3C286 on the 29th of July 2013, modelled by the ionFR program (Sotomayor-Beltran [2013]). This covers the KAT_DAT1 time range.
Figure 4.2: Ionospheric Faraday depths for the line of sight of KAT-7 observing 3C138 on the 16th and 17th of November, 2013, modelled by the ionFR program (Sotomayor-Beltran [2013]). This covers the KAT_DAT2 time range.

Figure 4.3: Ionospheric Faraday depths for the line of sight of KAT-7 observing 3C138 on the 18th and 19th of November, 2013, modelled by the ionFR program (Sotomayor-Beltran [2013]). This covers the KAT_DAT3 time range.
Another fact to take note of for future work, is that a TrigNet station is becoming online at the SKA core site, which enables TEC measurements to be made for lines of sight using the MeerKAT or SKA telescopes.

The work presented here can be extrapolated to all polarization observations and to a statistical analysis carried out for better characterising of the measurement of the TEC and its potential impact on the RM of polarized sources.
Bibliography


