Understanding factors that may contribute to changes in mathematics teachers' beliefs about mathematics teaching-learning.

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ABSTRACT

Considering the results of the Third International Mathematics and Science Study of 2011, South African learners have performed poorly in mathematics. From these results we realise that South African learners have a very limited understanding of mathematics. This limited understanding can be accredited to the mathematics teaching-learning that takes place in South African classrooms. South African teachers are the key to success in implementing a curriculum and deciding how mathematics should be taught and learned in the classroom.

There seems to be a widening gap between levels of ability in mathematics; it seems that learners are becoming less capable of solving problems in real-life context. In the last decade there has been a shift in the focus of the South African curriculum, starting with a teacher-centred approach, where the teacher was responsible for transferring knowledge to learners; to a learner-centred approach, where the teacher is the facilitator and learners have to take responsibility for their own learning. These shifts in teaching-learning approaches require mathematics teachers to make major changes in their own beliefs about mathematics teaching-learning. Beliefs are difficult to change, because it requires a teacher to move from a familiar way of acting and thinking, to a new, unknown way of acting and thinking.

The factors that could lead to changes in mathematics teachers' beliefs about mathematics teaching-learning were investigated in this study. A qualitative phenomenological approach was used to reach the aim of the study. The paradigm used was a interpretivist approach. Purposeful sampling was used to conduct semi-structured interviews that were tape-recorded and then analysed, using content analysis in order to better understand and describe changes that may have occurred in mathematics teachers' beliefs about mathematics teaching-learning.

The study engaged four mathematics teachers (n=4) with different years of teaching experience. The results describe the different teachers' experiences and factors that have an influence on their mathematical beliefs about mathematics teaching-learning.

The significance of the study lies in the fact that this study makes a contribution to South African and international literature on understanding teachers' mathematical beliefs and

the impact that a mathematics teacher education programme had on teachers' mathematical beliefs.

Keywords for indexing:

"mathematics teachers' beliefs", "mathematics teaching-learning", "mathematics beliefs", "changing beliefs about mathematics", "mathematics teacher education".

OPSOMMING

Die resultate van die TIMSS (Third International Mathematics and Science Study) 2011, dui daarop dat Suid-Afrikaanse leerders swak presteer in wiskunde. Vanuit die resultate, kom ons tot die besef dat Suid-Afrikaanse leerders 'n beperkte verstaan van wiskunde het. Hierdie swak begrip van wiskunde kan toegeskryf word aan die wiskundeonderrig-leer wat in Suid-Afrikaanse wiskundeklaskamers plaasvind. Suid-Afrikaanse wiskundeonderwysers bepaal hoe suksesvol die kurrikulum geïmplementeer word en besluit ook hoe wiskunde onderrig en geleer word in die klaskamer.

Dit kom voor asof die gaping tussen die vermoë en onvermoë van leerders om lewenswerklike wiskundeprobleme op te los, groter en groter word. In die laaste dekade was daar 'n verandering in fokus in die Suid-Afrikaanse kurrikulum. Aanvanklik was die fokus op 'n onderwysergesentreerde benadering, waar die onderwyser verantwoordelik was om kennis oor te dra aan die leerders. Die fokus het toe verander na 'n leerdergesentreerde benadering, waar die onderwyser as fasiliteerder optree en leerders verantwoordelikheid moet aanvaar vir hulle eie leer. Hierdie verandering in onderrig-leer benadering verwag van die wiskundeonderwyser om groot veranderings te maak in hulle eie oortuiging oor die onderrig-leer van wiskunde. Oortuigings is baie moeilik om te verander, omdat daar verwag word dat 'n persoon van 'n bekende manier van dink en optree, moet verander na 'n onbekende manier van dink en optree.

Hierdie studie het die faktore wat kon lei tot veranderings in wiskundeonderwysers se oortuigings oor wiskunde onderrig-leer ondersoek. Ten einde die uitkomste van die studie te bereik, is 'n kwalitatiewe fenomenologiese benadering gebruik, wat gebruik gemaak het van 'n doelbewuste streekproef, ten einde semi-gestruktureerde onderhoude te voer wat met 'n bandopname ingesamel is en toe geanaliseer was deur gebruik te maak van inhouds-analise. Die einddoel was om veranderings wat in onderwysers se oortuigings plaasgevind het, beter te kan verstaan en beskryf.

Die deelnemers het bestaan uit vier wiskundeonderwysers (n=4) met verskillende jare van ervaring. Die resultate beskryf elkeen van die onderwysers se ervarings en faktore wat hulle oortuigings oor wiskundeonderrig-leer kon beïnvloed.

Die belangrikheid en betekenis van hierdie studie lê daarin dat dit 'n bydrae maak tot Suid-Afrikaanse en internasionale literatuur oor die verstaan van wiskunde-oortuigings

en die impak wat ‘n wiskundeprogram op wiskundeonderwysers se oortuigings oor wiskundeonderrig-leer gehad het.

Sleutelwoorde vir indeksering:

“wiskundeonderwysers se oortuigings”, “wiskundeonderrig-leer”, “oortuigings oor wiskunde”, “verandering van oortuigings van wiskunde”, “wiskundeonderwysersopleiding”.

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1.1 Introduction

Considering the results of the Third International Mathematics and Science Study of 2011, South African learners have performed poorly in mathematics (Spaull, 2013:4). From these results, there is a realisation that South African learners have a very limited understanding of mathematics. This limited understanding can be accredited to the mathematics teaching-learning that takes place in South African classrooms. South African teachers are the key to success in implementing a curriculum and deciding how mathematics should be taught and learned in the classroom, therefore they have the greatest impact on the learners' results.

Mathematics teachers' beliefs about mathematics shape the way in which mathematics teaching-learning takes place. This research is an attempt to look into possible factors that may influence mathematics teachers' beliefs about mathematics teaching-learning and how these factors can contribute to a better understanding of mathematics teaching-learning in South Africa.

1.2 Background and Overview of the Study

Teachers' beliefs about mathematics have an influence on the way they teach it (Beswick, 2012:127). Holm and Kajander (2012:7) concluded that how teachers choose to teach a mathematics class is influenced by a combination of their beliefs about mathematics and their capabilities. Teachers develop certain characteristic patterns in their teaching practices and these patterns may be manifestations of consciously held beliefs that teachers bring to their classroom (Harbin & Newton, 2013:539).

Dionne (1984:225) suggests that beliefs about mathematics can be distinguished as three different views, called the traditional perspective, the formalist perspective and the constructivist perspective. Similarly, Ernest (1991b:250) recognised three main types of beliefs about the nature of mathematics that are held by teachers, namely the platonist view, the instrumentalist view, and the problem-solving view. Liljedahl (2008:3) identifies the three views as, the toolbox aspect, the system aspect and the process aspect respectively. All these different notions of the authors correspond more or less with each other, meaning that they refer to the same concepts, using different terms. In this study, Ernest's (1991b:250) beliefs will be used, because Ernest is one of the first pioneers in the field of views of mathematics education. In the following paragraphs each of these views and how it influences the teaching of mathematics will be described briefly.

The Platonist sees mathematics as a collection of true facts and correct methods which should be given by an absolute authority (Ernest, 1991a:114). In this view, mathematics is seen as a static body of knowledge that is held together through logic and meaning (Shilling-Triana & Styliandes, 2012:393). The focus of this type of belief, is that knowledge must be transferred, with an emphasis on knowledge rather than the process of doing mathematics (Beswick, 2012:113).

The second view also views mathematics as a set of facts, rules and knowledge, but gives the user the right to choose which method they wish to use (Ernest, 1991a:115). The instrumentalist view sees mathematics reduced to facts, rules and skills that are adapted for use. The usefulness of mathematics is overemphasised and mathematics is reduced to a tool (Nieuwoudt, 1998:69). In this view mathematics is learned through repetition and application of formulas without any understanding (Molefe, 2006:22). This view can be compared to the use of a computer - one might know how to use a computer, but not how it works internally. The same way that the use of a computer is reduced to a tool, mathematics is reduced to a single tool, with emphasis on outcomes, rather than the procedure of learning.

The problem-solving view focuses on the process of doing mathematics, encouraging active learning, creating your own knowledge and proofing your knowledge on your own (Ernest, 1991c:294). The problem-solving view refers to mathematics as a dynamic and creative human activity (Beswick, 2009:154). The learning of mathematics is seen as a process rather than a product, with emphasis on mathematical reasoning, proving and constructing of own mathematical knowledge (Molefe, 2006:23). In the problem-solving view, the mathematics classroom is learner-centred with the teacher acting as a facilitator of learning (Shilling-Triana & Styliandes, 2012:393).

According to Hernandez-Martinez and Williams (2013:45), the number of students who see mathematics as a subject learned by rote-learning, a subject that is in isolation of other subjects and have a disaffection for mathematics, is increasing. Webb and Webb (2008:41) found that South African teachers exhibit different levels of mathematical knowledge and pedagogical knowledge, but all seem to have difficulty in changing practices into more learner-centred approaches, because they hold on to a traditional way of teaching where knowledge is transmitted from teacher to learner (Plotz, 2007:2). Furthermore, Balfour (2014:18) comments that the South African government seems to continue to lower the pass mark bar in order to make results look better, but teachers have not transformed. This means that teachers keep teaching in the way they have been teaching and are not willing to change, which is unfortunately, the opposite of what research constitutes as good teaching (Liljedahl *et al.*, 2007:278). One could therefore question the necessity of understanding how to change the way in which teachers teach.

Most researchers come to the conclusion that beliefs about the nature of mathematics influence the way in which teachers teach mathematics (Beswick, 2009:98; Beswick, 2012:128; Cooney *et al.*, 1988:255; Ernest, 1991b:249; Nieuwoudt, 1998:68; Schoenfeld, 1992b:337; Thompson, 1992:108). A study done by Gazit and Patkin (2012:175) concluded that teachers and students are not able to connect mathematics to the real world, and even simple problems are solved incorrectly because the real-world context of the problem is not taken into consideration. A study done in South Africa by Pather (2012:254) found that mathematics education students had very negative perceptions of mathematics during their schooling. A quote by Schoenfeld (1992b:337) will be used to summarise the situation that is still a reality in South African mathematics classrooms today:

> "…the school mathematics experience has been uninspiring at best, and mentally and emotionally crushing at worst…Ironically, the most logical of the human disciplines of knowledge is transformed through a misrepresentative pedagogy into a body of precepts and facts to be remembered 'because 'the teacher said so'. Despite its power, rich traditions and beauty, mathematics is too often unknown, misunderstood and rendered inaccessible. The consequences of traditional

> mathematics teaching have been documented; they include lack of meaningful understanding, susceptibility to 'mathematical puffery and nonsense', low and uneven participation and personal dread..."

From the preceding arguments, it becomes more clear that beliefs have a direct impact on the practices of mathematics teachers (Beswick, 2012:128). With the above in mind, a rationale will be offered for the study that I wish to undertake.

1.2.1 Rationale

There seems to be a widening gap between levels of ability in mathematics. Learners seem to be getting less capable of solving problems in real-life context (Benadé, 2013:2). In the last decade there has been a shift in the focus of the South African curriculum, starting with a teacher-centred approach, where teacher was responsible for transferring knowledge to learners; to a learner-centred approach, where the teacher is the facilitator and learners have to take responsibility for their own learning (DoE, 2003:2). These shifts in teaching-learning approaches require the mathematics teacher to make major changes in their own beliefs about mathematics teaching-learning.

The problem we face in South Africa is not only restricted to our country, but can be found worldwide, including countries such as the United States (Liljedahl *et al.*, 2007), Australia (Beswick, 2012), Canada (Zazkis & Zazkis, 2011), Latvia (Sapkova, 2013), Spain (Gómez-Chacón *et al.*, 2014) and Turkey (Karatas, 2014) to name a few. Liljedahl *et al.* (2012:105) states that beliefs are difficult to change, because it requires a teacher to move from a familiar way of acting and thinking, to a new unknown way of acting and thinking.

Webb and Webb (2008:17) assert that South African teachers in the Eastern Cape expressed beliefs consistent with a problem-solving view, but that their mathematics teaching-learning practices do not correlate with their espoused beliefs. It is encouraging that teachers profess to hold a problem-solving view of mathematics teaching-learning, but we need to understand factors that can help support and promote permanent changes in mathematics teachers' beliefs about mathematics teaching-learning.

The rationale therefore is to contribute to the understanding of the beliefs of mathematics teachers in South Africa, and to make a contribution to national and international research.

1.3 Review of Literature

From existing literature, it appears that mathematics teachers' beliefs were explored in relation to practices, but not in relation to the understanding of how change in beliefs can occur. Beliefs form the basis of teachers' teaching, interpretations and perceptions in the mathematics classroom (Liljedahl *et al.*, 2007:278). Although some teachers have received the same type of training during their undergraduate study, their teaching of mathematics is very different (Sahin & Yilmaz, 2011:73). Prospective teachers do not come to teacher education as an empty slate. In contrast, long before they enrol in education courses, they have developed certain beliefs about mathematics teaching-learning (Ball, 1988:43). It is on these beliefs that mathematics teachers build the foundation from which they will eventually teach mathematics (Liljedahl *et al.*, 2007:278). These beliefs are in many cases, unfortunately, the opposite of what research constitutes as good teaching (Liljedahl *et al.*, 2007:278). Holm and Kajander (2012:7) found that teachers are likely to teach in the way that they were taught. Taking the above into consideration, we need to understand how different factors can contribute to changes in mathematics teachers' beliefs. Conner *et al.* (2011:484) assert that more research is needed on changing beliefs and assessing that change, because we do not know what the influence of the teaching experience will be on teachers' beliefs about mathematics teaching-learning and how teachers can differently be supported in teaching-learning mathematics.

Changing teachers' beliefs about mathematics teaching-learning is a challenging process, one which cannot be accomplished easily (Conner *et al.*, 2011:484). Changing teachers' beliefs about mathematics involves challenging beliefs, learning mathematics in a constructivist environment and involving teachers in mathematical discovery (Liljedahl *et al.*, 2007:280). There is an interdependency between teacher knowledge and beliefs (Holm & Kajander, 2012:9). Many times, beliefs impede any change in teaching practices (Roesken-Winter, 2013:160). In order for changes in beliefs to take place, teachers need to be in a teaching context that promotes mathematical discovery (Liljedahl *et al.*, 2007:280), where they can improve their knowledge of school mathematics as they work to improve and change their teaching of mathematics (Ma, 1999:147). This then needs to be combined with experiences that challenge and confront beliefs teachers have about the learning and teaching of mathematics, which cause cognitive conflict (Holm & Kajander, 2012:10).

The problem is that although research has shown interventions being very effective in producing changes in post-graduate teachers' beliefs about mathematics, it does not explain the factors behind this change (Liljedahl *et al.*, 2007). In this study I aim to understand what factors could contribute to changes in beliefs about mathematics and how knowledge of these factors could provide recommendations for mathematics teacher education. My research aims are stated in the section below.

1.4 Research Aims

The main aim of the study:
To understand factors that may contribute to changes in post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme**.**
Secondary aims for the study are:

i) To describe post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme.
ii) To identify factors that contribute to changes in post-graduate teacher's beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme.

In an attempt to achieve these aims, the following research questions were addressed:

The study was guided through the following primary research question:
How can we better understand possible factors that contribute to changes in post-graduate teachers' beliefs about mathematics teaching-learning in a in a BEd Hons (Mathematics) programme?
The following sub-questions were used to support the primary question:
i) What are post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme?
ii) Which factors may contribute to changes in post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme?

The findings were used to better understand factors associated with changes in teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme.

1.5 Literature Review

The literature study comprises of an in-depth study of the literature available on beliefs and views of mathematics, changes in beliefs in mathematics and mathematics teaching-learning. Literature was searched by means of EBSCOhost, SABINET and internet search engines.

The following keywords were used in searches: “beliefs”, “math* beliefs”, “beliefs in mathematics education”, “views of mathematics”, “the nature of mathematics”, “changes in math* beliefs”, “teaching and learning mathematics”, “teaching-learning mathematics”, “math* beliefs”.

1.6 Empirical Study

1.6.1 Research Design

The research design is a way of positioning the researcher in the world and showing how the research question and data will be connected (Punch, 2006:48). Nieuwenhuis (2007b:72) states that a research design is a plan or strategy which starts at the underlying philosophical assumptions and moves to the specifying of participants, tools and procedures for collecting data and how data will be analysed. The research design will depend on the researcher's assumptions, research experience and research practices (Nieuwenhuis, 2007b:72).

The research design can be described using Figure 1.1. This study used a qualitative phenomenological approach that made use of purposeful sampling in order to conduct semi-structured interviews that were tape-recorded and then analysed using content analysis in order to better understand and describe changes that may have occurred in mathematical beliefs.

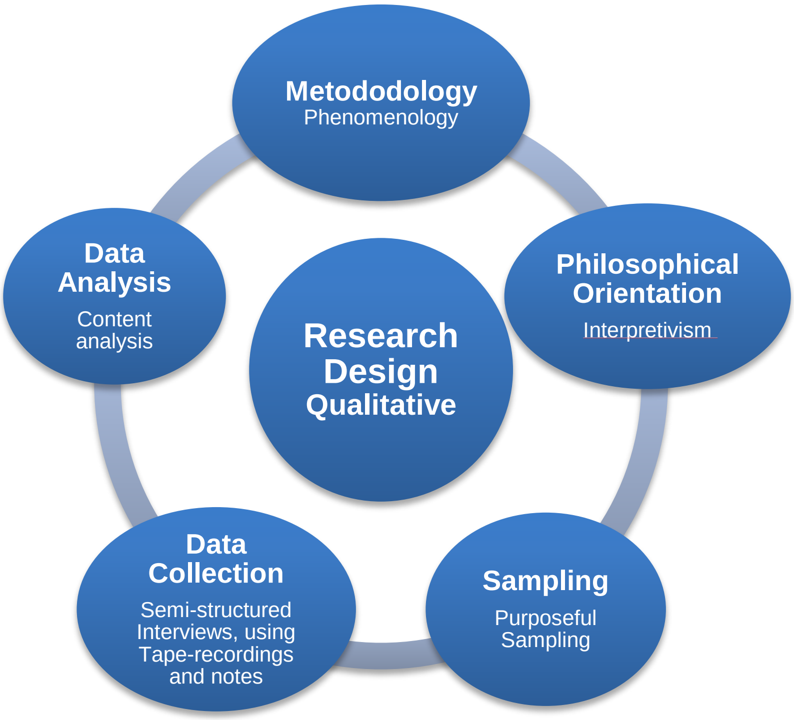

Figure 1.1 Research Design (Du Preez, 2013)

1.6.2 Research Procedures

The following research procedures were used:

An in-depth analysis of existing literature was done. Data were generated at two different stages in the research process. The first data generation took place at the start of the semester; the second data generation took place at the end of the semester. Questions were formed using data generated in the previous semi-structured interview which took place in the preceding interview. All data generation took place using individual semi-structured interviews.

The first interview was conducted to establish context of participants' experiences, specifically what beliefs they held about mathematics teaching-learning. The second interview allowed participants to reconstruct the details of their experiences of the BEd Hons (Mathematics) programme and encouraged participants to reflect on the meaning of their experiences.

1.6.3 Study Population and Sample

The study population of the research consisted of full-time and part-time students, enrolled for a BEd Hons in (Mathematics) Education at a South African University (N=8). The sample used for this study was four of the students enrolled for a BEd Hons in Mathematics Education who were willing to participate in the research.

The participants were selected because they were willing to take part in the study.

1.6.4 Researcher's Role

The researcher's role during the data collection was that of collector, analyser and the interpreter of the data. The researcher filled the role of interviewer and observer during data collection and also analysed and interpreted the data after collection.

1.6.5 Data Analysis

I chose content analysis because I had to analyse data from semi-structured interviews, which means I had to analyse transcriptions of data. This method allowed me to answer my research question in the best possible way, because I was able to critique and confirm existing theories on mathematics teachers' beliefs about mathematics teaching-learning. I also made use of inductive coding, because qualitative data analysis is a process of inductive reasoning, thinking and theorising (Schurink *et al.*, 2012:397).

1.6.6 Reliability and Validity

I made use of a variety of data sources, including interviews, journals, publications and books as well as websites (Nieuwenhuis, 2007b:113). I verified raw data by discussing gathered data with participants, to make sure that there was no misinterpretation from my part (Nieuwenhuis, 2007b:113). I sent a piece of raw data to an independent person, as well as the research aim and research question to see if her coding agreed with mine, to make sure that my coding is trustworthy.

I sent a copy of my findings, interpretations and conclusion to my supervisor as well as other people who are knowledgeable in the field of mathematics education to hear their comments and inputs on the research that was done. I controlled prejudice by getting input from my study leader and other knowledgeable people in the field of mathematics education (Nieuwenhuis, 2007b:114). I made sure not to generalise results, because I only worked with a small population group at a specific university. I could therefore not generalise about all mathematics teachers in South Africa and my conclusion focused on these specific teachers and factors that influenced them (Nieuwenhuis, 2007b:113).

1.6.7 Ethical Considerations

Voluntary participation means that participation should at all times be voluntary and that participants may not be forced to participate in the research (Strydom, 2012:116). Participants may at any time, have withdrawn themselves from the study without any consequences (Strydom, 2012:117).

Obtaining *informed consent* implies that all possible information on the goal of the investigation, the duration, the procedures and the potential advantages and disadvantages be discussed with participants before the research may take place (Strydom, 2012:117). I needed to obtain permission from participants before I could conduct my research.

Privacy and confidentiality should be maintained at every stage of the research process (Strydom, 2012:119). I did not make use of descriptors or names that could lead to the identification of participants. I made use of aliases to refer to participants.

Honesty must be a top priority in that no attempt is made to deceive or misinform participants (Strydom, 2012:123). I did not deceive participants in any way.

I also had to submit an application for ethical approval by the ethical board of the university, which was approved. I handed out consent forms to all participants to complete before any data was collected. I ensured the anonymity of the participants, by neither giving their names, nor describing them.

1.6.8 Measuring Instruments

A qualitative research methodology in the form of a phenomenological study was used in the research. The research instruments included semi-structured interviews with individual participants.

1.7 Chapter Outline

Chapter 1: Background and Overview of the Study

Chapter 2: Teachers' Beliefs about Teaching-Learning Mathematics

Chapter 3: Relationship between Beliefs about Mathematics Teaching-Learning and Practices.

Chapter 4: Research Design and Methodology

Chapter 5: Findings and Interpretation

Chapter 6: Conclusions and Recommendations

1.8 Value of the Research

This study made a contribution to South African and international literature on understanding teachers' mathematical beliefs and the impact that a mathematics teacher education programme had on teachers' mathematical beliefs. This study also contributed to the subject group Mathematics Education, by understanding the impact that a BEd Hons (Mathematics) programme had on participants' beliefs about mathematics teaching-learning.

CHAPTER 2: TEACHERS' BELIEFS ABOUT THE TEACHING-LEARNING OF MATHEMATICS

2.1 Introduction

Focusing on teachers' beliefs of mathematics education is a highly promising area of investigation (Beghetto & Baxter, 2012:2). Research has consistently demonstrated the role that a teacher's beliefs play in predicting learner achievement, often beyond prior knowledge and other related factors (Beghetto & Baxter, 2012:2). Studying the beliefs of teachers gives us an opportunity to shed light on how teachers may improve their teaching (Beghetto & Baxter, 2012:2).

Beswick (2012:127) states that one's conception of what mathematics is, affects how it should be presented. One can make the statement that teachers will present mathematics in a manner that will highlight what they believe is the most important. Conner *et al.* (2011:484) assert that more research is needed on changing beliefs and assessing that change, because teachers are still teaching in the way that they were taught, seeing mathematics as only facts and procedures which are fixed (Lofstrom, 2015:538). Teachers' beliefs about mathematics education are shaped by their experiences, the impact of continuous perceptions from the world around them, the dominant belief about mathematics as well as the beliefs of their own teachers (Lofstrom, 2015:538).

This chapter offers an overview of the nature of mathematics and where it started, and continues with an in-depth look at beliefs about mathematics teaching-learning.

2.2 Nature of Mathematics

Over the years many possible definitions have been given for what mathematics is. There has constantly been a debate about whether mathematics is abstract ideas or if it should only be used to solve real-world problems (Plotz, 2007:45). Mathematics can be viewed as a certain and static body of knowledge that is held together by logical and meaningful relationships (Nieuwoudt, 1998:69). Mathematics can also be viewed as a dynamic, creative, discovering human activity that is aimed at solving problems and is constantly being revised to find new patterns (Dossey, 1992:40). Taking the above into consideration, we need to unpack the nature of mathematics, because that determines how we define mathematics.

The nature of mathematics needs to be unpacked in order to determine how the teaching-learning of mathematics takes place (Plotz, 2007:43). The nature of mathematics can be taken as far back as four thousand years. The greatest contributors to this discussion were Plato and his student Aristotle (Dossey, 1992:40). Plato stated that mathematics already existed outside of the individual and it is the individual's responsibility to go and find it (Dossey, 1992:43). In contrast to Plato, Aristotle stated that mathematics has its origin inside the individual and is created when individuals interact with the world around them (Dossey, 1992:43).

From this early debate, many mathematics scholars have come up with their own understandings of the nature of mathematics. Ernest (1991b:250) described mathematics as a process of inquiry and coming to know. Schoenfeld (1992a:339) asserts that mathematics is an act of sense-making, which is constructed by interaction with others and the world. For Plotz (2007:43), mathematics is a human activity in which concepts are constructed, relationships are discovered, methods are invented and problems are solved that come from the real world. Beswick (2012:128) saw mathematics as something creative which uses strategies such as examples and counter-examples, patterns and justifications. The Curriculum Assessment Policy Statement (CAPS) defines mathematics as human activity that uses observation and investigation to discover relationships in a context (DBE, 2013).

Traditionally, mathematics has been viewed as certain knowledge, meaning that it exists out of logical structures that were seen as true and certain (Ernest, 1991a:4). Euclid created a logical structure nearly 2500 years ago, which was used as a paradigm for truth and certainty (Ernest, 1991a:4). According to Nieuwoudt (2002:2) after World War II, teaching-learning programmes were focused on the truth and certainty of mathematics, which is still the dominant view in some South African classrooms (Webb & Webb, 2008:13). The focus of mathematics was on rote memorisation, conformity and discipline (Benadé, 2013:12). Teaching-learning was seen as a linear process where mathematics was viewed as separate, unrelated pieces that should be memorised and passed from teacher to learners (Benadé, 2013:12).

Since the late 1980's there has been a change in the way mathematics is perceived (Plotz, 2007:43). There was a shift from knowing mathematics to doing mathematics in the last few years (Plotz, 2007:44). Benadé (2013:14) asserts that there is an increased awareness of and interest in teaching-learning mathematics in a meaningful way. Teachers have started to realise that there is more to teaching-learning mathematics than just transferring objective pieces of unrelated information (Benadé, 2013:14). This point is true for South African teachers, as found by Webb and Webb (2008:17), Benadé (2013), Plotz (2007), Stoker (2003) and Brodie (2001).

Ernest (1991a:114) distinguishes between two major views in mathematics, namely the multiplistic view and the relativistic view. In the multiplistic view, different methods and answers are seen as good and carry the same weight, and are a matter of personal choice, but a criteria for choosing a certain approach is missing (Ernest, 1991a:114). In the relativistic view of mathematics, multiple ways and answers are acknowledged, but the evaluation depends on the overall context (Ernest, 1991a:114).

Mathematics has to be an activity that makes sense to learners, if learners are to understand and use mathematics in a meaningful way. We cannot deny that mathematics has a logical structure of related ideas, relations and procedures, but the focus has shifted from knowing these ideas, to the process of building up these logical structures (Nieuwoudt, 1998:77; Plotz, 2007:43).

2.3 Teachers' Beliefs about Mathematics Teaching-Learning

Mathematics teaching-learning will refer to reported classroom actions and routines in a mathematics classroom. Teachers' beliefs about teaching-learning can determine how one approaches a problem, what techniques will be used or avoided, how long and hard one will work on it and what resources one will use (Roesken-Winter, 2013:159).

To understand a teacher's beliefs about mathematics teaching-learning, it is important to look at the influence of the paradigm that the teacher grew up in (Benadé, 2013:14). In the modern paradigm, mathematics is seen as information that needs to be transferred to learners by practice and repetition (Benadé, 2013:14). The modern paradigm is also noted by Ernest (1991a:114) as a multiplistic view of mathematics. As stated in the previous section, Benadé (2013:14) highlights that there has been a change in the way mathematics is viewed and emphasis is placed on the need for meaningful learning to take place and not just the transferring of content. In the current classroom, learners grow up in a post-modern paradigm, where the teacher is not expected to know everything and learners are seen as shareholders in the teaching-learning process (Benadé, 2013:12). This paradigm agrees with the relativistic view of Ernest (1991a:114), where teaching-learning is context dependent. Teachers who grew up in a post-modern paradigm are more willing and likely to change the way they teach, than teachers who grew up in a modern paradigm.

Holm and Kajander (2012:7) concluded that how teachers choose to teach mathematics is influenced by a combination of teachers' beliefs about mathematics and their capabilities as mathematics teachers. Teachers develop certain characteristic patterns in their teaching practices and these patterns may be manifestations of consciously held beliefs that teachers bring to their classroom (Harbin & Newton, 2013:539).

Dionne (1984:225) suggests that beliefs about mathematics can be distinguished as three different views, called the traditional perspective, the formalist perspective and the constructivist perspective. Similarly, Ernest (1991b:250) recognised three main types of beliefs about the nature of mathematics that are held by teachers, namely the platonist view, the instrumentalist view, and the problem-solving view. Liljedahl (2008:3) identifies the three views as, the toolbox aspect, the system aspect and the process aspect. All these different concepts of the different authors correspond with each other. All of them refer to the same ideas, just giving different names. In this study Ernest's (1991b:250) notions or views will be used, because he was one of the first researchers to do an in-depth study into mathematics teachers' beliefs about mathematics teaching-learning. These views are dynamic in nature and can change as individuals have new experiences that collide with their previously held views (Sahin & Yilmaz, 2011:74). In the following paragraphs, each of Ernest's (1991a:250) different views will be described.

2.4 Platonist View

The platonist sees mathematics as a collection of true facts and correct methods which should be given by an absolute authority (Ernest, 1991a:114). It found its origin in Plato's argument that mathematics already exists and needs to be discovered by man (Dossey, 1992:40). The platonist regards mathematics as a static body of knowledge that is held together through logic and meaning (Shilling-Triana & Styliandes, 2012:393).

The focus of this view is that knowledge must be transferred, with an emphasis on procedures rather than the process of doing mathematics (Beswick, 2012:113; Nieuwoudt, 1998:69). The teaching-learning of mathematics is seen as a predictable, passive exercise, where the teacher is the only one who has mathematical knowledge and the learners are empty vessels that have to be filled, topic by topic, by the teacher (Beswick, 2012:129). The different topics, of which mathematics is composed, are seen as unrelated pieces. Understanding is not necessary, only being able to do rote computations.

The teacher must explain everything to learners and learners should learn by listening and repetition (Nieuwoudt, 1998:357). The platonist view places emphasis on answers being correct, not on mathematical ideas, concepts or knowledge, and rewards the learning of rules, but not doing mathematics (Molefe, 2006:9). The teacher should see that learners master the curriculum by explaining, repeating and practicing rote computations (Benadé, 2013:12).

In the platonist classroom, the teacher is seen as the master that holds all the information and learners should receive this information only from the teacher (Benadé, 2013:15). Lessons start with a review of work covered during the previous day, homework is checked, then the teacher gives instructions on how learners should solve problems and learners are then given a large number of questions that require them to repeat this same procedure (Nieuwoudt, 2002:4).

A teacher with a platonist view of mathematics sees effective teaching as being able to competently explain, allocate tasks, monitor learners' work and give advice to learners (Benadé, 2013:15; Nieuwoudt, 1998:69). This view of mathematics is an example of the modernist paradigm, with a focus on the final product (correct answer), rather than the process of doing mathematics, where every teaching moment is predictable and planned for (Benadé, 2013:15; Beswick, 2012:129).

The platonist view believes that all mathematics can be broken into small pieces and transferred to the learner, who will assemble all these little pieces to create an understanding of the mathematics as a whole (Nieuwoudt, 2002:4). This type of teaching leads to learners developing unrelated pieces of mathematical knowledge and being unable to relate or understand what they are doing or learning (Nieuwoudt, 1998:164).

This manner of teaching can be related to a factory, where a “one-size-fits-all” approach is used; learners are seen as products that need to be shaped in a specific way, through a very specific process (Nieuwoudt, 2002:5). The platonist view makes the assumption that all learners are identical and will react in exactly the same way, but this deprives learners and teachers of their opportunity to realise their God-given potential and their own uniqueness (Nieuwoudt, 2002:5).
Beswick (2012:143) reported on the case of Jennifer, who viewed mathematics from a platonist view, emphasizing the usefulness of mathematics and teaching predominantly in the way that she was taught. Jennifer tried to change the way in which she taught, but failed to teach trigonometry in a meaningful way that would actively involve learners.

2.5 Instrumentalist View

The instrumentalist view sees mathematics reduced to facts, rules and skills that are adapted for use. Mathematics is viewed as a set of facts, rules and procedures, but gives the user the right to choose which method they wish to use (Ernest, 1991a:115).The usefulness of mathematics is overemphasised and mathematics is reduced to a tool (Nieuwoudt, 1998:69). Mathematics is learned through repetition and application of formula’s without any understanding (Molefe, 2006:22). This view can be compared to the use of a computer: one might know how to use a computer, but not how it works internally. Mathematics is reduced to a single tool in the same way that the use of a computer is reduced to a tool - with emphasis on outcomes, rather than the process of learning. Mathematics is used in an unreflective, pragmatic way (Ernest, 1991a:114).

The aim with this view, is that there is an end result to obtain and it doesn’t really matter how we obtain it, as long as we get the desired result (Benadé, 2013:17; Nieuwoudt, 1998:75). In this view the teacher has to demonstrate and explain formula’s in a way that learners understand it, and learners should be able to repeat exactly what the teacher demonstrated (Benadé, 2013:17; Beswick, 2012:130). In the instrumentalist classroom, learners are expected to know rules and find the correct answers, using the rule (Benadé, 2013:17; Webb & Webb, 2008:14).

The instrumentalist teacher focuses on increasing the number of fixed plans, which the learner can follow to get to the correct answer from start to finish (Benadé, 2013:28; Ernest, 1991b:250). The teacher has to constantly guide the learner with each new question and through each step.

Teaching according to an instrumentalist view, mathematics is reduced to a step-by-step guide with a lack of understanding. When learners only learn the rule, they might find it much easier than understanding where it came from (Benadé, 2013:28; Webb & Webb, 2008:14). The rewards for this type of approach is immediate, because the leaner is able to immediately solve similar questions, boosting their self-confidence (Benadé, 2013:29). The instrumentalist teacher saves a lot of time by not explaining where concepts come from and needs less understanding of the subject to teach difficult concepts, because they only teach learners steps to get to an answer, with little or no understanding (Benadé, 2013:29; Nieuwoudt, 1998:76).

Benadé (2013:30) argues that this type of view of teaching may prohibit learners from solving real-life problems or problems that are in different contexts (Nieuwoudt, 2002). We cannot remove the importance of procedures or steps in the process of doing mathematics, but it could lead to little or no understanding of mathematics and compartmentalising mathematics as unrelated boxes that should be kept separate.

The type of classroom is very similar to the Platonist classroom, the only difference being that the instrumentalist places much more emphasis on steps and very specific procedures.

The case of Kathy is an example of a instrumentalist view of mathematics, because she enjoys the step-by-step process of doing mathematics and the certainty of mathematics always being true and certain (Conner *et al.*, 2011:493).

2.6 Problem-Solving View

The problem-solving view focuses on the process of doing mathematics, encouraging active learning, creating your own knowledge and proving your knowledge on your own (Ernest, 1991c:294). The problem-solving view sees mathematics as a dynamic and creative human activity (Beswick, 2009:154). The learning of mathematics is seen as a process rather than a product, with emphasis on mathematical reasoning, proving and constructing own mathematical knowledge (Molefe, 2006:23). The mathematics classroom is learner-centred with the teacher acting as a facilitator of learning (Shilling-Triana & Styliandes, 2012:393). There is a constant process of discovery taking place in the mathematics classroom, where learners use their own knowledge to discover patterns and relationships, and create methods (Benadé, 2013:16).

The teacher has the role of facilitator and guides learners by posing questions and challenging learners to think and solve problems by using their own knowledge and information provided (Benadé, 2013:16). The focus of this view is on the process of doing mathematics, discovering relationships between different topics and being actively involved in creating meaning out of mathematics which is related to a real-world context.

The focus of the teacher from a problem-solving view of mathematics is on the utility and functionality of mathematics, learning mathematics so that it can be applied to real-life situations and problems (Benadé, 2013:12). There is an increased emphasis on the use of technology and making use of representations. This view of teaching is found in the post-modern paradigm, where the teacher doesn't admit to know everything, but makes learners part of the teaching-learning process (Benadé, 2013:12). The problem-solving view presents mathematics as a continuous process of exploration that is always open to revision (Handal, 2003:47).

The reflective process and exploratory learning is emphasised in a problem-solving view, with a focus on group learning, plenty of discussion and informal answers (Handal, 2003:47). A teacher with a problem-solving view of mathematics will structure their classroom in a way that promotes group work, discussion and interaction. The classroom will contain materials that can be manipulated by learners, because it promotes the understanding of mathematics teaching-learning. The teacher with a problem-solving view will make use of terminology and language that is on a level that the learners will be able to understand and enable them to share their ideas and thoughts (Nieuwoudt, 1998:107). A welcoming classroom that promotes investigation, creativity and taking mathematical risks is created by the teacher who has a problem-solving view of mathematics.

The case of David is an example of a teacher with a problem-solving view of mathematics; David was able to make connections between real life and mathematics (Conner *et al.*, 2011:492). He was able to connect his background knowledge, reflect on it and develop a new understanding.

The teacher with a problem-solving view of mathematics promotes the active involvement of learners, supports them in making discoveries and making knowledge their own (Benadé, 2013:16). Teaching mathematics from a problem-solving view not only leads to the fulfilment of learners' potential, but makes learners aware of social issues and a need for change (Ernest, 1991c:295).

2.7 Summary

In this chapter the nature of mathematics was discussed, and how it has changed over time to focussing on the process in recent years, rather than the product. Mathematics teaching-learning was then discussed and the different beliefs about mathematics were discussed. When looking at literature, it seems that mathematics teachers tend to hold different beliefs about mathematics teaching-learning and this has an influence on their classroom practices.

In the next chapter, a discussion will follow on the relationship between beliefs about mathematics teaching-learning and classroom practices, as well as how these beliefs about mathematics teaching-learning can possibly be changed.

CHAPTER 3: RELATIONSHIP BETWEEN BELIEFS ABOUT MATHEMATICS TEACHING-LEARNING AND PRACTICES

3.1 Introduction

Most researchers come to the conclusion that beliefs about the nature of mathematics influence the way in which teachers teach mathematics (Beswick, 2009:98; Beswick, 2012:128; Cooney *et al.*, 1988:255; Ernest, 1991b:249; Nieuwoudt, 1998:68; Schoenfeld, 1992b:337; Thompson, 1992:108). Webb and Webb (2008:41) found that South African teachers exhibit different levels of mathematical knowledge and tend to hold traditional beliefs about mathematics teaching-learning. South African teachers seem to have difficulty in changing practices into more learner-centred approaches, because they hold on to a traditional way of teaching while transmitting knowledge from teacher to learner (Plotz, 2007:2).

According to Hernandez-Martinez and Williams (2013:45), the number of students who see mathematics as a subject learned by rote-learning, a subject that is in isolation of other subjects and have a disaffection for mathematics, is increasing. Teachers keep teaching in the way they have been taught and are not willing to change, which is unfortunately the opposite of what research constitutes as good teaching (Liljedahl *et al.*, 2007:278). A study done by Gazit and Patkin (2012:175) concluded that teachers and learners are not able to connect mathematics to the real world, and even simple problems are solved incorrectly because the real-world context of the problem is not taken into consideration. A study done in South Africa by Pather (2012:254) revealed that learners had very negative perceptions of mathematics during their schooling. The following quote from Schoenfeld (1992b:337) summarises the situation that is still a reality in mathematics classrooms in South Africa today.

> …the school mathematics experience has been uninspiring at best and mentally and emotionally crushing at worst…Ironically, the most logical of the human disciplines of knowledge is transformed through a misrepresentative pedagogy into a body of precepts and facts to be remembered 'because the teacher said so.' Despite its power, rich traditions and beauty, mathematics is too often unknown, misunderstood, and rendered inaccessible. The consequences of traditional

> mathematics teaching have been documented; they include lack of meaningful understanding, susceptibility to 'mathematical puffery and nonsense', low and uneven participation and personal dread...

From the preceding arguments it becomes more clear that beliefs have a direct impact on the practices of mathematics teachers (Beswick, 2012:128). Liljedahl (2008:2) emphasises that mathematics teachers' teaching-learning practices are guided by what they believe to be true about mathematics. Balfour (2014:18) comments that the government in South Africa seems to continue to lower the pass mark bar in order to make results look better. These actions by the government lead to many teachers believing that their beliefs about mathematics teaching-learning is the most effective way to teach, because they are not forced to reflect on their own beliefs.

3.2 Relationship between Teachers' Beliefs about Mathematics and their Teaching-Learning of Mathematics

The relationship between teachers' beliefs about mathematics and their classroom practices has been investigated with varying results (Liljedahl *et al.*, 2012:1). In a review of literature, three different relationships between beliefs about mathematics teaching-learning practices have emerged, but most research concludes that beliefs about the nature of mathematics, influence the way in which teachers teach mathematics (Beswick, 2009:98; Beswick, 2012:128; Cooney *et al.*, 1988:255; Ernest, 1991b:249; Nieuwoudt, 1998:68; Schoenfeld, 1992b:337; Thompson, 1992:108).

Gujarati (2013:635) termed these relationships, the causal relationship, the dialectic relationship and the "sensible system" framework.

The causal relationship is one that maintains, that practice can always be shown to be in line with beliefs, regardless of contextual factors (Gujarati, 2013:635). This is supported by various studies, like the case of Jennifer (Beswick, 2012:143), who held a platonist view of mathematics, leading to teaching-learning practices being done in accordance of a platonist view of mathematics. In a study done by Oesterle and Liljedahl (2009:1256), the case of Harriet further supports the notion of a causal relationship. Harriet believes that mathematics is a creative subject, with a variety of methods to get to an answer as well as actively engaging students, corresponding with a problem-solving view of mathematics. Harriet's mathematics teaching-learning practices are consistent with her beliefs of mathematics teaching-learning (Oesterle & Liljedahl, 2009:1257).

The dialectic relationship is one of inconsistency between beliefs about mathematics teaching-learning and classroom practices (Gujarati, 2013:635). Harbin and Newton (2013:538) found inconsistencies between teachers' beliefs about mathematics and their actual teaching-learning practices by looking at a diverse group of elementary teachers. The inconsistency means that reported practices are not the same as teachers' expressed beliefs about mathematics teaching-learning. Beswick (2012:129) states that especially during the early stages of a teacher's career, beliefs about mathematics do not correspond to mathematics teaching-learning practices. Speer (2005:365) explains that these inconsistencies can be caused by a communication gap between the researcher and the teacher taking part in the research. She argues that participants might not be able to express their beliefs in a way that the researcher would have done, creating an inconsistency (Speer, 2005:365). Context also has a major impact on beliefs, a teacher may express certain beliefs about mathematics in a university classroom, but then enact completely opposing mathematics teaching-learning practices in front of a mathematics class (Beswick, 2012:130).

The "sensible system" framework sees teachers as complex, sensible people who have reasons for making certain decisions. Instead of viewing this as a dialectic relationship, the "sensible system" said that beliefs are structured in an orderly manner within the participant, in a way that makes sense to him/her, but does not necessarily make sense to the researcher. Gujarati (2013:636) makes the statement that a participant may try so hard to show the "right" belief of mathematics, that they eventually do not express their true own beliefs.

From the preceding literature, it becomes clearer that when making assumptions about a mathematics teacher's beliefs of mathematics teaching-learning, the researcher should verify results with the participant and make sure that the understanding of both the participant and researcher are the same. We see that many researchers that report on discrepancies between beliefs and teaching-learning practices is due to a lack of understanding between the researcher and the participant.

Next, a discussion on different types of classrooms, to better understand how beliefs about mathematic teaching-learning, relates to practices.

3.3 Traditional Classroom

A traditional classroom can be seen as a classroom where the teacher holds an instrumentalist view or a platonist view of mathematics (Nieuwoudt, 1998:90). In the traditional mathematics classroom, learners have to follow the example set by teachers. Welsch (1987:6) summarised what happens in a traditional mathematics classroom as starting with the marking of homework, then a brief explanation of new work and the rest of the period is given to repetition of what the teacher explained.

The teacher with an instrumentalist or platonist view on mathematics education focuses on getting the correct answer, the focus is therefore on getting results, not on the process of doing mathematics (Nieuwoudt, 1998:89). Nieuwoudt (1998:91) states that the role of the teacher in a traditional mathematics classroom is to structure mathematics, order steps, demonstrate how mathematics is done and assess how well learners can repeat what was demonstrated, while keeping learners quiet throughout the lesson.

In the traditional mathematics classroom the teacher is seen as the source of all knowledge and the teacher has to transfer this knowledge to the learner. If the learner is not able to receive this knowledge that the teacher transfers, the learner is labelled as unteachable (Benadé, 2013:12). Effective teaching in a traditional mathematics classroom is when a large amount of students is taught using the same strategy and they are all able to repeat exactly what the teacher demonstrated.

3.4 Problem-Solving Classroom

A teacher with a problem-solving view of mathematics has a learner-centred classroom, with all activities actively involving learners into constructing their own meaning from doing mathematics (Thompson, 1992:136). The teacher acts as facilitator and encourages learners to learn by posing interesting questions, creating challenging questions that challenge learners to think and arrive at their own conclusions (Benadé, 2013:16). The teacher takes into account that the learner is not just an empty vessel, but rather an individual with prior knowledge from previous experiences. The teacher with a problem-solving view of mathematics will model problem-solving, explore real-world mathematical contexts, value multiple solution strategies and give students time to create, discuss, hypothesise and investigate (Gujarati, 2013:634).

The teacher with a problem-solving view of mathematics will give learners enough time to discover, investigate and experience the why, when and how to use different methods for themselves (Nieuwoudt, 2002:17). The teacher will also encourage learners to reflect on their own understanding and methods in order to improve their ability to critically evaluate their own learning process. A discussion on why we would want to change mathematics teachers' beliefs about mathematics teaching-learning will follow.

3.5 Reasons for Changing Teachers' Beliefs about Mathematics Teaching-Learning

From the preceding arguments, we realise that beliefs about mathematics teaching-learning, have a direct impact on classroom practices. If we look at the quote from Schoenfeld (1992a:337), there is a problem with the way that mathematics has been and is being taught. The need for changes in the way mathematics is being taught, is found worldwide (Beswick, 2012; Carney, 2014; Gill, 2004; Harbin & Newton, 2013; Karatas, 2014; Liljedahl, 2010), including South Africa (Benadé, 2013; Nieuwoudt, 2002; Plotz, 2007; Webb & Webb, 2008). Lofstrom (2015:538) states that traditional beliefs about mathematics teaching-learning impede the understanding of mathematics, because there is a reliance on memorised formulae, performance-orientated outlook and a reliance on justification from authority. A traditional view of mathematics teaching-learning impedes teachers to give problems that they themselves struggle with, because of the belief that the teacher has to be authority and transmitter of knowledge (Lofstrom, 2015:538). Liljedahl *et al.* (2012:109) highlight the fact that participants in studies grew more conservative in their beliefs about mathematics teaching-learning, likely because of traditional workbooks and worksheets, so the problem is becoming bigger and bigger.

Teaching mathematics using traditional teaching methods leads to learners not being able to relate different mathematics topics to one another and unable to see the value of what they are doing and learning, causing even mathematics anxiety (Nieuwoudt, 2002:4). This type of teaching does not encourage meaningful learning beyond the lower levels of learning, which results in a lack of understanding (Nieuwoudt, 2002:6). Learners do not participate actively in the classroom, the teachers emphasise right and wrong answers and learners are restricted to passive receivers of knowledge (Handal, 2003:50). Traditional teaching cannot be a long-term way of teaching for South Africa or any other country (Nieuwoudt, 2002:6). A traditional view of teaching does not promote making sense of mathematics, but encourages rote learning (Schoenfeld, 2012:317).

Traditional beliefs about mathematics teaching-learning can also impede teachers from changing their mathematics teaching-learning approaches (Roesken-Winter, 2013:160). There has to be a realisation that the quality of an educational system cannot exceed the quality of its teachers, therefore to improve education, we need to empower teachers to teach mathematics in a more effective way (Sapkova, 2013:735). Improving the mathematical content knowledge for teaching (MCKfT) will lead to better teachers, because MCKfT involves taking complex subject matter and translating it into representations that learners can understand (Plotz, 2007:16).

Schoenfeld (2012:319) makes the statement that teachers should rethink the way that they teach mathematics, so that learners can make sense of mathematics. Teaching is effective when it enables the attainment of an intended outcome through learning (Nieuwoudt, 2002:11). The problem that education currently faces is that the focus is only on the outcomes and not on the development of processes that can help learners achieve the outcomes. Meaningful learning occurs when learners are able to make connections between prior knowledge and new knowledge (Benadé, 2013:34). Focusing only on outcomes makes learners believe that mathematics exists out of different topics that have nothing to do with each other and fails to equip learners with the ability to make sense of mathematics (Nieuwoudt, 2002:17).

Sapkova (2013:740) found that learners performed better when they were taught in a problem-solving manner. If a teacher does not get the opportunity to develop problem-solving mathematical beliefs about mathematics teaching-learning, it is likely they will fail when they start teaching in a problem-solving manner (Sahin & Yilmaz, 2011:74). It is critical that we understand which factors could lead to changes in teachers' beliefs about mathematics teaching-learning, because many of them hold misconceptions and negative attitudes toward a subject that they have to teach (Shilling-Triana & Styliandes, 2012:394).

3.6 Changing teachers' beliefs about mathematics teaching-learning

After a critical review, Liljedahl *et al.* (2012:105) make the statement that change in a teachers' beliefs about mathematics is possible, but very difficult. Different researchers (Conner *et al.*, 2011:484; Gill, 2004:164; Handal, 2003:53; Karatas, 2014:394) conclude that the difficulty in changing teachers' beliefs about mathematics teaching-learning, is because the teacher has to move from familiar ways of thinking and acting to new unexplored ways of thinking and acting (Liljedahl *et al.*, 2012:105).

Roesken-Winter (2013:160) found that even though teachers are willing to change their way of teaching mathematics, they might only add new experiences to their traditional style of teaching. An example of this, is the case of an experienced teacher, who participated in a problem-solving training course, but the teacher's beliefs about mathematics teaching-learning prevented him from successfully implementing new ideas (Roesken-Winter, 2013:160). We cannot assume that a single experience will drastically change teachers' learning-teaching practices (Liljedahl *et al.*, 2012:109). A study by Holm and Kajander (2012:10) found that a single experience of changing teachers' beliefs is unlikely to cause permanent changes in teachers' beliefs about mathematics teaching-learning.

Liljedahl *et al.* (2012:106) found that change can occur in mathematics teachers' beliefs about mathematics teaching-learning after intervention or exposure to new beliefs about teaching-learning mathematics, but once removed from this intervention or exposure, the participant will gradually return back to previous mathematics teaching-learning practices. An example is the case of Sally, who showed positive signs of changing her beliefs about mathematics to a problem-solving belief, but in the absence of assistance and professional learning, reverted back to positivist beliefs about mathematics teaching-learning (Beswick, 2012:145).

Teachers also tend to fall back on more traditional teaching, because they are not willing to do the additional preparation time that change requires or simply do not have the time (Benadé, 2013:20). Other obstacles that prevent changes in mathematics teaching-learning beliefs' of teachers, is a lack of cooperation among staff members, size of classrooms, availability of technology, resources, funding, non-supportive management and parents, length of class periods and prospects for growth (Handal, 2003:52; Sapkova, 2013:739).

Liljedahl *et al.* (2007:283) report that cognitive conflict is necessary for changes in mathematical beliefs about mathematics teaching-learning to occur. Cognitive conflict can be described as learning that creates a change in beliefs, because beliefs formed through new experiences are in conflict with beliefs formed through prior experiences (Shilling-Triana & Styliandes, 2012:395).

Liljedahl *et al.* (2007:283) suggest three methods of intervention to change beliefs, namely challenging existing beliefs, involving teachers in a problem-solving teaching environment and creating opportunities for mathematical discovery (Liljedahl *et al.*, 2006:280). Conner *et al.* (2011:502) found that challenging existing beliefs and also giving teachers time to reflect thoroughly on beliefs and experiences played a key part in changing teachers' beliefs about mathematics teaching-learning.

The problem then is that research has shown interventions being effective in producing changes in teachers' beliefs about mathematics, but does not explain the factors behind this change (Liljedahl *et al.*, 2007). A discussion on teacher knowledge will follow, because this has a big impact on teachers' beliefs about mathematics teaching-learning. If a teacher does not have adequate mathematical content knowledge, it is highly unlikely that a problem-solving approach to teaching mathematics will be used, because they do not have enough knowledge to guide learners.

3.7 Teacher Knowledge

In a longitudinal study by Blomeke *et al.* (2013:137), it was found that mathematical knowledge is related to teachers' later beliefs about teaching-learning mathematics and that beliefs do not necessarily determine mathematics teachers' later knowledge of mathematics. Teachers must know the subject they teach, because a teacher who does not know a subject very well is not likely to have the knowledge to help learners learn (Ball *et al.*, 2008:404). A certain level of MPCK is required before a teacher can value a problem-solving approach to teaching-learning of mathematics (Blomeke *et al.*, 2013:137).

In a study by Carney *et al.* (2014:25) there was a direct link between knowledge and beliefs. Participants in the study were part of a course that improved mathematical content knowledge and the results indicated that teachers, whose mathematical content knowledge increased, experienced a significant shift in beliefs about mathematics teaching-learning towards a problem-solving view.

From the preceding, there is a realisation that teacher knowledge plays a major role in mathematics teachers' beliefs of mathematics teaching-learning. Therefore a discussion on teacher knowledge will follow.

Research on teaching has led to a focus on the knowledge of teachers (Ben-Peretz, 2011:3; Plotz, 2007:7). The notion of teacher knowledge has been defined as a body of knowledge that enables teachers to teach subject matter using appropriate pedagogical strategies (Ben-Peretz, 2011:8). In order to understand teachers' knowledge, we need to break it down into its different components.

Teacher knowledge can be divided into different categories of teacher knowledge, namely content knowledge (CK), pedagogical content knowledge (PCK) and general pedagogical knowledge (GPK) (Blomeke *et al.*, 2013:131). Research has proven that it is a combination between CK and PCK that is related to learner achievement in mathematics (Baumert *et al.*, 2010:135; Carney *et al.*, 2014:10; Hill *et al.*, 2007:110), therefore the focus will be on PCK and CK.

Shulman (1986:9) defines PCK as subject-specific knowledge that is relevant for teachers, referring to ways of representing and formulating a subject that makes it understandable to others. It includes an understanding of what makes learning a specific topic, easy or difficult; the conceptions and the preconception that learners of different ages and backgrounds bring with them to the classroom (Blomeke *et al.*, 2013:131). PCK also includes curricular knowledge, which refers to the selection and arrangement of the material that is to be taught and learned (Shulman, 1987:5).

Ball and colleagues (2008:400) developed a framework for the specific knowledge needed to effectively teach mathematics (Ball *et al.*, 2008:400). The framework distinguishes between CK and PCK, within these two domains, additional types of knowledge necessary for effective mathematics teaching is distinguished (Carney *et al.*, 2014:9).

Chapman (2013:237), using the work of Ball, went and further analysed these two domains, and distinguished between six types of knowledge that have to be investigated when studying mathematics teachers, and knowledge for teaching mathematics. The CK would include: common mathematics content knowledge and specialised mathematical content knowledge, while the PCK would include: knowledge at the mathematical horizon, knowledge of content and students, knowledge of content and teaching and knowledge of the curriculum. Figure 1 gives a summary of the six types of knowledge using the work of Ball *et al.* (2008), Chapman (2013) and Plotz (2007).

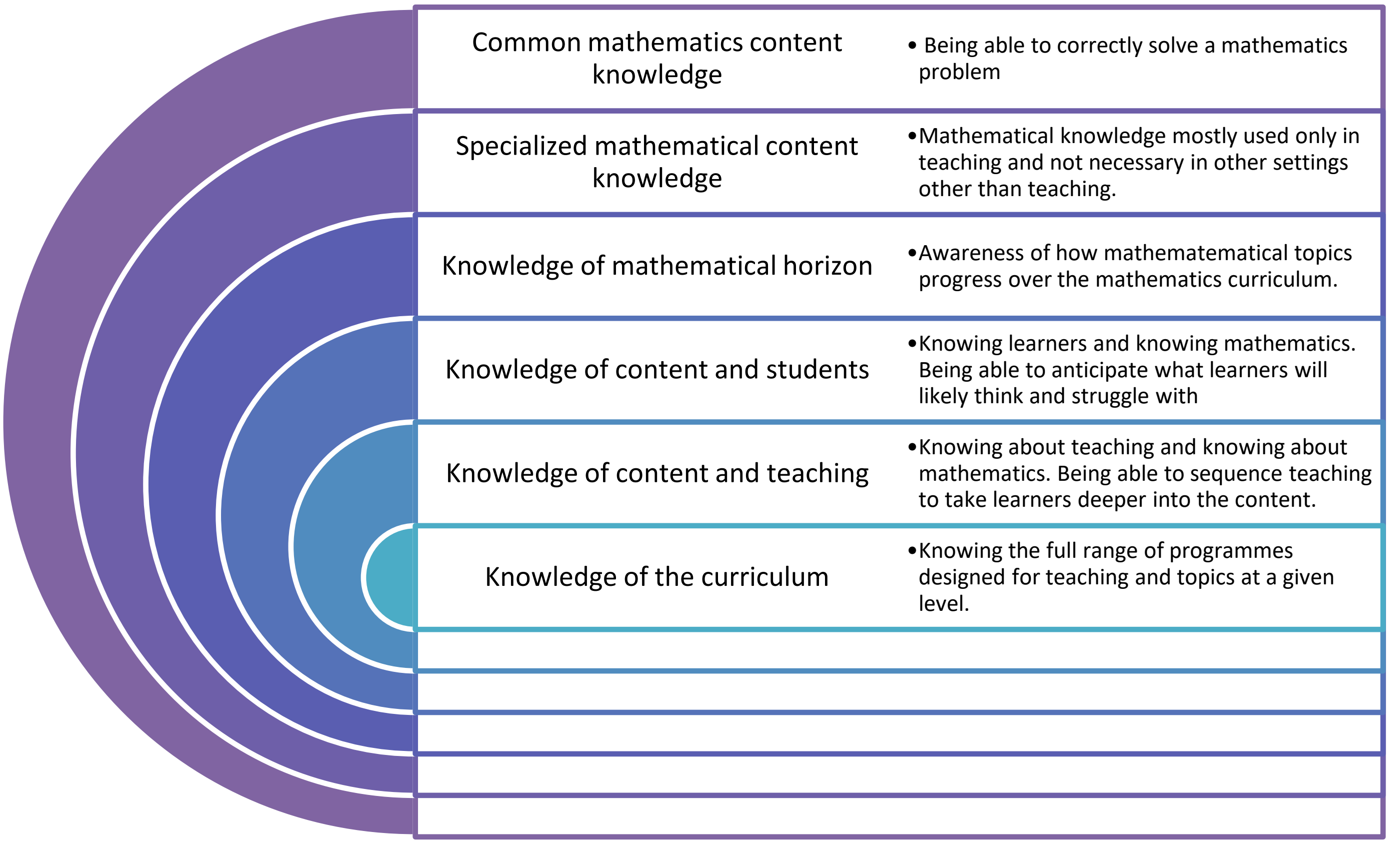

Figure 1: Types of Knowledge

Plotz (2007:15) found that it is necessary to combine the PCK and mathematical content knowledge (MCK), to form a distinct kind of knowledge that is necessary for teaching mathematics, namely mathematical content knowledge for teaching (MCKfT). MCKfT specifically refers to knowledge on how to present fundamental mathematical concepts to learners (Blomeke *et al.*, 2013:131) and also the distinct MCK that a teacher must have (Plotz, 2007:17). Before teaching takes place, the teacher must select the mathematics content and concepts, taking the learners' background knowledge into consideration, and using a variety of different teaching strategies to help learners learn (Blomeke *et al.*, 2013:131). The teacher should then use language that enhances learners understanding of mathematic concepts and be aware of misconceptions that learners may hold, as well as being able to use effective intervention strategies and give appropriate feedback to learners (Blomeke *et al.*, 2013:137). MCKfT should enable teachers to assist learners in the problem-solving process (Plotz, 2007:17) by combining the six types of knowledge, found in Figure 1.

A discussion on metacognition will follow, because the importance of reflective practices in changing beliefs was highlighted by Conner *et al.* (2011:502).

3.8 Metacognition

Reflective practices play an important part in the activation and development of MCKfT (Plotz, 2007:18). MCKfT should be developed and improved upon by teachers who should constantly be reflecting on their own MCK and the shortcomings of their current MCK (Plotz, 2007:20). Reflective practices will be referred to as metacognition and needs to be discussed because it plays such an important role in teachers' beliefs about mathematics teaching-learning.

Metacognition is defined as knowledge of a person's own cognitive processes and products (Demircioglu *et al.*, 2010). It also includes the ability to monitor, regulate and evaluate one's own and other's thinking (Nool, 2012:302). According to Esterhuyse (2015:23), metacognition can be seen as knowledge, the awareness of one's thinking processes and reflection on the whole process in order to solve a problem.

According to Demircioglu *et al.* (2010:494), metacognition is a necessary process in problem-solving. Metacognition has been researched very extensively, but limited research has been done on teachers' metacognition (Fransman, 2014:5). Metacognition in teachers, refers to knowing which instructional strategies are used, when they are used, why they are used and how they are used (Fransman, 2014). In a study by Demircioglu *et al.* (2010:495), it was found that metacognitive experiences are very important in becoming a good problem solver. In order for teachers to appreciate the value of a problem-solving approach to mathematics teaching-learning, it is necessary for them to have a high level of metacognition. Schoenfeld (1992a) states that metacognitive skills are elements that determine the success or failure in problem-solving. Taking the statement of Schoenfeld into consideration, a discussion on how metacognition is used in problem-solving will be discussed.

The first phase in solving a problem, is the orientation phase, which includes comprehension strategies, analysis of information, representation, assessment of difficulty and chances of success (Demircioglu *et al.*, 2010:494). The second phase is the organisation phase, which includes identifying goals and planning how a problem will be solved (Demircioglu *et al.*, 2010:494). The third phase is the execution phase, meaning to perform actions, monitoring progress and making decisions (Demircioglu *et al.*, 2010:495). The final phase is the verification phase, where actions of the other three phases are evaluated (Demircioglu *et al.*, 2010:495).

Teachers need to be aware of their own thinking and the process of how they solve problems, in order to teach learners to improve their metacognition (Esterhuyse, 2015:30). If a teacher uses the metacognition and reflects on practices, it has the potential to allow the teacher to continually reconstruct knowledge and beliefs in response to changing demands and expectations (Graham & Phelps, 2003).

We realise that the institution responsible for training teachers has a pivotal role to play in helping develop and expanding their knowledge, as well as improving metacognition, which all have an impact on teachers' beliefs of mathematics teaching-learning. Therefore a discussion on teacher education will follow.

3.9 Teacher Education

Teacher education can be defined as policies and procedures designed to equip prospective teachers with the knowledge, attitudes, behaviours and skills they require to perform tasks effectively in the classroom, school and wider community (Davis & Adler, 2006:272). Teacher education focuses on the development of teaching skills, pedagogical theory and professional skills (Kennedy, 1997:3).

In the past, teacher educators were trained to help produce teachers who could memorise formulae and textbook content with the only goal being to pass the exam or test (Van Aswegen, 2004:1). Teacher educators are now required to help facilitate teachers that are able to guide learners to be prepared for jobs that do not yet exist, in other words to become self-directed learners. This requires that teachers and teacher educators must constantly be in a process of reflection in order for MCKfT to be improved (Plotz, 2007:20).

Teacher education in South Africa is increasingly under the spotlight, because the quality of teaching is determined by the teachers, who are in turn trained and developed through teacher education (Adler *et al.*, 2009:28). Adler *et al.* (2009:29) assert that in the South African context, the constant change of the curriculum, has led to a change in content, practices and more interactional pedagogies. Teacher education is not simply a matter of increasing the content knowledge of a teacher, but developing ways in which teachers understand and can use the knowledge they have (Kennedy, 1997:5). Lourens and Brodie (2011:4) assert that knowing, is when knowledge becomes part of a teacher's practices. Knowledge can be possessed, while knowing implies using the knowledge (Lourens & Brodie, 2011:4)

Looking critically at teacher education in South Africa, the new curriculum has broadened the knowledge bases that teachers need to teach, making it very important for teacher educators to select the correct content to teach in their limited time working with teachers (Adler *et al.*, 2009:38). A challenge for the BEd programme, is the way it provides access to knowledge and how teachers are supported to attain the knowledge they need for teaching-learning (Adler *et al.*, 2009:38).

Spaull (2013:3) writes:

> As it stands, the South African education system is grossly inefficient, severely underperforming and egregiously unfair.

The shocking reality is that many South African mathematics teachers have below-basic levels of MCK, with high proportions of teachers being unable to answer questions aimed at their pupils (Spaull, 2013:5). We then have to look at how teacher education should take place.

Nieuwoudt (2002:16) argues that the perfect example of teaching is illustrated through Jesus Christ. Jesus increased his disciples' knowledge by teaching them at every opportunity. Likewise, Spaull (2013:58) suggests that a nation-wide system of teacher-training should be implemented to increase the MCK of teachers. Jesus taught his disciples by leading by example and not only speaking, but illustrating (1 Peter 2:21-24). Requiring teachers to teach in a certain way, without giving them an opportunity to discover, investigate and experience for themselves, will very likely be unsuccessful in leading to new ways of teaching-learning (Nieuwoudt, 2002:16).

Teacher education should prepare teachers to be able to deal with problematic real-life classroom situations in an effective manner (Nieuwoudt, 2002:17). Similarly, Jesus used examples that his disciples could relate to and learn from, to handle real-life situations (Matthew 18:23-35). Jesus also sent out his disciples on their own and afterwards they came back and reflected on their experiences and how they could further grow in their belief (Matthew 10). Similarly, teacher education should give teachers an opportunity to teach and reflect on their own mathematics teaching-learning and support them in growing as mathematics teachers, through mentoring, discussions and support.

3.10 BEd Hons (Mathematics) programme

The mathematics program that the participants were enrolled had a year duration. The mathematics program endeavoured to ground participants in the structures of mathematics education and curriculum development (Nieuwoudt & Roux, 2016:iv). It aimed for enrolled participants to be able to fundamentally discuss, critically evaluate and apply current perspectives, theories, models and practices regarding the teaching and learning of mathematics and the mathematics curriculum in different contexts. It required the integration of mathematics into real life situations and the ability to identify, motivate, explain and account for variables that influence effective curriculum development and teaching-learning of effective mathematics education (Nieuwoudt & Nieuwoudt, 2012:iv).

The program had problem-solving at its centre, with classes conducted in the form of discussions and creating problem-solving opportunities. Participants in the program were expected to participate actively and constructively in critical discussions based on the focus and study materials (Nieuwoudt & Nieuwoudt, 2015:xiii). Participants were also required to hand in formal assignments that covered topics such as perspectives of mathematics and mathematics education, teaching models (Nieuwoudt & Nieuwoudt, 2012:xii), effective learning and teaching of mathematics, mathematical knowledge for teaching, meaningful learning in mathematics and practical implications of theories on mathematics teaching (Nieuwoudt & Roux, 2016:xii).

3.11 Summary

How mathematics teachers choose to teach, is influenced by a combination of teachers' beliefs about mathematics teaching-learning and their mathematical knowledge (Holm & Kajander, 2012:10). To improve mathematics teaching-learning, teacher education should not only improve mathematical knowledge, but must give sustained professional support to mathematics teachers. Simply focusing only on improving mathematical knowledge will not necessarily change the way mathematics is taught (Philipp, 2007:263). A combination of improving mathematical knowledge; introducing beliefs about mathematics teaching-learning, to mathematics teachers, that is different from their own beliefs; giving them time to reflect on their own mathematical knowledge and beliefs, and giving continued support throughout, will possibly lead to a change in beliefs about mathematics teaching-learning (Holm & Kajander, 2012:10).

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

In the following chapter, the research design and methodology will be discussed. Firstly, the research aims and the literature review will be discussed, followed by the research design, which includes the methodology, philosophical orientation, procedure, population and sampling, data collection and data analysis. The validity and trustworthiness of the study will follow and then the researchers' role will be explained, followed by ethical considerations. A summary concludes this chapter.

4.2 Research Aims

In Chapter 1, we realised that it is necessary to understand the factors behind the change in mathematics teachers' beliefs about mathematics teaching-learning. This leads to the following discussion:

The main aim of the study was to:
Understand factors that may contribute to changes in post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme**.**

Secondary aims for the study are:

- To describe post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme.
- To identify factors that contribute to changes in post-graduate teacher's beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme.

In an attempt to achieve these aims the following research questions were addressed:

The study was guided through the following primary research question:
How can we better understand possible factors that contribute to changes in post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme?
The following sub-questions were used to support the primary question:

- What are post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme?

- Which might be factors that contribute to changes in post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme?

The findings were used to better understand factors associated with changes in teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme.

4.3 Literature Study

The literature study comprises of an in-depth study of the literature available on beliefs and views of mathematics, changes in beliefs in mathematics and mathematics teaching-learning. Literature was searched by means of EBSCOhost, SABINET and internet search engines.

The following keywords were used in searches: "beliefs", "math* beliefs", "beliefs in mathematics education", "views of mathematics", "the nature of mathematics", "changes in math* beliefs", "teaching and learning mathematics", "teaching-learning mathematics", "math* beliefs".

4.4 Research Design

The research design is a way of positioning the researcher in the world and showing how the research question and data will be connected (Punch, 2006:48). Nieuwenhuis (2007b:72) states that a research design is a plan or strategy which starts at the underlying philosophical assumptions and moves to the specifying of participants, tools and procedures for collecting data and how data will be analysed. The research design will depend on the researcher's assumptions, research experience and research practices (Nieuwenhuis, 2007b:72).

The research design can be described using Figure 1.1. This study used a qualitative phenomenological approach that made use of purposeful sampling in order to conduct semi-structured interviews that were tape-recorded and then analysed, using content analysis in order to better understand and describe changes that may have occurred in mathematical beliefs.

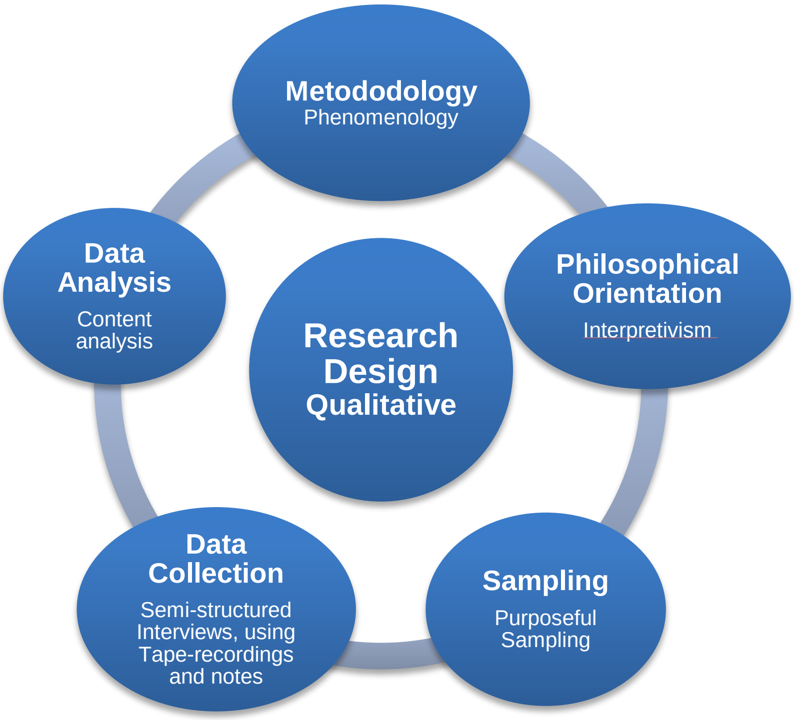

Figure 2

4.4.1 Methodology

In this study, I made use of a qualitative phenomenological study. A phenomenological study means that a researcher looks at a specific phenomenon and gives a description of the phenomenon through the experiences of people concerning the phenomenon (Nieuwenhuis, 2007b:78). The study focused on the experiences of individuals and was only able to fully capture participants' beliefs by using their own descriptions and explanations; therefore, a qualitative phenomenological study was used.

4.4.2 Philosophical Orientation

This study was done from an interpretivist paradigm. Interpretivism proclaims that reality is constructed socially and that the purpose of the researcher is to understand the meaning that people attach to reality (Nieuwenhuis, 2007b:58). Interpretivism is based on the assumptions that human life can only be understood from within, that reality is socially constructed, that the human mind is the origin of meaning, human behaviour is affected by knowledge and that the social world does not exist independently of human knowledge

(Nieuwenhuis, 2007b:60). In this study, I used interpretivism because I needed to understand the phenomena of mathematical beliefs and the factors that might be associated with change. This paradigm was most suited because I did not wish to change mathematical beliefs, but rather to understand mathematical beliefs and the factors that may or may not be associated with changes in mathematical beliefs of people who construct their own reality (Nieuwenhuis, 2007b:59).

4.4.3 Research Procedures

The following research procedures were used:

- An in-depth analysis of existing literature was done
- Data were generated at two different stages in the research process. The first data generation took place at the start of the semester, the second data generation took place at the end of the semester and questions were formed using data generated in the first semi-structured interview, which took place at start of the semester. All data generation took place by using individual semi-structured interviews.
- The first interview was done to establish context of participant's experiences, specifically what beliefs they held about mathematics teaching-learning.
- The second interview encouraged participants to reflect on the meaning their experience held for them and the impact of the course on them as mathematics teachers.

4.4.4 Study Population and Sample

Sampling refers to the process of selecting a portion of a population based on a characteristic that the participant has that is needed for the study (Nieuwenhuis, 2007b:79). According to Nieuwenhuis (2007b:79), qualitative research is generally based on non-probability and purposive sampling. In the study, I made use of purposive sampling, by not only participant selection, but also the selection of the setting and activities to be included in data collection (Nieuwenhuis, 2007b:79). The criteria for selection was that the participant be a BEd Hons (Mathematics) student, at this specific South-African University and willing to take part in the study. I studied participants' experiences of the BEd Hons (Mathematics) programme and tried to understand the impact on teachers' beliefs of mathematics teaching-learning.

The study population of the research consisted of full-time and part-time students enrolled for a BEd Hons (Mathematics) programme at a South African University. The sample used for this study was four of the students who were willing to take part, out of a group of 8 honours students. The participants ranged from full-time students with little teaching experience, to mathematics educators with more than 6 years of teaching experience.

4.4.5 Data Collection and Instruments

Data were collected at two different stages in the research process during the course of one semester. The first data collection took place at the start of the semester, the second data collection took place at the end of the semester and questions were formed using data collected in the first semi-structured interview. All data generation took place using individual semi-structured interviews. Nieuwenhuis (2007b:87) states that an interview is a two-way conversation in which the interviewer asks the participant questions, to generate data and learn about the ideas, beliefs, views and opinions of the participant. The aim was to understand participants' mathematical beliefs and to corroborate data emerging, with other data sources (Nieuwenhuis, 2007b:87). I used semi-structured interviews, because when the participant lost focus in the interview, I was able to guide them back on track, as well as asking for clarification and then probing further. I was of the opinion that this method would give me the richest data about the mathematical beliefs of the participants and factors that may or may not result in changes in mathematical beliefs.

Data were collected by means of tape-recording of interviews. Tape-recording interviews allowed me access to the original data at any time and I was able to concentrate on the interview, rather than having to make sure I got all the original data (Nieuwenhuis, 2007b:89). I had to obtain permission from the participants before I used a tape-recorder, because some participants might not have felt comfortable and may have withdrawn. As soon as I finished the interviews, I made a transcript of the interviews and identified gaps that needed to be explored in a follow-up interview where necessary (Nieuwenhuis, 2007b:89). Transcriptions are available if required.

4.4.6 Data Analysis

According to Nieuwenhuis (2007a:99), qualitative data analysis is a process, method or procedure through which the researcher extracts a type of explanation, interpretation or understanding from collected data. Data analysis tries to determine how participants attach meaning to a subject, by looking at their perceptions, attitudes, understanding, knowledge, values, emotions and experiences (Nieuwenhuis, 2007b:99). Qualitative data analysis is a process of inductive reasoning, thinking and theorising (Schurink *et al.*, 2012:397).

Qualitative data analysis tends to be a continuous process and not a linear process, meaning that data collection, processing, analysis and reporting, all form part of each other and cannot be done independently from one another (Nieuwenhuis, 2007a:101). For the purpose of this study, I made use of content analysis. According to Nieuwenhuis (2007a:101), content analysis is a systematic approach which identifies and summarises the main points of the data. Content analysis is a process where data is looked at from several points of view, with the goal of identifying keywords which will help to understand and interpret data more effectively (Nieuwenhuis, 2007a:101).

I chose content analysis, because I had to analyse data from semi-structured interviews, which means I had to analyse transcriptions of data. This method allowed me to answer my research question in the best possible way, because I was able to critique and confirm existing theories on mathematics teachers' beliefs about mathematics teaching-learning. I also made use of inductive coding, because qualitative data analysis is a process of inductive reasoning, thinking and theorising (Schurink *et al.*, 2012:397).

4.5 Validity and Trustworthiness

Nieuwenhuis (2007b:113) states that trustworthiness is a demonstration that proof for results are reported, is true and that the arguments are based on solid and clear results.

I made use of a variety of data sources, including interviews, journals, books as well as websites (Nieuwenhuis, 2007b:113). I verified raw data, by discussing gathered data with participants to make sure that there was no misinterpretation from my part (Nieuwenhuis, 2007b:113). I sent a piece of raw data to an independent person, as well as the research aim and research question to see if her coding agreed with mine, to make sure that my coding is trustworthy.

I sent a copy of my findings, interpretations and conclusion to my study leader as well as other people who are knowledgeable in the field of mathematics education, to hear their comments and inputs on the research that was done. I also controlled prejudice by getting input from my study leader and other knowledgeable people in the field of mathematics education (Nieuwenhuis, 2007b:114). I made sure not to generalise results, as I only worked with a small population group at a specific university. I could therefore not generalise about all mathematics teachers in South Africa and my conclusion focused on these specific teachers, and factors that influenced them (Nieuwenhuis, 2007b:113). I also made sure to keep participants anonymous, by not providing their names, nor describing them.

Nieuwenhuis (2007b:115) suggests that to improve validity and trustworthiness, the limitations of the study need to be presented beforehand. I am aware that my study only makes use of a few participants and that not all teachers in South Africa have the same background and experiences as my participants. I also have a restricted amount of time to finish my studies, which means that data will not be as rich as it could have been had I had ideal conditions and a longer time period.

4.6 Researchers Role

According to Nieuwenhuis (2007b:79) the researcher's involvement and immersion in qualitative research is essential, because the researcher has to make sense of the real life context and interpret it. I filled the role as collector, analyser and interpreter of data. In order to do this, I was the interviewer in the semi-structured interviews, as well as analysing and interpreting coded data. I also had to make sure that all ethical considerations were taken into account during the whole research process.

4.7 Ethical Considerations

The fact that human beings are the subjects of this study brings unique ethical considerations to the fore. According to Strydom (2012:113), research should be based on mutual trust, acceptance, cooperation and well-accepted conventions. In the study, the dominant ethical considerations were:

Voluntary participation - meaning that participation should at all times be voluntary and that participants may not be forced to participate in the research (Strydom, 2012:116). Participants may also at any time have withdrawn themselves from the study without any consequences (Strydom, 2012:117).

Obtaining informed consent - implying that all possible information on the goal of the investigation, the duration, the procedures and the potential advantages and disadvantages be discussed with participants before the research may take place (Strydom, 2012:117). I needed to obtain permission from participants before I could conduct my research.

Privacy and confidentiality - this should be maintained at every stage of the research process (Strydom, 2012:119). I did not make use of descriptors or names that could lead to the identification of participants. I made use of aliases to refer to participants.

Honesty – this must be a top priority in that no attempt is made to deceive or misinform participants (Strydom, 2012:123). I did not deceive participants in any way.

I had to submit an application for ethical approval by the ethical board of the university, which was approved. I also handed out consent forms to all participants to complete before any data were collected.

4.8 Limitations

Nieuwenhuis (2007b:115) suggests that to improve validity and trustworthiness, the limitations of the study need to be presented beforehand. I am aware that my study only makes use of a few participants and that not all teachers in South Africa have the same background and experiences as my participants. I also have a restricted amount of time to finish my studies, which means that data will not be as rich as it could have been, had I had ideal conditions and a longer time period. It is also impossible to generalise results, because I worked with a small population group with different experiences and backgrounds; therefore I cannot generalise results to the whole country or even the whole province.

4.9 Conclusion

As a starting researcher I acknowledge that the planning and execution of this study could have been done in other ways. This chapter explained the method I chose to follow, even though there are many other methods that could have been more effective. This chapter has to serve as an outline of how this research was done.

In the next chapter reporting of data will take place as well as an analysis of data.

CHAPTER 5: FINDINGS AND INTERPRETATION

5.1 Introduction

This chapter will start with a discussion of findings about participants' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme. An interpretation of the findings will follow the discussion. In this chapter I will be answering the first sub-question by doing a content analysis, of the data generated through semi-structured interviews. I will be discussing each participant individually and try to see the world through their eyes.

5.2 The Case of Sally

5.2.1 Description of Sally's background

Sally is a pre-service teacher with little experience teaching on a full-time basis. In the following paragraph, Sally's interview answers will be given.

5.2.2 Findings for Sally

Sally sees mathematics as a network of ideas that come together to form a unity. She said that mathematics should not be difficult, but consists of building blocks that have to be mastered in order to do more complicated work. She makes the statement that when one is not well grounded in basic structures of mathematics; there is no use in being taught more complicated mathematics. She sees mathematics as a procedure, where the user has to use different methods to solve a problem, and not only solving the problem, but also explaining ideas and communicating these ideas. She said mathematics is more than just steps; it goes hand in hand with an understanding of how mathematics is used in everyday life.

Sally makes the statement that you are successful in mathematics when you are able to understand how each step in solving a problem works. The emphasis shouldn't be on the answer, but rather understanding the steps and the process. Sally highlights the importance of using visual aids to explain mathematics; she wants her learners to understand the applications of mathematics outside of the classroom. She also mentioned that she would really like to incorporate technology into her classroom, specifically GeoGebra, because it enables learners to manipulate and investigate on their own. She makes the statement that the teacher should not be talking the whole time, but needs to involve learners in group work and give learners opportunities to exchange ideas.

Sally described her average mathematics period as follows; she begins with a revision exercise, such as practicing multiplication, then she starts with new work, by explaining the specific topic and then learners are expected to do specific exercises or worksheets. While learners are working, her role is to answer questions and monitor learners. The period ends with the marking of a few of the questions. She thinks that learners shouldn't get too much homework, so that parents do not do the homework for the learners.

She said that the length of mathematics periods and the amount of work to be done in a mathematics classroom makes it difficult for her to teach the way that she would like to teach. Her own planning also plays a big role and the discipline of learners influences how she teaches her lessons. She made the statement that she has changed her teaching-learning of mathematics during the course of her studies and experiences. She said that being exposed to different learning-teaching practices has led her to use more learner-centred teaching approaches, where learners play a more active part in the mathematics classroom. She also said that she would be willing to change her teaching-learning practices in her mathematics classroom, if her way of teaching was ineffective, if new methods were more successful, if she gained more experience, if colleagues expected her to change or if learners or colleagues supported her to teach in a specific way.

Her overall experience of the mathematics course was a very positive one. She enjoyed looking at how the theory in mathematics can be applied in a teaching context. When she understood the underlying principles of different mathematical ideas, she was able to understand and appreciate the practical part of doing mathematics much better. She started to realise, through being exposed to the theory, that mathematics is not only restricted to numbers, but is a much more complex and creative subject.

Her confidence as teacher also increased as the course progressed, because she started working with great mathematical ideas and started to understand where they come from and how they are applied in different topics in mathematics education. The course also required her to do extra reading and gave her much more insight into mathematics. The new information that she gained, motivated her to better understand theory, in order to explain to learners where mathematical concepts come from.

She said that some of the content wasn't relevant for school mathematics and that she would have appreciated a practical component in her studies, where she would be given an opportunity to test theories practically in a school environment. She was of the opinion that not applying the knowledge gained, would lead to forgetting the knowledge.

She said that through what she learned, she would now be able to easily answer any questions that learners might have and explain to them how to correctly solve a problem. She also said that she would like to create a teaching environment that encourages active learning, where the teacher teaches as little as possible and facilitates learners during the class.

5.2.3 Interpretation

The first part of the interpretation addresses the first sub-question (What are post-graduate teachers' beliefs about mathematics teaching-learning in a BEd Hons (Mathematics) programme?), followed by possible factors that influence beliefs and what possible factors could lead to change.

5.2.3.1 Describing Sally's Beliefs about Mathematics Teaching-Learning

When exploring literature and Sally's response, it contains many of the characteristics of a problem-solving view of mathematics. In contrast to an instrumentalist and platonist view, where mathematics is presented as unrelated pieces (Nieuwoudt, 1998:164). Sally defined mathematics as a network of ideas or building blocks that form a unity. Ernest (1991a:294) stated that a problem-solving view encourages building relationships between different mathematical ideas as well as discovering these relationships.

Benadé (2013:16) mentions the importance of relating mathematics to a real-world context in a problem-solving view and applying mathematics to solve real-life problems. Likewise, Sally repeatedly highlighted the importance of making mathematics realistic and relating it to the real-world context. Molefe (2006:23) writes that when mathematics teaching-learning is taught from a problem-solving view, it emphasises the process of doing mathematics, rather than the product of the mathematics. Sally repeatedly stated that understanding the process of doing mathematics, is much more important than being able to find the correct answer. She also wants to make sure that learners can make sense of mathematics and understand the origins of the different topics. This is in contrast to a platonist classroom, where learners are simply told to practice a formula without any understanding (Ernest, 1991b:114).

Benadé (2013:12) highlights that a teacher with a problem-solving view of mathematics, will emphasise the use of technology and use of representations so that leaners can relate their theoretical knowledge to a real-life context. Sally stated that different representations of mathematics are very important to her, specifically using technology to help learners manipulate and discover. Using technology and programmes such as GeoGebra, makes learners part of the teaching-learning process and gives them the responsibility for their own learning (Handal, 2003:47).

In a problem-solving classroom, the teacher acts as facilitator of learning (Shilling-Triana & Styliandes, 2012:393) and a classroom atmosphere that encourages investigation, creativity and taking risk is created (Nieuwoudt, 1998:107). Sally sees her role as facilitator and motivator, creating a classroom that encourages active learning. She also wants to encourage learners to work in groups and communicate ideas to the rest of the class, which is also a characteristic of a problem-solving view of mathematics (Gujarati, 2013:634; Handal, 2003:47; Nieuwoudt, 1998:107).

There is a discrepancy between Sally's beliefs about mathematics teaching-learning and her classroom practices. Although Sally expressed that she has a problem-solving view of mathematics, it seems that her reported classroom practices correlate with those of a platonist view of mathematics. Her lesson starts with a review of work covered, checking homework, explaining new work and giving learners a large number of questions that require them to repeat the same procedure, which is the predictable routine of a platonist classroom (Nieuwoudt, 2002:4). She also expressed that she feels more confident because she feels like she can easily answer questions and can correctly solve mathematical problems. This statement gives the idea that she thinks she must be the master (Ernest, 1991b:114) of the class and be able to do all the mathematics easily, which is in contrast to a problem-solving view, where the teacher must be willing to take mathematical risks (Nieuwoudt, 1998:107) and be able to admit that they don't know everything (Benadé, 2013:12).

Sally has a problem-solving view of mathematics, but it seems as if her mathematics teaching-learning practices do not correlate with her beliefs. She was very excited to change her classroom practices, by incorporating technology, using visual representations, relating mathematics to the real-world, incorporating group work, having discussions and taking the role of facilitator. After looking at the world through Sally's eyes, we need to discuss factors that could influence her beliefs.

5.2.3.2 Possible Factors that Influence Sally's Beliefs about Mathematics Teaching-Learning Practices

The discrepancy between Sally's beliefs and classroom practices can be explained by Holm and Kajander (2012:7), who found that teachers are likely to teach in the way they were taught. Sally has been in a platonist classroom most of her life and cannot imagine that a different way of teaching might be more effective. Sally is still young in her teaching career and hasn't been teaching for many years. Beswick (2012:129) states that during the early stages of a teacher's career, beliefs about mathematics do not correspond with mathematics teaching-learning practices. This could be one of the reasons why there is an inconsistency between Sally's beliefs and mathematics teaching-learning practices.

Another important point is the role that context plays in teachers' mathematics teaching-learning practices. Beswick (2012:130) writes that teachers may express certain beliefs about mathematics, but enact opposing mathematics teaching-learning practices in front of a mathematics class.

Sally did state what prevented her from teaching mathematics in a different way. She said that the length of mathematics periods and the amount of work to be done in the mathematics classroom, made it difficult for her to teach mathematics the way she would like to teach it. She also mentioned that the discipline of learners influenced how she taught. Research confirms the statements made by Sally, namely length of class periods, size of classrooms, as well as lack of cooperation from learners, which influence mathematics teaching-learning practices (Handal, 2003:52; Sapkova, 2013:739).

Taking the above into consideration, a discussion of possible factors that could have led to a change in Sally's beliefs about mathematics teaching-learning will follow.

5.2.3.3 Possible Factors that could Change Sally's Beliefs about Mathematics Teaching-Learning

The first possible factor that Sally named as one of the reasons why her beliefs on mathematics teaching-learning might have changed is being exposed to different mathematics teaching-learning practices. Liljedahl *et al.* (2012:106) found that changes can occur in mathematics teachers' beliefs about teaching-learning mathematics after exposure to new beliefs, that are different from their own, but he warns that once removed from this exposure, the teacher may gradually return back to previous mathematic teaching-learning practices. Being exposed to different practices opens the teacher's eyes to the possibilities of teaching in a more effective manner (Sapkova, 2013:740).

An understanding of the big ideas in mathematics, helped her understand how to use it and understand the content better. Improving mathematical teacher knowledge leads to a teacher who is better equipped to explain the process of doing mathematics and not just giving a formula or rule that has to be memorised without any understanding (Ball *et al.*, 2008:404). Sally even stated that teachers should not be allowed to teach mathematics without obtaining an Honours degree in Mathematics Education, because they need a deeper subject knowledge. An appreciation of a problem-solving approach to the teaching-learning of mathematics can only develop if teachers hold a certain level of MCKfT (Blomeke *et al.*, 2013:137). Sally also expressed the gratitude she had for all the extra reading she had to do, because it allowed her to improve her knowledge about mathematics teaching-learning and increased her confidence to try new approaches in her classroom.

The third factor that Sally indicated was the context of her teaching, specifically referring to colleagues. Beswick (2012:145) says that in the absence of assistance and support, most teachers will revert back to previously held beliefs about mathematics teaching-learning. Having the support of management and staff members also increases the likelihood that changes in beliefs about mathematics teaching-learning will occur and be lasting (Handal, 2003:52; Sapkova, 2013:739).

The critique that Sally had against the course was that she was not able to practically practice a new approach to the teaching-learning of mathematics. She felt that this might cause her to forget the theory she had learned. She would have liked to see more practical examples of how to teach in a problem-solving manner and how it would look in the real-world context, with all the demands of the school system.

5.3 The Case of John

5.3.1 Description of John's Background

John is a pre-service teacher with little experience in teaching full-time. In the following paragraph, John's interview answers will be provided.

5.3.2 Findings

John sees mathematics as a daily problem, for example, working with money. John said that mathematics has to involve real-life problems and that mathematics goes far deeper than figures and patterns. Mathematics is not defined as formulae, but formulae are a part of mathematics. John said that much of the mathematics he teaches, has never been used by him, outside of the formal school structure, such as trigonometry and pi. He said that unfortunately mathematics at school is restricted to steps and procedures that don't necessarily relate to the real world. He makes the statement, "*A lot of mathematics done inside the school setting is not applicable outside of the school context*."

John said that learners are successful in mathematics when they are able to have a problem put before them and they are able to independently solve the problem using their own logic and thinking. John also puts emphasis on knowing how you solved the problem and being able to repeat it again. John said that when learners learn using memorised steps, that they do not necessarily understand mathematics, but when they struggle with problems and discover their own way of solving a problem, they have a true understanding of mathematics.

John thinks that mathematics should be learned by discovery and that it is the teacher's responsibility to give the learner meaningful problems that lead them to discovering concepts and formulae by themselves. John also said that there are different ways of solving a problem and learners should not be restricted to only use a certain method or procedure, but rather be encouraged to make use of multiple methods.

John said that when a learner answers a question and the answer is incorrect, he will ask the learner how he got to the answer to determine whether the learner made a calculation error or if the learner has a misconception. When a learner has a misconception, he will assist the learner and make sure to correct any misconceptions.

John said that his usual class periods will start with making sure that everybody in the classroom is calm and ready to work, followed by marking of homework. Then he will start with the new work and sometimes, if there is time available, do a self-discovery exercise. Then he will give learners exercises to do in the classroom, where he will try and involve learners as much as possible, followed by giving them homework for the next day.

John said the length of mathematics periods has a very big impact on how he teaches. He also said that the placement of periods is a very important factor to consider, because learners tend to be more restless before a break or at the end of the day. He also said that he loses a lot of time because of sport matches, because many of the learners participate in sport and are then absent for the mathematics period. He also said that when the administration period at the start of the day is longer, all other periods become shorter, which makes it very difficult to get through the mathematics curriculum.

John said he would change his mathematics teaching-learning if it resulted in better discipline in the classroom or if it made teaching content easier. He also said that he would be encouraged to change mathematics teaching-learning if he acquired more subject knowledge and was able to see that an approach was more successful than his previous approach. He recalled a specific incident where he taught a specific lesson twice, the first time he just told learners what to do and how to do it; in the second lesson he interacted more with the learners and gave them opportunity to discover for themselves. He noted that in the second method, the learners had fewer questions and understood much better than with the first method. He also says that he would change his mathematics teaching-learning if he copied a successful mathematics teacher and it was successful. He also makes the statement that if the new method doesn’t work, he will stay with his current way of mathematics teaching-learning.

John said that he found his studies challenging, but very rewarding. He learned a tremendous amount from doing his mini-dissertation. He also thinks differently about his teaching now and incorporates information learned into his teaching on a daily basis. He realised that he has to be adaptable to the learner’s needs and that mathematics teaching-learning is about accommodating the learner and not about the teacher teaching in a comfortable way.

5.3.3 Interpretation

The first part of the interpretation addresses the first sub-question, followed by possible factors that influence beliefs and what possible factors could lead to change.

5.3.3.1 Describing John's Beliefs about Mathematics Teaching-Learning

The words John uses to describe mathematics teaching-learning, give a strong indication of a problem-solving view of mathematics. John links mathematics with real-world problems and explains that mathematics goes far beyond figures and patterns. Benadé (2013:16) states that in a problem-solving classroom, the teacher will relate the mathematics teaching-learning to a real-world context. Mathematics in a problem-solving classroom is applied to real-life situations and problems (Benadé, 2013:12). John emphasises that mathematics should be useful and that mathematics learned, should be useful outside of the classroom, this correlates with Benadé's findings.

John thinks that learners are successful when they are able to independently solve mathematical problems using their own logic and thinking. Ernest (1991c:294) writes that in a problem-solving view of mathematics teaching-learning, leaners are encouraged to create their own knowledge. In a problem-solving classroom, there is an emphasis on mathematical reasoning, proving and construction of own mathematical knowledge (Molefe, 2006:23). John also highlights the importance of discovery and giving learners meaningful problems to do. Making discoveries is part of a problem-solving view of mathematics teaching-learning, where learners should be able to connect mathematics to real life (Conner *et al.*, 2011:492).

John takes learners' background knowledge into consideration when teaching. When a learner answers a question incorrectly in John's classroom, he makes sure to guide the learner to the correct answer through a discussion, rather than just telling the learner that the answer is incorrect. This is a characteristic of a teacher with a problem-solving view of mathematics teaching-learning, where the teacher takes on the role of facilitator and guides learners by providing knowledge and asking challenging questions (Benadé, 2013:16).

John said that he tries to involve learners in the classroom as much as possible, which creates an environment where learners are actively involved in the classroom. A problem-solving mathematics classroom should be learner-centred, with the teacher actively involving learners in the learning process (Shilling-Triana & Styliandes, 2012:393). John also said that it was very important for him to be able to adapt to the need of his learners. Nieuwoudt (1998:107) states that a teacher with a problem-solving view of mathematics will teach on a level that the learners will be able to understand and also gives them opportunity to express and share their ideas and thoughts.

It becomes quite clear that John holds a problem-solving view of mathematics, because his statements are supported by literature. His description of mathematics and how he teaches is the same as a teacher with a problem-solving view of mathematics. A discussion on what factors might influence John's beliefs about mathematics teaching-learning will follow.

5.3.3.2 Possible Factors that Influence John's Beliefs about Mathematics Teaching-Learning Practices

John highlights the importance of the length of mathematics periods as being a big factor in determining the way he teaches. An obstacle that prevents teachers from teaching in the way that they would like to teach is the length of class periods (Handal, 2003:52; Sapkova, 2013:739). John also mentioned the length of the administration period at the start of the day, because it also takes away some of his teaching time.

John mentioned that sport had a big impact on his teaching, because learners constantly miss his mathematics class, because they have to drive to sport matches that are at neighbouring schools. This factor is related to the length of mathematics periods, because learners have missed lessons taught and the teacher has to help them catch up on work not done.

Another factor that John named as a big influence on his mathematics teaching-learning practices was the mathematics curriculum. The mathematics curriculum is full of content that learners need to master each day, this knowledge is then tested through an annual national assessment, where all schools are rated, based on their performance in these tests. This puts a lot of pressure on mathematics teachers, because if their learners don't perform, the teachers are held responsible. This corresponds to Handal (2003:52) and Sapkova's (2013:739) findings, that pressure from parents and management have a direct impact on teachers' mathematics teaching-learning practices.

Taking the above into consideration, a discussion of possible factors that could have led to a change in John's beliefs about mathematics teaching-learning will follow.

5.3.3.3 Possible Factors that could Change John's Mathematics Teaching-Learning Practices

John, like the other participants, expressed that increasing his subject knowledge would contribute to his changing beliefs about mathematics teaching-learning. As John's subject knowledge grew through the course, he started to realise that he could teach in a problem-solving way. Blomeke *et al.* (2013:137) found that as teachers MPCK grew, they were able to appreciate the value of a problem-solving approach to mathematics teaching-learning. When John was confronted with different beliefs, he started to re-evaluate his own beliefs about mathematics teaching-learning. John started to realise how important it is to teach at a level that learners can understand.

The second factor that was identified through the interviews with John, was his personal motivation. John said that if he used a specific approach and realised that learners understand better and are enjoying mathematics more, he would definitely use that specific approach. Teachers cannot be motivated to keep teaching mathematic in a certain way, if there are no prospects for growth for them or the learners (Handal, 2003:52; Sapkova, 2013:739). John needs to see the results of his teaching, otherwise he will not teach in a specific way.

5.4 The Case of Margaret

5.4.1 Description of Margaret's Background

Margaret is an in-service teacher. She has been teaching mathematics for more than seven years in a High School. The context of the school is one where it is expected of learners and teachers to excel in all spheres of schooling. The pressure to excel from parents on teachers and learners is very high. The school has very good funding and resources are easily acquired and available to teachers. All classrooms have access to the internet, and white boards and projectors are found in most classrooms. In the following paragraph, Margaret's interview answers will be discussed.

5.4.2 Findings

Margaret sees mathematics as a logical and precise approach to solving problems. She said that mathematics has both practical and technical components; and that mathematics has many different fields, but most rely on a good theoretical background, combined with practical applications. She said that mathematics is built on key principles and requires a lot of creativity.

She said that how mathematics is used, depends on your family and if they use mathematics in their daily lives or not. She said that if a learner is constantly shown real-life applications of mathematics, they are going to see the meaning and worth of it. She said that mathematics at school level is only part of mathematics as a whole and that the teacher has to teach these specific parts to learners. She said that mathematics can be a cold and isolated subject which is the view of many learners that she teaches.

She said that a scholar of mathematics can never reach a point where they know everything of mathematics. She said that mathematics is a powerful tool through which learners can develop certain skills that can be used in their daily lives. She said learners are successful in mathematics when they are able to present their thoughts logically and effectively. She also said that learners need to be able to make sense of problems and be able to solve them by using their own thoughts and methods.

Margaret is of the opinion that mathematics cannot be learned by using a single approach, but rather that different parts of mathematics should be learned by different approaches. She said that the problem-solving approach is very effective in teaching volume, area and trigonometry, while the study of shapes and their properties is restricted to paper, which is two dimensional. She emphasises that teachers shouldn't use only one approach, because then they reduce mathematics to a one-dimensional subject that should be taught to learners that are complex and dynamic, and definitely not one-dimensional.

Margaret made the following statement, *"Mistakes are proof that you are trying"*. She said that when learners answer a question incorrectly, she first has to determine whether the learners have made a calculation error, weren't paying attention or have a misconception. She likes to play the devil's advocate in her classroom by making learners question their own answer and this leads to learners having confidence in their own answers.

Margaret said that she doesn’t like a typical mathematics period, but unfortunately South Africa’s school system is typical. She said that the whole system is typical, starting with bells indicating when what happens, leading to learners gaging themselves only for the amount of time required for completing each day. She said the curriculum makes it very difficult to use your own initiative, as it gives specific criteria that have to be met. She said that not only are the learners caught in this typical system, but the teachers are too.

She said that her mathematics periods will begin by going through the homework and using it to identify problems. She will then mark the homework and start with a new topic or repeat work if necessary. She will then give learners similar problems to ones explained and then give them homework. Margaret said that when she starts a new topic, she tries to make it as practical as she can, but it is not always possible, due to the short time available.

She said that if schools would do away with their bells, it would help her to teach in the way that she would like to. She stated that if she was given more time to teach, it would make it much easier to give time to learners, to discover and take responsibility for their own learning. She would also need materials that have examples and worksheets with problem-solving exercises, to help her to teach in the way that she would like to.

She said that she would change her beliefs about mathematics teaching-learning if she saw that learners responded positively towards her teaching. She also created a less traditional classroom atmosphere that encourages participation and teaches out of a caring relationship. She changed her beliefs about mathematics teaching-learning through positive experiences, being introduced to new teaching methods and also by doing research and gaining more knowledge.

5.4.3 Interpretation

The first part of the interpretation will be answering the first sub-question, followed by possible factors that influence beliefs and what possible factors could lead to change.

5.4.3.1 Describing Margaret’s Beliefs about Mathematics Teaching-Learning

While analysing Margaret’s interview, it becomes apparent that she has a problem-solving view of mathematics teaching-learning. Margaret describes mathematics as a creative subject that has both practical and technical components. Benadé (2013:12) highlights that a teacher with a problem-solving view of mathematics will focus on the utility and functionality of mathematics. Beswick (2009:154) comments that in a problem-solving view, mathematics is seen as a dynamic and creative human activity. Nieuwoudt (1998:107) writes that a problem-solving classroom promotes investigation, creativity and taking mathematical risks.

According to Gujarati (2013:634), a teacher with a problem-solving view of mathematics will model problem-solving and explore real-world mathematical contexts. Margaret expressed that if learners were shown real-life applications of mathematics, they would be more motivated to do mathematics.

Margaret made the statement that as a scholar of mathematics; you can never reach a point where you know everything about mathematics. Benadé (2013:16) states that a teacher with a problem-solving view of mathematics, believes that teaching-learning mathematics is a constant process of discovery. A teacher with a problem-solving view of mathematics will present mathematics as a continuous process of exploration that is always open to revision (Handal, 2003:47).

Margaret highlights that learners are successful in mathematics when they are able to present their thoughts logically and effectively. One of the key aspects of a problem-solving view, is to allow learners to construct their own mathematical knowledge (Benadé, 2013:16; Nieuwoudt, 2002:17) and give learners time to explain their approach (Molefe, 2006:23).

Margaret highlights the importance of not only using a single approach to teaching, but using different approaches. In a problem-solving classroom, the teacher sees each learner as unique and having their own prior knowledge and experiences that they bring to the class with them. Nieuwoudt (2002:1) writes that because of the unique way that God created each human, it is impossible to use only a single way to teach every learner. Margaret expressed the same feelings, because she wants to teach out of a caring relationship and encourage learners.

It becomes quite clear that Margaret holds a problem-solving view of mathematics, because her type of teaching, and how she thinks of mathematics, is similar to that of a teacher with a problem-solving view of mathematics teaching-learning. Although Margaret expressed a desire to always teach in a problem-solving manner, her mathematics classroom practices correlate with those of a more traditional view of mathematics.

A discussion on what factors might influence Margaret's beliefs about mathematics teaching-learning will follow.

5.4.3.2 Possible Factors that Influence Margaret's Beliefs about Mathematics Teaching-Learning Practices

All Margaret's answers correlated with a problem-solving view of mathematics teaching-learning, but she taught the learners in the mathematics classroom in a traditional way. Taking a closer look, she expressed a major concern about the time available for her to teach. Teachers often tend to fall back to more traditional beliefs about mathematics teaching-learning, because they simply do not have the time to incorporate new ways of teaching (Benadé, 2013:20). Margaret expressed her desire to have more time to teach learners and give them time to take responsibility for their own learning.

Margaret also said that the curriculum made it very difficult for her to teach in a way that she would like to. The curriculum prescribes exactly what should happen each day and what content should be covered daily. A lack of cooperation between management and teachers makes changes in mathematics teaching-learning almost impossible (Handal, 2003:52; Sapkova, 2013:739). Management in this case, does not necessarily refer to the school's management, but rather the education department that prescribes the curriculum.

Margaret also highlighted that she receives very little assistance in the form of problem-solving work sheets and exercises that could help her. In the absence of assistance and professional learning, teachers tend to fall back on more traditional beliefs about mathematics teaching-learning (Beswick, 2012:145).

Taking the above into consideration, a discussion of possible factors that could have led to a change in Margaret's beliefs about mathematics teaching-learning will follow.

5.4.3.3 Possible Factors that could Change Margaret's Beliefs about Mathematics Teaching-Learning

Margaret, like the other participants, expressed that increasing her subject knowledge would contribute to changing her beliefs about mathematics teaching-learning. As

Margaret was introduced to new teaching methods, and by doing research, she started to realise that she wanted to teach in a different way. Blomeke *et al.* (2013:137) found that as teachers' MPCK grew, they were able to appreciate the value of a problem-solving approach to mathematics teaching-learning. Even before enrolling for her honours degree in mathematics education, Margaret wanted to teach in a different way, but did not know how or where to start.

Margaret, like John, highlights the importance of the length of mathematics periods as being a big factor in determining the way she teaches. An obstacle that prevents teachers from teaching in the way that they would like to teach is the length of class periods (Handal, 2003:52; Sapkova, 2013:739). Margaret even suggested doing away with bells that she could then teach in a more effective manner.

Similar to John, Margaret found that the mathematics curriculum prevented creativity. The mathematics curriculum is full of content that learners need to master each day; this knowledge is then tested through an annual national assessment, where all schools are rated based on their performance in these tests. This puts a lot of pressure on mathematics teachers, because if their learners do not perform, the teachers are held responsible. This corresponds to Handal (2003:52) and Sapkova's (2013:739) findings that pressure from parents and management have a direct impact on teachers' mathematics teaching-learning practices.

Margaret also indicated that she would change her beliefs about mathematics teaching-learning if she was given support and materials. Liljedahl *et al.* (2012:106) found that change can occur in mathematics teachers' beliefs about mathematics teaching-learning, after intervention or exposure to new beliefs about teaching-learning mathematics, but once removed from this intervention or exposure, the participant will gradually return back to previous mathematics teaching-learning practices. In the absence of assistance, teachers reverted back to positivist beliefs about mathematics teaching-learning (Beswick, 2012:145).

5.5 The Case of June

5.5.1 Description of June's Background

June is a pre-service teacher with little experience in teaching on a full-time basis. In the following paragraph, June's interview answers will be given.

5.5.2 Findings

June sees mathematics as patterns that are observed by people that can be explained using numbers and figures. She said that mathematics at school does not encompass everything and that mathematics taught at school makes up a part of mathematics as a whole. She said that mathematics includes everything and is a tool and the means by which we understand and learn about the world.

She defines success in mathematics in two ways, the first being in the formal sense and then in general. She said the formal sense is when a learner receives a good mark for the subject and in the general sense when a learner is able to use mathematics effectively in his or her life. She said she is successful in mathematics when she is able to understand the origins, the uses and the implementations of mathematics.

June does not have as much practical experience and feels that when dealing with an incorrect answer, she would probably be prepared for all the possible responses or she would ask another learner to assist in answering questions. June feels that mathematics is learned and taught by using different approaches, but has seen that a problem-solving approach seems to be the most effective. She said that the approach that she would use would depend on the context of the learners. She also thinks that when she teaches, she will probably teach in a traditional way, because of the time and the requirements of the curriculum. She is also not sure how she would implement different teaching methods and would need help to teach in a problem-solving way.

She said that if she would have more time, she would be able to teach in the way that she would like to teach. She also mentioned that if the department had more leniency and restructured the curriculum in a way that gave more time for mathematics, she would be able to teach in the way that she would like to. She said that she would need the cooperation of parents, as well as cooperation of teachers from different subjects. She said that she would be discouraged to teach in a certain way if other teachers were not happy with her teaching or put pressure on her to get certain results. She would also change her teaching if learners' attitudes were negative.

Over the course of her studies, she realised that she wants to teach in a different way, but she is not sure if she will. She also realised that explaining content is not good enough, but that discovery, context, problems and planning play a major part in teaching. She stated that discussions with her professor led to a change in the way that she perceived mathematics and how it should be presented. June makes the statement that she would change her beliefs about mathematics teaching-learning if she gained more knowledge through trial and error, because of the context of the school, as well as her personal motivation and own attitude.

She found her studies very insightful and felt she had grown a lot as a teacher who now thinks more critically about mathematics education. She commented that her knowledge of mathematics has grown, because she had to do reading outside of the prescribed material and came to realise that teachers need to be constantly growing in their understanding of mathematics teaching and learning. She realised that there is not a perfect method to teach mathematics and not an "ultimate approach" to teaching mathematics, rather, that mathematics is a dynamic and growing subject. She said the information gained during the course would definitely influence the way she will teach and relate to learners.

June was also constantly given time to reflect on her own beliefs and ideas. She was given the opportunity to learn and grow independently, while still being accountable for her own performance. Her perspective about what it means to be a "good" mathematics teacher has changed and the critical reflection has caused her to re-evaluate her own values, beliefs and ideas.

5.5.3 Interpretation

The first part of the interpretation will address the first sub-question, followed by possible factors that influence beliefs and what possible factors could lead to change.

5.5.3.1 Describing June's Beliefs about Mathematics Teaching-Learning

June's responses indicate that she has a more traditional view of mathematics teaching-learning, specifically an instrumentalist view of mathematics. June expressed that mathematics can include everything and is a tool and the means by which we understand and learn about the world. This statement correlates to an instrumentalist view of mathematics teaching-learning, because the usefulness of mathematics is overemphasised (Nieuwoudt, 1998:69) and mathematics is reduced to skills, rules and facts that are adapted for use (Ernest, 1991a:115).

She defines success in mathematics in two ways, the first is to achieve good marks and the second is to be able to use mathematics effectively in life. This corresponds with an instrumentalist view of mathematics, because the focus is on the outcomes and product, rather than the process of doing mathematics (Benadé, 2013:17; Nieuwoudt, 1998:75). The focus of an instrumentalist view on mathematics, is increasing the number of fixed plans that a learner can follow to solve mathematics problems effectively (Benadé, 2013:28; Ernest, 1991b:250), which is very similar to what engineers have to be able to do, namely, to fix a problem. June seems to agree with this type of thinking, because even when referring to a problem-solving approach, she would follow it because it is a more effective way to solve problems. This indicates an overemphasis on the pragmatic (Ernest, 1991a:114) and being able to get good results, rather than developing skills and understanding.

June stated that to understand mathematics, is understanding the origins, uses and implementations of mathematics. This is in contrast to an instrumentalist view of mathematics, because this goes deeper than just getting the correct answer, but rather understanding the process of doing mathematics (Molefe, 2006:23). June also has a sense of obligation towards her learners, because she will teach in a way that promotes their understanding and improves their results in mathematics. This relates to making learners part of the teaching-learning of mathematics and not excluding them from the process of learning (Nieuwoudt, 1998:107).

There are some discrepancies between June's beliefs about mathematics teaching-learning and some of her responses. A discussion will follow on possible factors that may contribute to some of June's statements relating to a problem-solving view of mathematics. Although some of her statements indicate that her beliefs about mathematics teaching-learning might be in the process of changing, at the time of the interviews, her responses indicated that she held an instrumentalist view of mathematics teaching-learning.

5.5.3.2 Possible Factors that Influence June’s Beliefs about Mathematics Teaching-Learning Practices

The discrepancy can be explained by Holm and Kajander (2012:7), who found that teachers are likely to teach in the way they were taught. June has been in a traditional classroom most of her life and would find a different way of teaching mathematics strange. June, like Sally, is still young in her teaching career. Beswick (2012:129) states that during the early stages of a teacher’s career, beliefs about mathematics do not always correspond with mathematics teaching-learning practices.

June might have tried to show the “right” belief of mathematics, that may not be her own belief (Gujarati, 2013:636). The discrepancy might also be caused, because she is starting to move from a traditional belief towards a problem-solving belief. She might have difficulty expressing her beliefs, because she might be moving from familiar ways of thinking and acting, to new unexplored ways of thinking and acting (Liljedahl *et al.*, 2012:105). She also expressed her concerns as to how she would implement different teaching-learning methods in her classroom.

June’s beliefs are also influenced to a large extent because of the limited time she has to work with learners to cover a wide range of topics. Research has proven that constraints such as length of class periods and resources are major obstacles that influence teachers’ beliefs about mathematics teaching-learning (Handal, 2003:52; Sapkova, 2013:739).

June’s beliefs are also heavily influenced by expectations of colleagues and management; because of pressure from them to obtain immediate results, she might be teaching in a specific way. An instrumentalist approach to teaching can lead to immediate success, because the learner does not need time to understand a concept, they only need to apply a rule or formula with little or no understanding and immediately find the correct answer (Benadé, 2013:28; Webb & Webb, 2008:14).

Next a discussion will follow on possible factors that could lead to change in June’s beliefs about mathematics teaching-learning.

5.5.3.3 Possible Factors that could Change June's Beliefs about Mathematics Teaching-Learning

The first possible factor that could change June's beliefs about mathematics teaching-learning would be more support from colleagues and parents. June says that if colleagues were to encourage and support her in teaching in a different way and also show her how to teach differently, she would very likely change her beliefs about mathematics teaching-learning. Liljedahl *et al.* (2012:106) support this notion, because they found that when teachers are exposed continuously to interventions and support, changes in beliefs about mathematics teaching-learning will likely occur.

The second factor that influenced her beliefs was when her MPCK increased. June read up on information outside of the prescribed material and in the process realised that teachers need to be constantly growing in their understanding of mathematics. Plotz (2007:16) found that as teachers' subject knowledge increased, they were able to appreciate the value of a problem-solving approach. When new information that does not fit into existing beliefs is received, it is possible that it will lead to cognitive conflict (Holm & Kajander, 2012:10), meaning that the teacher realises that existing beliefs need to be changed to make sense of new knowledge. In discussions with her professor, June also started to realise that her current beliefs about mathematics teaching-learning are not the most effective way to teach. In informal discussion with her professor, June's beliefs about mathematics teaching-learning were challenged (Liljedahl *et al.*, 2006:280) without confronting her beliefs directly, but rather explaining his own beliefs about mathematics teaching-learning.

The third factor June mentioned, that caused a change in her beliefs about mathematics teaching-learning, was reflection. Reflection can be seen as metacognition, referring to the ability to monitor, regulate and evaluate one's own thinking (Nool, 2012:302). June highlighted the importance of being able to reflect on her own beliefs and ideas, which gave her the opportunity to grow independently from the expectations of lecturers. Conner *et al.* (2011:502) found that challenging existing beliefs and also giving teachers time to reflect thoroughly on beliefs and experiences, played a key part in changing mathematics beliefs about teaching-learning. It is extremely important to allow teachers ample time to reflect on their own beliefs after challenging their existing beliefs.

In the final chapter a discussion and recommendations of the study will be presented.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

In reviewing literature, it became apparent that mathematics teachers' beliefs about mathematics teaching-learning have a direct impact on the way that mathematics teachers teach mathematics. It was also found, through literature, that beliefs about mathematics teaching-learning can be changed, but there is a gap in understanding the factors that could lead to change. This study explored these factors and answered the primary research question. In the following paragraphs, the results will be summarised and a conclusion given.

6.2 Conclusion

6.2.1 Factors that Impede Changes to Beliefs about Mathematics Teaching-Learning

Although changes in mathematics teachers' beliefs about mathematics teaching-learning is possible, Liljedahl *et al.* (2012:106) found that changes might be temporary. Once removed from intervention, teachers may gradually return back to previous mathematic teaching-learning practices.

The factors that participants noted in this context, that presented the biggest challenge to them in creating a lasting change in their mathematics teaching-learning practices, was available time. All participants said that the amount of time that they have to teach is very limited. Although research has shown that teaching in a problem-solving manner is more effective, it requires teachers to move from familiar ways of teaching-learning to new unexplored ways of teaching-learning. This mind shift in itself, takes a tremendous amount of time and energy, which participants said that they do not have.

The second factor that most participants noted as being a major barrier is the prescribed curriculum. Teachers are given very little room to use creativity in the teaching-learning of mathematics and have to stick to a strictly prescribed curriculum that tells them exactly what to do. This in itself presents a challenge to teachers to try a new way of teaching-learning mathematics, because it might cause them to fall behind on their planning and schedule.

The third factor that participants identified as a challenge was that there is a lack of support and supporting materials to help them teach in a different way. Teachers have also not been trained on how to create their own materials. This creates a problem, because teachers lack the time and resources to create something in which they have no experience. There is also very limited support in the schools from other teachers and management, as well as virtually no contact with lecturers once degrees are completed. This makes it possible that teachers may just add new experiences to their traditional style of teaching, even though teachers are willing to change their way of teaching mathematics.

The fourth factor that only one teacher highlighted as being a barrier, is sport matches. This factor is very specific to the South African context, where sports are practiced at schools and not privately at clubs. This factor could fall under time constraint because sport matches take away valuable teaching time from teachers; but because it is only a portion of the class that leave for sport, it will be classified on its own.

6.2.2 Factors that could Lead to Changes in Beliefs about Mathematics Teaching-Learning

The following factors have to be present and kept in mind before and after in teacher education, in order to make changes in mathematics teachers' beliefs about mathematics teaching-learning last.

The first factor that is necessary for any changes to possibly occur in mathematics teachers' beliefs about mathematics teaching-learning is that teachers must be exposed to mathematical knowledge. This knowledge should include information about mathematics teaching-learning practices and why these specific practices were developed. Furthermore teachers need to learn more about beliefs about mathematics and how these beliefs form part of teaching-learning practices. Teacher educator must also present beliefs to teachers that are different from their own beliefs, giving teachers an opportunity to identify their own beliefs about mathematics teaching-learning. Mathematics teachers should also be taught about the possible great ideas in mathematics and how they form the basis of mathematics. Increasing teachers' mathematical knowledge, will lead to an increase in confidence, making teachers more open to practices that require more insight and preparation.

The second factor that could possibly contribute to changes in beliefs about mathematics teachers' beliefs about mathematics teaching-learning would be the ongoing support and assistance to teachers. Teachers need someone to continually guide and support them in teaching mathematics in a different way. It requires time and energy to teach in a way that is not familiar, and without the support of others, it is almost impossible that change will occur. For any lasting changes to occur, mathematics teachers need the support not only of colleagues, but also of management and even lecturers. Nearly all participants said if they were given regular support, in the form of motivation and training, they would likely change their beliefs about mathematics teaching-learning and make use of new innovative ways of teaching mathematics. Teachers also need to be motivated by others to continue to strive for improvement and excellence. If learners are not motivated and do not respond to different teaching-learning practices, it will be likely that the teacher will stop trying and revert back to previously held beliefs about mathematics teaching-learning.

The third factor that only one of the participants mentioned, that is crucial to any change taking place, is to be given time to reflect on everything. Teacher educators should continually encourage teachers to reflect on how new knowledge affects their current beliefs about mathematics teaching-learning. It is impossible for any changes to take place if teachers have not really thought about their own mathematics teaching-learning beliefs and how these beliefs might or might not be the best for teaching mathematics. Cognitive conflict will also never occur if teachers do not reflect on new knowledge and current knowledge.

To improve mathematics teaching-learning, teacher education should not only improve mathematical knowledge, but must give sustained professional support to mathematics teachers. Simply focusing only on improving mathematical knowledge will not necessarily change the way mathematics is taught (Philipp, 2007:263). A combination of improving mathematical knowledge, introducing mathematics teachers to beliefs about mathematics teaching-learning - different from their own - giving them time to reflect on their own mathematical knowledge and beliefs and giving continued support throughout, will possibly lead to changes in beliefs about mathematics teaching-learning (Holm & Kajander, 2012:10).

6.3 Recommendations

6.3.1 Recommendations for Teacher Education

Based on the literature study and the empirical results, the following recommendations can be useful for teacher education:

- Mathematics teacher educators need to start modelling problem-solving lessons.
- Mathematics teacher educators must ensure that they give teachers time to reflect and guide them in the process of reflection by making use of metacognition.
- Mathematics teacher educators should include individual informal discussions with all teachers, to encourage them to think about their own beliefs.
- Mathematics teacher educators must look at how support is given to teachers, after the teacher has completed a degree.
- Mathematics teacher educator programmes must create a platform where graduates are put in touch with other graduates, so that they can support one another.
- Mathematics teacher educators should be aware of their own beliefs and how it affects their own teaching-learning practices.

6.3.2 Recommendations for Mathematics Teachers

Based on the literature study and the empirical results, the following recommendations can be useful for teachers:

- Teachers should gain as much knowledge as possible about mathematics teaching-learning and be lifelong learners.
- Teachers should allow themselves enough time to reflect on their own beliefs about mathematics teaching-learning.
- Teachers should consciously look for a mentor that can support them in teaching-learning mathematics in a different way.
- Teachers should take the initiative to never stop learning and continually reflect on their own mathematical beliefs and practices.

6.4 Limitations of this Study

I am aware that my study only makes use of a few participants with the majority of the participants being pre-service teachers. I am also aware that not all teachers in South

Africa have the same background and experiences as my participants. I also have a restricted amount of time to finish my studies, which means that data will not be as rich as it could have been had I had ideal conditions and a longer period of time. It is also impossible to generalise results, because I worked with a small population group, with different experiences and backgrounds; therefore I cannot generalise results to the whole country or even the whole province. The number of teachers enrolled for the postgraduate course was very small and not everybody enrolled was willing to participate in the research.

6.5 Contribution of the Study

This study made a contribution to South African and international literature on understanding teachers' mathematical beliefs and the impact that a mathematics teacher education programme had on teachers' mathematical beliefs. This study also contributed to the subject group Mathematics Education, by understanding the impact that a BEd Hons (Mathematics) programme had on participants' beliefs about mathematics teaching-learning and understanding factors that could lead to lasting change in mathematics teachers' beliefs about mathematics teaching-learning.

6.6 A Final Note

The perfect example of how to create lasting changes in beliefs can be found in the example of Jesus Christ. Jesus illustrated how the process of changing beliefs must happen. The first thing He did was to build a relationship with His disciples. Secondly, He provided them with knowledge that was in conflict with their own beliefs and knowledge. He not only presented knowledge, but illustrated and explained what he meant practically and wonderfully. He also mentored a small group of people, supporting them throughout the whole process. He gave His disciples a chance to go out and teach people on their own, afterwards coming back and reflecting on experiences, giving them a chance to ask questions. After they were equipped to go out and preach the gospel, they also followed the same process, by mentoring somebody else and creating communities that supported them and each other.

I would like to point out the miraculous results this method had on the world. This type of mentoring and teaching led to the greatest growth seen in any religion in the world. I believe that it is only because of the power of God that this was possible, but I believe that using the same approach to train teachers, will lead to great results. It is clear that Jesus Christ already perfected teaching others, we must follow His example and not only teach mathematics, but true wisdom.

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