

# **The development of an alternative rebalancing strategy for a pension fund**

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**MSc (Business Mathematics and Informatics)**

**Thesis submitted for the degree Doctor of Philosophy at the  
Potchefstroom Campus on the North-West University**

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**November 2006**

**Potchefstroom Campus**

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# ACKNOWLEDGEMENTS

This study would not have been successful without the contribution of various individuals. I would like to express my gratitude to the various people who have assisted me throughout my research.

- Prof. J.H.P. van Heerden, my supervisor, for the fact that he always gave the highest priority to my thesis and developing my critical way of thinking.
- My parents and family, for their love, support and continuous encouragement.
- The successful completion of such a mammoth task is always dependant on the sacrifices of others. I truly admire, and am grateful to my wife, Suné, for her support, inspiration and love that carried me through.
- To my two colleagues, Dr E.P.J. Kleynhans and Mr J.J.A. Pearson, for their positive energy and support.
- Mr R. Taylor, for the revision of the grammar and style of the thesis.
- Prof. Lessing, for the revision of the thesis' references.

Above all I would like to thank my heavenly Father for the ability, guidance and strength that He has given me and for answering my prayers.

The Author

Potchefstroom

November 2006

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# ABSTRACT

Portfolio managers typically have two choices: let the portfolio drift with the markets or give direction to the portfolio by rebalancing according to a target allocation. Simply allowing the portfolio to drift with the markets without rebalancing is no longer appropriate. This leaves portfolio managers with the second option of giving direction to the portfolio according to a target allocation. This can be achieved by implementing an existing rebalancing strategy. Portfolio managers typically rely on ad hoc rebalancing strategies that are either calendar-based, such as monthly or quarterly rebalancing, or volatility-based, such as rebalancing whenever the asset ratios are more than 5% from the target.

In the volatility-based rebalancing strategies, the percentage from the target indicates when rebalancing should occur, and the percentage relies on rule of thumb or the use of historical data. The problem with using historical data is that it will not necessarily predict appropriate ranges for the future. These ranges give the indication of when rebalancing should occur. This indicates a need to develop a well-defined rebalancing strategy that assists the portfolio manager to manage a portfolio. Such a rebalancing strategy should be easy to implement.

The aim of this research was to develop and implement a well-defined rebalancing strategy that is adjustable over time to assist portfolio managers to maintain their portfolios in line with the objectives and risk aversions of the trustees' of a pension fund.

The study introduced the concept of a portfolio drift strategy to set the scene for the different rebalancing strategies. This was to emphasize the importance for a

portfolio manager to adopt a rebalancing strategy for a pension fund. An overview is provided of the five broad rebalancing strategies followed by some advantages and disadvantages of each.

Certain rebalancing strategies were used to explain the benefit of rebalancing and the cost of rebalancing. The study investigated different methodologies used to identify when the portfolio manager should rebalance and how far back the portfolio manager should rebalance.

The study focused on Masters' range rebalancing strategy and this strategy was used as the benchmark strategy for the study. The study summarises the benchmark strategy that will be used to evaluate the alternative rebalancing strategies. A selective number of performance measurement tools were used to evaluate the benchmark strategy. This criterion was used to compare the alternative rebalancing strategies.

The study introduced a new decision-making method that has the same return as the benchmark strategy but the risk of the portfolio is lower. The decision-making method is called the second difference method (SD-method). The SD-method builds on Masters' range rebalancing strategy by eliminating certain implicit assumptions made by Masters. The elimination of Masters' implicit assumptions led to an increase in the complexity of the SD-method of the procedure to rebalance. The complexity was eliminated using a computer program formulating the rules of the SD-method.

The study developed a new method called the Cusum Test method (CT-method) with its own assumptions and risk specifications to identify when rebalancing should occur. The new method discussed in the study was less complex to implement and, in contrast to Masters' range rebalancing, the ranges were adjustable over time. The CT-method outperformed Masters' range rebalancing strategy and the SD-method on a risk-adjusted return basis.

## UITTREKSEL

Portefeuljebestuurders het tipies twee keuses: laat die portefeulje met die markte dryf of gee rigting aan die portefeulje, deur middel van herbalansering na gelang van 'n teikentoeuwsing. Om die portefeulje bloot sonder herbalansering te laat dryf is nie meer van toepassing nie. Dít laat portefeuljebestuurders met die tweede keuse, nl. om rigting aan die portefeulje na gelang van 'n teikentoeuwsing te gee. Dít kan bereik word deur 'n bestaande herbalanseringstrategie toe te pas. Portefeuljebestuurders maak tipies staat op strategieë dienaangaande herbalansering wat òf kalendergegrond, soos maandelikse of kwartaallikse herbalansering, òf volatiliteitsgegrond is, soos herbalansering wanneer die bateverhoudings meer as 5% van die teiken af is.

Met die volatiliteitsgegronde herbalanseringstrategieë, dui die persentasie vanaf die teiken aan wanneer herbalansering moet plaasvind, en die persentasie is afhanklik van die praktykreël of die gebruik van historiese data. Die probleem met die gebruik van historiese data is dat dit nie noodwendig toepaslike reekse vir die toekoms sal voorspel nie. Hierdie reekse gee die aanduiding van wanneer herbalansering moet plaasvind. Dit dui 'n behoefte aan om 'n goed gedefinieerde herbalanseringstrategie te ontwikkel wat die portefeuljebestuurder help om 'n portefeulje te bestuur. 'n Herbalanseringstrategie van hierdie aard behoort maklik te wees om toe te pas.

Die doelwit van hierdie navorsing is om 'n goed gedefinieerde herbalanseringstrategie te ontwikkel en toe te pas, wat met die verloop van tyd aangepas kan word, om portefeuljebestuurders te help om hulle portefeuljes in ooreenstemming met die doelwitte van die trustees van 'n pensioenfonds en risikoafkerings te handhaaf.

Die studie het die konsep van 'n portefeuljedrifstrategie voorgestel om die agtergrond van die verskillende herbalanseringstrategieë te skep, om sodoende die belangrikheid daarvan vir 'n pensioenfonds se portefeuljebestuurder om 'n herbalanseringstrategie aan te neem, te beklemtoon. 'n Oorsig van die vyf breë herbalanseringstrategieë het gevolg, asook sommige voor- en nadele van die verskillende herbalanseringstrategieë.

Sekere herbalanseringstrategieë is gedoen ten einde die voordeel van herbalansering en die koste daarvan te verduidelik. Die studie het verskillende metodologieë ondersoek wat gebruik is om te identifiseer wanneer die portefeuljebestuurder moet herbalanseer en hoe ver terug die portefeuljebestuurder moet herbalanseer.

Die studie is op Masters se reeksherbalanseringstrategie gerig en hierdie strategie is as die toonaangewende strategie vir die studie gebruik. Die studie som die toonaangewende strategie op wat gebruik sal word om die alternatiewe herbalanseringstrategieë te evalueer. 'n Selektiewe aantal werkverrigtingsmeetinstrumente is gebruik om die toonaangewende strategie te evalueer. Hierdie maatstaf is gebruik om die alternatiewe herbalanseringstrategieë te vergelyk.

Die studie het 'n nuwe besluitnemingsmetode voorgestel wat dieselfde rendement as die toonaangewende strategie het maar die risiko van die portefeulje is laer. Die besluitnemingsmetode is die tweede verskilmetode ("SD"-metode). Die SD-metode bou op Masters se reeksherbalanseringstrategie deur sekere vanselfsprekende veronderstellings van Masters uit te skakel. Die uitskakeling van Masters se vanselfsprekende veronderstellings het gelei tot 'n toename in die ingewikkeldheid van die SD-metode van die prosedure om te

herbalanseer. Die ingewikkeldheid is uitgeskakel deur 'n rekenaarprogram te gebruik wat die reëls van die SD-metode formuleer.

Die studie het 'n nuwe metode, die Cusum-toetsmetode (CT-metode), met sy eie veronderstellings en risikospesifikasies ontwikkel, om te identifiseer wanneer herbalansering moet plaasvind. Die nuwe metodes wat in die studie bespreek is, is nie so ingewikkeld om te implementeer nie en, in vergelyking met Masters se reeksherbalanseringstrategie, is die reekse met verloop van tyd aanpasbaar. Die CT-metode het Masters se reeksherbalanseringstrategie en die SD-metode op 'n risiko-aangepaste rendementsgrondslag uitpresteer.

# CHAPTER 1

## PROBLEM DESCRIPTION, AIM OF STUDY AND METHODOLOGY

### 1.1. Introduction

It is usual for portfolio managers, in conjunction with the trustees of a pension fund, to set an appropriate initial asset allocation strategy for their pension fund. This determines the weight, in percentage, of the contribution of each asset class to the pension fund. According to Alexander Forbes (2004:2) this asset allocation strategy is determined according to the long-term asset allocation mix to optimise the risk-adjusted return required to achieve objectives of the trustees' of a pension fund. The initial asset allocation mix is assumed to be the fixed target allocation of the pension fund. This assumption must hold to allow valid analysis on pension funds.

Over time, market dynamics, performance differences, changes in the risk appetite of the portfolio managers, or cash inflows and outflows may cause the asset allocation to differ from the target allocation. The difference between the actual asset allocation and the target allocation may create new risks or new opportunities. This may leave the trustees of a pension fund exposed to more risk than they desired or intended.

According to Bradfield and Swartz (2003:3), a portfolio tends to become more risky when allowed to change over time. The further the portfolio moves away from the target allocation, the higher the risk of the portfolio, and vice versa. In the early 1990s, for example, a portfolio manager may have started with an asset allocation of 50% in equity and 50% in bonds. If no rebalancing had occurred along the way, the asset allocation was probably unbalanced by 1999, possibly 75% in equity and 25% in bonds. According to Oswald (2004:53), equity, and especially technology equity, suffered significant losses as the equity market declined steeply from 2000 to 2002. If rebalancing had occurred during the period from the 1990s, the fund's risk and losses would have been significantly lower.

Implementing a well-defined passive rebalancing strategy (see section 2.3) may assist portfolio managers in managing the portfolio along the target allocation. According to Goodsall and Plaxco (1996:1), passive rebalancing is a risk control mechanism. These rebalancing strategies are powerful in their ability to enhance returns as well as to reduce risk.

## **1.2. Problem description**

Portfolio managers typically have two choices: let the portfolio drift with the markets, or give direction to the portfolio by rebalancing according to a target allocation. Simply allowing the portfolio to drift with the markets without rebalancing is no longer appropriate. The reason for this is that, according to Goodsall and Plaxco (1994), "Those who allowed their mix to drift paid the price of the higher volatility than any of the rebalancing disciplines." This indicates that allowing a portfolio to drift will increase the risk of the portfolio, because the volatility of the portfolio is greater.

This leaves portfolio managers with the second option of giving direction to the portfolio according to a target allocation. This can be achieved by implementing an existing rebalancing strategy. Portfolio managers typically rely on ad hoc rebalancing strategies that are either calendar-based (such as monthly or quarterly rebalancing) or volatility-based (such as rebalancing whenever the asset ratios are more than 5% from the target).

In the volatility-based rebalancing strategies, the percentage from the target indicates when rebalancing should occur, and the percentage relies on rule of thumb or the use of historical data (see section 3.2, page 47-48). The problem with using historical data is that it will not necessarily predict appropriate ranges for the future, indicating when rebalancing should occur. According to Goodsall and Plaxco (1996:3), a particular set of bands will cause rebalancing to occur at certain times, but might then allow drift over the entire existing time span if the volatility decreases slightly. Donohue and Yip (2003:49) state that a considerable amount of research has been done in developing optimal asset allocations for the objectives and risk aversion of the trustees of a pension fund, but relatively little research has been done on developing optimal implementation strategies that guide portfolio managers to improve the management of a portfolio. This indicates a need to develop a well-defined rebalancing strategy that assists the portfolio manager to manage a portfolio. Such a rebalancing strategy should be easy to implement.

### **1.3. Aim of the study**

In the problem description, two possible shortcomings were identified in existing rebalancing strategies. The first shortcoming is the use of historical data to calculate the ranges for the different asset classes, indicating when rebalancing should occur. These ranges should be adjustable or flexible as time passes. The second shortcoming is the ease with which the rebalancing strategy might be implemented. The aim of this research is to develop and implement a well-defined rebalancing strategy that is adjustable over time to assist portfolio managers to maintain their portfolios in line with objectives and risk aversions of the trustees of a pension fund.

In order to determine whether the study is viable, certain objectives must be satisfied:

#### **1.3.1. Investigate and implement Masters' rebalancing strategy**

Masters' range rebalancing strategy (see section 3.2) will be investigated to identify possible shortcomings in range rebalancing strategies. Masters' range rebalancing strategy, according to Bradfield and Swartz (2003:18), outperforms existing rebalancing strategies in terms of risk and return. Masters' range rebalancing strategy will be used as a benchmark to measure the performance of proposed rebalancing strategies. One of the performance measurement tools that will be used is the risk-adjusted return. The risk-adjusted return is the ratio between the returns and the risks of the portfolio (see section 3.3.3.3).

### **1.3.2. Development of a well-defined rebalancing strategy**

In order to develop a well-defined rebalancing strategy, certain criteria must be met.

- The rebalancing strategy must be tailor-made to satisfy the needs of the trustees of a pension fund.
- The rebalancing strategy should be easy to implement.
- The rebalancing strategy should be adjustable, so that the ranges indicating when rebalancing should occur can continually be adjusted with time.

### **1.4. Methodology**

A literature review will investigate the five broad categories into which most rebalancing strategies fall. These five categories are: Periodic Rebalancing, Threshold Rebalancing, Range Rebalancing, Volatility-Based Rebalancing and Tactical Rebalancing. The final part of the literature review will focus on Range Rebalancing and, more specifically, the Masters' range rebalancing strategy. Masters' range rebalancing strategies will be the benchmark for this research.

Data obtained from portfolio managers of a pension fund will be used to determine which rebalancing strategy outperforms the rest in terms of risk-adjusted return. In most of the literature, weekly or quarterly data is used. It was decided to use daily data in this study to stay up to date on a daily basis. The

data source has requested anonymity and that the data be considered as classified information.

In the development of an alternative rebalancing strategy, certain statistical inference methods and regression analysis methods will be used. In chapter 5, these methods will be used some integrated literature on these methods will be presented.

### **1.5. The outline of the study**

In chapter 2, an overview will be presented of the five broad categories into which most of the rebalancing strategies fall. The focus will then shift to the benefits and the cost of the different rebalancing strategies. The chapter will conclude by investigating the literature on how far back the portfolio manager should rebalance.

Chapter 3 will investigate and implement Masters' range rebalancing strategy. Certain performance measurement tools will be used to obtain a benchmark for comparison purposes in chapters 4 and 5.

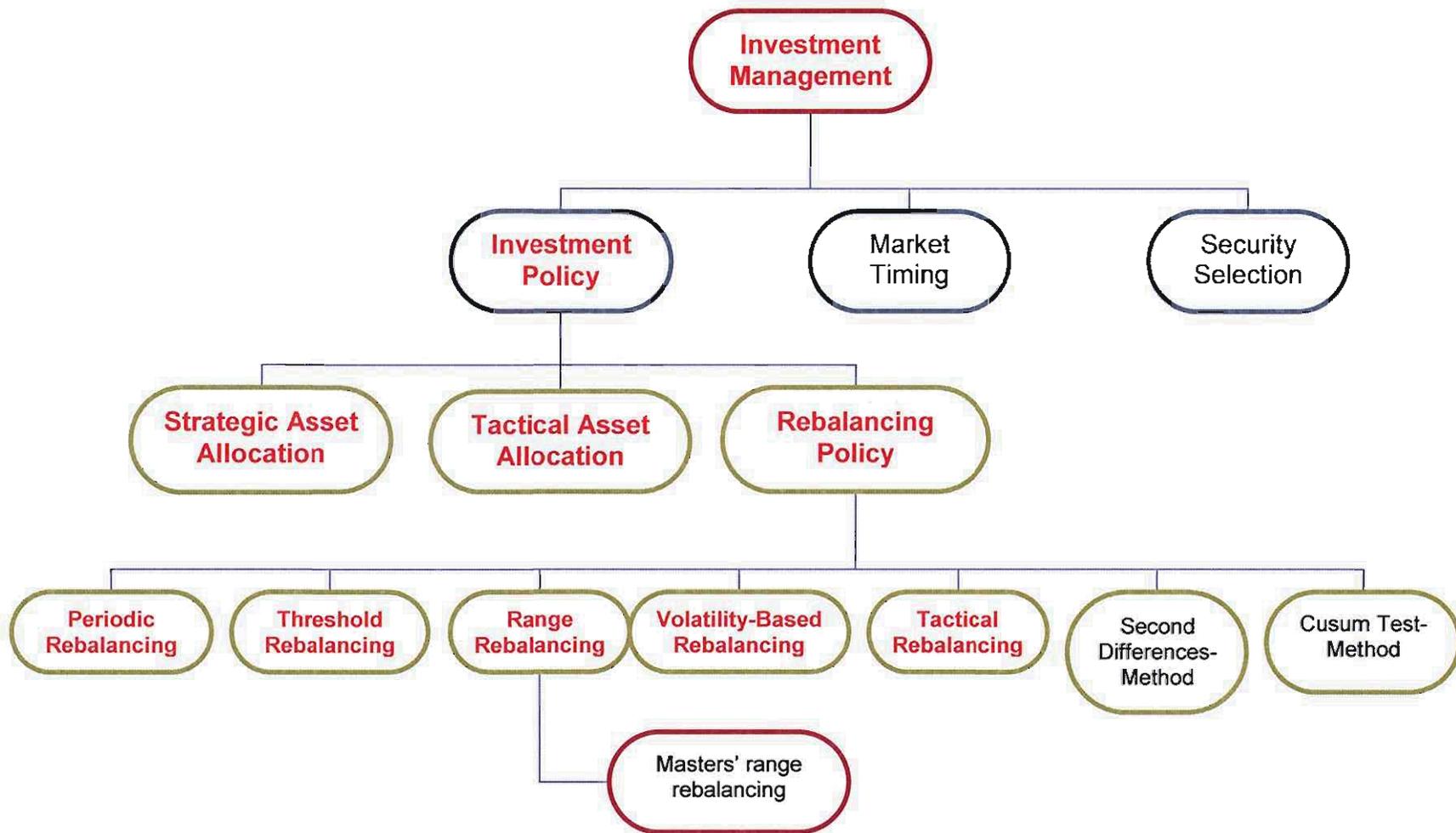
Chapter 4 will introduce an alternative method that a portfolio manager may use to maintain a portfolio according to the target allocation. Chapter 4 will use the calculation that Masters used in his rebalancing strategy, but the chapter will add extra criteria to monitor the portfolio.

Chapter 5 will develop an alternative method to determine when rebalancing should take place. Certain statistical inference methods and regression analysis methods will be introduced to develop the alternative rebalancing strategy.

The last chapter will conclude the thesis and make recommendations for further research opportunities.

The flow diagram in front of each chapter represents the framework of the study. The red highlighted nodes give an indication of what will be discussed in the chapter. The top-down approach represents the activities that comprise the investment management process which include setting an investment policy, market timing (when to move in and out of asset classes) and security selection (which specific securities to buy or sell). The investment policy flows into strategic asset allocation, tactical asset allocation and rebalancing policy. The focus of the study shift solely on the five broad rebalancing strategies but specifically Masters' range rebalancing strategy and expanding the rebalancing strategies by proposing two new rebalancing strategies: second differences method and the cusum test method.

The thesis consists of a glossary pertaining regular used definitions for specific concepts. The glossary is a fold out page at the back of the thesis to assist the reader with regular used concepts throughout the thesis.



# CHAPTER 2

## LITERATURE STUDY

### 2.1 Introduction

Portfolio managers and trustees of a pension fund are the key role players in a pension fund environment. In this study, portfolio managers will refer to managers, and trustees of a pension fund will refer to investors. The main goal of the trustees of a pension fund is to determine an investment policy statement. The investment policy statement<sup>1</sup> should explain the objectives of the trust to guide the trustees of a pension fund to achieve their goals. The portfolio managers use the investment policy statement to determine the long-term asset allocation mix, called strategic asset allocation. Sharpe (1992:7) defines asset allocation as the allocation of an investor's portfolio among a number of specific asset classes. According to Chan (1998), the process of asset allocation can be divided into three stages:

- Strategic asset allocation
- Tactical asset allocation
- Portfolio rebalancing

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<sup>1</sup> For further insight in the investment policy statement, refer to the Trustee Act 2000, Investment Guide for Trustees.

Strategic asset allocation is the process whereby pension funds derive a long-term asset allocation mix to optimise the risk-adjusted return to achieve the investment objectives of the trustees of a pension fund (Alexander Forbes, 2004:2). The strategic asset allocation is the initial asset allocation of the pension fund and is called the target allocation (see section 1.1).

The initial asset allocation changes over time as market conditions change. These changes in market conditions can create new opportunities and/or new risks. This may leave the trustees of a pension fund exposed to more risk than they desired or intended in their investment policy statement. According to Firer *et al.* (2003:17) this exposure will leave the portfolio manager with at least three alternatives: allowing the portfolio to drift, buy more of the best performing asset class at the expense of the lesser performing asset classes, or rebalance back to a target allocation. Rebalancing is the process of making adjustments to the asset allocation to ensure that the long-term asset allocation satisfies the objectives of the trustees of a pension fund. A survey of 401 000 participants by the Investment Company Institute (2000) found that only 25% of investors made any changes in the fund's allocation since they first enrolled in their investment. Of the rest, most had made only one or two changes over the lives of their accounts.

Tactical asset allocation should not be confused with rebalancing. Rebalancing and tactical asset allocation are related strategies but not identical. Tactical asset allocation actively changes the asset allocation mix based on market conditions, while rebalancing changes the asset allocation mix based on calendar-based rebalancing strategies or volatility-based rebalancing strategies (see section 2.3). Tactical asset allocation is also known as market timing and is based on the belief that markets are inefficient. This belief allows investors to increase investment returns by timing the markets' cycles in order to make

certain moves between asset classes. For further insight in tactical asset allocation refer to Firer *et al* (1992) and Waksman *et al* (1997). This study focuses solely on passive rebalancing strategies. Passive rebalancing strategies assume that markets are efficient.

Most passive rebalancing strategies fall into one of five broad categories:

- Periodic rebalancing
- Threshold rebalancing
- Range rebalancing
- Volatility-based rebalancing
- Tactical rebalancing

Chapter 2 (section 2.3) will present an overview of the five different passive rebalancing strategies. The next section of chapter 2 will discuss the benefit of rebalancing using the studies of Plaxco and Arnott (2002:9-22), Buetow *et al* (2002:23-32) and Masters (2003:52-57). The discussion of the cost of rebalancing will follow the benefit of rebalancing focusing on Plaxco and Arnott (2002:9-22), Buetow *et al* (2002:23-32) and Masters (2003:52-57).

In order to understand the five different passive rebalancing strategies, it is necessary to discuss the drift of a portfolio.

## 2.2 Portfolio Drift

Portfolio managers typically have three alternatives:

- Let the portfolio drift with the market.
- Buy more of the best performing asset class at the expense of the lesser performing asset classes.
- Give direction to the portfolio by rebalancing according to a target allocation.

Portfolios tend to drift away from their target allocation because that the asset classes evolve differently over the investment period. Trustees of a pension fund may have emotional attachments to the portfolio, especially when the asset classes appreciate and add value to the portfolio. When the portfolio manager decides to rebalance the portfolio, the portfolio manager will typically buy additional assets in an asset class that loses value and sell assets for the asset classes that add value. This will led to a dispute between trustees of a pension fund and portfolio managers, because trustees of a pension fund won't agree to selling the assets in an asset class that adds value and buying assets in an asset class that loses value (Nersesian, 2005:109-110). According to Goodsall and Plaxco (1996:1): "A drifting strategy will ensure a minimum position before a rally and a maximum position when a market falls, which will minimise gains and maximise losses."

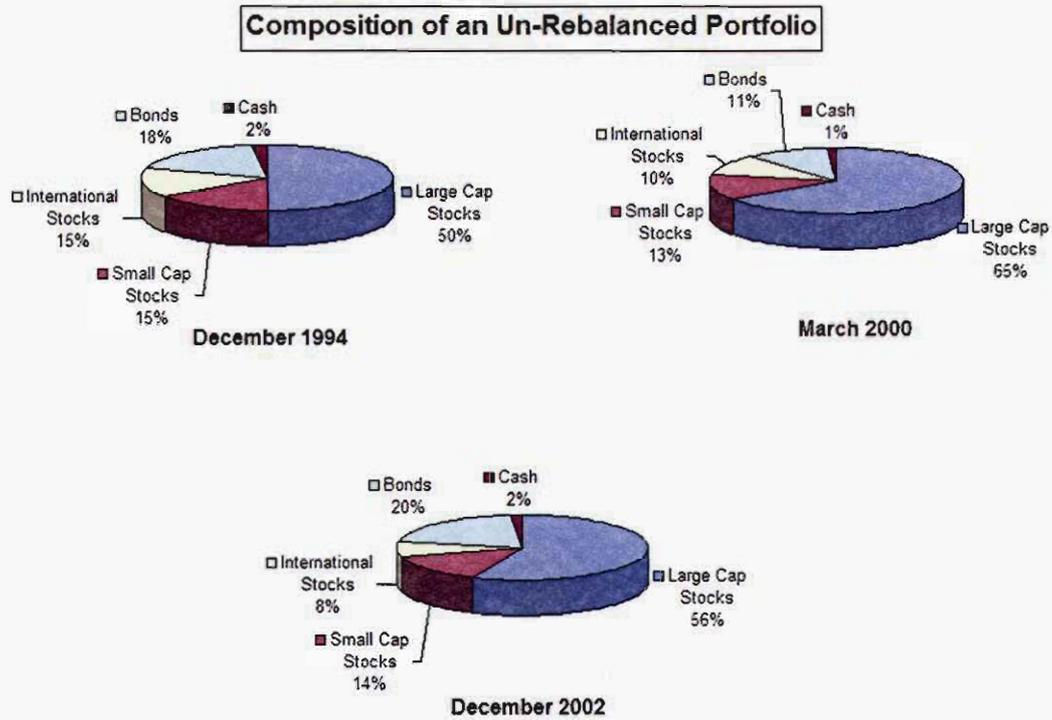
By allowing a portfolio to drift with a market, the asset allocation changes as time passes by and moves away from the strategic asset allocation mix. Therefore the time and effort spent on developing a strategic asset allocation mix would be unnecessary. As time passes, the actual allocation may well move far away from

the strategic asset allocation mix, allowing potentially risky results. On the other hand, portfolio drift allows much more volatility in the rate of return than in the case of rebalancing. Higher volatility implies higher risk. There are three separate findings on portfolio drift:

- “On a risk-adjusted basis there is no period in which drift has beaten rebalancing.” (Arnott & Lovell, 1993)
- “Those who allowed their mix to drift paid the price of the higher volatility than any of the rebalancing disciplines.” (Goodsall & Plaxco, 1994)
- “In allowing a portfolio to drift, one typically finds that the portfolio tends to become more risky.” (Bradfield & Swartz, 2003:3)

Two different examples by Montague (2006:1-5) and Nersesian (2005:109-113) will illustrate these three findings. Figure 2.2.1 shows how the strategic asset allocation mix can change, allowing a portfolio to drift with the market. Montague (2006:1) tracked a hypothetical portfolio consisting of five different asset classes from December 1994 to December 2002. The portfolio was allowed to drift with the market and no cash inflows or outflows occurred during that period.

The strategic asset allocation mix consists of: 50% in U.S. large cap stocks, 15% in U.S. small cap stocks, 15% in international stocks, 18% in bonds and 2% in cash.



**Figure 2.2.1: Composition of an Un-Rebalanced Portfolio**

In the late 1990s, the equity market experienced an exceptional period moving into a bull market. This resulted in almost a 90% composition of total equity share (large cap stocks, small cap stocks and international stocks) of the portfolio at the end of March 2000. Bonds and cash made up the remaining portion of the portfolio.

The shift to equities led to an increase in the returns of the portfolio compared to the strategic asset allocation mix. If the market conditions change to a down trend, it may lead to a decrease in the returns of the portfolio. This position will increase the risk formulated by the trustees of a pension fund in their long-term objectives.

The study by Nersesian (2005:109-113) introduced a hypothetical portfolio divided into five asset classes over a twenty-year period. The return on the portfolio drifting along the market was \$8,874,742 with a volatility (risk) of 13.6%. The same portfolio was rebalanced on an annual basis. The return on the portfolio was \$9,324,229 with a volatility of 11%. This indicates that allowing a portfolio to drift along the markets may introduce lower return subject to higher risk.

The following section discusses the five broad rebalancing strategies: periodic rebalancing, threshold rebalancing, range rebalancing, volatility-based rebalancing and tactical rebalancing.

### **2.3 Different Rebalancing Strategies**

The goal of the trustees of a pension fund is to determine an investment policy statement. The investment policy statement should explain the long-term objectives of the trust. Rebalancing takes place to meet the long-term objectives of the trust. The term portfolio rebalancing refers to the continuous adjustments made to the asset allocation of the portfolio to keep the long-term strategic asset allocation mix in line with the long-term objectives. According to Firer *et al* (2003:18) rebalancing is a simple process but powerful in its ability to bring discipline to the asset allocation process. This discipline aligns the asset allocation of the portfolio with the desired strategic asset allocation mix, eliminating the probability of allowing the portfolio to drift with the markets.

The following two sections will present the five broad rebalancing strategies: periodic rebalancing, threshold rebalancing, range rebalancing, volatility-based rebalancing and tactical rebalancing. The next section presents underlying

information on the following rebalancing strategies: periodic rebalancing, threshold rebalancing and range rebalancing.

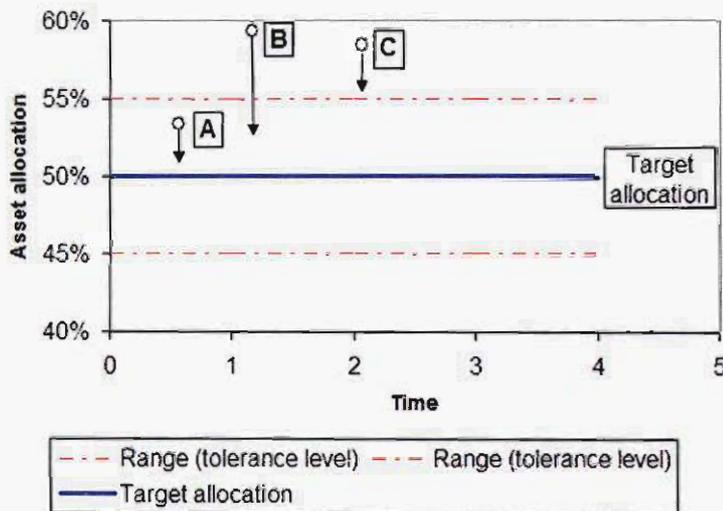
### **2.3.1 Periodic rebalancing, threshold rebalancing and range rebalancing**

Arnott and Lovell (1993) suggested that three different rebalancing strategies could be considered. As discussed in section 2.1 there are five different rebalancing strategies. The last two strategies were introduced after 1993.

The first rebalancing strategy to consider is periodic rebalancing. Periodic rebalancing is when the actual asset allocation is rebalanced back to the target allocation on a regular schedule, such as monthly, quarterly, semi-annually or annually.

Some advantages of periodic rebalancing are its simplicity and lower transaction costs if the rebalancing periods are longer. The disadvantage for longer rebalancing periods is that this allows for a higher tracking error. Another disadvantage is the importance of market timing to enhance the performance of the strategy, because the strategy is independent of market performance.

In figure 2.3.1.1 at point A, under periodic rebalancing, the portfolio should be rebalanced back to the target allocation at the end of the specific schedule (e.g. monthly).



**Figure 2.3.1.1: Different Rebalancing Strategies for a specific asset class**

The second rebalancing strategy is known as threshold rebalancing. Threshold rebalancing occurs when the portfolio is rebalanced back to the target allocation whenever a particular asset class has shifted by more than a pre-defined percentage. For example, suppose the target allocation for equity is 50% and the pre-defined percentage is 5%. Whenever equity breached 55% on the upside or 45% on the downside, rebalancing would take place to bring equity back to 50% (target allocation).

Threshold rebalancing is more flexible than periodic rebalancing, but if the market is very volatile then rebalancing may take place too often and that may lead to unnecessary transaction costs and therefore decreasing returns.

In figure 2.3.1.1 at point B, under threshold rebalancing, the portfolio should be rebalanced back to the target allocation whenever the pre-defined percentage is breached.

Range rebalancing is the third rebalancing strategy suggested by Arnott and Lovell (1993). This range rebalancing strategy is similar to threshold rebalancing, but a range system is based on pre-defined triggers. The pre-defined percentage of threshold rebalancing is generated by rule of thumb. Range rebalancing uses historical data for each asset class to determine the pre-defined triggers which are also percentages. These trigger points can be monitored daily, monthly or even only yearly. Chapter 3 will explain range rebalancing, and particularly Masters' range rebalancing strategy. Masters' range rebalancing strategy will to be used as the benchmark strategy.

There are a number of advantages in range rebalancing. Range rebalancing is easy to implement. Wider ranges will reduce transaction costs and enhance tracking error costs and vice versa. Range rebalancing incorporates market performance, meaning the reaction of asset performance to the market performance will trigger rebalancing. Some shortcomings in range rebalancing can be identified. In a portfolio consisting of a wide range of target allocations, setting an effective percentage range is difficult. The frequency of rebalancing would increase if an asset class has greater volatility and a higher target allocation.

In figure 2.3.1.1 at point C, under range rebalancing, the portfolio should be rebalanced back to the range whenever the pre-defined triggers are intersected.

Section 2.3.2 will conclude with the last two rebalancing strategies: volatility-based and active rebalancing.

### **2.3.2 Volatility-based rebalancing and active rebalancing**

The last two rebalancing strategies are volatility-based rebalancing and active rebalancing. Volatility-based rebalancing is based on the relative volatility of the different asset classes. If the volatility of an asset class increases, the ranges will more widely spread around the target allocation. The ranges for small-cap stocks would be considerably wider than the ranges for short-term bonds, because stocks are more volatile than bonds. Active rebalancing strategy is based on expected market conditions. Active rebalancing is similar to tactical asset allocation, which attempts to forecast short-term market trends. Tactical asset allocation actively changes the asset allocation mix based on market conditions.

To put the theory into reality, the next section will discuss the benefits of rebalancing by means of comparing the different rebalancing strategies. The study will focus on three different studies by Plaxco and Arnott (2002:9-22), Buetow *et al* (2002:23-32) and Masters (2003:52-57) to illustrate the benefit of rebalancing.

## **2.4 Benefit of Rebalancing**

In the investment policy statement, the trustees determine the main objectives of the portfolio. The portfolio managers use this investment policy statement to determine the target allocation of the portfolio. The target allocation should be maintained to achieve the main objectives of the portfolio. When the portfolio's actual asset allocation differs from the target allocation, the so-called tracking error will occur, implying higher risk. The key benefit of rebalancing is to reduce this tracking error. By reducing the tracking error the risk of the specific asset

class will decrease. Rebalancing is the adjustment of the portfolio's actual asset allocation to the target allocation.

### 2.4.1 Periodic rebalancing versus Portfolio drift

Plaxco and Arnott (2002:10) review the passive rebalancing strategies (discussed in section 2.3) used for the analysis on their different portfolios. The study compares the passive rebalancing strategies in different environments and addresses the benefits of rebalancing from an alpha perspective<sup>2</sup> and a risk control perspective.

The study begins by comparing the performance of the various passive rebalancing strategies with a portfolio drift on a global portfolio<sup>3</sup> over the last twenty-one years. Table 2.4.1.1 summarises the results of the analysis on the global portfolio using two passive rebalancing strategies. The table consists of two passive rebalancing strategies and portfolio drift presented in the columns. Periodic rebalancing strategy consists of three different rebalancing frequencies: monthly, quarterly and annually. Range rebalancing consists of two components indicating to what level the portfolio manager should rebalance. Refer to section 2.6 for the explanation as to what level the portfolio manager should rebalance. The rows consist of three performance measurement tools used to compare the different strategies. The three performance measurement tools are: annualised return, standard deviation and risk-adjusted return. Refer to section 3.3.3 for the calculations of the different performance measurement tools. The standard deviation is the estimate for the risk of a portfolio.

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<sup>2</sup> According to Banz (1981), alpha refer to the excess returns. Refer to Grinold and Khan (2000:52) for the formal definition of alpha.

<sup>3</sup> Global portfolio consists only of international asset classes.

Portfolio drift has a lower return compared to the return of the two passive rebalancing strategies and underperforms significantly on a risk-adjusted return basis (risk-adjusted return = 1.23). This indicates that the portfolio drift has a greater risk for lower returns. For further insight in the risk-adjusted return refer to section 3.3.3.3. Table 2.4.1.1 indicates that the two rebalancing strategies outperform the portfolio drift according to their risk-adjusted returns. Note the cost of rebalancing was implemented in the analysis of Plaxco and Arnott (2002:10-14). Refer to section 2.5 for the calculation of the cost of rebalancing.

**Table 2.4.1.1: Different rebalancing strategies on a global portfolio**

**GLOBAL REBALANCING STRATEGIES**  
January 1980 - December 2000

	Rebalancing Strategies					Portfolio Drift
	Periodic Rebalancing			Range Rebalancing		
	Monthly	Quarterly	Annual	to the Range	to Benchmark	
<b>Annualized Return</b>	13.44%	13.50%	13.50%	13.58%	13.36%	13.12%
<b>Standard Deviation</b>	9.31%	9.31%	9.47%	9.72%	9.42%	10.64%
<b>Risk-Adjusted Return</b>	1.44	1.45	1.43	1.40	1.42	1.23

Ranges are  $\pm 5\%$

Global Portfolio: 60% equity and 40% bond

Source: Plaxco and Arnott (2002:11)

The study of Plaxco and Arnott (2002:11-12) further explored the various passive rebalancing strategies with a portfolio drift on domestic US rebalancing<sup>4</sup> and global asset class rebalancing<sup>5</sup>. The columns of table 2.4.1.2 consist of the performance measurement tools and the equity allocation. The performance measurement tools are used to compare the various passive rebalancing strategies with one another. The equity allocation is divided into two sub-

<sup>4</sup> Domestic US portfolio consists only of US asset classes.

<sup>5</sup> Global asset classes refer to a portfolio consisting of international asset classes.

columns: end and average. This is an indication of the percentage allocation of equity at the end of the period as well as the average allocation of equity. The first two sub-rows are the two asset classes used in the study of Plaxco and Arnott (2002:11) and the rest are the various passive rebalancing strategies. Table 2.4.1.3 contains the same information. The results are different because the study of Plaxco and Arnott (2002:11) used a global asset class rebalancing.

**Table 2.4.1.2: Different rebalancing strategies on a U.S. domestic portfolio**

**DOMESTIC US REBALANCING**  
January 1968 - December 2000

		Annualized Return	Standard Deviation	Risk-Adjusted Return	Equity Allocation	
					END	AVERAGE
<b>Asset</b>	<b>Equities</b>	12.13%	15.29%	0.79	100.0%	100.0%
<b>Classe</b>	<b>Bonds</b>	8.55%	10.42%	0.82	0.0%	0.0%
<b>Portfolio drift</b>		10.79%	11.23%	0.96	74.5%	58.2%
<b>Rebalancing</b>	<b>Monthly</b>	10.63%	10.59%	1	50.0%	50.0%
	<b>Quarterly</b>	10.68%	10.57%	1.01	50.0%	50.1%
	<b>Annual</b>	10.64%	10.65%	1	50.0%	50.7%
	<b>To the Range</b>	10.62%	10.63%	1	47.8%	51.4%
	<b>To Benchmark</b>	10.56%	10.64%	0.99	47.6%	50.6%

Transaction costs of 10 basis points  
Ranges  $\pm 5\%$

Source: Plaxco and Arnott (2002:12)

Table 2.4.1.2 shows that the annualised returns for quarterly rebalancing are 11 basis points lower from 10.79% to 10.68% than portfolio drift, as well as 66 basis points reduction in risk. Quarterly rebalancing only cost 11 basis points to reduce the standard deviation (risk) to 66 basis points from 11.23% to 10.57%. This leads to the conclusion that quarterly rebalancing outperforms the portfolio drift on a risk-adjusted return basis.

**Table 2.4.1.3: Different rebalancing strategies on global asset classes**

**GLOBAL ASSET CLASS REBALANCING**  
January 1980 - December 2000

		Annualized Return	Standard Deviation	Return/ Risk	Equity Allocation	
					END	AVERAGE
<b>Asset</b>	<b>Equities</b>	11.80%	14.34%	0.82	100	100
<b>Classe</b>	<b>Bonds</b>	9.43%	7.81%	1.21	0	0
<b>Portfolio drift</b>		10.81%	9.78%	1.11	66	56
<b>Rebalancing</b>	<b>Monthly</b>	10.87%	9.19%	1.18	50	50
	<b>Quarterly</b>	10.96%	9.17%	1.2	50.1	50.2
	<b>Annual</b>	10.90%	9.21%	1.18	50.7	51.8
	<b>To the Range</b>	10.89%	9.24%	1.18	50.9	53.9
	<b>To Benchmark</b>	11.00%	9.22%	1.19	50.8	51.7

Transaction costs of 10 basis points  
Ranges  $\pm$  5%

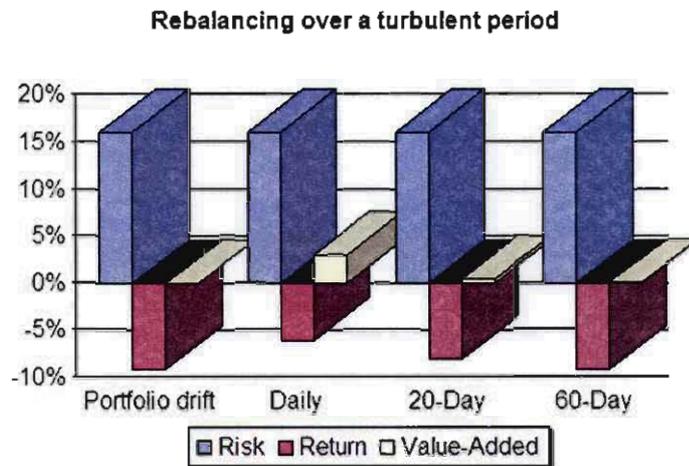
Source: Plaxco and Arnott (2002:12)

Table 2.4.1.3 shows the results of global asset class rebalancing. The portfolio drift strategy allows an overweighing of equity (66% at the end and 56% average) according to the target allocation of equity (50%). This will result into a more risky strategy (9.78%) than quarterly rebalancing (9.17%) if markets turn. Over the period, quarterly rebalancing outperforms portfolio drift in both return and risk.

The study of Plaxco and Arnott (2002:12-15) continued by discussing the benefits of rebalancing from an alpha perspective and a risk control perspective. Rebalancing gives direction to streamline the portfolio according to the target allocation. Rebalancing addresses the risks that can occur in a portfolio, but the challenge comes in converting rebalancing into a profit centre (alpha). According to Plaxco and Arnott (2002:12), two alternatives can be identified to enhance the return while maintaining a constant risk on the portfolio. These alternatives are

daily rebalancing and tactical rebalancing. Tactical rebalancing is not part of the scope of this study (see section 2.1).

The benefits of daily rebalancing occur when markets frequently revert to the mean and when there is less correlation among the global markets. Over the long term, the benefits of various rebalancing strategies converge, making the field even (Plaxco & Arnott, 2002:12). Conversely, in the short term, certain rebalancing strategies outperform the other rebalancing strategies depending on the state of the market. Figure 2.4.1.1 compares the results of different rebalancing strategies to illustrate the benefit of daily rebalancing during a volatile market. Across the world, the equity markets in the year 2000 experienced significant upward and downward trends at different times. During this period, most of the rebalancing strategies performed the same in the return and risk environment except for daily rebalancing. Figure 2.4.1.1 illustrates that the daily rebalancing strategy earned an extra 300 basis points after transaction costs.



**Figure 2.4.1.1: Rebalancing over a turbulent period**

Source: Plaxco and Arnott (2002:13)

On a risk basis, the benefit of rebalancing is to compensate for any sudden and dramatic movements in the market. Allowing a portfolio to drift with the market, the portfolio manager will have no control on the asset allocation.

Table 2.4.1.2 shows the result of the portfolio drift finishing with a weight of 74.5% in equity (see column 4). In comparison with the quarterly rebalancing strategy that finished with 50% of the portfolio in equity (see column 4). The portfolio drift is overweighted in a risky asset class that puts the investor at far greater risk than they intended. In this comparison, an additional 11 basis points is added by taking on an additional 66 basis points of risk. The extra compensation for risk is not worthwhile for 11 basis points in return, but this will depend solely on the investment policy statement of the investors. For further

insight into periodic rebalancing refer to the studies by Arnott and Lovell (1990, 1993).

#### **2.4.2 Threshold rebalancing combined with frequency monitoring versus portfolio drift**

The study of Buetow *et al* (2002:24) combines a threshold deviation interval with frequent monitoring of the portfolio (chapter 4). The study focused on controlling the risk of the portfolio but, in doing so, adds value to the portfolio that will lead to enhanced returns. Buetow *et al* (2002:25-30) went further to implement their strategy on a two-asset case, a four-asset case using simulation, and a four-asset case using bootstrap. This section will only give an overview on the two-asset case.

The analysis begins with a simple two-asset case using U.S. stocks (S&P 500 index) and bonds (J.P. Morgan All Government Index). It was assumed that the asset allocation is 60% in U.S. stocks and 40% in bonds. The study found that the daily returns of the two classes of assets are best described by a logistic distribution. Table 2.4.2.1 shows the returns of the two classes of assets over the period January 1987 to August 2000. Table 2.4.2.1 presents the average equity and bond weightings at different threshold intervals using daily frequency monitoring and 10 000 simulations. The columns of table 2.4.2.1 consist of the different rebalancing intervals (1%, 5%, 10% and 20%). The first column in figure 2.4.2.1 is the correlation between equity and bonds where -1 is the indication that equity and bonds are not correlated and +1 indicates that equity and bonds are correlated. For different correlations between equity and bonds, different average exposures exist according to different rebalancing intervals using daily simulations. The last row of table 2.4.2.1 presents the average asset exposures at different rebalancing intervals using actual daily data.

**Table 2.4.2.1: Simulated asset exposures for a two-asset portfolio****Simulated asset exposures for a two-asset portfolio**

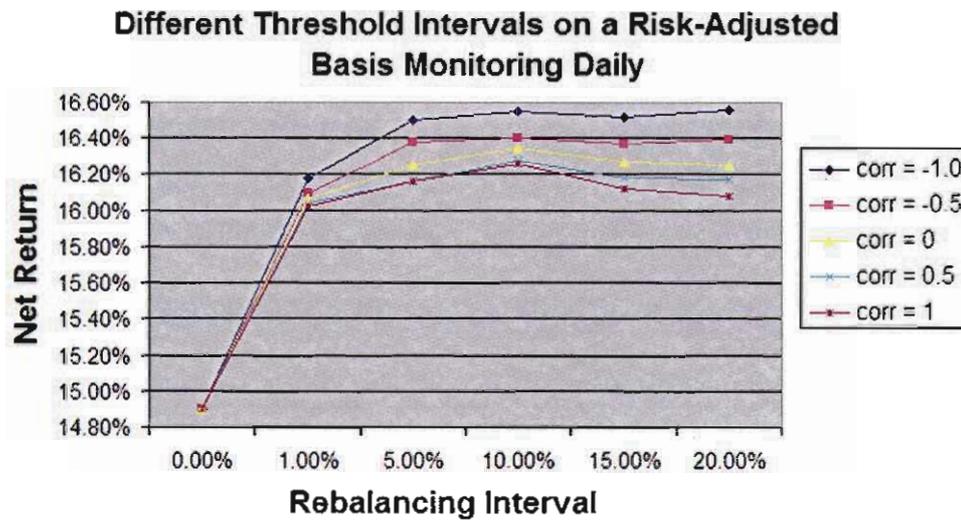
	Average Asset Exposures at Different Rebalancing Intervals using Daily Simulations							
	1%		5%		10%		20%	
Correlation	Equity	Bonds	Equity	Bonds	Equity	Bonds	Equity	Bonds
-1.0	60.01%	39.99%	60.11%	39.89%	60.41%	39.59%	61.60%	38.40%
-0.5	60.01%	39.99%	60.13%	39.87%	60.52%	39.48%	62.01%	37.99%
0.0	60.01%	39.99%	60.17%	39.83%	60.67%	39.33%	62.44%	37.56%
0.5	60.01%	39.99%	60.24%	39.76%	60.84%	39.16%	63.13%	36.87%
1.0	60.02%	39.98%	60.42%	39.58%	61.47%	38.53%	64.09%	35.91%

	Average Asset Exposures at Different Rebalancing Intervals using Actual Daily Data							
0.1462	60.07%	39.93%	61.02%	38.98%	62.33%	37.67%	64.76%	35.24%

Source: Buetow *et al.* (2002:25)

Figure 2.4.2.1 compares the different threshold intervals (see section 2.3.1) on a risk-adjusted return basis, but assuming fixed (daily) frequency monitoring. From figure 2.4.2.1 it can be concluded that a 5% threshold interval for the different correlations between the asset classes outperforms the other threshold intervals on a risk-adjusted return basis. If the threshold interval is larger than 5%, the returns in most of the different correlations between the asset classes are higher but with much greater underlying risks. Thus figure 2.4.2.1 illustrates that for a 10% rebalancing interval, the return is 16.55% if the correlation between equity and bonds is -1. For a 5% rebalancing interval the return is 16.5% if the correlation between equity and bonds is -1. This leads to the conclusion that, for a 5% increase in the rebalancing interval (risk) in this scenario, the return will only increase by 0.05%, meaning that the trustees of a pension fund will be more exposed to risk (5%) for only 0.05% extra return on their investment.



**Figure 2.4.2.1: Different Threshold Intervals on a Risk-Adjusted Basis Monitoring Daily**

Source: Buetow *et al* (2002:27)

The study concluded the two-asset class case by fixing the threshold interval (5%) over the different frequency monitoring periods. Table 2.4.2.2 summarises the results, fixing the threshold interval over different frequency monitoring periods. The study concluded that the 5% threshold interval with daily monitoring outperformed the rest with a return and risk ratio of 1.5. As mentioned before, the study of Buetow *et al* (2002:28-30) went further on the four-asset case with simulation and the four-asset case with bootstrapping, but this is not part of the scope of this study.

The study of Plaxco and Arnott (2002) implemented and compared periodic rebalancing, range rebalancing and portfolio drift. The study of Buetow *et al* (2002) went further and explored the combination of different rebalancing strategies. The combination is threshold rebalancing with frequency monitoring.

In contrast, Masters' (2003) range rebalancing strategy uses historical data to calculate the ranges to indicate when rebalancing should occur.

**Table 2.4.2.2: Net Return and Risk Results for a 5% Rebalancing Interval**

**Actual Net Return and Risk Results for a 5% Rebalancing Interval (1987 - 2000)**

	Constant Mix	Daily	Monthly	Quarterly	Semiannual
Rebalanced Portfolio					
Average Net Return	13.46%	13.81%	12.32%	12.30%	12.22%
Standard Deviation	9.20%	9.20%	9.20%	9.20%	9.20%
Return/Risk	1.46	1.50	1.34	1.34	1.33

Source: Buetow *et al.* (2002:29)

Masters' range rebalancing strategy quantifies the benefit of rebalancing as a quadratic function taking into consideration tracking error, percentage deviation from the target allocation, and the risk tolerance of the portfolio manager. The net benefit of rebalancing is defined as the difference between the benefit of rebalancing and the cost of rebalancing. For further explanation refer to chapter 3. Masters defines the net benefit of rebalancing as the trigger point that indicates when rebalancing should occur. This trigger point is calculated for each asset class determining the ranges around the target allocation. Table 2.2.4.3 compares the Masters' range rebalancing strategy against portfolio drift over the period January 1997 to December 2001. Table 2.2.4.3 consists of different performance measurement tools presented in the columns and different strategies presented in the rows. The different strategies used in table 2.2.4.3 are the un-rebalanced strategy (portfolio drift) and Masters' range rebalancing strategy, rebalancing back to the target and rebalancing only halfway back to the target. Masters' range rebalancing, rebalancing back to the target (annualised return = 7.75% and volatility = 11.92%) and rebalancing only halfway back to the

target (annualised return = 8.04% and volatility = 11.90%) outperform the un-rebalanced strategy (annualised return = 7.48% and volatility = 12.82%) in terms of return and risk.

**Table 2.4.2.3: Masters' Range Rebalancing versus Portfolio Drift**

Comparison of Rebalancing Strategies - January 1997 - December 2001

<b>Strategy</b>	<b>Annualized Return</b>	<b>Tracking Error</b>	<b>Volatility</b>	<b>Number of Rebalancings</b>	<b>Annualized Cost of Rebalancing (bp)</b>
Unrebalanced	7.48%	1.38%	12.82%	0	0
Rebalance to Target	7.75%	0.49%	11.92%	6	5
Rebalance Halfway	8.04%	0.52%	11.90%	10	5

Source: Masters (2003:56)

A study by Bradfield and Swartz (2003:14) implemented Masters' range rebalancing strategy on a fund in South Africa. The study of Bradfield and Swartz (2003:14-18) went further by comparing periodic rebalancing to Masters' range rebalancing strategy. Table 2.4.2.3 summarises the results of the different strategies used:

- A portfolio drift (un rebalanced)
- A portfolio that is rebalanced back to the target when the trigger point is intersected
- A portfolio that is rebalanced halfway back between target allocation and the trigger point when the trigger point is breached
- Four portfolios based on periodic rebalancing back to the target allocation.

Table 2.4.2.4 consists of performance measurement tools presented in the columns and the different rebalancing strategies presented in the rows. In table 2.4.2.4 Masters' range rebalancing strategies halfway back to the target (risk-adjusted return = 1.56) outperforms quarterly rebalancing (risk-adjusted return = 1.40) and portfolio drift (risk-adjusted return = 1.42) on a risk-adjusted return basis. Refer to section 3.3.3.3 for the definition and calculation of the risk-adjusted return.

**Table 2.4.2.4: Masters' Range Rebalancing versus Periodic Rebalancing**

**Masters' range rebalancing versus Periodic Rebalancing**

Results		Annualized Return	Annualized Standard deviation	Risk-Adjusted Return	Tracking Error	Average Number of Share	Turnover per annum	
Rebalancing Strategies	Range Rebalancing	1/2 Trigger	18.03%	11.54%	1.56	0.12%	5.7	7.26%
		To Target	16.79%	11.60%	1.45	0.31%	5.4	9.28%
		Drift	17.33%	12.17%	1.42	0.84%	0	0.00%
	Periodic Rebalancing	Monthly	12.50%	11.72%	1.07	1.26%	12	19.76%
		Quarterly	16.08%	11.45%	1.40	0.97%	3	8.88%
		Bi-Annually	17.02%	11.33%	1.50	1.06%	2	5.98%
	Annually	16.84%	11.41%	1.48	1.05%	1	2.19%	
Benchmarks	Equities	14.87%	21.49%	0.69	11.44%	0	0.00%	
	Bonds	19.61%	8.79%	2.23	11.44%	0	0.00%	
	50:50 Equity Bonds	17.98%	11.58%	1.55	0.00%	0	0.00%	

Source: Bradfield and Swartz (2003:18)

## 2.5 Cost of rebalancing

If a portfolio manager decides to rebalance an asset class, transaction cost will occur. The transaction cost is referred to as the cost of rebalancing. The cost of rebalancing consists of trading cost and market impact cost.

Trading cost differs from one asset class to the other, because of the different calculation methodologies of trading costs used in the different markets. Market impact cost differs because the demand and supply of the asset classes differ from one another depending on the market conditions. The demand and supply for a specific asset class is not the only determinant of market impact cost. There are a number of determinants affecting the market impact cost. For further insight into market impact cost refer to Torre (1997). Each asset class has a certain cost when rebalancing occurs. The cost for the different asset classes cannot be quantified as a single amount using a formula, because of the difference in trading cost and market impact cost. The calculation of the cost of rebalancing depends on the proxy used by the portfolio manager.

The study of Plaxco and Arnott (2002:22) assumes that all asset classes are traded via the Commodity and Futures Trading Commission-approved futures markets at an average cost of 0.1% per transaction. The experience of Plaxco and Arnott (2002:22) on trading \$500 billion in futures in the past decade supports the assumption that the cost of rebalancing an asset class is 0.1%.

The study of Buetow *et al* (2002:32) assumes different transaction costs of rebalancing that depend on the asset class and whether the market is liquid or illiquid. Table 2.5.1 illustrates the different transaction costs of rebalancing different asset classes.

**Table 2.5.1: Transaction Cost Assumptions****TRANSACTION COST ASSUMPTIONS**

<b>Asset Class</b>	<b>Transaction Costs of Rebalancing</b>	
	<b>Cost in Illiquid Market</b>	<b>Cost in Liquid Market</b>
U.S. Large-Cap	0.050%	0.250%
U.S. Small-Cap	0.075%	0.375%
Bonds	0.050%	0.250%
International Equities	0.150%	0.750%

Source: Buetow *et al.* (2002:28)

Table 2.5.1 illustrates the cost of rebalancing for U.S. Large-Cap in an illiquid market is 0.05% and in a liquid market it is 0.25%. The difference in the cost of rebalancing between the liquid market and illiquid market can be the market impact effect on the cost of rebalancing. In the study of Buetow *et al.* (2002), costs are compiled with input from a number of large Wall Street investment firms.

The study of Masters (2003:53), on the other hand, suggests that the cost of rebalancing is linear (see section 3.2). This indicates that selling twice as much of an asset will cost roughly twice as much. Table 2.5.2 illustrates Masters' cost of rebalancing of a particular asset class.

**Table 2.5.2: Masters' Cost of Rebalancing****MASTERS' COST OF REBALANCING**

<b>Asset Class</b>	<b>Costs of Rebalancing</b>
U.S. Growth	1.0%
U.S. Value	1.0%
U.S. Bonds	1.0%
International Equities	1.5%

Source: Masters (2003:55)

The cost of rebalancing U.S. Growth is 1%. Masters' cost of rebalancing will be discussed in section 3.2.

The cost of rebalancing a specific asset class will depend on the portfolio manager's calculation of transaction cost. This will depend on the trading cost and market impact cost of a specific asset class. Each rebalancing strategy has a specific way of calculating the cost of rebalancing.

In this study, the analysis will be done on a pension fund assuming Masters' cost of rebalancing function. Masters' range rebalancing strategy is the benchmark strategy that will be used to compare the other proposed rebalancing strategies.

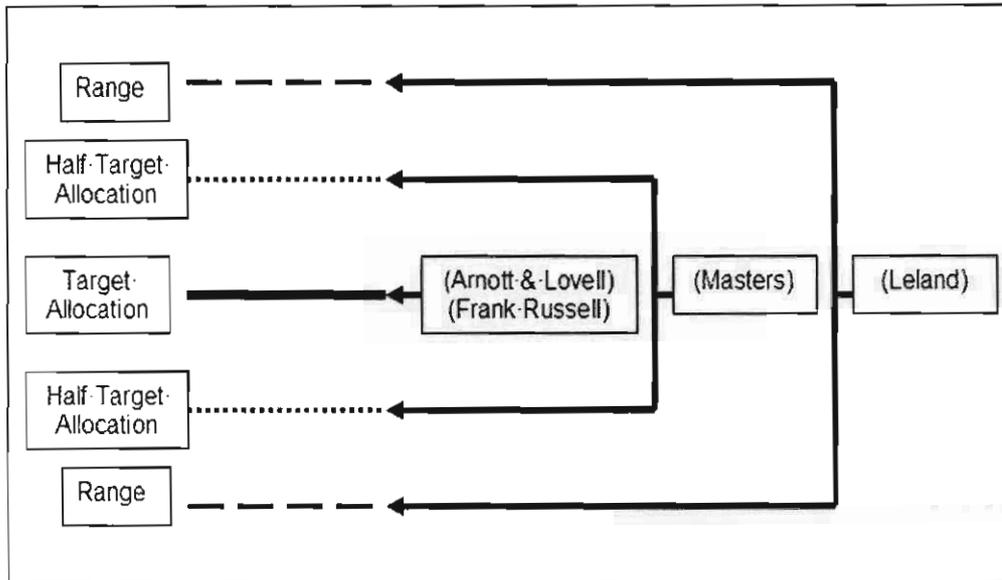
Chapter 2 introduced portfolio drift to describe the five broad rebalancing strategies. The chapter began with the discussion on the five broad rebalancing strategies: periodic rebalancing, threshold rebalancing, range rebalancing, volatility-based rebalancing and tactical rebalancing. The chapter then focused on the studies by Plaxco and Arnott (2002:9-22), Buetow *et al* (2002:23-32) and

Masters (2003:52-57) to distinguish between the benefit of rebalancing and the cost of rebalancing. The next section will discuss to what level the portfolio manager should rebalance a specific asset class and after this section, concluding remarks will be presented.

## **2.6 To what level should the portfolio manager rebalance?**

This study focuses solely on developing an alternative rebalancing strategy, indicating when rebalancing should occur. For the empirical assessment of the different rebalancing strategies, the study should take into consideration how far back the portfolio manager should rebalance the different asset classes.

The study of Arnott and Lovell (1993) found that periodic monthly rebalancing back to the target allocation is optimal. They assumed an asset mix of 50% in United States (US) Equity and 50% in US bonds over the period 1986 to 1991. The periodic monthly rebalancing produced a higher annual turnover than the other periodic rebalancing strategies. By comparing the difference in annual turnover between rebalancing back to the target allocation with rebalancing back to the range, the study of Arnott and Lovell (1993) found that rebalancing back to the range incurs the lowest annual turnover. A study by Frank Russell; (2000) complemented Arnott and Lovell (1993), concluding that rebalancing back to the target allocation is optimal when the asset allocation mix drift outside a tolerance range.

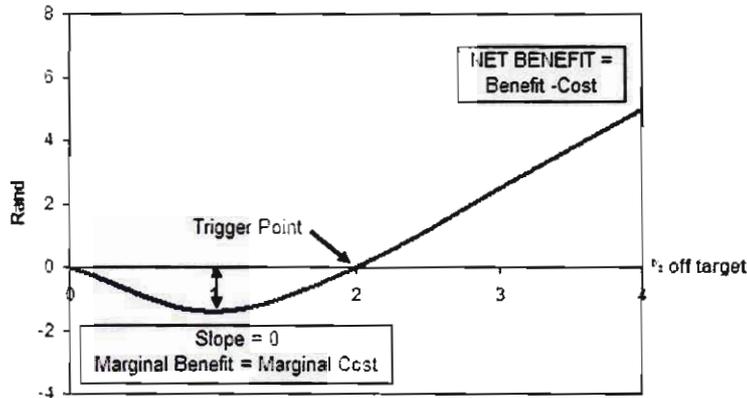


**Figure 2.6.1: To what level the portfolio manager should rebalance**

The theoretical work of Leland (1996:11) suggests that rebalancing back to the range is optimal and as long as the actual asset allocation stays within the tolerance range, no rebalancing is required. The study of Leland (1996:1) is based on an asset allocation mix of 60% in equities and 40% in bonds. Leland (1996:11-12) also found that, taking transaction costs into consideration, periodic rebalancing to the target allocation will not be optimal. However, the study of Leland (1996) assumed that any purchases of equities must be financed by selling bonds and vice versa.

Masters' (2003:55-56) study found that rebalancing only halfway back to the target allocation is optimal. The study of Masters (2003:53-55) developed an alternative range rebalancing strategy that takes into consideration the volatility, correlations, risk tolerance and cost of rebalancing the class of assets. Masters' range rebalancing strategy focuses on the benefits and costs of rebalancing.

Whenever the benefits are more than the costs, then rebalancing can occur (see section 3.2).



**Figure 2.6.2: Net Benefit Curve**

Source: Masters (2003:56)

Figure 2.6.2 illustrates Masters' net benefit that is equal to the cost of rebalancing subtracted from the benefit of rebalancing (see section 3.2). At the point  $T/2$  (halfway from the target allocation) where the slope of the net benefit curve is zero (see appendix D). That is where the marginal benefit is equal to the marginal cost. On the left hand side of point  $T/2$ , if rebalancing takes place towards the target allocation, the marginal costs of rebalancing outweigh the marginal benefit, and vice versa.

## 2.7 Conclusion

This chapter introduced the concept of a portfolio drift strategy to set the background to the different rebalancing strategies. This was to emphasise the importance for a portfolio manager of a pension fund to adopt a rebalancing strategy. An overview of the five broad rebalancing strategies followed, with some advantages and disadvantages of the different rebalancing strategies.

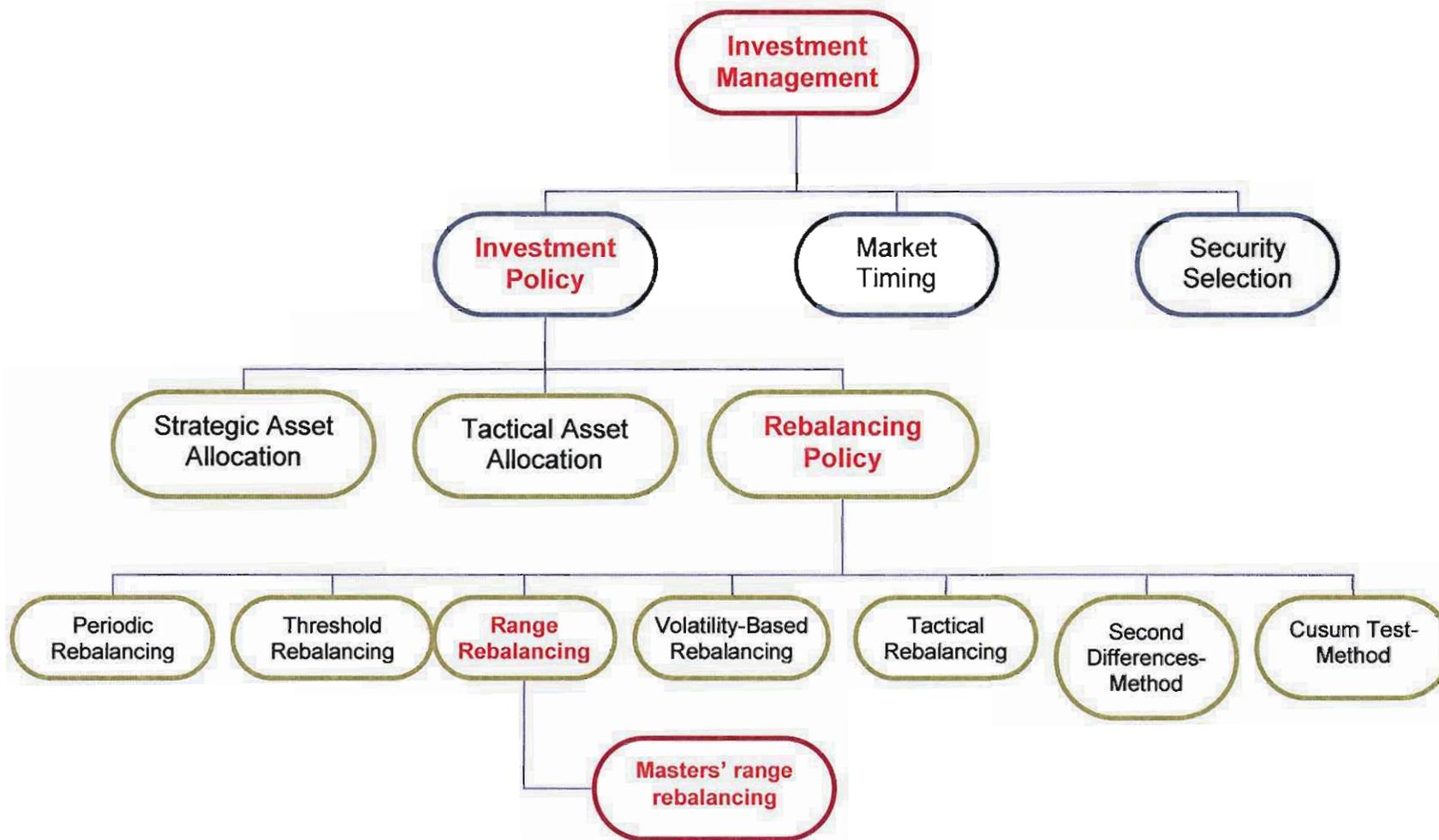
The study went further to identify three different articles on rebalancing strategies:

- Plaxco and Arnott (2002:9-22)
- Buetow *et al* (2002:23-32)
- Masters (2003:52-57)

The purpose of this was to explain the benefit of rebalancing and the cost of rebalancing.

The chapter concluded with different methodologies to identify to what level the portfolio manager should rebalance.

Chapter 3 will continue with Masters' (2003:52-57) study defining when rebalancing should occur. Masters (2003:53-55) used both the concepts of the benefit of rebalancing and the cost of rebalancing to identify when rebalancing should occur. Masters' study will be used as the benchmark study for the thesis.



## CHAPTER 3

### AN ASSESSMENT OF MASTERS' APPROACH

#### 3.1 Introduction

Chapter 2 introduced the working mechanism of different rebalancing strategies. This was to distinguish between the benefit of rebalancing and the cost of rebalancing (see section 2.4 & 2.5). A portfolio manager should weigh the benefit of rebalancing against the cost of rebalancing. According to Masters (2003:53), the optimal range for rebalancing a portfolio is where the difference between the benefit of rebalancing and the cost of rebalancing is zero (see section 3.2). Masters' range rebalancing strategy is used as the departure point for chapter 3.

The benefit of Masters' range rebalancing strategy over the other rebalancing strategies is that it is sensitive to market changes and eliminates the portfolio manager's temptation to delay rebalancing. Masters' range rebalancing strategy is tailor-made for each manager's portfolio according to the manager's risk preferences, portfolio compositions and the degree of risk of the different assets in the portfolio (Bradfield & Swartz, 2003:3).

Chapter 3 evaluates the calculation of Masters' trigger point and Masters' benefit of rebalancing as well as the cost of rebalancing and uses Masters' range rebalancing strategy as the benchmark strategy. When evaluating Masters'

range rebalancing, periodic rebalancing and portfolio drift, Bradfield and Swartz (2003) found that Masters' range rebalancing outperforms periodic rebalancing and portfolio drift according to the risk-adjusted return.

The first part of this chapter summarises Masters' range rebalancing strategy. The chapter continues by shifting the focus to the implementation of Masters' range rebalancing strategy.

The rebalancing strategies are compared by calculating performance measurement tools, such as return, risk and risk-adjusted return<sup>6</sup>. These performance measurement tools are used to evaluate the performance of the rebalancing strategies. This chapter will conclude with the results of the performance measurement tools on Masters' range rebalancing strategy so that it may be used as the benchmark against which the alternative developed rebalancing strategies are assessed (see chapter 4 & chapter 5).

### **3.2 Masters' approach**

Masters' theory is based on range rebalancing. Range rebalancing takes into consideration that each asset class may have a different range in which the value of the asset class is allowed to drift. This approach sets bands around the target allocation for each asset class in the portfolio and rebalances whenever those ranges are exceeded.

In order to use Masters' range rebalancing strategy as the benchmark, it is necessary to explain the calculation of Masters' trigger point. The trigger point is expressed as a percentage; this percentage is used as the range of a specific

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<sup>6</sup> The industry standard for calculating risk-adjusted return: the *Sharpe Ratio* (Grant, 2004:65).

asset class. Rebalancing occurs whenever these ranges are exceeded. The following will underpin the calculation of Masters' trigger point.

To explain the calculation of the trigger point two underlying determinants, the benefit of rebalancing and the cost of rebalancing, should be explained.

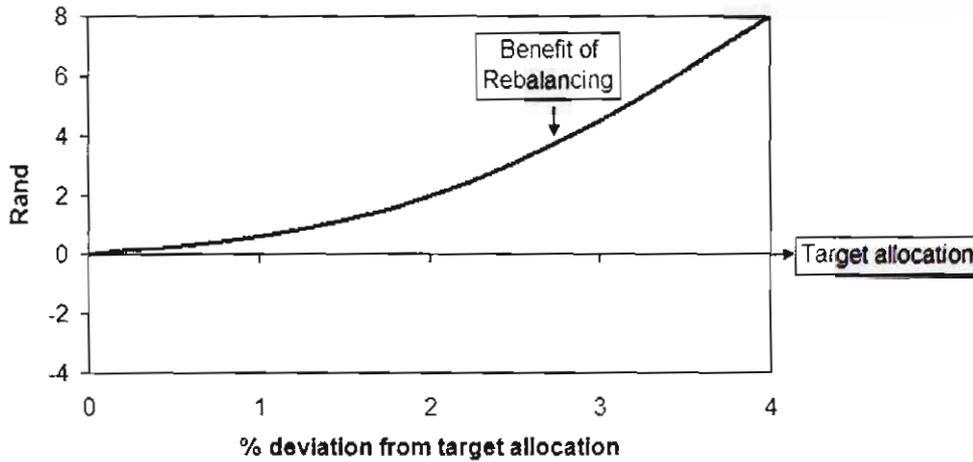
The benefit of rebalancing consists of three important concepts: tracking error, percentage deviation from the target allocation ( $\Delta$ ) and the portfolio manager's risk tolerance ( $K$ ), expressed as a percentage. The risk tolerance of portfolio managers differ because of their different attitudes towards risk. Tracking error occurs when a portfolio's actual allocation deviates from its target allocation; the deviation is expressed as a percentage (Bradfield & Swartz, 2003:4). The key benefit of rebalancing is to reduce this tracking error (Masters, 2003).

Masters (2003: 53) put these concepts together to quantify the benefit of rebalancing a particular asset class:

$$\text{Benefit of rebalancing} = \frac{(\text{Tracking Error})^2 \Delta^2}{2K}$$

The benefit of rebalancing a particular asset class is the percentage of the value of that particular asset class that the portfolio manager would gain.

Figure 3.2.1 illustrates the benefit of rebalancing a particular asset class versus the percentage the asset class moves off target.



**Figure 3.2.1: Benefit of rebalancing**

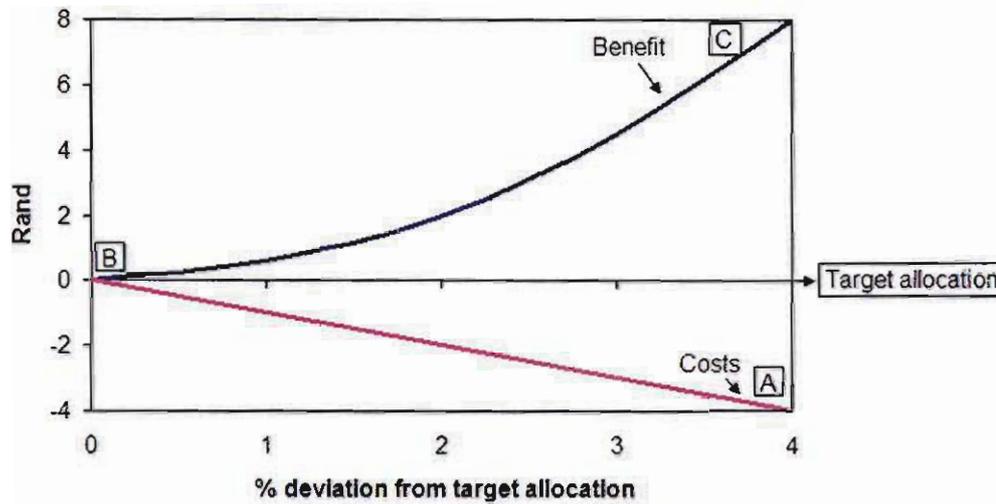
Source: Masters (2003:53)

Rebalancing a portfolio or an asset class incurs transaction cost. Prior literature (Dumas & Luciano, 1991; Leland, 1996; Liu, 2001) shows the impact of transaction costs on aspects of portfolio management for optimal rebalancing strategies to minimise the transaction cost and tracking error cost. The cost of rebalancing a portfolio or an asset class should be taken into consideration. The cost of rebalancing can be summarised as the direct trading costs as well as the indirect market impact costs. Masters (2003:53) assumes that the costs of rebalancing are linear and can be formulated as:

$$C\Delta$$

where  $C$  is the cost of rebalancing a particular asset class and the cost is expressed as a percentage of the value of an asset class, and  $\Delta$  is the percentage off-target for a specific asset class.

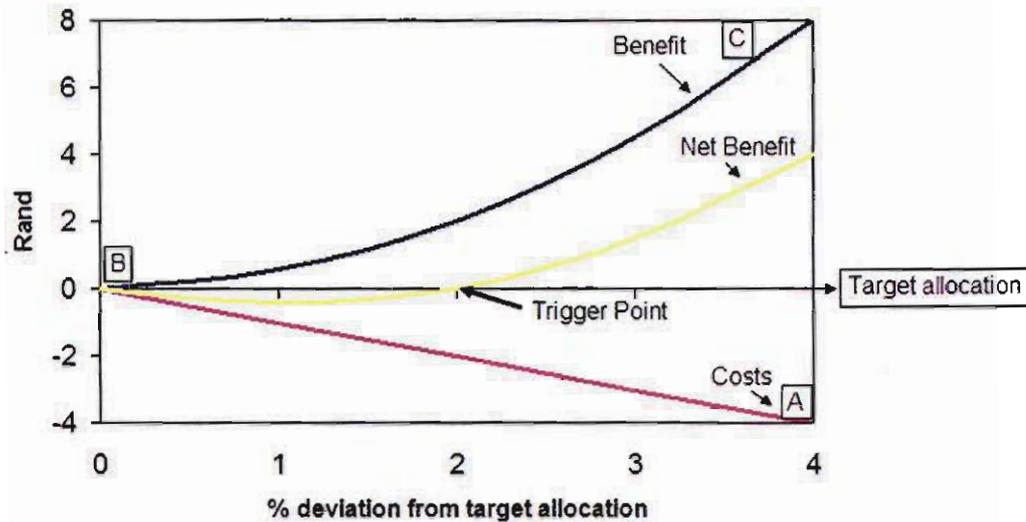
Figure 3.2.2 shows the benefit of rebalancing which is quadratically linked to  $\Delta$  and the costs of rebalancing are linearly linked to  $\Delta$ .



**Figure 3.2.2: The Benefit of rebalancing is quadratic whilst Costs are linear**

Source: Masters (2003:53)

Masters' definition of the trigger point is where the linear cost function (BA) is equal to the benefit of rebalancing (BC) (see Figure 3.2.3). The net benefit of rebalancing is determined by subtracting the cost function from the benefit of rebalancing.



**Figure 3.2.3: Net Benefit of rebalancing – showing the trigger point**

Source: Masters (2003:54)

The theory around the calculation of this trigger point is an optimisation problem introduced by Leland (1996:3-8).

The trigger point is calculated as follows:

Note the trigger point  $T$  for asset class  $i$  is where:

$$\frac{(\text{Tracking Error}_i)^2 T_i^2}{2K} - C_i T_i = 0$$

By rearranging

$$T_i = \frac{2KC_i}{(\text{Tracking Error}_i)^2} \quad (1)$$

The tracking error, by definition (see section 3.2), is the deviation between the portfolio's actual asset allocation and the target allocation (benchmark), where the deviation is expressed as a percentage.

By treating the rest of the portfolio as asset j (benchmark):

$$(\text{Tracking Error}_t)^2 \equiv \text{Var}(X_i, X_j) = \sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j \quad (2)$$

Where

$X_i$   $\equiv$  returns of the asset class in consideration and

$X_j$   $\equiv$  returns of the rest of the portfolio

See Appendix A for the formal proof of (2).

Tracking error is constant over a fixed period  $(t_1; t_2)$ . If the period changes  $(t_1; t_2 + h)$ , it may result in a different tracking error (variance),  $h$  is an integer. Tracking error therefore depends on time and, if time changes, the tracking error may also change. Masters' range rebalancing strategy (time dependence) uses historical data to determine ranges that are applicable over a future period. This shortcoming will be addressed in chapter 5 when developing the Cusum Test-method, which determines the ranges that are adjustable over time.

By substituting (2) in (1) the trigger point can be expressed as:

$$\begin{aligned}
 T_i &= \frac{2KC_i}{\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j} \\
 &= \frac{2KC_i}{\sigma_i^2 + \sigma_j^2 - 2\sigma_j} \quad \text{because } \rho_{ij} = \frac{\sigma_j}{\sigma_i\sigma_j}
 \end{aligned} \tag{3}$$

Where

$\sigma_i^2$  is the variance of the returns of the asset class under consideration,

$\sigma_j^2$  is the variance of the returns of the rest of the portfolio,

$\rho_{ij}$  is the correlation between the asset class under consideration and the rest of the portfolio,

$T_i$  is the trigger point for the specific asset class,

$K$  is the risk appetite of the specific portfolio manager,

$C_i$  is the cost of rebalancing a specific asset class.

From the above analysis, the implicit assumptions of Masters' range rebalancing strategy can be formulated as:

- $K$  (Risk Tolerance) is inversely proportional to the risk tolerance parameter  $\lambda$ <sup>7</sup>.
- $K$  is determined only by the portfolio manager.
- The cost function that is used to rebalance an asset class is linear.
- The tracking error is calculated by using only the particular asset class under consideration and the rest of the portfolio.
- Tracking error is calculated over a fixed interval. This interval must not change during the period of analysis.

This concludes the calculation of Masters' trigger point. As mentioned in section 3.1, Masters' range rebalancing strategy will be the benchmark strategy for this study. The benchmark strategy will be used to evaluate the alternative rebalancing strategies that are based on less strict assumptions developed in chapter 4 and 5. Performance measurement tools should be identified to analyse the rebalancing strategies (see section 3.3.3). The next section will discuss the implementation of Masters' range rebalancing strategy.

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<sup>7</sup> Lambda measures the aversion to total risk (Grinold & Kahn, 2000:96-99)

### 3.3 Empirical assessment

#### 3.3.1 Data and Input assumptions

In order to assess the Masters' range rebalancing strategy, the portfolio of a pension fund is used over the period September 2003 to December 2005. Table 3.3.1 shows the resulting trigger points on daily data for the five asset classes: South African (SA) equities, South African (SA) bonds, South African (SA) cash, global equities and global bonds.

**Table 3.3.1.1: Input assumptions and resulting trigger point**

Asset Class	Target Allocation	Sample assumptions					Trigger Point	Rebalance when	
		$K$	$C_i$	$\sigma_i$	$\sigma_j$	$\sigma_{ij}$		below	above
SA Equities	54.61%	5.00%	0.50%	1.04%	0.67%	0.00001898	4.34%	50.27%	58.95%
SA Bonds	26.06%	5.00%	1.00%	0.34%	1.24%	0.00001340	7.28%	18.78%	33.35%
SA Cash	9.28%	5.00%	0.50%	0.03%	1.38%	0.00000060	2.65%	6.63%	11.92%
Global Equities	6.14%	5.00%	1.50%	0.49%	1.23%	0.00000787	9.39%	-3.25%	15.53%
Global Bonds	3.81%	5.00%	1.50%	0.24%	1.36%	0.00000066	7.95%	-4.15%	11.76%

Daily index data is used to assess the off-target positions daily. A specific index is used as a proxy for the movement of each asset class over the period. The reason for using index data is that it is accepted in practice as a proxy for the value of an asset class. The pension fund consists of five different indices, Shareholder Weighted All Share Index (J403), All Bond Index (ALBI), Alexander Forbes Short Term Fixed Interest Composite Index (STeFI), MSCI World Index and Salomon Smith Barney World Government Bond Index (WGBI). For the calculation purposes of the different indices refer to FTSE and JSE (2004), BESA

and ASSA (2000), Alexander Forbes (2000), Morgan Stanley (2006) and City Group (2003), respectively.

The Salomon Smith Barney World Government Bond Index (WGBI) is only distributed once a week. The problem persists that the portfolio that is used as an example contains daily index data. The following transformation is used to transpose the weekly data to daily data. Assume there are five trading days:

$$D_n = \frac{W}{(1+i)^{5-n}} \quad n = 1, 2, 3, 4, 5$$

where  $D$  is the placeholder for a specific day and  $W$  is the placeholder for a specific week. This formula holds only for one week. The following formula holds for  $z$  weeks:

$$D_{ns} = \frac{W_s}{(1+i)^{5-n}} \quad s = 1, \dots, z$$

where

$$W_{s-1}(1+i)^5 = W_s$$

$$i = \sqrt[5]{\frac{W_s}{W_{s-1}}} - 1$$

The transformation serves to compare the different asset classes and to satisfy the condition for calculation purposes. The following paragraphs will discuss the input assumptions used in table 3.3.1.1.

There are two input assumptions, risk tolerance of the portfolio manager ( $K$ , see the first column in table 3.3.1.1), and the cost of rebalancing the particular asset class ( $C_i$ , see second column in table 3.3.1.1). The following paragraphs will address the two input assumptions.

According to Masters (2003), a standard estimate for the risk tolerance of portfolio managers is 5%. A 5% risk tolerance level for portfolio managers is assumed in the empirical assessment. In section 3.2, it is shown that the risk tolerance is inversely proportional to  $\lambda^8$ . Thus, for a 5% risk tolerance level, the corresponding lambda is 20%. Lambda indicates whether the portfolio manager is risk averse, risk neutral or risk seeking. The closer lambda is to zero, the more risk averse the portfolio manager will be. The risk appetite of the trustees of a pension fund will typically be low, thus the lambda of 20% may be acceptable for the empirical assessment.

The second input assumption is the cost of rebalancing a particular asset class. The trading cost for the different asset classes differs according to the accessibility and cost to trade in that specific market (see chapter 2). Table 3.3.1.1 above shows that it is more expensive to trade abroad than to trade in the local market (see the  $C_i$ 's). Masters (2003) assumes specific trading costs for the different asset classes given in table 3.3.1.1.

The next section discusses the two important questions the portfolio manager should answer: When rebalancing should occur, and to what level the portfolio manager should rebalance?

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<sup>8</sup> Lambda measures the aversion to total risk (Grinold & Kahn, 2000:96-99).

### **3.3.2 Formulating rebalancing rules**

To assess Masters' range rebalancing strategy, certain rebalancing rules should be formulated based on when rebalancing should occur and to what level the portfolio manager should rebalance the particular asset class.

#### **3.3.2.1 When rebalancing should occur.**

The trigger points for Masters' range rebalancing strategies should be calculated using the data and input assumptions discussed in section 3.3.1. These trigger points are calculated using one month's historical data on a daily basis for each asset class. The trigger points are valid for one year and after a year the trigger points are recalculated using one month's historical data. The trigger points for the different asset classes indicate the bands for the asset classes and, whenever the band limits are exceeded, rebalancing should occur. Note that the trigger points cannot be recalculated on a frequent basis. One of the reasons may be that the tracking error (variance see (2) on p 46) may change on a continuous basis.

#### **3.3.2.2 To what level the portfolio manager should rebalance**

Some literature (Arnott & Lovell, 1993; Russell, 2000) has advocated rebalancing back to the target and others have suggested rebalancing back to the trigger point (Donohue & Yip, 2003). Masters, on the other hand, argues that the key is to rebalance only halfway back to the target (compare figure 2.6.1). See Appendix B for the formal proof. Table 3.3.2.2.1 shows the resulting trigger points (see columns 3 & 5, respectively) that indicate when rebalancing should occur for the different asset classes for Masters' range rebalancing strategies.

Columns 4 and 6 specify the bands to which the asset classes should be brought back when rebalancing occur.

**Table 3.3.2.2.1: To what level the portfolio manager should rebalance**

Asset Class	Target Allocation	Rebalancing rules			
		When below	Rebalance to	When above	Rebalance to
SA Equities	54.61%	50.27%	52.44%	58.95%	56.78%
SA Bonds	26.06%	18.78%	22.42%	33.35%	29.71%
SA Cash	9.28%	6.63%	7.96%	11.92%	10.61%
Global Equities	6.14%	-3.25%	1.45%	15.53%	10.83%
Global Bonds	3.81%	-4.15%	-0.17%	11.76%	7.79%

### 3.3.3 Performance measurement tools

To compare the rebalancing strategies, certain performance measurement tools should be defined. These performance measurement tools are calculated for the rebalancing strategies to assess the performance of each portfolio. The following sections will address the performance measurement tools used in this study.

### 3.3.3.1 Returns

Return is the amount of profit made by the portfolio. Returns over a period  $t$ , of length  $\Delta t$ , which runs from  $t$  to  $t + \Delta t$  where the portfolio's value at time  $(t; t + \Delta t)$  is denoted by  $(P(t); P(t + \Delta t))$  and the distribution<sup>9</sup> over the period total is denoted by  $d(t)$ :

$$R(t) = \frac{P(t + \Delta t) + d(t)}{P(t)}$$

The portfolio's total rate of return is:

$$rr(t) = \left[ \frac{P(t + \Delta t) + d(t) - P(t)}{P(t)} \right] \times 100$$

In the performance analysis, the compound returns are used. According to Grinold and Kahn (2001:483), the compound returns provide an accurate measure of the final value of the portfolio. The compound total return on portfolio  $Z$  over periods 1 through  $T$  denoted as  $R_z(1, T)$ :

$$R_z(1, T) = \prod_{t=1}^T R_z(t)$$

---

<sup>9</sup> It is assumed that if the period  $\Delta t$  is relatively long, and a cash flow  $d(t)$  occurs in mid-period, reinvestment of this cash flow  $d(t)$  over the rest of the period can take place at the risk-free rate of return. An example of such a cash flow is dividends.

### 3.3.3.2 Risk

According to Grinold and Kahn (2001:47) the definition of risk must satisfy the following criteria: it must be universal, symmetric, flexible and accurately forecastable. Markowitz' (1959) definition of risk may be defined as the standard deviation. This definition of risk meets the criteria stated by Grinold and Kahn. Thus the performance measurement tool that will to be used in the analysis of risk is the standard deviation. If  $R_z$  is the total return of a portfolio over the period  $t$  to  $t + \Delta t$ , then the standard deviation of return (risk of the portfolio) can be denoted by:

$$\sigma_z = \text{std}(R_z) = \sqrt{W^T V W}$$

where  $W$  is the weight of the different asset classes and  $V$  is the variance/covariance matrix of these asset classes.

The target allocation for a specific asset class is the weight assigned to that asset class. According to the study, the weights of the asset classes are (see target allocation in table 3.3.2.2.1):

$$W^T = [0.5461 \ 0.2606 \ 0.0928 \ 0.0614 \ 0.0381]$$

One of the characteristics of the standard deviation is that it does not have the portfolio property (Grinold & Kahn, 2001:47). Thus the standard deviation of a

portfolio is not the weighted average of the standard deviations of the different asset classes.

### 3.3.3.3 Risk-adjusted Return (Sharp ratio)

According to Grinold and Kahn (2001:33), the Sharp ratio ( $SR_Z$ ) is defined for any risky portfolio  $Z$  ( $\sigma_Z > 0$ ) as the ratio of the excess return ( $f_Z$ ) on portfolio  $Z$  and the risk of portfolio  $Z$   $\sigma_Z$ :

$$SR_Z = \frac{f_Z}{\sigma_Z}$$

where

$$\sigma_Z = \sqrt{W^T V W} \text{ and}$$

$$f_Z = rr(t)$$

The excess return is the difference between the average rate of return for portfolio  $Z$  and the average rate of return on risk-free assets during the same period. For calculating excess return, a constant 4% risk-free rate of return is assumed.

$SR_Z$  is a measure of how much an investment returned in relation to the amount of risk it assumed. It is often used to compare a high-risk, potentially high-return investment with a low-risk, lower-return investment.

### 3.4 Results

Masters' range rebalancing strategy can be assessed using the data and information in section 3.3. Before moving on to the results of Masters' range rebalancing strategy, the criteria used to assess the range rebalancing strategy should be reviewed:

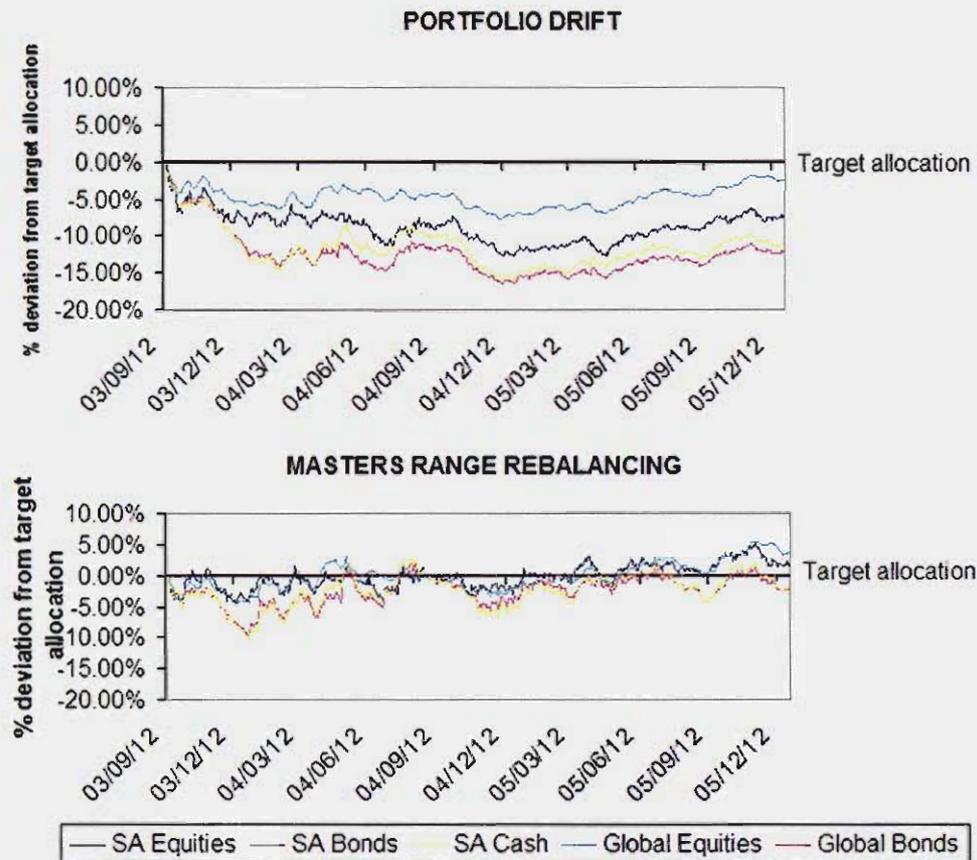
- Pension fund over the period September 2003 to December 2005.
- The portfolio consists of five asset classes.
- The risk tolerance level of the portfolio manager is 5%.
- Cost of rebalancing varies between the asset classes (see table 3.3.1.1).
- The portfolio manager rebalances halfway back to the range (see table 3.3.1.2).
- Performance analysis is measured using return, risk and risk-adjusted return.

**Table 3.4.1: Summarised Results of Masters' Range Rebalancing**

Summarised results over the period September 2003 - December 2005

Results	Performance Measurement Tools			
Rebalancing Strategy	Portfolio Return	Portfolio Risk	Risk-adjusted Return	Number of Trades
Range Rebalancing	30.77%	2.11%	14.58	14

Table 3.4.1 summarises the results of Masters' range rebalancing strategy. Column 2 of table 3.4.1 shows the portfolio returns generated using Masters' range rebalancing over the period September 2003 to December 2005. The percentage returns on the portfolio yield 30.77%. This percentage of return was subject to the risk ( $\sigma_z = 2.11\%$ ) of keeping the portfolio along the target allocation. The third column in table 3.4.1 refers to the risk-adjusted return of the portfolio. The risk-adjusted return indicates the amount of return subject to the risk of the portfolio. A ratio larger than unity indicates that the return in percentage of the portfolio is higher than the risk in percentage of that specific portfolio. Table 3.4.1 indicates the risk-adjusted return of Masters' range rebalancing on the portfolio is 14.58. The return of the portfolio over that period is 14.58 times higher than the risk of the portfolio over the same period. There were a total of 14 trades during the period September 2003 to December 2005.



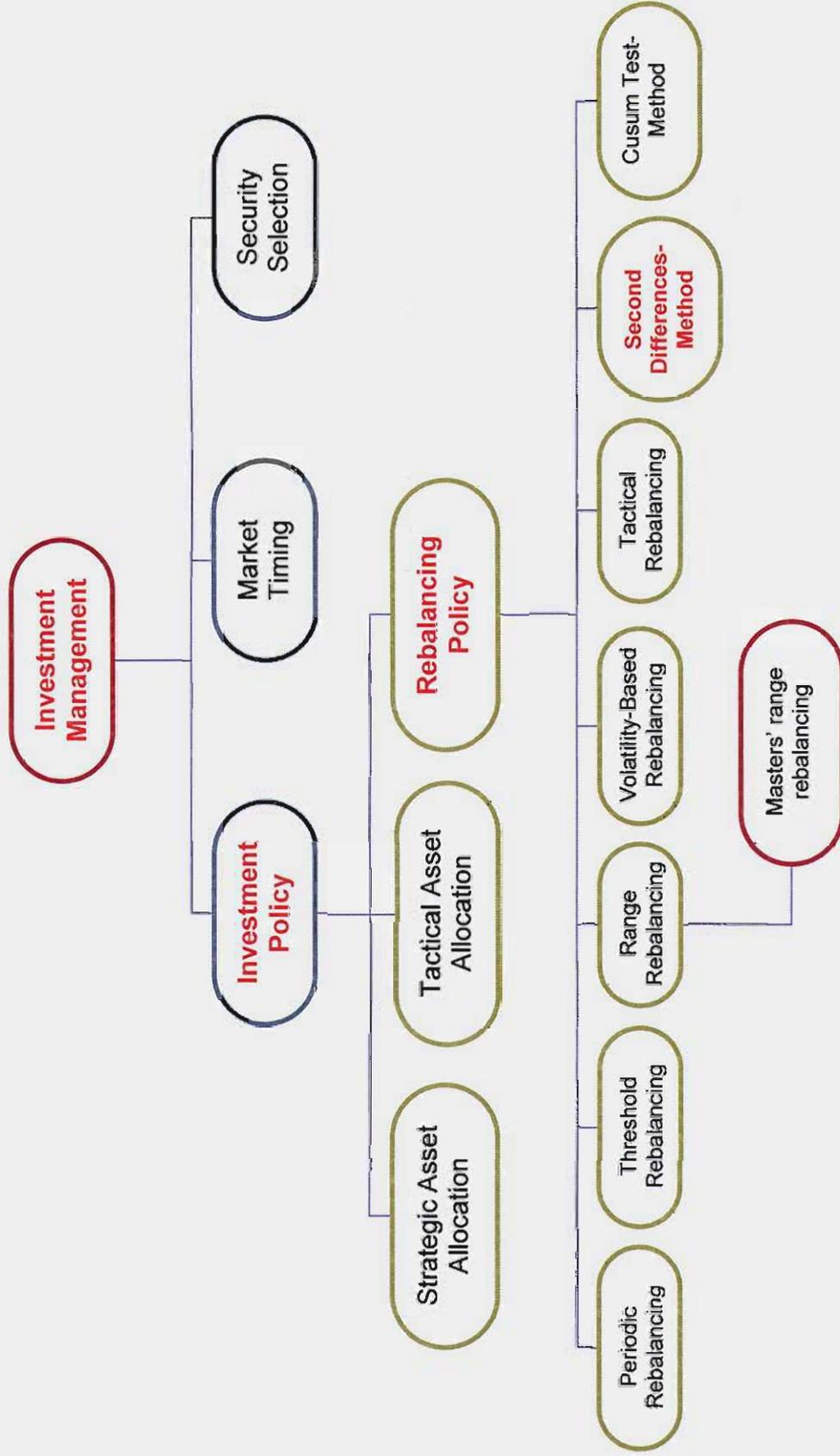
**Figure 3.4.1: Masters Range Rebalancing versus Portfolio Drift**

Figure 3.4.1 illustrates the percentage deviation from the target allocation that can occur when allowing a portfolio to drift compared to using Masters' range rebalancing to keep the portfolio aligned with the target allocation. Taking the pension fund without adjusting it over the period will result in the asset classes deviating from the target allocation (see figure 3.4.1, Portfolio drift). This drift away from the target allocation will result in overweighting the risky asset classes. Whenever there is a sudden downside change in market condition, the trustees of a pension fund will incur greater risk through portfolio drift than with Masters' range rebalancing.

These results in section 3.4 will be used to compare the alternative rebalancing strategies with one another. The performance of the alternative rebalancing strategies will be measured against the benchmark strategy.

### **3.5 Conclusion**

This chapter summarises the benchmark strategy (see table 3.4.1) that will be used to evaluate the alternative rebalancing strategies. A number of performance measurement tools were used to evaluate the benchmark strategy. These criteria will be used to compare the alternative rebalancing strategies. In chapter 4, a new decision-making method will be developed that may improve the return of a portfolio subject to certain risk constraints. The decision-making method is called the second difference method (SD-method).



# CHAPTER 4

## SECOND DIFFERENCE METHOD

### 4.1 Introduction

Masters' range rebalancing strategy was analysed in chapter 3. This was so that Masters' range rebalancing strategy could be used as the benchmark rebalancing strategy. A number of performance measurement tools were used within the benchmark strategy. These performance measurement tools are returns, risks and risk-adjusted returns. The results obtained in section 3.4 with the use of the benchmark strategy will be compared in chapter 4 with the alternative rebalancing strategy. The study of Buetow *et al* (2002:24) found that combining threshold rebalancing with frequent monitoring can result in a superior rebalancing strategy. Chapter 4 will develop a rebalancing strategy using Masters' range rebalancing combined with additional risk constraint. These risk constraints must be monitored on a daily basis.

Buying and holding a portfolio without rebalancing is no longer appropriate. According to Arnott and Lovell (1993): "On a risk-adjusted basis, however, there is no period in which drift has beaten rebalancing" and Bradfield & Swartz (2003): "In allowing a portfolio to drift, one typically finds that the portfolio tends to become more risky". By implementing a well-defined rebalancing strategy, an improvement in the risk-adjusted return for the pension fund may be obtained.

To improve the risk-adjusted return of a portfolio, a new decision-making method, namely the second difference method (SD-method), is developed in chapter 4. This is to evaluate the movement of a specific asset class in terms of the portfolio's return subject to certain risk constraints. This method gives, for example, trustees of a pension fund the flexibility to construct their own risk profile for the specific portfolio. The objective of the method is to enhance the return of the portfolio according to the risk specifications of the trustees of a pension fund. The SD-method consists of three components:

- Range rebalancing
- Retracement levels
- The second difference test (SD-test).

Masters' range rebalancing is used to indicate minimum and maximum ranges according to the minimum and maximum risk tolerance of the portfolio managers (see section 4.2.1). The maximum ranges are used to indicate when rebalancing must occur. The retracement levels are used to add certainty when a specific asset class will get support or will get resistance (see section 4.2.2). The SD-test monitors the speed at which the asset classes value are changing on a daily basis (see section 4.2.3).

Chapter 4 will introduce the SD-method explaining the three components of the SD-method, range rebalancing, retracement levels and SD-test. The chapter continues by focusing on an empirical illustration of the SD-method. Chapter 4 concludes by comparing the SD-method with the benchmark strategy (Masters' range rebalancing strategy).

The next section will explain the three components of the SD-method.

## 4.2 Second difference method (SD-method)

### 4.2.1 Range rebalancing

Rebalancing strategies assist portfolio managers in making decisions about when rebalancing should occur. There are a number of rebalancing strategies assisting the portfolio manager in making these decisions. One of these is range rebalancing or, more specifically, Masters' range rebalancing (see section 3.2).

Masters defines a range between an upper band and a lower band. Masters uses these bands to determine the upper and lower trigger points respectively (see section 3.2) which indicate when rebalancing should occur. Denote the upper and lower trigger points respectively as  $T_{iUP}$  and  $T_{iLOW}$  for the  $i$ -th asset class.

The SD-method, on the other hand, introduces two separate ranges indicating the minimum and maximum risk appetite of the portfolio managers of a pension fund. These minimum ( $K_1$ ) and maximum ( $K_2$ ) risks are substituted in the Masters' approach formula (3) discussed in section 3.2, to get the minimum and maximum ranges for a specific asset class. The minimum risk ( $K_1$ ) defines one range (between two bands) and the maximum risk ( $K_2$ ) defines the other range (between two other bands). Denote the two bands subject to the minimum risk ( $K_1$ ) as  $(T_{iUP_{min}}; T_{iLOW_{min}})$  and for the two bands subject to the maximum risk ( $K_2$ ) as  $(T_{iUP_{max}}; T_{iLOW_{max}})$ . The vertical axis of figure 4.2.1.1 indicates the percentage deviation from the target allocation of the  $i$ -th asset class. Denote  $t_{c_1}$  as the point at which the value of the asset class intersects  $T_{iUP_{max}}$  band and  $t_{c_2}$  as the point at which the value of the asset class intersects  $T_{iLOW_{max}}$  band. The minimum (associated with  $K_1$ ) and maximum (associated with  $K_2$ ) ranges are the first risk constraints that must be satisfied in order to enhance return.

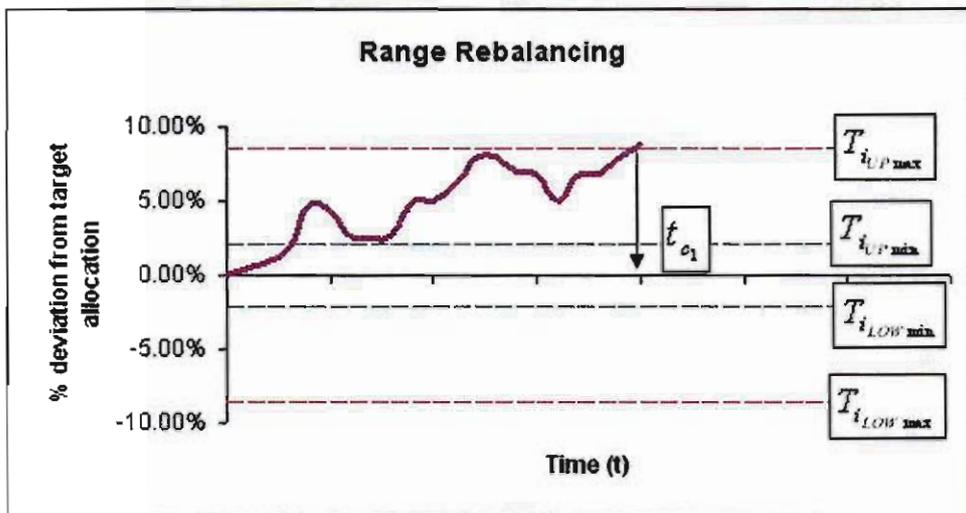


Figure 4.2.1.1: The minimum and maximum ranges with an upward trend

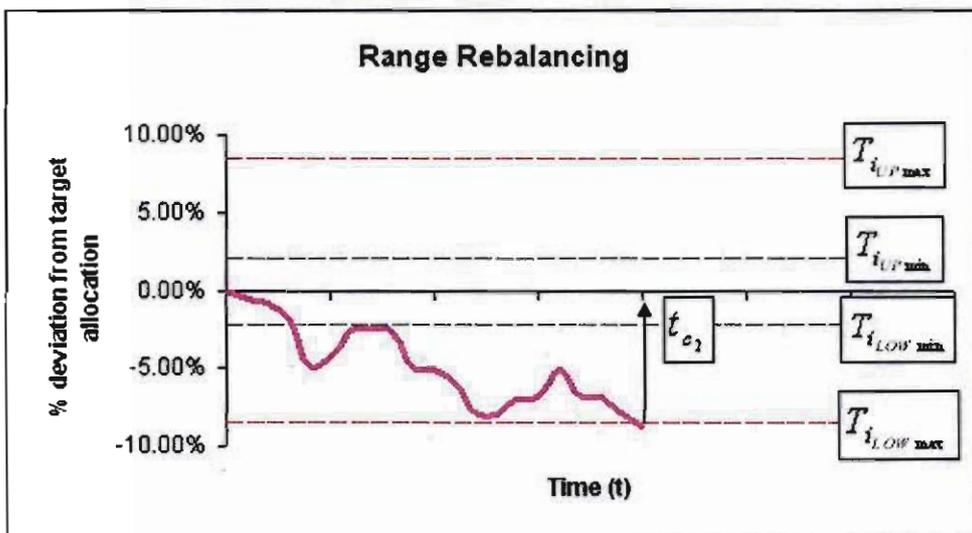


Figure 4.2.1.2: The minimum and maximum ranges with a downward trend

By using only the minimum (associated with  $K_1$ ) and maximum (associated with  $K_2$ ) ranges, it may happen that inadequate signals for rebalancing a specific asset class may occur (see section 4.2.2). To add more certainty to decision making, another risk constraint is proposed, namely retracement levels. This is an additional range that is incorporated into the process to indicate whether the

asset class will get support or resistance. Adding additional risk constraints will provide the portfolio manager with greater certainty when making decisions.

#### **4.2.2 Retracement levels.**

Retracement level is a value movement in an asset class that is counter to a previous trend. For example, in the value of a rising asset class, a 25% retracement would indicate a value decline equal to 25% of the previous advance.

Moving along with the trend of a market allows the portfolio manager<sup>10</sup> to enhance returns. Allowing an asset class to drift too far away from its target allocation becomes too risky. Markets may make a drastic and sudden turns. (Masters, 2003)

There is a probability that this movement in the market could occur when the value of the asset class is drifting between the calculated minimum range and the maximum range. To minimise the effect of a sudden turn in the market, resistance and support levels are built in to warn the portfolio manager when the market may turn. A support level is a value level that behaves as a "floor", that may indicate the value of an asset class going below that level. Resistance level, on the other hand, is a value level that acts as a "ceiling", that may indicate the value of an asset class rising further.

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<sup>10</sup> Portfolio managers manage and monitor the portfolio's movement on behalf of the trustees of a pension fund.

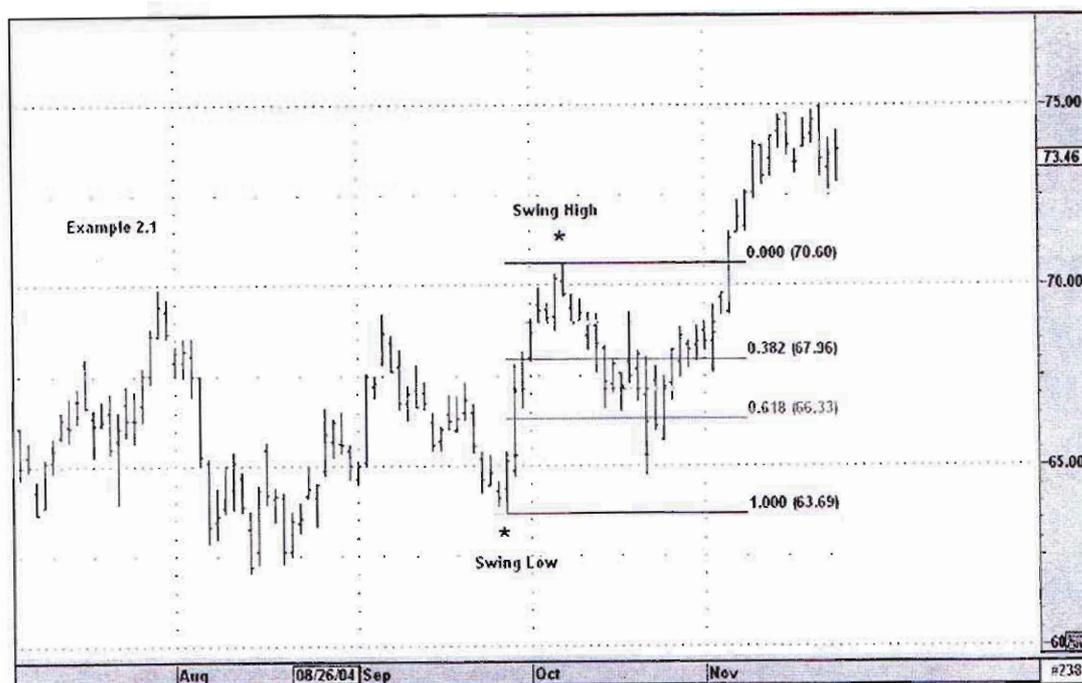
These resistance and support levels are based on a numerical sequence made popular by Leonardo Fibonacci in the thirteenth century<sup>11</sup>.

An important relationship of this sequence is that if the ratio of two successive numbers in the Fibonacci series is taken, the ratio converges to 0.6180345. This ratio is called "the golden ratio". If you also calculate the ratios using alternate numbers in the Fibonacci series the resulting ratios converge to 0.38196.

Many technicians use Fibonacci numbers in their Technical Analysis when trying to determine support and resistance levels, and commonly use 0.382 and 0.618 retracements (see section 4.2.2). A 0.382 retracement from a trend is commonly thought to imply a continuation of the trend. A 0.618 retracement implies that a trend change may be in the making (Poulos, 2004). Figure 4.2.2.1 illustrates that this particular asset class found support at a 0.618 retracement. The main objective is to use these ratios to determine when rebalancing should occur.

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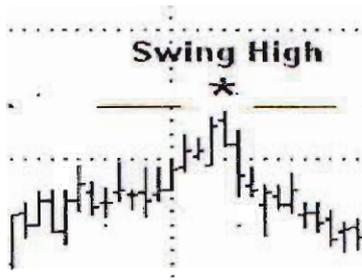
<sup>11</sup> This numerical sequence can be derived by the sum of the previous two numbers. This sequence is called the Fibonacci series given by: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...



**Figure 4.2.2.1: The asset class found support at a 0.618 retracement level**

In order to apply Fibonacci numbers to value charts, it is necessary to identify Swing Highs and Swing Lows. These value charts are constructed out of value bars that represent the value of an asset class on a daily basis<sup>12</sup>. A Swing High is a short term high bar with at least two lower highs on both the left and right of the high bar. Figure 4.2.2.2 illustrates a Swing High. A Swing Low is a short term low bar with at least two higher lows on both the left and right of the low bar. (Poulos, 2004) Figure 4.2.2.3 illustrates a Swing Low.

<sup>12</sup> The length of a value bar is determined using the distance between the maximum and minimum value traded on that day. The bar consists out two horizontal lines indicating the opening value and closing value on that day.



**Figure 4.2.2.2: Swing High is a short term high bar with at least two higher bars**



**Figure 4.2.2.3: Swing Low is a short term low bar with at least two lower bars**

In a downward trend (upward trend) of an asset class, the general idea is to go short (long) in the market on a retracement to a Fibonacci resistance level (support level). This is to minimise the consequences of a sudden downward (upward) movement in the market. The price retracement levels can be applied to the value bar chart when the significant Swing High and Swing Low are identified. Once the market starts to turn upward (downward) after the Swing Low (Swing High), the next high (low) may be projected at 0.382, or 0.618 (see section 4.2.2). The value of the asset class may find resistance (support) at 0.618 indicating that rebalancing should occur if the value of the asset class is between the minimum range (associated with  $K_1$ ) and the maximum range

(associated with  $K_2$ ). Denote respectively the intersection between the value of the asset class and the resistance level (support level) as  $t_{c_3}$  ( $t_{c_4}$ ).

Two risk constraints, range rebalancing and retracement levels, were added to improve the decision-making process for the portfolio managers of a pension fund subject to the investment objectives. The third risk constraint is added to evaluate the strength of the movement of the value of a specific asset class. The risk constraint is added to assist the portfolio manager to monitor the asset classes on a daily basis. The three risk constraints combined are used to assist the portfolio manager in making a decision as to when rebalancing should occur. The third risk constraint, the second difference test, is introduced in the next section. The second difference test assists the portfolio manager to identify the speed at which the value of an asset class is changing as well as to determine the trend of the specific asset class. It will assist the portfolio manager in knowing the speed at which the value of an asset class is changing as well as to determine the trend of the specific asset class in order to make a decision when the value of the asset class intersects the range. This may give an indication whether or not rebalancing should take place at the range, depending on the speed at which the value of an asset class intersects the range.

### 4.2.3 Second difference test (SD-Test)

Second difference of the value of an asset class measures the speed at which the values of the asset class are changing. The SD-Test has the ability to determine the strength of an upward or downward trend as well as the swiftness of value changes in the trends' direction. The portfolio manager may use the SD-Test to clarify the speed at which the change in the value of the asset class is moving as well as the direction in which it is moving.

In order to draw conclusions from the SD-Test, the assumption that an asset class should be quoted for at least 21 days must hold. The reason for using 21 days is that it is the nearest Fibonacci number which is in line with the number of trading days in a month. If the asset class satisfies the condition with regard to the number of trading days assumption, the SD-test can be performed. If the assumption does not satisfy the condition, the SD-Test is not applicable.

Whenever the minimum range (associated with  $K_i$ ) is intersected by the value of an asset class, a point is defined from which the portfolio manager can monitor the movement of the asset class. Monitoring is done with the SD-Test. The starting point for the SD-Test is  $a(p_i)$  and is defined as:

$$a(p_i) = (1 - p_i)T_i$$

where  $p_i$  is a certain percentage for the  $i$ -th asset class and  $T_i$  is the trigger point for the  $i$ -th asset class.

From the starting point, the second differences are calculated to monitor the speed at which a specific asset class is moving. The time interval over which the second differences should be calculated must be defined. Denote  $k$  as a Fibonacci number which defines the length of a fixed interval:  $[t_{a(p_i)} - k ; t_{a(p_i)}]$ . Denote  $s$  as a integer with which the fixed interval moves:  $[t_{a(p_i)} - k + s ; t_{a(p_i)} + s]$ . The move occurs after calculating the second difference over the previous interval:  $[t_{a(p_i)} - k + (s - 1) ; t_{a(p_i)} + (s - 1)]$ .

Where  $a(p_i) = (1 - p_i)T_i$ ,  $k$  is a Fibonacci number and  $s = 0, 1, \dots, n$  while  $t_{a(p_i)}$  is the point in time that corresponds with  $a(p_i)$ .

There are three scenarios that can occur over a continuous time interval and/or discrete time interval when applying the SD-Test when  $\frac{dV}{dT} > 0$  or  $\frac{dV}{dT} < 0$ .

Denote  $V$  as the value of the asset class,  $T$  as time and let

$\frac{d}{dT} \left( \frac{dV}{dT} \right) = \frac{d^2V}{dT^2} = \lim_{\Delta T \rightarrow 0} \frac{\Delta^2 V}{\Delta^2 T}$ . One of the following 3 scenarios may occur over a

continuous time interval:

(i) **Scenario 1:**  $\frac{d}{dT} \left( \frac{dV}{dT} \right) > 0$ , over the interval  $(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$

while  $\frac{dV}{dT} > 0$ .

(ii) **Scenario 2:**  $\frac{d}{dT} \left( \frac{dV}{dT} \right) < 0$ , over the interval  $(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$

while  $\frac{dV}{dT} > 0$

(iii) **Scenario 3:**  $\frac{d}{dT} \left( \frac{dV}{dT} \right) > 0, < 0$  or  $= 0$ , over the interval

$(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$  while  $\frac{dV}{dT} > 0$

(iv) **Scenario 4:**  $\frac{d}{dT} \left( \frac{dV}{dT} \right) > 0, < 0$  or  $= 0$ , over the interval

$(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$  while  $\frac{dV}{dT} < 0$

However, the only information available is a set of discrete data points. These discrete data points consist of an interval, with length  $k$ -units, of dates. In the discrete case, the following three scenarios may occur if  $\frac{\Delta V}{\Delta T} > 0$  or  $\frac{\Delta V}{\Delta T} < 0$  and when applying the SD-Test:

(i) **Scenario 1:**  $\frac{\Delta^2 V}{\Delta^2 T} > 0$ , over the interval  $(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$  while  $\frac{\Delta V}{\Delta T} > 0$ .

(ii) **Scenario 2:**  $\frac{\Delta^2 V}{\Delta^2 T} < 0$ , over the interval  $(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$  while  $\frac{\Delta V}{\Delta T} > 0$

(iii) **Scenario 3:**  $\frac{\Delta^2 V}{\Delta^2 T} > 0, < 0$  or  $= 0$ , over the interval  $(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$  while  $\frac{\Delta V}{\Delta T} > 0$

(iv) **Scenario 4:**  $\frac{\Delta^2 V}{\Delta^2 T} > 0, < 0$  or  $= 0$ , over the interval  $(t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$  while  $\frac{\Delta V}{\Delta T} > 0$

The computation of the second differences is given in table 4.2.2.1:

**Table 4.2.3.1: The first and second divided differences estimate the first and second derivatives, respectively.**

Data	First divided difference	Second divided difference
$T_1 \quad V_1$	$\frac{V_2 - V_1}{T_2 - T_1}$	$\frac{\frac{V_2 - V_1}{T_2 - T_1} - \frac{V_3 - V_2}{T_3 - T_2}}{T_3 - T_1}$
$T_2 \quad V_2$		
$T_3 \quad V_3$	$\frac{V_3 - V_2}{T_3 - T_2}$	

Note that time increases by increments of one, but in the SD-Test the increments equal  $k$  (see section 4.2.3). The second differences can be calculated as follows because the time interval is fixed ( $k$ ):

$$\Delta^2 V_i = (V_{i+2} - V_{i+1}) - (V_{i+1} - V_i) \quad i = 1, 2, \dots \text{ over the interval } (t_{a(p_i)} - k + s ; t_{a(p_i)} + s)$$

Scenario 1 indicates that the change in value of a specific asset class is increasing with a positive rate of change over a specific interval. On the other hand, scenario 2 indicates that the change in value of a specific asset class is increasing at a decreasing rate of change over a specific interval. Both of these

scenarios are included as a combination in scenario 3. Scenario 1 and scenario 2 stipulate the speed at which the value of the asset class is changing and the direction in which it is moving. A problem can occur in determining the speed and direction of the value of an asset class when scenario 3 or 4 is present. To deal with this problem the exponential weighted moving averages are calculated on the values of the SD-Test of a specific asset class.

Exponential smoothing is a method to produce a smooth time series. In the Single Moving Averages, all the observations are weighted equally. By using the Exponential smoother, recent observations are given a greater weight than the previous observations. This model is called the exponential weighted moving average.

The exponential function is:  $f(t) = \delta e^{\beta(t-1)}$  with  $\delta$  and  $\beta$  the parameters. According to Pindyck and Rubinfeld (2001) if the exponential function is used to determine weights  $f(t_1), f(t_2), \dots, f(t_e)$  subject to the constraint that the sum of the weights equals unity, the weights are given by the following formula:

$$\alpha(1-\alpha)^{\tau} \quad \tau = 0, 1, \dots \quad (1)$$

Here  $\alpha$  is a number between 0 and 1 that indicates how heavily the portfolio manager will weight recent values relative to older ones. The eight item exponential weighted moving average is introduced where the exponential weights are calculated using (1):

$$\alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \dots, \alpha(1-\alpha)^7$$

The length of the interval ( $k$ ) over which the second differences are taken is 8 because, according to the Elliott Wave Theory, the Fibonacci number 8 is suggested for calculations over a short interval. The concern of a portfolio manager is to time the market before any sudden movements in the market may occur. The calculations of the SD-Test are done over a short interval ( $k = 8$ ). The closer to unity the number of items in a moving average with weights according to an exponential function, the closer the moving averages will be to the original series. By doing this it will not make any contribution in the analysis.

To assist the portfolio manager to determine the speed at which the value of the asset class is changing, and in which direction this change occurs, the eight-item exponential weighted moving average is introduced to smooth out the noise of the second differences. This gives a better idea what is happening with the value of the asset class value. The second differences are taken over a fixed time interval  $[t_{a(p_i)} - k ; t_{a(p_i)}]$  with  $k = 8$ . After the second difference of a fixed interval is calculated, the next second difference is determined. This fixed time interval moves one unit for  $s$  times  $[t_{a(p_i)} - k + s ; t_{a(p_i)} + s]$ . Note that  $s > 16$  to have at least enough moving averages to smooth out the noise. The first eight second differences (between the interval  $[t_{a(p_i)} - k ; t_{c_1}(t_{c_2})]$ ) will be respectively assigned the following exponential weights  $\alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \dots, \alpha(1-\alpha)^7$ . Denote  $\underline{W}$  as the vector of weights assigned to the second differences to smooth out the noise:

$$\underline{W}^T = [\alpha(1-\alpha)^7 ; \alpha(1-\alpha)^6 ; \dots ; \alpha]$$

Denote  $\mathbf{v}_1$  as the first vector consisting out of the values of the first eight second differences calculated over the subinterval with length 8  $[t_{a(p_1)} - k; t_{c_1}(t_{c_2})]$ .

$$\mathbf{v}_1 = [SD_1 \quad SD_2 \quad \dots \quad SD_8]$$

The last value of the second difference in the first vector (second difference ( $SD_8$ )) calculated over the time interval  $[t_{a(p_1)} - 1; t_{a(p_1)} + 7]$ ) will be assigned the exponential weight  $\alpha$ . The first value of the second difference (second difference ( $SD_1$ )) calculated over the time interval  $[t_{a(p_1)} - 8; t_{a(p_1)}]$ ) will be assigned the exponential weight  $\alpha(1-\alpha)^7$ . The calculation of the first exponential smoothed second differences is:

$$sd_1 = \mathbf{v}_1 \tilde{W}$$

In general:

$$sd_n = \mathbf{v}_n \tilde{W} \quad n = 1, 2, \dots$$

This process continues by assigning exactly the same weights ( $\underline{W}$ ) to each successive vector. The second differences in the vector move respectively one unit at a time. This is done by adding the next second difference observation and dropping the first second difference observation. These vectors are:

$$\mathbf{v}_1 = [SD_1 \quad SD_2 \quad \dots \quad SD_8]$$

$$\mathbf{v}_2 = [SD_2 \quad SD_3 \quad \dots \quad SD_9]$$

$$\mathbf{v}_s = [SD_s \quad SD_{s+1} \quad \dots \quad SD_{s+7}]$$

where  $s$  is the total number of data points, which is 7.

This process stops when the last second difference for the last vector is calculated in the point  $t_{c_1}(t_{c_2})$ .

$$t_{sd_{s+7}} = t_{c_1}$$

$$\text{or } t_{sd_{s+7}} = t_{c_2}$$

In general, the exponential weighted moving average on the second differences can be calculated as:

$$[sd_1 \quad sd_2 \quad \dots \quad sd_k] = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_s] \begin{bmatrix} \alpha(1-\alpha)^7 \\ \alpha(1-\alpha)^6 \\ \cdot \\ \cdot \\ \cdot \\ \alpha \end{bmatrix}$$

where

$$\tilde{W} = \begin{bmatrix} \alpha(1-\alpha)^7 \\ \alpha(1-\alpha)^6 \\ \cdot \\ \cdot \\ \cdot \\ \alpha \end{bmatrix}$$

and

$$\mathbf{v}_1 = [SD_1 \quad SD_2 \quad \dots \quad SD_8]$$

$$\mathbf{v}_2 = [SD_2 \quad SD_3 \quad \dots \quad SD_9]$$

$$\mathbf{v}_s = [SD_s \quad SD_{s+1} \quad \dots \quad SD_{s+7}]$$

and  $s$  is the total number of data points minus the integer 7.

where the last second difference for the last vector is calculated in:

$$t_{sd_{s+7}} = t_{c_1}$$

$$\text{or } t_{sd_{s+7}} = t_{c_2}$$

An empirical assessment will illustrate the procedure that the portfolio manager should put in place to use the SD-method. The data for the following two sections will be the same as the data which the study used in section 3.3.1.

### **4.3 Empirical Illustration of the SD-method**

The empirical illustration is divided into two separate sections (see section 3.3):

- Data and Input assumptions
- Formulating rebalancing rules

These two sections give an outline of the empirical illustration that must hold to get results (see section 4.4) using the SD-method. These two sections define the procedure the portfolio manager should put in place to use the SD-method.

#### **4.3.1 Data and Input assumptions**

The data the study uses to assess the SD-method is the same that was used to illustrate Masters' range rebalancing strategy (see section 3.3.1). The data used to assess the SD-method:

- Pension fund consisting of five different asset classes
- Data over the period September 2003 to December 2005
- Daily index data
- Indices used: J403, ALBI, STeFI, MSCI and WGBI.

As discussed in section 3.3.1, there are two input assumptions. The second input assumption is the cost of rebalancing. In order to compare the SD-method with the benchmark strategy, the cost of rebalancing a particular asset class will be same as discussed in section 3.3.1 (see table 3.3.1.1 column 4). The first input assumption, risk tolerance of the portfolio manager, differs from the assumption made in chapter 3. As discussed in section 4.2.1, the SD-method introduced a minimum ( $K_1$ ) and maximum ( $K_2$ ) risk tolerance level for the portfolio manager. The two risk tolerance levels that will be used in the SD-method for the portfolio manager of a pension fund are 5% and 7.5%, respectively. The 5% risk tolerance level is selected because the study wants to compare the SD-method with Masters' range rebalancing strategy. Masters' range rebalancing strategy uses a 5% risk tolerance level. The 7.5% risk tolerance level was arbitrarily selected as the maximum ( $K_2$ ) risk tolerance level for the portfolio manager. The corresponding set of bands for the minimum ( $K_1$ ) and maximum ( $K_2$ ) risk tolerance levels are  $(T_{t_{UP\ min}}; T_{t_{LOW\ min}})$  and  $(T_{t_{UP\ max}}; T_{t_{LOW\ max}})$ . Table 4.3.1.1 in the last four columns shows the resulting set of bands (trigger points) over three different periods (see section 3.3.2.1).

Table 4.3.1.1: Resulting Trigger Points for Different Risk Tolerance Levels

Asset Class	Target Allocation	$T_{\%diff_{max}}$	$T_{\%diff_{min}}$	$T_{\%P_{max}}$	$T_{\%P_{min}}$
<b>2003/08/11 - 2003/09/12</b>					
SA Equities	54.61%	48.11%	50.27%	58.95%	61.11%
SA Bonds	26.06%	15.14%	18.17%	33.35%	36.99%
SA Cash	9.28%	5.30%	6.63%	11.92%	13.25%
Global Equities	6.14%	-7.94%	-3.25%	15.53%	20.22%
Global Bonds	3.81%	-8.12%	-4.15%	11.76%	15.74%
<b>2004/08/11 - 2004/09/13</b>					
SA Equities	54.61%	50.60%	51.60%	57.62%	59.12%
SA Bonds	26.06%	14.33%	18.24%	33.88%	37.79%
SA Cash	9.28%	4.46%	6.07%	12.48%	14.09%
Global Equities	6.14%	-4.44%	-0.92%	13.19%	16.72%
Global Bonds	3.81%	-11.75%	-6.57%	14.18%	19.37%
<b>2005/08/11 - 2005/09/12</b>					
SA Equities	54.61%	39.79%	44.73%	64.49%	69.43%
SA Bonds	26.06%	14.71%	18.50%	33.63%	37.41%
SA Cash	9.28%	5.02%	6.44%	12.11%	13.53%
Global Equities	6.14%	-35.93%	-21.91%	34.18%	48.21%
Global Bonds	3.81%	-9.77%	-5.25%	12.86%	17.39%

The next section will discuss when rebalancing should occur and to what level the portfolio manager should rebalance when using the SD-method.

#### 4.3.2 Formulating rebalancing rules

To assess the SD-method, certain rebalancing rules should be formulated based on when rebalancing should occur and to what level the portfolio manager should rebalance the particular asset class.

### 4.3.2.1 When rebalancing should occur

The SD-method consists of: range rebalancing, retracement levels and the SD-test discussed in section 4.2. To indicate when rebalancing should occur, the portfolio manager should take all three parts of the SD-method into consideration.

The SD-method uses Masters' range rebalancing strategy formula (3) to determine the trigger points. The SD-method uses two different risk tolerance levels ( $K_1$  and  $K_2$ ) for the portfolio manager to calculate the trigger points of a specific asset class. Using two different risk tolerance levels will result in two sets of bands ( $(T_{iLP_{min}}; T_{iLOW_{min}})$  and  $(T_{iLP_{max}}; T_{iLOW_{max}})$ ) indicating when rebalancing should occur. For the values of the two sets of bands refer to the last four columns of table 4.3.1.1. Rebalancing should occur whenever:

- $T_{iLP_{max}}$  on the upside or  $T_{iLOW_{max}}$  on the downside is intersected.

Before an asset classes percentage movement reaches  $T_{iLP_{max}}$  or  $T_{iLOW_{max}}$  this percentage movement must intersect  $T_{iLP_{min}}$  or  $T_{iLOW_{min}}$ .  $T_{iLP_{min}}$  or  $T_{iLOW_{min}}$  gives a warning signal that rebalancing may be considered. From now on, the tempo in the rate of change is monitored. Whenever these two bands are intersected, the SD-Test is activated.

The resistance and support levels discussed in section 4.2.2 should be taken into consideration to add certainty as to when rebalancing should occur. Note that when applying the resistance and support levels, intraday data should be

available. In this study only one asset class (equity) consists of intraday data. Rebalancing should occur whenever:

- The second differences are positive,  $\frac{\Delta^2 V}{\Delta^2 T} > 0$  and  $\frac{\Delta V}{\Delta T} > 0$  (or second differences are negative,  $\frac{\Delta^2 V}{\Delta^2 T} < 0$  and  $\frac{\Delta V}{\Delta T} < 0$ ) and intersect the resistance (or support) level. Remember that the resistance level or support level is a risk constraint if, and only if, the resistance level (or support) level is below (or above)  $T_{i,P_{max}}$  (or  $T_{i,LOW_{max}}$ ).

The last constraint use to indicate when rebalancing should occur is the SD-test. The SD-test is activated whenever the  $T_{i,P_{min}}$  or  $T_{i,LOW_{min}}$  is intersected. If the SD-test is activated for a specific asset class, then the portfolio manager will monitor the asset class on a daily basis. The starting point for the SD-test is denoted by  $a(p_i) = (1 - p_i)T_i$ . The study assumes an arbitrary 25% value for  $p_i$ . Three scenarios can occur when determining the SD-test - refer to section 4.2.3. Rebalancing should occur whenever:

- The second differences indicate that the change in value of a specific asset class is increasing (decreasing) with an increasing (decreasing) rate of change above (below) the target allocation.

To summarise when rebalancing should occur using the SD-method, refer to Appendix C. The next section discusses to what level the portfolio manager should rebalance.

### 4.3.2.2 To what level to rebalance

The study by Buetow *et al* (2002:25) suggests rebalancing back to the target allocation. The study by Masters (2003) suggests rebalancing only halfway back to the target allocation (see section 2.6 & 3.3.2.2). The SD-method assumes rebalancing halfway back to the target allocation, because the study wants to compare the two methods with one another.

The performance of the SD-method will be presented in the next section putting all the rebalancing rules in place.

## 4.4 Results

The SD-method can be assessed using table 4.3.1.1 in section 4.3.1. Before continuing, the criteria used to assess the SD-method should be reviewed:

- Pension fund over the period September 2003 to December 2005.
- The portfolio consists of five asset classes.
- The risk tolerance level of the portfolio manager is 5% and 7.5%, respectively.
- Cost of rebalancing varies between the asset classes (see table 3.3.1.1).
- The portfolio manager rebalances halfway back to the range (see table 3.3.1.2).
- Performance is measured using return, risk and risk-adjusted return.

**Table 4.4.1: Summarised Results of Different Rebalancing Strategies**

Summarised results of different rebalancing strategies over the period September 2003 - December 2005

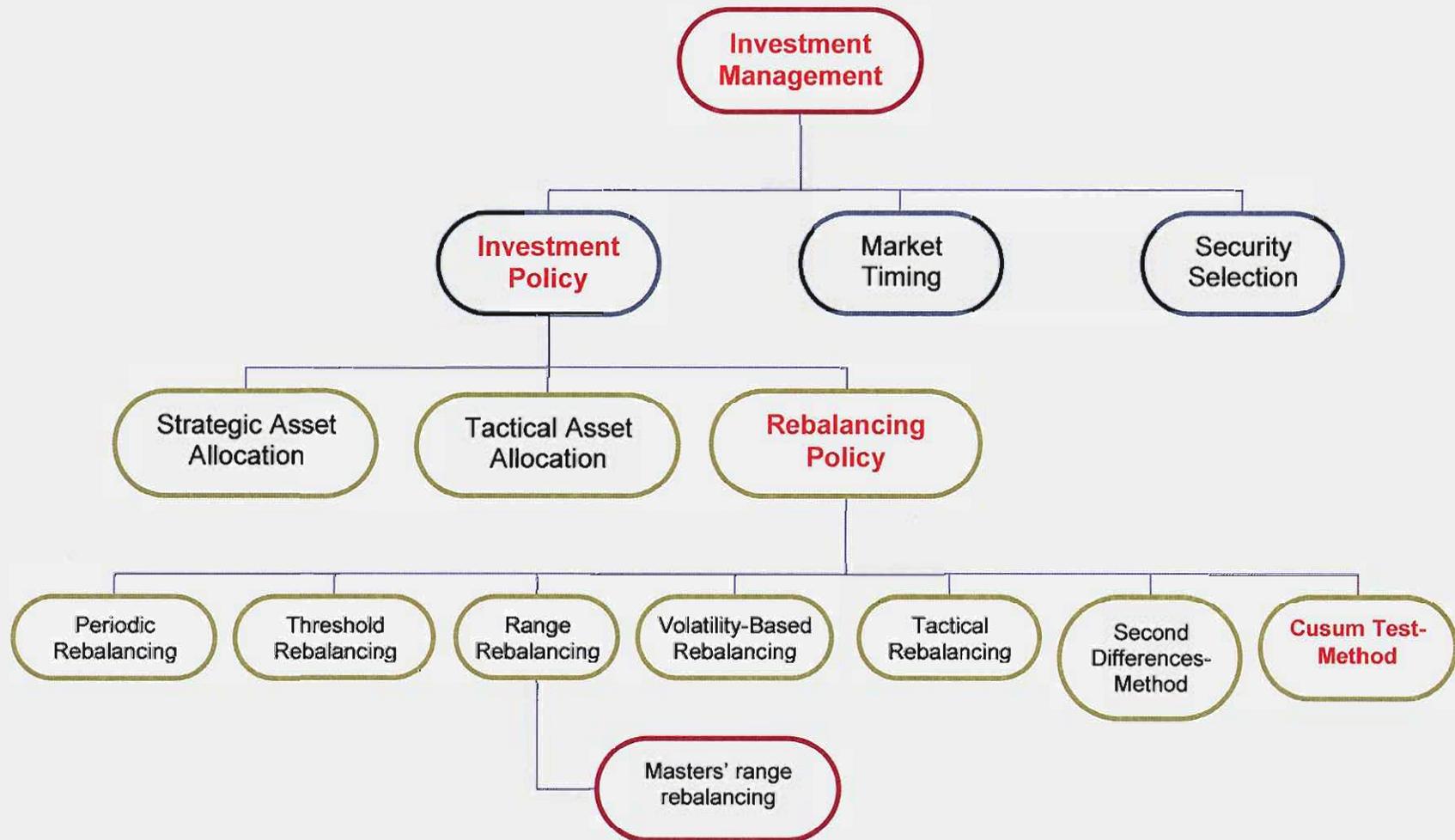
Results Rebalancing Strategies	Performance Measurement Tools			Number of Trades
	Portfolio Return	Portfolio Risk	Risk-adjusted Return	
<b>Range Rebalancing</b>	30.77%	2.11%	14.58	14
<b>SD - Method</b>	22.66%	1.40%	16.19	11

Table 4.4.1 compares the two rebalancing strategies, Masters' range rebalancing and the SD-method on the basis of their results using the performance measurement tools. The table consists of the performance measurement tools along the columns and the different rebalancing strategies along the rows. To compare the two rebalancing strategies the portfolio manager should analyse the risk-adjusted return. The SD-method (risk-adjusted return = 16.19) outperforms Masters' range rebalancing strategy (risk-adjusted return = 14.58) on the basis of the risk-adjusted return. This indicates that the SD-method risk-adjusted return is much larger than Masters' range rebalancing risk-adjusted return. The SD-method portfolio returns are 16.19 times larger than the risk of the portfolio where Masters' range rebalancing portfolio returns are 14.58 times larger than the risk of the portfolio. The last column of table 4.4.1 represents the number of trades per rebalancing strategy over the period September 2003 to December 2005. The total number of trades for the SD-method was 11 to the 14 trades of Masters' range rebalancing strategy and it still outperforms Masters' range rebalancing strategy on a risk-adjusted return basis.

#### 4.5 Conclusion

This chapter built on Masters' range rebalancing strategy by introducing the SD-method to eliminate certain implicit assumptions made by Masters (see section 3.2). The SD-method was introduced to enhance the return of a portfolio according to the risk specifications of the trustees of a pension fund. The SD-method consists of three components, range rebalancing, retracement levels and the SD-Test. These components assisted the portfolio manager in making rebalancing decisions, for example, when rebalancing should occur. From the discussions in chapter 4, some of the assumptions of Masters were relaxed, but this increased the complexity of the procedure to rebalance. The computer program eased the implementation of the SD-method but the ranges were not adjustable over time.

In Chapter 5, a new method is introduced to identify when rebalancing should occur, and which has its own assumptions and risk specifications. The new method discussed in chapter 5 is less complex to implement and its ranges are adjustable over time.



# CHAPTER 5

## CUSUM TEST METHOD

### 5.1 Introduction

Chapter 4 built on existing range rebalancing strategies (Masters' range rebalancing strategy), but added two components to assist the portfolio manager in making decisions as to when rebalancing should occur. The new method introduced in chapter 4 was called the second differences method (SD-method). The SD-method was developed to enhance the return of the portfolio according to the risk specifications of the trustees of a pension fund. The complexity of the implementation of the SD-method increased as a result of additional risk constraints. The complexity of the implementation of the SD-method is a problem according to this study's objectives. One of the study's objectives is that the rebalancing strategy should be easy to implement (see section 1.3). The computer program eased the implementation of the SD-method but the ranges of the SD-method were not adjustable over time.

The study of Donohue and Yip (2003:49) found that little research has been done on developing optimal implementation strategies that will assist the portfolio manager in managing a portfolio. These optimal rebalancing strategies are too complex to implement. The complexity of the implementation of a new rebalancing strategy postulates to a proposed rebalancing strategy, discussed in this chapter. The proposed rebalancing strategy is based on the technique of Brown, Durbin and Evans (1975:149-163) and is called the Cusum Test (CT-method). The CT-method is presented as an alternative rebalancing strategy

because the statistical methods may satisfy both of the main objectives (see section 1.2):

- The rebalancing strategy should be easy to implement.
- The rebalancing strategy should be adjustable, so that the ranges that indicate when rebalancing should occur are able to fluctuate with time.

The new method has its own less strict assumptions to identify the point when rebalancing should occur.

The first part of chapter 5 will set the scene for the CT-method. This chapter continues by presenting background information on how the proposed rebalancing strategy has been evolved. The second part of this chapter will introduce some literature on the proposed rebalancing strategy. The literature will focus on the working mechanics of the statistical interference methods and regression analysis methods. An empirical illustration of the proposed rebalancing strategy will follow. The empirical illustration will be used as the guideline for the final results on the CT-method. The last part of this chapter will conclude by making final remarks.

The next section will set the scene as to why the CT-method will be used as an alternative rebalancing strategy.

## 5.2 Background

The objective of this study is to guide the portfolio manager in deciding when rebalancing should occur. A well-defined rebalancing strategy is the guidance tool used by the portfolio manager to enhance the returns and reduce the risk of a portfolio. There are three focus points the portfolio manager should apply to achieve this objective:

- A well-defined rebalancing strategy
- maximising the returns
- minimising the risk

A variety of studies have shown that no single rebalancing strategy exists that is optimal for all investors (Alexander Forbes, 2004). To identify or develop a well-defined rebalancing strategy, the type of portfolio should be known in advance. The focus area is on pension funds where the trustees, as the committee, specify the risk appetite of the portfolio. This chapter will develop a tailor-made rebalancing strategy to satisfy the needs of a pension fund according to the trustees' risk specifications.

"The mechanism by which rebalancing adds value and overcomes the effects of a large returns differential between the asset classes is straightforward. The discipline that it imposes of buy low / sell high / book profit, is remarkably powerful, acting like a ratchet mechanism, to compound a series of small benefits over time." (Goodsall & Plaxo, 1994)

According to Goodsall and Plaxo (1994), implementing a well-defined rebalancing strategy will add value to the portfolio. Thus the tailor-made rebalancing strategy will enhance the returns of the portfolio according to the risk

appetite of the trustees. Risk and return have a direct relationship to one another (e.g. the higher the risk, the higher the return and vice versa).

The key point of this tailor-made rebalancing strategy is to keep the portfolio focused on the long-term objective. The long term objective is to keep the risk appetite of the trustees of a pension fund intact. By keeping the long-term objective intact, the portfolio will also increase in value according to Goodsall and Plaxo (1994). This led to the development of a tailor-made rebalancing strategy that can be used to monitor and manage the portfolio according to the long-term objective.

### 5.2.1 F-test method

According to Goodsall and Plaxco (1996:1), passive rebalancing is a risk control mechanism. The risk of an asset class should be monitored by a portfolio manager. Linear regression analysis estimates a straight line through data points so that the residuals are minimised. These residuals can be used to calculate the risk of an asset class. Risk is thus measured by variance and variance is defined in terms of the residuals  $\left(\sigma^2 = \frac{1}{n-1} \sum (\text{Residuals})^2\right)$ . If there is a significant difference in the risk measured against the previous risk, rebalancing should occur to keep the risk in line with the historical risks. The variance is a measurement for risk. If the risk  $(\sigma_r^2)$  is compared with the historical risk  $(\sigma_i^2)$  and  $(\sigma_r^2)$  differs significantly from  $(\sigma_i^2)$  then rebalancing should occur.

A method which uses the F-test was developed to monitor and manage the risk of each asset class in a portfolio. The risk of each asset class was encapsulated

in the variance of that specific asset class. This is to quantify the risk of an asset class to assist the portfolio manager in deciding when rebalancing should occur.

Given the following simple linear regression formula:

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \quad i = 1, \dots, n$$

with the  $\varepsilon_i \sim N(0, \sigma^2)$  and  $Y_i \sim N(\beta_1 + \beta_2 X_i, \sigma^2)$ .

The variance can be calculated as:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

where  $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$  and  $\hat{\alpha}$  and  $\hat{\beta}$  are least squares estimates for  $\alpha$  and  $\beta$ .

It can be shown that  $Y_i$  and  $\hat{Y}_i$  are standard normal random variables.

The proposed F-test method is:

$$F_r = \frac{\sum_{t=k+1}^r s_t^2}{\sum_{t=k+1}^r s_t^2} \quad r = k+1, \dots, T$$

The sum of squares of independent standard normal random variables are Chi-square distributed (Rice). Thus

$$F_r = \frac{\sum_{t=k+1}^r s_t^2}{\sum_{t=k+1}^T s_t^2} \sim \frac{\frac{\chi_{n_1-1}^2}{n_1-1}}{\frac{\chi_{n_2-1}^2}{n_2-1}} \sim F_{n_1-1, n_2-1}$$

See Rice(1995:181-182) for the formal proof.

The number of data points ( $n$ ) should be evaluated to see whether a smaller number of data points have or do not have a significant effect on the variance. Remember that the F-test method encapsulates the variance of a specific asset class. This is to quantify the risk of a specific asset class to assist the portfolio manager in deciding when rebalancing should occur.

To prove that the variance is significant for a small number of data points ( $n$ ), the properties of the variance of  $\chi^2$  distribution should be known. If  $T \sim \chi_n^2$  distributed, the following holds:

- $E(T) = n$
- $Var(T) = 2n$

Thus the variance of a  $\chi_{n-1}^2$  distribution is:

$$\begin{aligned} \text{Var}\left(\frac{(n-1)s^2}{\sigma^2}\right) &= 2(n-1) \\ \frac{(n-1)^2}{\sigma^4} \text{Var}(s^2) &= 2(n-1) \\ \text{Var}(s^2) &= \frac{2\sigma^4}{n-1} \\ &\approx \frac{2\left(\frac{1}{n-1} \sum (Y - \bar{Y})^2\right)^4}{n-1} \\ &\approx \frac{2\left(\sum (Y - \bar{Y})^2\right)^2}{(n-1)^5} \end{aligned}$$

The formula above shows that the variance still has a significant effect for a small number of data points ( $n$ ). The problem with the F-test method is that it assumes that the data is identically and independently distributed. The definition of the F-distribution states that if there are two stochastic variables  $U \sim \chi_m^2$  and  $V \sim \chi_n^2$  and  $U$  and  $V$  are independent then  $W = \frac{U/m}{V/n} \sim F_{m,n}$ . The problem with the F-test method continues because the two chi-squared variables are not independent from one another. The study uses time series data that are dependent on one another. To solve this problem, the study moves towards regression analysis, more specifically the study of Brown *et al* (1975).

### 5.2.2 Brown, Durbin and Evans

Statistical tests exist to determine whether a regression relationship remains the same over the entire period of observations. These tests may show that the regression relationship changes over time and a solution for this problem is to introduce dummy variables when the change in the relationship is suspected, or to split the sample period at that point and perform the exact F-test devised by Chow (1960). Chow's F-test is subject to certain assumptions:

- The assumptions underlying the test must be fulfilled
- The Chow test will tell only if two regressions are different, without telling whether the difference is because of the intercepts, the slopes, or both.
- The Chow test assumes that the possible point(s) of structural break(s) are known in advance.

Based on portfolio management and specifically rebalancing asset classes in a portfolio, using F-values would become very useful for determining when the variance relationship of the specific asset class is not constant. Rebalancing would occur whenever the variance shifts relative to the previous variances of a specific asset class.

The problem when using dummy variables or the Chow's F-test is that both these procedures require a priori knowledge of the possible time when the function shifts (Khan, 1965:1208). Alternative tests devised by Goldfeld and Quandt (1972) and Fair and Jaffee (1972) do not require a priori information. The implementation of these tests is, in general, difficult to apply (Khan, 1974: 1208). A much simpler method of determining whether a regression relationship changes over a period of time has been developed by Brown *et al* (1975:149-

163). The following section discusses one technique proposed by Brown, Durbin and Evans.

### 5.3 Brown, Durbin, Evans techniques

The following section will discuss one technique, the Cusum Test, proposed by Brown *et al* (1975:149-163) for the testing of whether the regression relationship changes over time (stable).

#### 5.3.1 Cusum Test

Consider the standard regression model:

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + \varepsilon_t \quad t = 1, \dots, T$$

where

$y_t$  is the observation on the dependent variable,

$\mathbf{x}_t$  is the column vector of observations on  $k$  regressors,

$\boldsymbol{\beta}_t$  is the regression coefficients that can vary with time, and

$\varepsilon_t$  is representing the error term where  $\varepsilon_t \sim N(0, \sigma_t^2)$  and linear independent for  $t = 1, \dots, T$

The hypothesis tests whether  $\boldsymbol{\beta}_t$  is constant up to time  $t = t_0$  and differs from this constant value from then on.

The hypothesis to be tested is:

$$H_0 : \beta_1 = \beta_2 = \dots = \beta_t = \beta$$

Let  $b_r$  be the least-squares estimate of  $\beta$  from the sample of the first  $r$  observations, and let

$$w_r = \frac{y_r - \mathbf{x}'_r \mathbf{b}_{r-1}}{\sqrt{1 + \mathbf{x}'_r (\mathbf{X}'_{r-1} \mathbf{X}_{r-1})^{-1} \mathbf{x}_r}}, \quad r = k+1, \dots, T, \quad (5.3.1)$$

where  $w_r$  is the transformed residuals according to Khan (1974) and  $\mathbf{X}'_{r-1} = [\mathbf{x}_1, \dots, \mathbf{x}_{r-1}]$ .

According to Khan (1974:1209), it can be shown that these transformed residuals,  $w_{k+1}, \dots, w_t$ , are normal independent variables with zero means and variances,  $\sigma_{w_{k+1}}^2, \dots, \sigma_{w_t}^2$ , if  $\varepsilon_i \sim N(0, \sigma^2)$  and  $\varepsilon_i$ 's are linearly independent.

Brown, Durbin and Evans suggest plotting the following variables against time:

$$W_r = \frac{1}{\hat{\sigma}} \sum_{j=k+1}^r w_j, \quad r = k+1, \dots, T$$

where  $\hat{\sigma}$  denotes the estimated standard deviation determined by  $\hat{\sigma}^2 = S_r / (T - k)$ .

$$S_r = (\mathbf{Y}_r - X_r \hat{\boldsymbol{\beta}}_r)' (\mathbf{Y}_r - X_r \hat{\boldsymbol{\beta}}_r)$$

$$S_r = S_{r-1} + w_r^2 \quad r = k+1, \dots, T$$

Draw a pair of lines symmetrically above and below the mean value line  $E(W_r) = 0$  such that the probability of crossing one or both lines is  $\alpha$ , where  $\alpha$  is the required significance level. The pair of straight lines go through the points  $\{k, \pm a\sqrt{T-k}\}, \{T, \pm 3a\sqrt{T-k}\}$ , where  $a$  is a parameter. According to Brown, Durbin and Evans (1975:154), useful pairs of values of  $a$  and  $\alpha$  are:

$$\alpha = 0.01, a = 1.143$$

$$\alpha = 0.05, a = 0.948$$

$$\alpha = 0.10, a = 0.850$$

#### 5.4 An Empirical Illustration of the CT-method

The empirical illustration is used to convert the regression method (Cusum Test) to the proposed rebalancing strategy. Remember that the Cusum Test is not a rebalancing strategy. This will be the first time that these econometric / statistical procedures are used in the context of rebalancing. In section 3.2 Masters' range rebalancing strategy was assessed. Masters used three components to calculate the trigger point, whenever these intersected, the portfolio manager should rebalance that specific asset class. These three components are:

- Risk tolerance of the portfolio manager ( $K$ ).
- Cost of rebalancing a specific asset class ( $C$ ).
- Tracking error (variance between a specific asset class and the rest of the portfolio excluding that specific asset class).

If these regression methods are used as rebalancing strategies, the three bullets above should be addressed. The CT-method will be used as a rebalancing strategy. The following section will address the three bullets mentioned above around the CT-method. This is to illustrate the procedure so that the portfolio manager should be able to use it to determine when rebalancing should occur.

#### 5.4.1 Empirical Illustration of the CT-method

The study uses time series data of the returns of five asset classes to assess the rebalancing strategies. The rates of return for each asset class are used to make the time series stationary<sup>13</sup>. To adjust a time series from non-stationary to stationary time series, the growth rates are taken. The first step is to construct a time series model for the CT-method.

According to Fama (1965), the random walk process is often used as a model for the movement of stock market prices. The random walk process is used to encapsulate the variance of the stock market prices over sub periods. The variance is a measurement for risk (see section 5.2.1). If the risk ( $\sigma_t^2$ ) is compared with the historical risk ( $\sigma_t^2$ ) and ( $\sigma_t^2$ ) differs significantly from ( $\sigma_t^2$ ) then rebalancing should occur. The random walk process can be combined with the autoregressive models. Consider the following general time series model:

$$Y_t = \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_n Y_{t-n} + \varepsilon_t \quad t = 1, 2, \dots$$

<sup>13</sup> A time series is stationary if the empirical observations fluctuate around a relatively constant mean (Bails & Peppers, 1982:382).

The best model will be chosen according to the partial autocorrelation function on the data of the specific asset class. After determining the best model for the data, the Cusum Test will be implemented. Before the Cusum Test can be implemented, the portfolio manager should decide between three significance values ( $\alpha$ ). These three significance values will be used in the CT-method as the risk tolerance level of the portfolio managers. The chosen  $\alpha$  values are dependent on the risk appetite of the portfolio managers. The risk-seeking portfolio managers will choose a 10% significance value, whereas a risk-averse portfolio manager will choose a 1% significance value. This study focuses only on pension funds and a 1% significance value is assumed for the CT-method. After deciding the significance values, the portfolio manager can implement the Cusum Test and determine when rebalancing should occur.

When should rebalancing occur according to the CT-method? The following variables should be plotted against time  $\left( W_r = \frac{1}{\hat{\sigma}} \sum_{j=k+1}^r w_j, r = k+1, k+2, \dots, T \right)$ . The procedure continues by drawing a pair of lines symmetrically above and below the mean value line  $E(W_r) = 0$ . This pair of straight lines goes through the points  $\{k, \pm a\sqrt{T-k}\}, \{T, \pm 3a\sqrt{T-k}\}$  where  $a$  is the parameter for a given significance level ( $\alpha$ ). The risk tolerance level of the portfolio manager is where  $\alpha = 0.05$  and, according to Brown, *et al* (1975), the corresponding  $a = 0.948$ . Thus the pair of straight lines goes through the points  $\{k, \pm 1.143\sqrt{T-k}\}, \{T, \pm 3(1.143)\sqrt{T-k}\}$ . Rebalancing should occur whenever the variable ( $W_r$ ) intersects the pair of straight lines for a  $\alpha$ -value of 0.05.

## 5.5 Results

The CT-method can be assessed using the data in section 3.3. Before discussing the results of the CT-method, the criteria used to assess the CT-method should be reviewed:

- Pension fund over the period September 2003 to December 2005.
- The portfolio consists of five asset classes.
- The risk tolerance level of the portfolio manager is 5%.
- The cost of rebalancing varies between the asset classes (see table 3.3.1.1).
- The portfolio manager rebalances halfway back to the range (see table 3.3.1.2).
- Performance is measured using return, risk and risk-adjusted return.

**Table 5.5.1: Summarised Results of Different Rebalancing Strategies**

Summarised results of different rebalancing strategies over the period September 2003 - December 2005

Results Rebalancing Strategies	Performance Measurement Tools			Number of Trades
	Portfolio Return	Portfolio Risk	Risk-adjusted Return	
Range Rebalancing	30.77%	2.11%	14.58	14
SD - Method	22.66%	1.40%	16.19	11
CT - Method	63.37%	3.55%	17.85	1

Table 5.5.1 compares the three rebalancing strategies, Masters' range rebalancing (see section 3.4), the SD-method (see section 4.4) and the CT-method on the basis of their performance. The table consists of the performance measurement along the columns and the rebalancing strategies along the rows.

To compare the three rebalancing strategies, the portfolio manager should analyse the risk-adjusted return. The CT-method (risk-adjusted return = 17.85) outperforms both Masters' range rebalancing (risk-adjusted return = 14.58) and the SD-method (risk-adjusted return = 16.19) on the basis of the risk-adjusted return. The total number of trades using the CT-method was 1, compared to the 11 and 14 trades for the SD-method and Masters' range rebalancing strategy respectively. The CT-method still outperforms the SD-method and Masters' range rebalancing strategy on a risk-adjusted return basis.

### **5.6 Conclusion**

The study developed a new method, the Cusum Test method (CT-method) with its own assumptions and risk specifications to identify when rebalancing should occur. The new method discussed in this study was simpler to implement and, in contrast with Masters' range rebalancing, the ranges were adjustable over time. The CT-method also outperformed the SD-method and Masters' range rebalancing strategy on a risk-adjusted return basis.

## CHAPTER 6

### CONCLUSION AND RECOMMENDATIONS

#### 6.1 Introduction

The aim of this research was to develop and implement a well-defined rebalancing strategy that is adjustable over time to assist portfolio managers in maintaining their portfolios in line with the objectives and risk aversions of the trustees of a pension fund. The study went further and put two objectives in place that should have been satisfied to determine whether the study was viable. The first objective was to investigate and implement Masters' range rebalancing strategy (see section 3.2). Masters' range rebalancing strategy was used as the benchmark rebalancing strategy. The benchmark strategy was used to measure the performance of the proposed rebalancing strategies (see sections 4.3 & 5.4). The second objective was to develop a well-defined rebalancing strategy that satisfies certain criteria:

- The rebalancing strategy must be tailor-made to satisfy the needs of the trustees of a pension fund.
- The rebalancing strategy should be easy to implement.
- The rebalancing strategy should be adjustable, so that the ranges indicating when rebalancing should occur can adapt with time.

In order to reach the aim and objectives of the study, a literature overview of the five broad rebalancing strategies was given, focusing on the benefits and the cost of rebalancing as well as to what level the portfolio manager should rebalance. The study continued by investigating and implementing Masters' range rebalancing strategy. Lastly, the study covered the development and implementation of the proposed rebalancing strategies the SD-method and the CT-method. The next section draws conclusions based on the literature and the results of the study (chapters 2, 3, 4 and 5).

## 6.2 Conclusions

Chapter 2 introduced the concept of a portfolio drift strategy to set the scene for the rebalancing strategies. This was to emphasize the importance of adopting a rebalancing strategy to the portfolio manager of a pension fund. There are three separate findings to show the advantage of rebalancing over portfolio drift.

- "On a risk-adjusted basis, however, there is no period in which drift has beaten rebalancing." (Arnott & Lovell, 1993)
- "Those who allowed their mix to drift paid the price of the higher volatility than any of the rebalancing disciplines." (Goodsall & Plaxco, 1994)
- "In allowing a portfolio to drift, one typically finds that the portfolio tends to become more risky." (Bradfield & Swartz, 2003)

The study went further to identify the benefits and cost of rebalancing. This concluded that there are a number of rebalancing strategies a portfolio manager can adopt, and there is no final formula to calculate the cost of rebalancing. The literature also presents the combination of rebalancing strategies to determine when rebalancing should occur. These strategies adopt their own procedure to calculate the cost of rebalancing. The study of Plaxco and Arnott (2002)

concluded that quarterly rebalancing outperforms the other periodic rebalancing strategies as well as outperforming portfolio drift. The study of Buetow *et al* (2002) went further by combining threshold rebalancing with quarterly rebalancing and found that a 5% threshold interval with daily monitoring outperforms the other rebalancing strategies as well as outperforming portfolio drift. Lastly, Masters' (2003) range rebalancing strategy outperformed quarterly rebalancing and portfolio drift.

Chapter 2 determined to what level the portfolio manager should rebalance an asset class. Arnott and Lovell (1993) suggest that the portfolio manager should rebalance back to the target allocation if quarterly rebalancing is used. Leland (1996) suggests that the portfolio manager should rebalance back to the range if threshold rebalancing or range rebalancing is used. Lastly, Masters (2003) suggests that the portfolio manager should rebalance half way back to the target allocation if range rebalancing is used. This indicates that it depends on the rebalancing strategy the portfolio manager uses to determine how far back the portfolio manager should rebalance.

As mentioned in the introduction, one of the objectives of the study was to investigate and implement Masters' range rebalancing strategy. This was to use Masters' range rebalancing strategy as the benchmark strategy to evaluate the proposed rebalancing strategies. A number of aspects arose in the investigation of Masters' range rebalancing strategy.

- An increase in the risk tolerance level will widen the rebalancing ranges.
- This increase in the risk tolerance level will imply less frequent rebalancing.
- The more volatile asset classes tend to have higher trigger points (ranges).

- Asset classes with higher trading cost will have wider ranges to minimise the transaction costs.
- More correlated asset classes should have similar ranges. As the correlation increases or decreases, the ranges should widen or narrow respectively.

The complexity of the implementation of Masters' range rebalancing strategy over optimal rebalancing strategies (e.g. Donohue & Yip, 2003) is less. This condition satisfies the objectives of the study.

The second objective was to develop a well-defined rebalancing strategy that must meet certain criteria:

- The rebalancing strategy must be tailor-made to satisfy the needs of the trustees of a pension fund.
- The rebalancing strategy should be easy to implement.
- The rebalancing strategy should be adjustable, so that the ranges that indicate when rebalancing should occur can fluctuate with time.

If a rebalancing strategy fulfils these conditions and it outperforms the benchmark strategy, it can be considered as a new rebalancing strategy. The last two chapters developed rebalancing strategies that satisfy these criteria. Chapter 4 developed the SD-method that built on Masters' range rebalancing strategy. The SD-method went further by eliminating certain implicit assumptions made by Masters (see section 3.2). The SD-method was developed to enhance the returns of a portfolio according to the risk specifications of the trustees of a pension fund. The SD-method consists of three components, range rebalancing, retracement levels and the SD-Test. These components assist the portfolio manager in making decisions, such as when rebalancing should occur. The

chapter concluded by comparing the SD-method against Masters' range rebalancing strategy. The SD-method outperformed Masters' range rebalancing strategy on the risk-adjusted return basis (see section 4.4) over the same period. The SD-method is a new rebalancing strategy that was developed in this study and it can be used to assist the portfolio managers to comply with the needs of the trustees of a pension fund. The SD-method lacks the last two criteria from the second objective of the study. The reason for the increased complexity of the implementation of the SD-method was due to the relaxation of some of the assumptions made by Masters. The complexity of the implementation of the SD-method was addressed by developing a computer program to ease the implementation of the SD-method, but the ranges were not adjustable over time.

Chapter 5 dealt with the second objective by developing the CT-method. The CT-method is easy to implement. These methods address one of the problems of rebalancing strategies, which is to adjust the ranges over time. The CT-method (risk-adjusted return = 17.85) outperformed Masters' range rebalancing (risk-adjusted return = 14.58) and the SD-method (risk-adjusted return = 16.19) on the basis of the risk-adjusted return.

### 6.3 Recommendations

Masters' range rebalancing made certain implicit assumptions and can be formulated as (see section 3.2):

- The cost function used to rebalance an asset class is linear.
- The tracking error is calculated by using only the particular asset class under consideration and not the rest of the portfolio.
- The tracking error is calculated over a fixed interval.

- The portfolio manager should rebalance only halfway back to the target allocation.

The study assumed Masters' implicit assumptions and used bullet 1 and 4 in the implementation of the SD-method and the CT-method. Further, the CT-method addressed bullet 3 by relaxing the assumption so that the variance of an asset class can be calculated on a variable interval.

Further research can determine whether the cost function used in Masters' range rebalancing strategy is linear. This study assumed that the cost function for the SD-method and the CT-method are linear. Further research can evaluate whether a linear cost function is appropriate for the SD-method as well as for the CT-method.

Masters proved that it is optimal to rebalance only halfway back to the target allocation (see appendix B). This study assumed that the portfolio manager should only rebalance halfway back to the target for the SD-method and the CT-method. Further research can determine to what level the portfolio manager should rebalance using the SD-method and the CT-method.

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# APPENDIX A

CALCULATION OF THE TRACKING ERROR.....A.2

## CALCULATION OF THE TRACKING ERROR

Calculation of the tracking error presented in the Masters' (2003) paper lacks justification.

It is assumed that asset class  $i$  is off-target by  $\Delta$ , while the rest of the portfolio excluding the  $i$ -th asset class is denoted by  $j$ , with relative weight  $-\Delta$ . The following vector is introduced between the portfolio and the benchmark:

$$w = \begin{pmatrix} +\Delta \\ -\Delta \end{pmatrix}$$

The tracking error variance between the portfolio and the benchmark can now be determined.

$$\begin{aligned} \sigma_{TE}^2 &= w' \Omega w \\ &= \begin{pmatrix} +\Delta \\ -\Delta \end{pmatrix}' \begin{pmatrix} \sigma_{ii} & \sigma_{ij} \\ \sigma_{ji} & \sigma_{jj} \end{pmatrix} \begin{pmatrix} +\Delta \\ -\Delta \end{pmatrix} \\ &= \Delta^2 \begin{pmatrix} +1 \\ -1 \end{pmatrix}' \begin{pmatrix} \sigma_{ii} & \sigma_{ij} \\ \sigma_{ji} & \sigma_{jj} \end{pmatrix} \begin{pmatrix} +1 \\ -1 \end{pmatrix} \\ &= \Delta^2 (\sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}) \end{aligned}$$

This expression between brackets is simply the tracking error between the asset class  $i$  and the rest of the portfolio excluding asset class  $i$  (asset  $j$ ). Thus,

$$\sigma_{TE}^2 = \Delta^2 (\text{Tracking Error})^2$$

# APPENDIX B

HOW FAR BACK THE PORTFOLIO MANAGER SHOULD REBALANCE .....B.2

## HOW FAR BACK THE PORTFOLIO MANAGER SHOULD REBALANCE

According to Masters (2003), the net benefit of rebalancing can be defined as the cost of rebalancing subtracted from the benefit of rebalancing. The rebalancing cost can be formulated as:

$$C \times \Delta$$

where  $C$  is the total two-way cost of rebalancing and  $\Delta$  is the percentage off-target for a specific asset class.

The benefit of rebalancing a particular asset class can be quantified as:

$$\text{Benefit of rebalancing} = \frac{(\text{Tracking Error})^2 \Delta^2}{2K}$$

The net benefit of rebalancing a particular asset class is:

$$\text{Net Benefit of rebalancing} = \frac{(\text{Tracking Error}_i)^2 \Delta_i^2}{2K} - C_i \Delta_i = 0$$

Masters (2003) states to rebalance only as far as the marginal net rebalancing benefit is zero:

$$\frac{\partial \text{Net rebalancing benefit}}{\partial \Delta} = 0$$

Replacing Net rebalancing benefit with its equation:

$$\frac{\partial(\text{TrackingError})^2 \Delta^2 / 2K - C\Delta}{\partial \Delta} = 0$$

Differentiating

$$2(\text{TrackingError})^2 \Delta / 2K - C = 0$$

$$\Delta = \frac{KC}{(\text{TrackingError})^2}$$

This is exactly half the trigger point.

# APPENDIX C

WHEN REBALANCING SHOULD OCCUR USING THE SD-METHOD.....C.2

WHEN REBALANCING SHOULD OCCUR USING THE SD-METHOD

Asset classes value movement +/-	SD-METHOD										Rebalancing should occur	
	Range Rebalancing				Retracement Levels		SD-Test					
	Minimum range (associated with $K_1$ )	Maximum Range (associated with $K_2$ )	Support level (0.618)	Resistance Level (0.618)	Second derivative >0	Second derivative <0	Second derivative >0 and Exponential smoother >0	Exponential smoother <0	Exponential smoother <0	Exponential smoother <0		
+ intersection	√	x	x	x	x	√	x	x	x	x	x	NO
+ intersection	√	x	x	x	√	x	x	x	x	x	x	YES
+ intersection	√	x	x	x	x	x	x	x	√	x	x	NO
+ intersection	√	x	x	x	x	x	x	x	√	x	x	YES
+ intersection	√	x	x	√	x	√	x	x	√	x	x	NO
+ intersection	√	x	x	√	x	x	√	x	x	x	x	YES
+ intersection	√	x	x	√	x	x	√	x	x	x	√	NO
+ intersection	√	x	x	√	x	x	√	x	x	√	x	YES
+ intersection	√	√	x	x	x	√	x	x	x	x	√	YES
+ intersection	√	√	x	x	x	x	x	x	x	x	√	YES
- intersection	√	x	x	x	x	√	x	√	x	x	x	NO

APPENDIX C

- intersection	√	x	x	x	x	x	x	√	x	x	YES
- intersection	√	x	x	x	x	x	x	x	√	x	NO
- intersection	√	x	x	x	x	x	x	x	x	√	YES
- intersection	√	x	x	√	x	√	x	x	x	x	NO
- intersection	√	x	x	√	x	√	x	√	x	x	YES
- intersection	√	x	x	√	x	√	x	x	√	x	NO
- intersection	√	x	x	√	x	√	x	x	x	√	YES
- intersection	√	√	x	x	√	√	x	x	x	x	YES
- intersection	√	√	x	x	x	√	x	x	√	x	YES

## Glossary

**Asset Allocation:** The process of dividing investments among different kinds of assets, such as stocks, bonds, real estate and cash, to optimize the risk/reward tradeoff based on an individual's or institution's specific situation and goals. This is a key concept in financial planning and money management.

**Bootstrapping:** Is a method for estimating the sampling distribution of an estimator by re-sampling with replacement from the original sample.

**Covariance:** A measure that determines the degree of co-movement between a portfolio's assets, and is derived by multiplying the standard deviations of the assets by the correlation coefficients of the assets.

**Equity:** Synonymous with share and stock, and used interchangeably.

**Market Timing:** Attempting to predict future market directions, usually by examining recent price and volume data or economic data, and investing based on those predictions. also called timing the market.

**Mean Reversion:** Future returns tend to be closer to their mean. Any significant movement away from the mean can be expected to be closer to the mean in future time periods.

**Passive Investments:** Investments that seek to replicate the performance of an index or an appropriate benchmark with the minimum of activity.

**Pension fund:** Fund set up for a pension plan.

**Portfolio Manager:** An individual who controls the assets of a mutual fund. The portfolio manager chooses and monitors appropriate investments and allocates funds accordingly.

**Rebalancing:** The process of maintaining an asset allocation percentage of a portfolio over time, or the adjustment thereof.

**Risk-Adjusted Return:** A measure of how much an investment returned in relation to the amount of risk it took on. Often used to compare a high-risk, potentially high-return investment with a low-risk, lower-return investment.

**Sharpe Ratio:** A statistical performance measure which calculates a risk-adjusted return.

**Standard Deviation:** A statistical measure that measures volatility of a dataset compared to its average, and serves as a measure of total risk.

**Tracking Error:** When the portfolio's actual asset allocation differs from the target allocation, the so-called tracking error will occur, implying higher risk.

**Trustee:** An individual or organization which holds or manages and invests assets for the benefit of another. The trustee is legally obliged to make all trust-related decisions with the trustee's interests in mind, and may be liable for damages in the event of not doing so. Trustees may be entitled to a payment for their services, if specified in the trust deed. In the specific case of the bond market, a trustee administers a bond issue for a borrower, and ensures that the issuer meets all the terms and conditions associated with the borrowing.

**Volatility:** The relative rate at which the price of a security moves up and down. Volatility is found by calculating the annualized standard deviation of daily change in price. If the price of a stock moves up and down rapidly over short time periods, it has high volatility. If the price almost never changes, it has low volatility.