Metacognitive locale: a design-based theory of students' metacognitive language and networking in Mathematics

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It all starts here ™

DECLARATION

I the undersigned, hereby declare that the work contained in this dissertation / thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

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Signature

30 April 2015

Date

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> *"The White Rabbit put on his spectacles. "Where shall I begin, please your Majesty?" he asked. "Begin at the beginning," the King said gravely, "and go on till you come to the end: then stop."*
>
> *(C.S. Lewis, Alice in Wonderland)*

Summary

The purpose of this study was to design a local theory explaining the relationship between metacognitive language and networks as constructs of a local instructional theory in the context of a fourth-year intermediate phase mathematics education methodology module. The local instructional theory was designed to facilitate an adapted lesson study through a problem-based learning instructional philosophy. A problem-based learning task was then designed outlining the education needs and resources of a South African primary school, characteristic of schools in a rural area. In particular the task describes a fictitious teacher's concern for teaching a Grade 6 mathematics class the concept of place value. Two groups of students, who volunteered to participate in this research, collaboratively designed and presented research lessons across two educational design-based research cycles for two rural schools in North West, as a form of service learning. In implementing the local instructional theory phases, participants were required to follow the lesson study approach by investigating, planning, developing, presenting, reflecting, refining and re-presenting the research lesson and its resources. These design sessions were videorecorded, transcribed and then coded in Atlas.ti through interpretivistic and hermeneutic analysis. The coded data were then imported into NodeXL to illustrate embedded networks. Not only social network data but also metacognitive network data were visualised in terms of metacognitive networks. The results show that across the local instructional theory phases, constructs of metacognition, metacognitive language and networking emerged on a social (stratum 1), interpersonal (stratum 2) and social-metacognitive (stratum 3) level. Collectively, these strata form the architecture of the theory of metacognitive locale that explains the relationship between the constructs. The findings suggest that when students express their metacognitive processes through a metacognitive language (e.g. I am thinking or feeling), their interpersonal metacognitive networks develop into shared metacognitive experiences which foster their metacognitive locale, a dimension of their metacognitive language and networking.

Opsomming

Die doel van hierdie studie was om ’n plaaslike teorie (PT) te ontwerp wat lig kan werp op die verhouding tussen metakognitiewe taal en netwerke as konstrukte van ’n plaaslike onderrigteorie (POT) in die konteks van ’n wiskunde-onderrigmodule vir die intermediêre fase vir vierdejaarstudente. Die POT is ontwerp om ’n aangepaste lesstudie te fasiliteer deur ’n onderrigfilosofie vir probleemgebaseerde leer (PBL). ’n PBL-taak is vervolgens ontwerp volgens die onderwysbehoeftes en hulpbronne van ’n Suid-Afrikaanse laerskool wat verteenwoordigend is van skole in ’n plattelandse gebied. Die taak beskryf spesifiek ’n denkbeeldige onderwyser se opdrag om ’n graad 6-wiskundeklas die begrip van plekwaarde te leer. Twee groepe studente wat vrywillig aan dié studie deelgeneem het, het gesamentlik oor twee opvoedkundige, ontwerpgebaseerde navorsingsiklusse navorsingslesse ontwerp en aangebied by twee plattelandse skole in Noordwes, as ’n vorm van proefonderwys. Tydens toepassing van die POT-fases moes deelnemers die LS-benadering volg deur die navorsingsles en sy hulpbronne te ondersoek, te beplan, te ontwikkel, aan te bied, daaroor te besin, dit te verfyn en weer aan te bied. Video-opnames is gemaak van dié ontwerpsessies, wat daarna getranskribeer is en met interpretatiewe en hermeneutiese ontleding in Atas.ti gekodeer is. Die gekodeerde data is daarna in NodeXL ingevoer om ingebedde netwerke te demonstreer. Data van sosiale netwerke sowel as metakognitiewe netwerke is gevisualiseer by wyse van metakognitiewe netwerke. Die resultate toon dat konstrukte van metakognisie, metakognitiewe taal en netwerking op ’n sosiale (stratum 1), interpersoonlike (stratum 2) en sosiaal-metakognitiewe (stratum 3) vlak in al die POT-fases ontstaan het. Dié strata vorm kollektief die argitektuur van die teorie van metakognitiewe plek, wat die verhouding tussen die konstrukte verklaar. Die bevindings dui daarop dat wanneer studente hulle metakognitiewe prosesse in metakognitiewe taal (bv. ek dink of voel) uitdruk, hulle interpersoonlike metakognitiewe netwerke tot gedeelde metakognitiewe ervarings lei, wat hul metakognitiewe plek bevestig as ’n dimensie van hulle metakognitiewe taal en netwerking.

Keywords

Problem-based learning

Mathematics education

Lesson study

Metacognition

Metacognitive language

Social network

Social network analysis

Metacognitive network

Design-based research

NodeXL

Reflection

Stratum

Strata

Interpersonal metacognitive network

Socially shared metacognitive network

Local theory

Local instructional theory

Metacognitive locale

List of abbreviations and symbols

Abbreviations	**Description**
CK	Conditional knowledge
DK	Declarative knowledge
E	Evaluation
HLT	Hypothetical learning trajectory
KP	Knowledge of the person
KS	Knowledge of strategies
KT	Knowledge of the task
LIT	Local instructional theory
LS	Lesson study
LT	Local theory
M	Monitoring
P	Planning
PBL	Problem-based learning
PK	Procedural knowledge

Symbol	
§	refer to section, paragraph, Figure or Table

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Chapter 1

Introduction, background and orientation

1.1 Introducing the conditions of theories in mathematics education

Since their introduction, social networking sites such as Facebook and MySpace have allowed network users to witness, share experiences and collaborate on daily practices (Bergs, 2005; Ellison, 2007). These virtual communities mirror the social principles of lesson study theory, which balances moral frameworks and social philosophies about society in the context of education. Similarly, these technological affordances have developed alongside a wave of metatheories that have been emphasised in social science research (for example metamemory, metacognition and metarepresentation) and stand in addition to network theories across the last century. To facilitate discussions and movements towards the future of societies and education, society is immersed either in a virtual environment or in a real-world setting. While examining people's connections with others in a shared environment, theorists have developed gestalt theory (Kohler, *In* Atwater et al., 2013), field theory (Lewin, 1951), sociometry (Moreno, 1956), and micro-triad analysis (Leinhardt, 2013) as ways of interpreting and visualising the lived experiences of society before the onset of social network media. These contributions led to the development of the more recent premise of social network analysis, which caters to both the real world and virtual communities' understanding and visualisations (Freeman, 2000).

Theorising about and visualising such networks allows educational researchers to explore the phenomenon of teaching and learning in terms of the language of learning and sociolinguistics (Bergs, 2005:31; Papacharissi, 2011). Also metacognition, or thinking about thinking, in problem-solving contexts increasingly receives attention.

When reflecting on mathematics education for learning and improved metacognition, Foster *et al.* (2013) elaborate on higher education's purpose as the fostering of students' ability to communicate and think critically to develop into lifelong learners who live with diversity, gain broader interests and become aware of their moral reasoning when accepting social responsibility. These essential goals of educating students to communicate about the questions in mathematics and with mathematics stress the importance of metacognition for teaching and learning within theories of teaching methodology for mathematics education.

This includes theories that scrutinise how communities of inquiry develop networks to function in the education environment. George Polya, a mathematician and teacher, expressed the aim of mathematics education as "to develop all the inner resources of the child" (Polya, 1981:3).

Voicing one's thoughts through metacognitive language and the interactive networking between these thoughts (Papaleontiou-Louca, 2008; Vygotsky, 1987) describes what and how we think or reflect. Externalising this metacognitive language is a powerful means of exploring the relationships that exist between and within individuals of the teaching-learning and doing of mathematics communities. As some researchers of mathematics education explore the link between language and thinking (Jones, 2007), others describe and discuss the underlying principles and methods that brought about a change in how to perceive thoughts (Bergs, 2005), reflect on experiences (Hudlicka, 2005) or elaborate on the differences of the mind (Richardson, 2010). Metacognition research (Flavell, 1979)[1] has a rising association with language (Vygotsky, 1987[2]; Anderson, 2008) and mathematics education and has now, more than ever, evolved through the relationships between research on both mathematical content and so-called *mathematical power*[3] (Sparks & Malkus, 2013).

More recently, an emerging *meta-metacognitive curriculum* (Rowse, 2009) questions whether curriculum content is adequate (Nicolao *et al.*, 2009) and whether higher education institutions can deliver both schooling and education (DoBE, 2012) in response to the needs of a changing society, a society professed to exist within either an online (virtual) or offline (real word or reality) state (Natile, 2013). Papacharissi (2011) states that metacognition research calls for an understanding of networks and claims that communication between individuals in the network is necessary for developing a country's education. Richardson (2010) fears that teachers are not rethinking and restructuring their classroom practice in any significant way. In addition, Maree *et al.* (2012) claims that learning processes and learning networks within teaching-learning and doing mathematics are often approached within the classroom walls as a mere side issue. From the theories and paradigms of networks, language

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[1] Although this source can be seen as outdated, Flavell is considered as the *father* of metacognition research (Papaleontiou-Louca, 2008).
[2] The Vygotskian perspectives are considered groundbreaking theoretical approaches in educational research (Boero, 2011) and are therefore included, not as an outdated source, but as an indication of the relevant nature of Vygotsky's theories.
[3] Sparks and Malkus (2013) considers the content strands as those learning outcomes in mathematics, while, the mathematical power refers to the metacognitive aspects of mathematics.

development and the prominent goals of mathematics education, it seems as though modern-day theorists gather for a séance to call up Flavell's (1979) legacy in metacognition, Bloomfield's (1933) language of learning and Whorf's (2012) networked association with the language of the mind.

The research study proposes to explore, explain and understand the nature of metacognitive language and networking in the development of metacognition and shared metacognition to contribute a local theory regarding mathematics students' metacognitive locale. With social network analysis and constructs of problem-based learning (PBL), metacognition will be explored within the adapted lesson study theory. The researcher proposes to understand if, how and why metacognition facilitates networks of knowledge for teaching-learning and doing mathematics.

1.1.1 The movement towards 21st century mathematics education

The conditions described in the introduction comment on the status quo of mathematics education and metacognition research as a dynamic and changing field. The importance of learning environments, curricula, and subject policies has grown immensely over the past twenty years, together with research focus on society's needs in various educational settings (Amiel & Reeves, 2008:30). With the introduction of Common Core State Standards in Mathematics (CCSM) in the United States of America, teacher involvement in research became very important (Erickson *et al.*, 2015). The CCSSM's concern about curriculum improvement axis (revolves around) school mathematics and teachers' ideas about teaching and learning. Curriculum movements in other countries such as Turkey (Koc, Isiksal & Bulut, 2007), Cyprus (De Bock, Deprez, Van Dooren, Roelens & Verschaffel, 2011), Spain (Desha *et al.*, 2009), the United Kingdom (Wiseman, 2010), Australia (Darling-Hammond & Lieberman, 2012) and South Africa (DoBE, 2012) appear to progress towards an international concern regarding mathematics and its education. Research on the teaching and learning of mathematical sciences portrays multiple perspectives and research paradigms, emphasising a broad and unified goal towards the development of mathematics education (Barab & Squire, 2004). Universities and colleges concerned with teacher education have recognised these changes and joined in the movement towards preparing students to preserve and uphold teaching and learning throughout this process and beyond.

The dedication to creating taxonomies to understand, define and measure metacognition and facilitate metacognitive experiences within the field of mathematics, in particular the education of mathematics, does not go unnoticed (Young, 2010:1). The knowledge of one's own cognition provokes and regulates aspects of cognition, behaviour, confidence as well as affective thinking and beliefs in mathematics (Johns, 2009). Often, teachers and students of mathematics and its education develop teaching and learning environments, skills and knowledge, to some extent, with different understandings of what skills and knowledge are important. These meanings are used to clarify, interpret, communicate, manipulate and reflect upon abstract mathematical concepts before, during and after teaching and learning. This change and movement in mathematics and teacher development programmes is driven by a dire need to turn out equipped and knowledgeable teachers, in response to a nation's cry (Carl, 2012).

Many teachers do not uphold, model or sufficiently promote the implementation and practice of metacognitive strategies, even though they must develop into lifelong learners (Chatzipanteli *et al*., 2014 & DoBE, 2012:3). Engelbrecht *et al.* (2010) and Siyepu (2013) assert that undergraduate mathematics education students are underprepared for effectively teaching mathematics. Furthermore, the World Economic Forum (Krook, 2013) has ranked South Africa 54th when compared to the tertiary enrolment of countries such as India and Morocco.

According to Van Eerde (2013) and Siyepu (2013), the developmental theory of Vygotsky suggests that improved self-regulation (metacognition) is initiated by the teacher–learner social interaction. This movement towards 21st century mathematics education necessitates metacognitive awareness and calls for an educational reform in universities within teaching and teacher research and encompasses a need for a metacognitively based mathematics curriculum (Siyepu, 2013). Such a curriculum must generate the necessary skills for development and implementation of, in particular, mathematics, metacognition and networking as the roles and responsibilities of teachers and students.

1.1.2 Encompassing the need for a metacognitively based curriculum

Practising and preservice teachers occasionally see mathematics as a collection of symbols or objects and may even ignore or be unaware of the interrelated concepts, processes and networks involved in the modelling and constructing of new knowledge (Lee *et al.*, 2013). Kieran *et al.* (2013:8) assert that: "It is the teacher who can affect to the greatest extent the achievement of one of the main purposes of the research enterprise; that is, the improvement of students learning of mathematics."

Sriraman and English (2007) propose powerful ideas that will contribute to the demands of the 21st century. They include the following:

- A social constructivist view of problem-solving, planning, monitoring and communication
- Effective and creative reasoning skills
- Analysing and transforming complex data sets
- Applying and understanding school mathematics
- Explaining, manipulating and forecasting complex systems through critical thinking and decision-making.

These ideas seem to emphasise the importance of metacognition, communication and networking across the teaching-learning and doing of mathematics. Albarracin *et al.* (2014) recommend the following for teacher development: "… training and on-going support to help capitalise on their mathematics program materials, or supplement them as evidence suggests and help make research based instructional decisions".

These ideas and recommendations scrutinise the need for metacognition in present and future teacher educator programmes, emphasising communication and networking among teachers, students and learners concerning mathematics and metacognitive aspects. To realise these ideas requires intensive training for preservice teachers and an acquaintance with research in mathematics education and metacognition. This includes understanding mathematical concepts and fostering an awareness of their implications in preparation and instruction of the contents and modelling of mathematics and metacognition in the mathematics classroom. Transforming skills, social networks and knowledge, as embraced by all disciplines within the national curriculum, promote higher-order skills as a vital part of development and empowerment through proficiency and competence in mathematics (DOBE, 2012). This progression towards the advocacy of an announced meta-metacognitive curriculum aims to improve not only one's life, but also one's chances in study and work by scaffolding various networks set in students' teaching-learning and doing of mathematics.

1.1.3 Paucity in the literature as motivation for this study

In spite of the sentiments described above, there is a paucity of literature on South Africa's preparation of mathematics education students for their roles as teachers and lifelong learners. According to Rock and Wilson (2005), lesson study as a teaching method has a *severely* limited system in place for professional development of teachers. Reflecting on this scarcity on the literature, they propose that teaching should change and not teachers. Additionally, “lesson study has not been used widely as professional development model in South Africa” (Posthuma, 2012:54) and is in itself a scarce topic among teachers in both primary and secondary schools. As a result, there is a lack of existing research to support the use of lesson study in South Africa (Posthuma, 2012; Rock & Wilson, 2005). The structure of lesson study enables teachers to engage in a research-based teaching practice with limited resources. It also offers possibilities to adapt and put into effect the resources they already have to improve not only their own teaching-learning and doing of mathematics, but also that of their students or peers (Taylor, 2008). Rock and Wilson (2005) agree that there exists a need to design and sustain high-quality professional development of teachers to expand students’ learning.

1.2 Background to the problem statement and rationale for the study

Reflecting on the knowledge society, Toffler and Butz (1990) and Scaradmalia, Brandsford, Kozma and Quellmalz (2012) agree that, in effect, health, cultural, financial and educational institutions play an ever-growing role in the lifelong learning, innovation and development of skills to solve realistic problems of both the present and future. Toffler and Butz (1990:6) further commented on the fate of education, stating: ‘‘The illiterate of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and relearn.”

In the scientific disciplines, such as philosophy, cybernetics, linguistics and psychology, the learning theorists Flavell, Dewey, Von Glasserfeld, Piaget, Vygotsky, Pask, Lave and Wenger have approached teaching and learning phenomena from a variety of learning paradigms. The paradigms comprise metacognition, experiential education, constructivism, genetic epistemology, the zone of proximal development, social anthropology, and discovery learning within existing communities of educational practice. These theories explain the infusing processes and relationships that exist between the teacher and the learner, cognition’s constructive role in the experiential world and the groups or individuals (teachers, students, and learners) who interact in a dialectic framework. Collectively these theories seem to suggest an understanding of the kind of knowledge and skills education, particularly in South Africa, has to produce. Higher education is expected to fulfil this demand (Anderson,

2008) even though the overloading of educational policies neglects asking the necessary questions that could enhance classroom practice (Maree *et al.*, 2012).

Sustaining such knowledge environments raises a concern about the establishment, educating and development of metacognitively able mathematics teachers. This encompasses a need to understand what mathematics education students of the present and the teachers of the future's metacognition entails of the mathematics education students of the present and the teachers of the future.

More than a decade ago, Brass (2002) identified a paradigm shift from computer-centred technologies to network-embedded societies. On the opportunity that these shifts provide for research on different forms of networks, particularly in the landscape of metacognition, Eagle *et al.* (2009:3) comment:

> When people engage in daily activities … they leave … breadcrumbs …When pulled together [the breadcrumbs] … offer increasingly comprehensive pictures of both individuals and groups, with the potential of transforming our understanding of our lives, organisations, and society in a fashion that was barely conceivable [in the past].

Van Staden (2012) claims that these traces, or crumbs, are necessary for the communication and sharing of information, to improve the networking and monitoring among individuals. It seems as though these traces in the network define the scope of this research and emphasises the importance of theorising metacognition within educational practice, particularly its networking and language.

1.2.1. Statement of the problem

In higher education, mathematics lecturers must establish a metacognitive community and reflect upon a meta-metacognitive curriculum within their courses which could help bridge the difficulties within mathematics education, as well as the challenges within curricula (Veenman *et al.*, 2006), to foster students' metacognition and to improve the present and future schooling of the country (Evans & Jones, 2009). A metacognitive locale is proposed to help provide the necessary scope for the development of classroom practice, particularly when educating for lifelong learning. Lovett *et al.* (1990) highlighted this as an important issue, stating that teaching metacognitively would result in improved and effective learning.

It is necessary to develop an understanding of the theoretical dimension of metacognition and the relationship between constructs that are metacognitive in nature.

Mathematics performance remains a concern in South Africa, a country beset by unemployment (Bhorat, 2007) and a reduced number of professionals possessing scarce skills. Based on the problem statement the following research question(s) are asked in this study.

1.2.2 Research question(s)

The primary research question in this study is: In what way(s) do(es) understanding of the identified metacognitive language and metacognitive networks contribute to theory building concerning students' metacognitive locale in teaching-learning and doing mathematics?

This main research question is divided into three secondary research questions.

1.2.3. Secondary research questions

- Question 1: What do students' metacognitive networks entail when teaching-learning and doing mathematics?

- Question 2: What do students' metacognitive language entail when teaching-learning and doing mathematics?

- Question 3: How can metacognitive language and metacognitive networking foster shared metacognitive experiences?

The purpose of the primary and secondary research questions purpose is outlined in the next section.

1.2.4 Purpose of the research

The central purpose of this research was to develop a local theory, scrutinising the role of metacognitive experiences such as those emerging from the language of individuals' thought and communication processes and experiences. Such a theory would aim to improve the teaching-learning and doing of mathematics using a lesson study model. Metacognitive language for teaching-learning, lesson study and the instruction of mathematical concepts was reflected upon and articulated to contribute to the explaining, exploring and understanding of prospective teachers' metacognitive networking. This study recognises the influence of experiences in understanding of concepts and the role metacognition plays in conceptualising, communicating and representing mathematical knowledge. The building of such networks of thought scrutinises social roles across participants' theory of mind. Similar to the research questions, the primary and secondary purposes are highlighted next.

1.2.4.1 Primary purpose of the study

Metacognitive language (Peskin & Astington, 2004) and metacognitive networks are proposed concepts that foster students' metacognitive locale, towards development of a local theory. The primary purpose of this study then is to develop a local theory of fourth year Intermediate phase mathematics education students' metacognition in order to theorise about and explain the relationship between the constructs of metacognition, metacognitive language and metacognitive networking.

1.2.4.2 Secondary purposes of the study

The following secondary purposes reflect the aim of the secondary research questions.

- Metacognitive language identified in the literature appears to have the following goals: to (a) categorise, (b) classify and (c) identify words that indicate an individual's expression of their metacognitive thinking. The study aimed to elaborate on students' use of this metacognitive language within the context of their teaching-learning and doing of mathematics and its relationship with biographical, metacognitive and social networking aspects.

- Social networking and community networks are considered in the fields of social linguistics. The study aimed to explore and understand the nature of students' social networking in terms of the metacognitive strategies they employ and whether these strategies are associated with the metacognitive language used within the social network.

- Students' metacognitive networking and metacognitive language were explored to establish their association with their shared metacognition when teaching-learning and doing mathematics.

The above purposes were synthesised to seek the nature of students' metacognitive locale with reference to the language and networking of their metacognition. The intention of the primary and secondary research purposes was to make a worthy contribution, emphasised in the next section.

1.2.5 Possible contributions of the study

The study proposed to contribute to the scholarly field through attempts to:

- bridge the gap identified in the literature to produce knowledge on the national and international publications front (Maree *et al.*, 2012; Lazer *et al.*, 2009; Chatzipanteli *et al*., 2014);
- examine, explain and understand the nature of prospective mathematics teachers' metacognitive language and networking to theorise, facilitate and model classroom practices for the identified fourth-year mathematics methodology course;
- challenge the theoretical and practical views of research in metacognition and mathematics education, such as the use of multiple methods (Creswell & Clarke, 2007) within design-based research to inquire metacognitive language (Deed, 2009) and metacognitive networks (Pasquali *et al.*, 2010);
- foster an awareness of prospective mathematics teachers' progress toward lifelong learning and intellectual wellbeing through an understanding of their metacognitive locale.

To understand and elaborate on the orientation of the research background and locale, a brief overview of the keywords and their definitions is given.

1.3 Definition and overview of keywords

Fink (2013) describes the defining of a concept not as a narrowing down of possible meanings, but as a breach of new landscapes and new implications. To interpret the title of this study correctly, it is necessary to explain the keywords and to, at the very least, restrict the concepts by the epistemological underpinnings relevant to this study. In the current study, the views of Sriraman and English (2010), Van den Akker *et al.* (2006), Papaleontiou-Louca, (2008), Vygotsky (1987) and Flavell (1979) will be referred to when defining the keywords. Their views are associated with socio-constructivist approaches, theory of mind and language of learning, research methodologies and metacognition within mathematics education environments.

1.3.1 Mathematics

Richardson (2010) claims that different groups define mathematics according to their times, background and the role mathematics plays across societies. Vassiliou (2013) defines mathematics as the literacy of numerical operations that represent the real world.

Mathematics is known to have a dynamic and disciplined nature (Nieuwoudt, 2006) and the Department of Basic Education (DoBE, 2012) views this nature as a distinctly human activity practised by all cultures. Mathematics is based on observing patterns, with rigorous logical thinking that leads to theories of abstract relations. Wolcott (2013) emphasises that mathematics is a cultural art. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change. For the purpose of this study, the above views of mathematics are synthetically defined as the study of the relationships that exist between objects and/or individuals in the world, representing reality to make well-founded and informative judgments that meet the needs of society, who models these relationships as responsible and reflective citizens in their social cultural contexts as an act of sense-making.

1.3.2 Mathematics education

According to DOBE (2012), mathematics is seen as a language of symbols and notations, which describes the relationships between numerical, geometrical and graphical relationships. It focuses on observing and representing physical and social phenomena and enhances mental processes for logical and critical thinking; above all, its focus is on problem-solving in both the teaching and learning of mathematics. As an educational phenomenon, this aim towards teaching and learning mathematics can be referred to as mathematics education.

Table 1.1 captures various definitions of mathematics education in the literature.

Table 1.1 Definitions of mathematics education

Description	Source
Mathematics education has more in common with social science than with mathematics.	Rowland (2001)
Students learn to transfer skills and ways of thinking learnt in one area of mathematics to other domains.	Gonzales & Herbst (2006)
Mathematics education is the facilitation of mathematics learning by mathematics teachers embedded within humans' cultural context.	Wolcott (2013:79)
A need to understand what mathematics education students of the present and the teachers of the future's metacognition entails. The rank attributed to mathematics within the respective sets of social and cultural values, related to the function of instruction, is mathematics education.	Schubring *et al.* (2006)

However, for the purpose(s) of this study, the definition of mathematics education provided by Godino *et al.* (2013) will be used. This definition states that mathematics education is a science that aspires to the design of practices and resources, thereby improving the teaching and learning of mathematics. This is the main hub of the mathematics education enterprise. With the above descriptions of mathematics and mathematics education in mind, it is noteworthy that research on mathematics education considers aspects of cognition and metacognition (Stack & Bound, 2012; Lai, 2011a).

1.3.3 Metacognition

Metacognition is fundamentally different from cognition. According to Akyol and Garrison (2011), metacognition research is supported by terminology such as executive functions and denotes metacognition as self-assessment and self-management concepts (Rivers, 2001). Metacognition is a critical factor, which involves reflecting on one's own learning process, and optimises the depth of learning and the ability to transfer knowledge to new contexts (Foster *et al*., 2013). Metacognition includes self-regulation – the ability to orchestrate one's learning, to plan, monitor success and correct errors when appropriate – all necessary for effective intentional learning. Metacognition also refers to the ability to reflect on one's own performance. As Ballera *et al*. (2013) state, metacognition refers to critical thinking, an awareness of one's own thinking and reflection on the thinking of the self and others. From this view, metacognition is defined in this study as a construct of the assumptions of one-and-other as cognitive development. It refers to the knowledge and management, thus regulation, of thinking (Lai, 2011b).

1.3.4 Key terms and concepts with regard to research methodology

Because of their complexity, a brief overview of the keywords associated with the research methodology is drawn upon in the following sections.

1.3.4.1 Educational design research

According to Dai (2012) the traditional methods of researching teaching and learning contexts (e.g. quantitative and/or qualitative case studies) cannot handle the complexity and interactivity of studies involving multiple data layers. The architecture for educational research design fitted well for this study and was strengthened in the argument that denotes lesson study to have similar principles, reflecting the design process within its series of iterative cycles (McKenney & Reeves, 2013). The process of this design is outlined in the flowchart below.

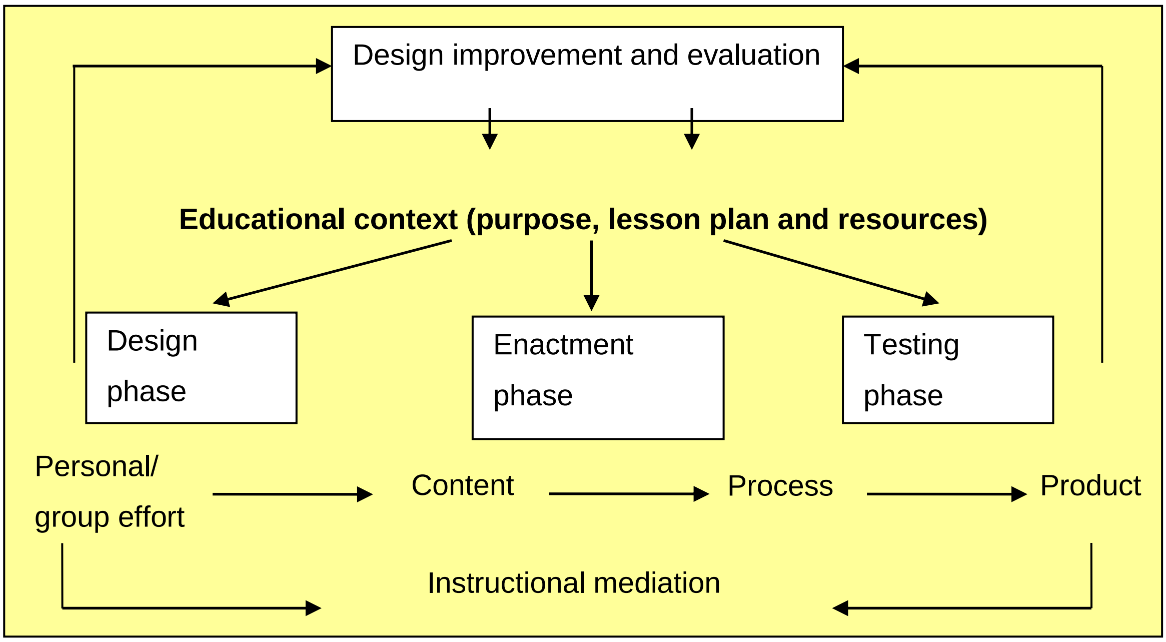

Figure 1.1 Architecture of the educational research design

Source: Adapted from Dai (2012)

The flowchart in Figure 1.1 illustrates the educational context as a framework in which the purpose, the planning of effective lessons and available resources are interacting with the design of teaching, learning and problem-solving experiences (Dai, 2012). In the design phase, the individual or group will typically reflect on the purposes and goals for designing a lesson that will be researched in terms of educational practice or effectiveness (McKenney & Reeves, 2013). This lesson is also referred to as a research lesson (Lee *et al.*, 2013:10).

1.3.4.2 Problem-based learning embedded within educational design research

According to Goos, Galbraith and Renshaw (2002) research on problem-solving is declining, but there are unresolved issues that require attention. Lester and Kehle (2003) identified one such issue as the established relationship between metacognition and problem-solving, while Schoenfeld (1992) claimed that students must be equipped to provide their own *point of view* when thinking mathematically. The zone of proximal development (Vygotsky, 1987) is a standard sociocultural theory that explains the functioning of these constructs of problem-solving, communicating and the role of the self, encapsulated in the PBL approach.

Moreover, the study of McMahon and Luca (2007) promotes the exploration of metacognition in terms of teamwork in an online environment. Focussing on three components of metacognition (planning, monitoring and evaluating) McMahon and Luca (2007) identified self-assessment, team monitoring, group reporting and reflecting as metacognitive strategies that students carry out collaboratively when learning online. Metacognitive knowledge was not included in McMahon and Luca's (2007) study as Pasquali, Timmermans and Cleeremans (2010) suggest for research on metacognition. This is an important aspect, as Papaleontiou-Louca (2008:28) asserts that such interactions that are aimed at the individual and social settings that relate to planning, monitoring and evaluating are indicators of metacognitive experiences and activate strategies deemed necessary to reach cognitive or metacognitive goals (for e.g. to improve one's knowledge or problem-solving skills). These goals are associated with the primary principles of PBL: constructive, self-directed, collaborative and contextual (de Jong, Verstegen, Tan, & O'Connor, 2013). Online PBL has many advantages for adult learners (Lehmann, Hahnlein & Ifenthaler, 2014) and enables communication for promoting and acquring knowledge. It also builds an understanding of the social perspective of a learning environment (also refer to section 3.2).

1.3.4.3 Lesson study

Lesson study (or *jugyoukenkyu*) is an inquiry model for developing professionalism of teachers, established in Japan (Rock & Wilson, 2005). Lately, the model has been incorporated in the United States (US) and London (UK) schools with slight adaptions made to the initial cyclic nature of lesson study (Rock & Wilson, 2005). Minar and Crown (2001) describe lesson study as a process which serves as an aid for teachers to design and develop new and innovative best practices to improve the quality of both teaching and learning experiences. Woodruff *et al.* (2013) promote lesson study as a fruitful and focused approach towards professional development in mathematics teaching and learning, through the collaboration of colleagues and experts to negotiate the design and delivery of meaningful lessons through a reflective discourse. With the aim of improving learning opportunities, Ylonen and Norwhich (2013) suggest that a methodology with a realistic approach to linked *contexts, mechanisms and outcomes* is necessary to develop lifelong learners. For the purpose of the current study, the researcher has combined these idealistic views of lesson study, and defines it as a teaching methodology that promotes reflective practices about one's own teaching and learning and that of one's peers with the aim of improving the quality of learning experiences. According to McDowell (2010), researching one's own teaching (a lesson study initiative) is a metacognitive construct based on an adopted theory of the teaching and learning experience that may be expressed openly or remain a hidden theory.

Lesson study involves teachers who gather regularly to design, implement, reflect and improve a single lesson or series of lessons (Rock & Wilson, 2005:78). In the current study, the PBL experience required participants to engage in lesson study "in design group sessions". In these sessions, participants were required to follow the cyclic process of lesson study to design, adapt and reflect upon a research lesson in a collaborative way.

Participants were asked to design a research lesson based upon a fictitious scenario that resembles a case study of a rural school as indicated in the PBL task (see Addendum A). Participants taught these lessons to their own learners during the practical teaching and work-integrated learning (WIL) components of their undergraduate course as part of service learning. In this study, before, during and after the process of designing the research lesson, the following roles of the participants identified by Rock and Wilson (2005) were emphasised:

- Identifying and collaborating on a specific generated problem obtained from reading and discussing the PBL task.
- Identifying and discussing the goal or vision of the pedagogical practice.
- Carefully planning, on their own and collaboratively with one or more participants.
- Videorecording their presented lessons for reflecting on them afterwards and discussing their experiences on what worked and why they thought it worked, and on what issues emerged from the presentation that needed adapting or redesigning in the online discussions and during the next cycle.
- Observing, recording and critically reflecting on the lesson(s) presented by one or more members of the design group, to analyse the lessons collaboratively online and during the next design group session(s).

Figure 1.2 illustrates the process that was followed regarding lesson study, embedded within the educational design research cycles.

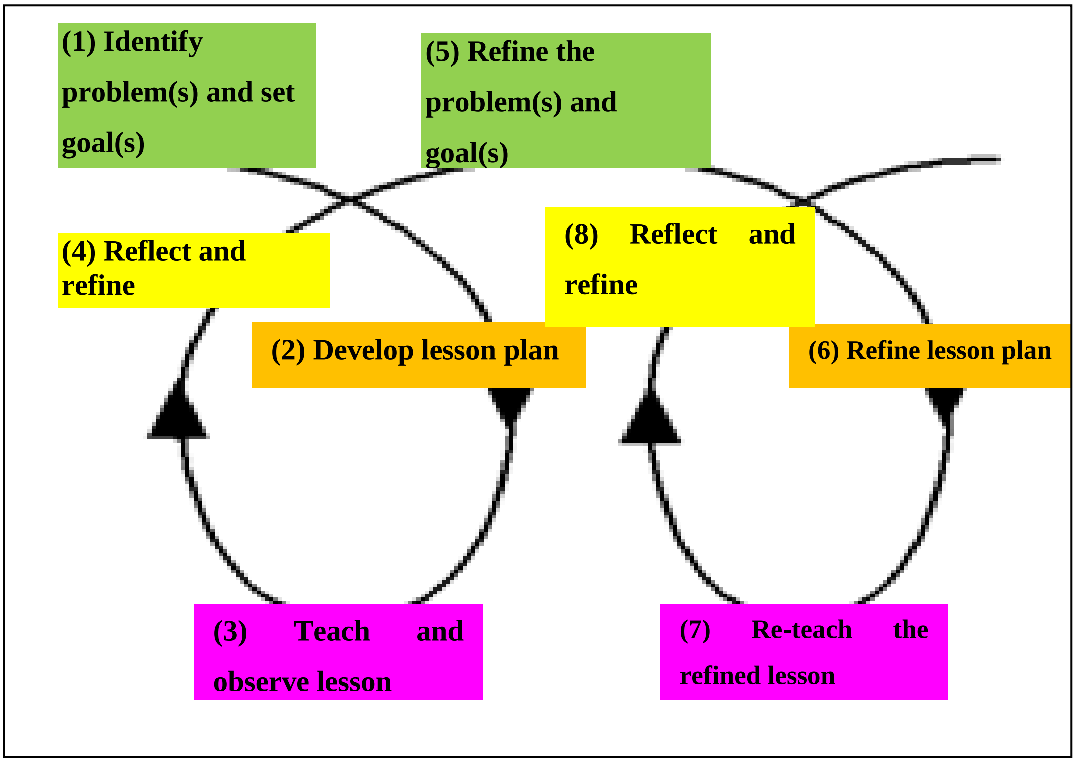

Figure 1.2 **Lesson study cycles**

Source: *Adapted from McKenney and Reeves (2013)*

The design of the lesson consists of a literature overview and an analysis of the context for which the lesson is prepared. In the current study, this was done through the PBL task. The participants identified the underlying problem(s) and goal(s) of the PBL task to solve, collaboratively, the problem of designing a lesson suited for the described fictitious scenario. This collaboration also drew upon their problem-solving skills as is required by PBL tasks. The participants developed a lesson plan and resources as an attempt to solve the problem, afterwards teaching and observing the lesson(s) to reflect on and redefine their goals. In the second cycle they acquired not only the theoretical knowledge discussed in the design group sessions, but also had more contextual knowledge and experience acquired from the first teaching and observation of the lesson. In the second cycle they re-visited the PBL task with newly required insight and refined the lesson plan, based on the goal(s) and problem(s). They re-presented the refined lesson and reflected upon their teaching in journals[4], which were used, again, for discussing what they would have done differently.

[4] The journals do not serve as a means to collect data, but to provide a personal space (Rock & Wilson, 2005) for the individual to reflect. The journals were mainly used to gather biographical data of the participants throughout the study (see Table 6.1) and to provide them with the opportunity to reflect on what happened in the design sessions.

1.4 Preliminary review of literature regarding the concepts relevant for this study

According to Maree *et al.* (2012), the initial literature review elaborates on the known and unknown states of the current literature and proposes how the current study might contribute to the existing body of scholarship. Within the preliminary review of the literature the conceptual and theoretical underpinnings of the keywords in this study are reflected upon and argued against the backdrop of the philosophical, methodological, ontological and epistemological implications for this study. First, a discussion follows regarding some philosophies of theory development in mathematics education.

1.4.1 Philosophies on theory development

The philosophies and theories that shaped the researcher's conceptual framework for this proposed study are discussed briefly. The current study utilised Vygotsky's (1987) zone of proximal development, Frith and Happe's (1999) theory of mind, Flavell's (1979) theory of metacognition and Moreno's (1956) social network analysis as the standard theories drawn upon to understand, explore and examine metacognitive networking and metacognitive language against the backdrop of social constructivism as a metatheory in lesson study.

1.4.2 Teaching and learning versus the teaching-learning paradox

The term *teaching and learning* is associated with a paradox (Dubin & Taveggia, 1968[5]) and refers, in this study, to the moment(s) where student teachers teach and learning occurs. In other words, it explains the phenomenon whereby the act of teaching leads to not only learning by the learners, but also activates learning by the teacher. The teacher may teach a concept in fractions, but at the same time realise (learn) that the learners struggle with other concepts. The teacher may realise that a particular approach to a problem was not the best choice, since many learners still do not understand. In this case, the teacher learnt what their prior knowledge is. The term *teaching-learning* does not refer to teaching and learning phenomena. Instead, in this study it refers to an "aha" moment (Schon, 1983) whereby teachers can experience a flash of acquired insight and becomes aware of their own and others' knowledge and skills, as a consequence of his/her teaching. Teaching-learning is a

[5] Since no references to teaching-learning as a concept could be found during the time of this preliminary literature review, and since the terminology fits into educational studies, the concept coined by Dubin and Taveggia (1968) will be used in this study as it emphasises the teacher's taking on the role of a researcher in the classroom (Czarnocha, 2008).

complex process whereby a general teaching method is brought into question, reflected upon and proven inconclusive at that moment (Dublin & Tavegga, 1968).

1.4.3 Cognition and metacognition

Higher education students get information and are required to solve problems in which cognitive and metacognitive activities are not necessarily clearly distinguished. Vos (2001) argues that the characteristics of metacognition must be formulated in such a way that it is clearly distinguished from cognition. Vos (2001) continues to elaborate on cognition as the source of memory, reading, language, problem-solving and social cognition. Metacognition can be seen as cognition about cognition. Flavell (1979:86) explains metacognition as "simply knowledge about knowledge" and therefore metacognition consists of two dimensions. The first domain is referred to as metacognitive knowledge and the second as metacognitive regulation. Reflection constantly takes place between the two domains (Garcia, Sanchez & Escudero, 2007).

Vos (2001:35) further explains that in metacognition "deliberate, planned, intentional, goal-directed, and future-oriented mental actions that are used to accomplish cognitive tasks". Metacognitive strategies are identified as reflective and regulative actions taken to draw upon cognitive knowledge to solve problems. However, the metacognitive information can relate to cognitive skills such as presenting a strategy to monitor goals. Therefore, metacognitive knowledge is seen as a personal and stationary construct while metacognitive regulation (strategies) consists of dynamic and personal constructs, indicating that metacognition is a self-orientated theory (Vos, 2001). This seems to be in contrast to studies done in the past by Brown and Coles (1997), who claimed that teaching to create an awareness of metacognition and metacognitive strategies would result in improved problem-solving approaches towards a successful outcome. This indicates that a social and collaborative dimension is associated with metacognition, requiring a shared language through which students and their lecturers can communicate meaningfully and reflectively. In their study on lesson study for the professional development of teachers, Rock and Wilson (2005) emphasise the components of metacognition associated with the engagement in lesson study activities. Some of these principles include reflecting, understanding and evaluating.

For the purpose of exploring students' metacognition their metacognitive knowledge is defined as "an awareness of the self" (Stack & Bound, 2012:34). Metacognitive knowledge refers to the knowledge and ability to select applicable strategies to enhance performance through problem-solving.

1.4.3.1 Socially shared metacognition

A more recent definition of metacognition by Papaleontiou-Louca (2008) suggests that the concept of metacognition is a social construct and not merely an expression of an individual's awareness of their mental state (Flavell, 1979). Social metacognition refers then to the thinking about one's own thinking and the thinking of others' thinking.

1.4.4 Networking

Theories of networks usually include the characteristics that arise from thinking. To convey this thinking, a language is needed to theorise about within a domain of knowledge embedded in the network that these ideas are expressed in and constitute both cognitive and metacognitive theories (Vos, 2001). According to McKetcher (2005) networks allow researchers to ask such questions as *Who is linked with whom?* and *What is the nature of the relationship?*

1.4.5 Language and communication

Words such as *right* and *wrong* are used in everyday language, and represent an own grammatical understanding of a process or approach. According to Vos (2001) language, together with its syntax and semantics, is associated with the metacognitive knowledge domain and can be expressed in mathematics as a verbal, algebraic, schematic, symbolic or communicative representation of one's own thinking. Once this internal dialogue is exposed among peers or colleagues, the characteristics, roles, conditions and relations are expressed and shared collaboratively (Vos, 2001). Such an inquiry must ensure a dialogue between oneself and others (Stack & Bound, 2012). As Van Eerde (2013:19) states:

> Through personal conversation, we turn ourselves about and converge or come together ... we become transformed as our differing views converge on what is presently beyond us ... and the situation changes or becomes transformed as we go through this convergence process.

In terms of language for understanding mathematics, Van der Walt, Maree and Ellis (2008:490) argue that South African learners possibly experience low performance in mathematics due to a limited "*technical vocabulary*" of the subject. From these statements and others (Goos, Galbraith & Renshaw, 2002) it appears that language plays an important part in mathematics education and relates to the Vygotskian theory of the zone of proximal development (Vygotsky, 2012). The researcher acknowledges that the technical vocabulary of mathematics may influence an individual's understanding and communication of mathematics. The emphasis was therefore on the language used in the social interactions to communicate about ideas in mathematics. Vygotsky (1987) emphasised that social interactions drawn upon the sociocultural basis of self-regulation which is, according to Brown and Coles (1997) and Papaleontiou-Louca (2008:24), the fuel that kindles "the transference from other-regulation to self-regulation".

1.4.5.1 The language of learning

Students seem to express and exchange ideas among one another through listening and sharing, thus creating a mutual understanding within an inquiring community to establish mathematics practices. Bermudez-Otero (2006) suggests that language is a cultural tool, similar to mathematics, and serves as one of the instruments for communication. According to Richardson (2010:508), "… words are carriers of meaning, and the worst offense that could be committed against reason was to misuse or misrepresent those meanings".

1.4.5.2 Metacognitive language

On describing the value of language in teaching and learning, Wolcott (2013:78) stated: "Experiences have an intersubjective reality. You and I share common experience, and we should think about this and talk about it together." Communication and language is placed in the centre of Vygotsky's theory of language and thought when he states that "thought is completed in words" (Vygotsky, 1987:249–250). Siyepu (2013) supports this view when iterating classroom activities, especially those that involve problem-solving, with opportunities for students and learners to reflect upon their work and interact, communicate about the decisions made and comment on the written and oral reflections.

1.5 Research design and methodology

This study is conceptualised as a real-world research design (McKenney & Reeves, 2013). As such, the research was conducted in a real-life setting with complex personal and contextual relations, including personal and social metacognition. The aim is not to control the social aspects of language or engagement in a community of inquiry. Rather, the study allows the researcher to explore and understand the phenomena of lesson study and metacognition as a developmental research approach for mathematics education students (De Jong *et al.*, 2013) to not only explain the phenomena but also to generate a local theory.

1.5.1 The paradigm complexity: embedding a series of design sessions within design-based research

A qualitative approach in this study was considered a useful method of data collection. The researcher incorporated the results obtained from the multiple data collections to make meaningful decisions about the progress and role of the design group (interview) sessions, which was the primary data collection method. Instead of obtaining the data directly from the participants in a solitaire study, a series of such design sessions were embedded in the design-research approach. The results obtained from the multiple design sessions informed the status of students' metacognition as part of their metacognitive locale, to inform and provide background of participants for the qualitative study.

According to Creswell and Plano Clark (2007), such multi-layered designs are sequential in nature, similar to the different cycles of design-based research (McKenney & Reeves, 2013) and the collection of supporting data with separate data analysis before, during and after data collection procedures supports this approach. The purpose of the design sessions were to explore, understand and explain concurrent as well as sequential findings within this study. The worldviews most suited for these designs must support the various interactive levels within the design-based methodology. The data collection approach was therefore of a qualitative nature and called for an interpretive view. However, as this view did not solely contribute to the overall emergence of the data, the findings or approaches to theorise about how this design can facilitate a metacognitive locale, a pragmatic world view was adopted (Creswell & Plano Clark, 2007). A brief discussion about the pragmatic perspective now follows.

1.5.2 The paradigmatic assumptions and perspectives

The worldview that contains the assumptions and beliefs necessary to depict the development and outline of the current research is that of a pragmatic paradigm. This philosophical foundation shapes the process and conduct of the inquiry, particularly because its nature shares the view that includes the development and improvement of and for education (Creswell & Plano Clark, 2007). A pragmatic philosophy of research in education, like that associated with Taatilla and Raij (2012), "focuses on learning by developing" and was regarded appropriate for this study as it relates to the interpretive and functionalist paradigms of social sciences (Taatila & Raij, 2012: 832).

The functionalist paradigm views the social world as a place where natural phenomena can be studied objectively and is often defined as realism (Ardalan, 2003). Parallel to functionalism, interpretivism views the social world as an ever-changing place, or location, where every situation can be interpreted by acknowledging the complex, iterative nature of social interactions, such as teaching-learning and doing mathematics (Taatila & Raij, 2012).

In the light of this philosophical foundation, the pragmatic paradigm calls for a research design that will incorporate the social reality, involving the teaching-learning and doing of mathematics, and interpreting such a reality. In terms of the current research, the design had to include the establishment and use of metacognitive language and metacognitive networks sought to foster a locale in which students could grow metacognitively, which is the main aspiration of the movement in paragraph 1.1. Creswell and Plano Clark (2007) claim that this decision must be based on the association of pragmatism with multiple research methods. In essence, the focus must be on the consequences of the research, the research questions and how the data collection informs the problems under scrutiny. Pragmatism fits into such a pluralistic-orientated purpose and needs to be supported by a design that will provide a way of finding out what works in practice and supply insight into why it works. With this in mind, the suitability of a research methodology such as design-based research was rooted within this study for theory development and as discussed in the following discussion.

1.5.3 The pragmatic paradigm necessity: aspiring to use an educational design research approach

Social and educational science researchers mainly conduct studies that investigate the effectiveness of the instruction medium, namely the teaching-learning situations, rather than focussing on instructional strategies or tasks, those appropriate for self-directed learning. For example, Amiel and Reeves (2008:30–31) quoted a report by Farley that states:

> …the value of basic research in education is severely limited, and here is the reason. The process of education is not a natural phenomenon of the kind that has sometimes rewarded scientific investigation. It is not one of the givens in our universe. It is man-made, designed to serve our needs. It is not governed by any natural laws. It is not in need of research to find out how it works. It is in need of creative invention to make it work better.

This calls for socially responsible research methods in a design that will satisfy the need to develop and support best practices of teaching-learning and doing mathematics. In the context of the current study, an example of such practices includes participants' involvement in designing and developing a mathematics lesson within a PBL experience that will extend or establish their own and possibly their learners' metacognition. Particularly, such a design must model and facilitate sufficient metacognitive language and metacognitive networking to foster a localised metacognitive culture, a shared group identity such as a learning environment that fosters students' metacognitive locale.

Anderson (2008:16) asks: "Why is it so difficult to implement research findings into practice?" Schoenfeld (2013:14) may have the answer: "Part of the reason is that the traditions of educational research are not themselves strongly aligned with effective models linking research and practice." However, in this study, the findings may contribute to the sustainability of the module under scrutiny.

From the issues that evolve from the above statements, the iterative nature of design-based research was selected as the suitable research approach (McMahon & Luca, 2007) and was considered in this study for local theory development.

1.5.4 Educational design research as an approach to develop a local theory

According to Mosham (1995:7), "a metacognitive theory must have a relatively organised structure of knowledge" that may be drawn upon to explain, understand or predict a broad range of cognitive and metacognitive phenomena. In the current study, the metacognitive locale of pre-service mathematics teachers was explored in an attempt to explain how metacognitive language and networking foster the development of self-directedness and, as such, contribute to the establishment and development of students' metacognitive locale.

Amiel and Reeves (2008) argue that educational design research's cycles are dynamically compound, exploratory, constructive and/or empirical research phases that call for a multiple design and developmental approach. The current study proposes to employ a qualitative approach throughout these cyclic phases. This idea is supported by the design-based research methodology which has a series of consecutive cycles (McKenney & Reeves, 2013) and contains three major stages namely: needs and context analysis, design development and formative evaluation and a semi-summative evaluation stage.

1.6 Research methodology

In this section, the researcher sheds light on the keywords relevant to the literature study as well as the pragmatic philosophy that drove the conduct of the data collection and analysis throughout this educational design research approach (McKenney & Reeves, 2013) towards metacognitive theory development.

1.6.1 The data collection procedures

Within the different layers of this study, data collection took place through a series of videorecordings across all design sessions. The study followed a sequential data collection plan.

1.6.2 Data collection instruments

Students were required to design and present lessons for a practical teaching component of their undergraduate course. A PBL task is designed for a whole class activity. To contribute to a local theory and to allow for a fair and open invitation to all students in the class, the whole class was briefly introduced to the PBL task by means of a Powerpoint presentation and the PBL task. The sampling of participants for this study's purpose took place during this whole class meeting (class session). Thereafter, participants organised themselves in groups to work collaboratively on the PBL task, to design, refine and present a research lesson across a series of design sessions.

1.6.3 Population and sampling

The population for this study is a group of fourth year mathematics education students (pre-service teachers), studying full-time at the Faculty of Educational Sciences at a rural university in the north-west province in South Africa [n = 60]. During the first design cycle, all students enrolled for the Mathematics Methodology course for Intermediate and Senior Phase teachers received the PBL task. After students have grouped themselves, a power point slide show was presented to briefly explain and share some background about the context of the PBL task, its relation to the content in their coursework and its relevance for this study. Three groups volunteered to take part in the study; however, the third group withdrew at a later stage. The participants in this study therefore consisted of two groups of five/six students (N = 11) enrolled for the Mathematics Methodology course for Intermediate and Senior Phase teachers. The criteria for selecting these participants included the factors of convenience, access and willingness to participate as suggested by Wellman *et al.* (2005).

1.6.4 Data analysis

Two types of data analysis were used: conversation analysis and coding of verbatim transcriptions.

1.6.4.1 Conversation analysis

Analysis was done after each design group session. The content of the conversations was also used to describe, summarise and display social network analysis.

First, the body of scholarship was reviewed to conceptualise and identify the main themes and categories for *a priori* codes necessary to answer the secondary and ultimately the primary research question. Because codes were used, this study envisaged a deductive-inductive approach to answering the research questions. Field notes, during and after observations, were made of participants' ideas and contributions. Their expressing of ideas or strategies used to solve the PBL task was noted. The design sessions were videorecorded and verbatim transcriptions were organised and categorised for analysis. The categories and codes that emerged with quotations were entered into Atlas.ti to explore and organise the underlying associations between the codes and the participants. A social network analysis software package/application was then used to illustrate the connections between the codes (with quotations) and the participants to illustrate the networking between concepts of the individual participant and the group in a holistic picture.

1.6.4.2 Verbatim transcriptions analysis

The initial coding of McKetcher (2005) was adapted for this study to emphasise the level of metacognitive reflection in the discussions. Using the NodeXL package, three levels of social network analysis were constructed (Borgatti & Halgin, 2011). Verbatim transcriptions were then analysed according the metacognitive components, networks and metacognitive language to determine the possible links (McKetcher et al., 2009) between different categories and to explore networks (connections) of the metacognitive components of networking and language.

Once the discussions were analysed according to the themes, the software package Atlas.ti was used for content analysis. In this case, the content of the discussions was categorised according to components of metacognition and metacognitive language. Atlas.ti allowed the researcher to code the networks of the different components of the themes to explore possible underlying associations between the constructs (categories). According to McKetcher *et al.*

(2009) this data set can be saved as an Excel output file to export the data into NodeXL. In addition, software packages with robust methodological tools were utilised throughout. These include:

- Microsoft Office Word for verbatim transcribing the design group sessions
- Atlas.ti for the use of storing, retrieving, and manipulating the qualitative data
- Excel for exporting the tabled information from Atlas.ti to NodeXL
- NodeXL to analyse and visualise how participants and the categories in the *a priori* codes network are linked for social network analysis.

The use of these software packages for analysis of the qualitative data and conversion into social network analysis was inspired by McKetcher *et al.* (2009). The outline of the data analysis procedures is depicted in the adapted Figure 1.3.

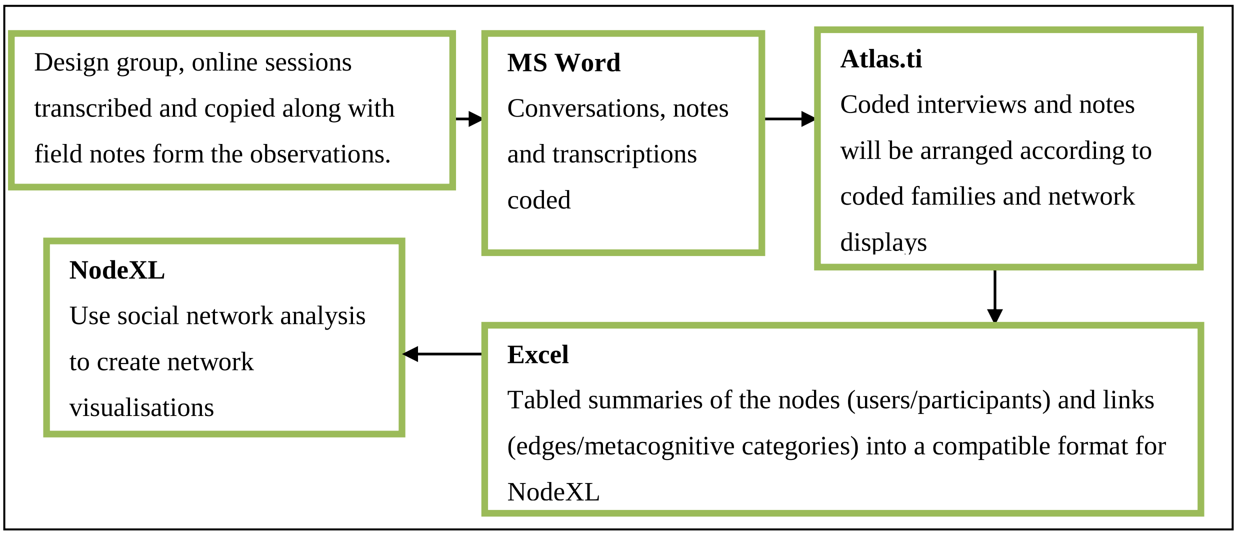

Figure 1.3 Outline of the data analysis plan

Source: Adapted from McKetcher et al. (2009)

This data analysis process was employed after every design group and scheduled online discussion to explore a possible increase (as is the case with PBL tasks) in participants' application communication of metacognitive thinking.

1.6.5 Validity and trustworthiness

There are a number of ways through which validity and trustworthiness were ensured in this study. In Chapter 6 these have been dealt with extensively (see 6.12).

1.6.6 Crystallisation

Besides the abovementioned issues of validity, crystallisation will also be employed as a validation strategy (Creswell & Plano Clark, 2007). To enhance the paradigmatic interpretations in this study, crystallisation will be used as it relates to multiple data sources and theoretical lenses. The aim is not to assume a more valid truth, as is the case with triangulation, but to open up a more complex and thorough understanding of the issues explored. In addition, multivocality ensured that the participants' views were recognised along with the members' reflections (Tracy, 2010). The multitude of participants' voices across the design sessions called for an understanding of the ethical issues to consider in this study.

1.7 Ethical issues

The participants were invited to take part in the study after gaining approval from the Dean of the Faculty of Education at the North-West University and the ethical committee[6] of the North-West University (see Addendum J1). Permission and consent were obtained before the students were invited to participate in the interviews of the design group sessions (see Addendum J2).

Lazer *et al.* (2009) advise that the use of social networking in research must protect the privacy of the participants and preserve the necessary data for research purposes.

Stack and Bound (2012:40) claim that applying metacognitive processes may challenge beliefs or other affective issues and should therefore be done in a "*culture of an ethic of care*". Some possible considerations include: creating a supportive community of practice, building relationships between participants over time, ensuring that critical reflection was constructive with enhancing practices, and encouraging a mindful attitude among participants.

The following ethical issues derived from Creswell and Plano Clark (2007), Maree *et al.* (2012) and Welman *et al.* (2005) were also considered (Table 1.2).

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[6] The certificate of ethical clearance is in Addendum J.

Table 1.2 Ethical issues

Ethical issues	**Considered approaches**
Anonymity	Participants' right to privacy was protected by using pseudonyms – thus keeping them anonymous at all times. The University's name was not mentioned. Although participants will be visible, the videorecordings were not used for any other purposes than for this study. Only the participants and the researcher have access to the videorecording.
Confidentiality	All information regarding the respondents, participants, schools and University is kept anonymous and confidential
Biased usage of language	Language or words within the study will not be used against any person or organisation
Emotional / physical risk	No respondent or participant was at risk of any physical or emotional harm
Rights of participants	Respondents and participants were not disempowered or marginalised against and were free to withdraw from the study at any time. Three groups were invited to take part in this study. Since one group withdrew, the study focused only on two groups.
Generalisation of results	Care was taken not to generalise the findings, results and discussions to the population from which the sample was drawn.

The following section deals with the limitations and possible problems that arose in the course of this study.

1.8 Limitations and potential problems: Issues with the PBL experience

The use of PBL as a learning experience posed some challenges for participants (McKenney & Reeves, 2013) and the researcher as facilitator. The participants experienced difficulty, at first, when attempting to decipher the PBL task. These expressions were treated with support and not as an expectation of the facilitating researcher to solve the problem for them; instead, participants' problems were assigned back to them through metacognitive prompting. It is important to note that participants were allowed to withdraw from the study at any time.

1.9 Structure of the thesis

The thesis consists of nine chapters as indicated in Table 1.3.

Table 1.3 Overview of the chapters' layout

Chapter	Thesis	Description
Chapter 1	Introduction, background and orientation	Introduction, orientation, purpose, motivation and methodology of the study.
Chapter 2	The nature and design of instructional theories and local theories in mathematics education	The level and way of theory construction for the synthesis and derivation of metacognition within social network analysis for mathematics education was reviewed.
Chapter 3	Mathematics education: PBL and lesson study	This literature review oversees the epistemological theories in the scholarly field PBL and lesson study in mathematics education. In addition this chapter offers a hypothetical learning trajectory, a local route for implementing Lesson study and a conceptualised local instructional theory to produce three constructs imbedded in the research problem to inform the development of local theory.
Chapter 4	Metacognition, its language and reflections	This literature review considers theories in the scholarly field of metacognitive networking and language in mathematics education. The chapter conceptualises the development of theories involving metacognition, its language and reflections. In part, a conceptual-theoretical framework is formed through which conclusions can be drawn to interpret and explain the findings.
Chapter 5	Metacognitive locale: networks and their conceptual-theoretical framework	This chapter offers a review of social network analysis, focussing on social roles and relationships, and indicates a way in which metacognitive networks can be presented and interpreted.
Chapter 6	Educational design based research methodology	The research design, paradigmatic assumptions and methodologies of the cycles within the design based research approach were dealt with. The nature of the instruments, interviews and strategies involved in the data collection and analysis stages are described.
Chapter 7	Presentation of the findings of Group A	The chapter reports on the findings of the Group A as they emerged throughout the design sessions.
Chapter 8	Presentation of the findings of Group B	The chapter reports on the findings of the Group B as they emerged throughout the design sessions.
Chapter 9	A narrative of the theory of metacognitive locale: discussion, limitations and recommendations	The final chapter discusses the results and provides a narrative of the theory of metacognitive locale. Limitations and recommendations are also argued.

First, Chapter 2 reviews the background on theory and theory development in mathematics education in terms of local instruction theories and their emerging constructs as a pre-product of local theories. Chapter 3 then elaborates on the development of PBL as an instructional approach and LS as a facet of mathematics education's teaching and learning, integrated in the module. Guidelines are discussed in terms of how the local instruction theory's principles facilitate the use of PBL to teach, learn and apply LS in practice. These instructional theory guidelines are based on the principles of theory design in the literature review of Chapter 2. Chapter 4 reviews metacognition, its language and networking as emerging constructs through PBL and LS's design. Chapters 4 and 5 contextualise these constructs to design a local theory of students' metacognitive locale. Chapter 6 elaborates on the educational design research methodology employed in this study to answer the research question(s). Chapters 7 and 8 present and analyse the data and Chapter 9 discusses the findings in terms of the conceptual-theoretical framework.

Chapter 2

The nature and design of instructional theories and local theories in mathematics education

2.1 Introduction

This chapter reviews the literature on the philosophical nature of theory design. In the following sections, a discussion follows the terminology of theory approaches to creating new theory and key regulators such as the scope and range of the elements and strategies of local and instructional theory construction. First, the researcher explains the epistemological preconceptions of this study to clarify the purpose and nature of theories in mathematics education. Second, local instructional theory and local theory is explained in the epistemological framework of mathematics education.

Figure 2.1 illustrates how the literature reviews in chapters 3 and 4 unfold and serves as a conceptual map or framework for designing a local instruction theory and a local theory as discussed in Chapter 1.

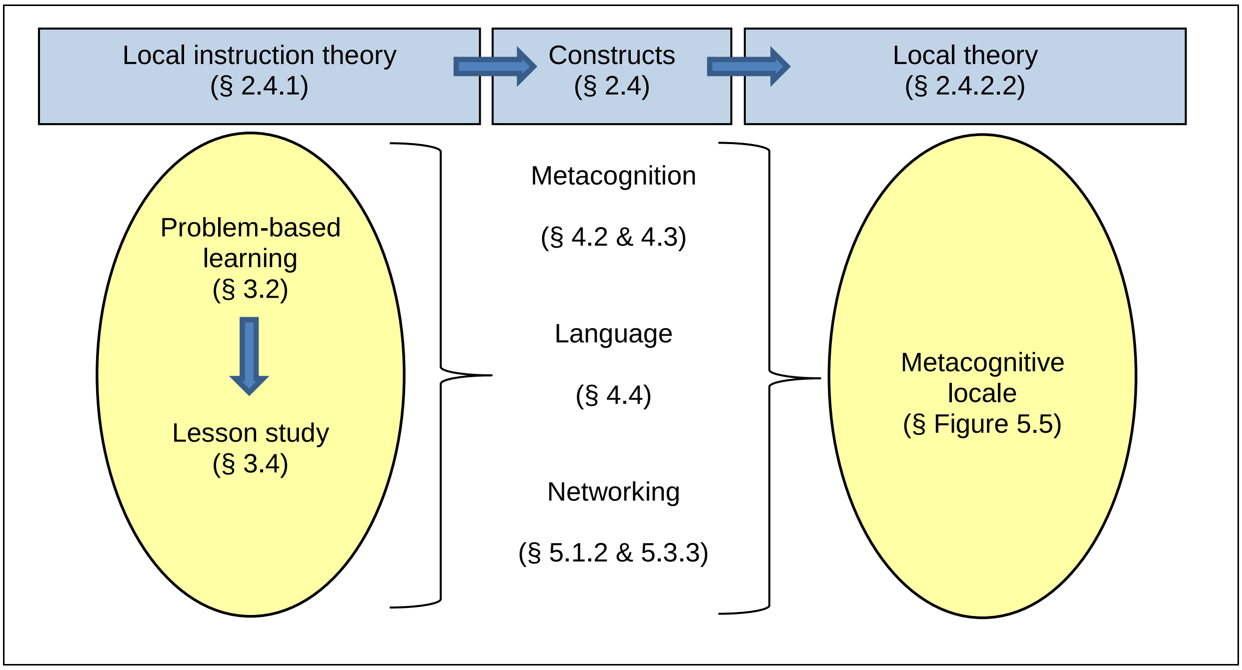

Figure 2.1 Sequence of literature reviews

Figure 2.1 shows how the literature review unfolds. The current study lies at the nexus of two fields of ontology, that of mathematics and its teaching and learning practices. The researcher noticed that there are various branches of understanding educational phenomena labelled as teaching and learning theories and considers this in the following discussions. However, the review excludes the different disciplines and theoretical underpinnings that do not contribute to the contextual, conceptual and philosophical foundations of this study, as portrayed in the problem statement (see 1.2.1). Instead, the researcher considers the theoretical framework of scholarship of mathematics education that typifies the constraints of the methodological and epistemological questions proposed in Chapter 1 in this conceptual and theoretical review. In addition, the following discussion on theory frames the foundations of philosophy in mathematics education and expresses the value, purpose and need for a theory.

2.1.1 Some epistemological preconceptions of theory

Research within theory construction is often warned against and cautioned because of its complexity and sometimes its abstractive nature (Walker & Avant, 2005). The term "theory" stems from the Greek word *theorio/a,* which means to consider, assess or reason through viewing, speculating or thinking about life or a phenomenon (Rodowick, 2007). By bringing together *thea* (sight) and *theoros* (the spectator), theory is associated with a reflective nature.

The value of education for the future of a country is beyond dispute and outlines the ethos of knowledge improvement in the knowledge economy (Temple, 2012). Improving knowledge could be a way of finding meaning or value for one's own life and in turn unveiling the purpose of education. When this meaning accompanies education theories and explains or changes cognition or behaviour, that realisation of self and others can, through theory, support or challenge existing theories. Therefore, teaching and learning theories need to be developed for a particular subject or discipline (Lowyck, 2014) with a particular context. In mathematics education, however, the subject matter and the types of theories involved in modelling knowledge can have a marked influence on theory development. Cohen *et al.* (2004) explain this process as provisional in the sense that it must be evaluated and tested constantly as new knowledge or theory emerges.

2.1.2 The nature of theory

According to Kuhn (2015), science is a constellation of facts, theories and methods that reflect the trials and errors of scholarly contributions across different disciplines. Gradually, the views on this relationship between understanding knowledge and its application in practice are changing as some scholars (Anderson, 2008; Borgatti & Halgin, 2011) agree that theories cannot exist alone, and that philosophy that overarches theories is therefore in higher demand as a scholastic contribution. Yet, the elements of observation, theory and philosophy predicate the design of new theories and suggest that theories always have, *and always will*, evoke responses from specialists in theory and practice (Carr, 2006:12). Creating new theory is then not only an increment of what is already known; rather, it remodels and re-evaluates existing theories to open debates about the value of theory and its implications for practice. In the discipline of mathematics education, Cobb (2009), who claims to create new theories of mathematics and its teaching and learning phenomenon, argues that theory cannot be separated from practice and stresses the value of the theory-and-practice relationship. In addition, Realistic Mathematics Education (RME) as an emerging field positions theory construction at the heart of the teaching and learning enterprise (Dai, 2012).

As the theoretical field grows and researchers revisit the relationship between theory and practice, the subject matter of mathematics and its teaching and learning becomes a cascade with different features of development and psychology theory in practice and everyday life. In mathematics education the content, skills and context must engage the teaching and learning relationships, thus recognising associating mathematical teaching and learning theories and their implications and value for practice (Foster *et al.*, 2013). Adams (2013:32) portrayed emerging educational theories as contributors to the quality of the teaching and learning experiences:

> When education began to be recognised as a subject in University curriculums, it was only natural that lecturers in education should look out through world literature for great names with which to adorn their lists of prescribed reading. Naturally, Socrates, Plato and Aristotle were seized on at the start and a good deal of ingenuity was shown in bringing out educational principles from their work.

Throughout the last century, these educational principles have transformed curriculums through introducing new theories. The existing theories then serve as a foundation to create opportunities to justify and build onto the vocabulary, intent, understanding and value of new theory. Mathematics education theory is concerned about those theories that aim to understand and improve teaching and learning of mathematics in various contexts (Sriraman & English, 2010). Ertmer and Newby (2013:46) list seven definite questions that distinguish mathematics education theories from one another:

- How does learning occur?

- Which factors influence learning?
- What is the role of memory?
- How does knowledge transfer occur?
- What types of learning does the theory explain?
- What basic assumptions or principles of this theory are relevant to instructional design?
- How does instruction promote learning?

Besides these guiding questions for theory development, Eisenhardt (1989) suggests that the first step to designing new theory requires a defining purpose of theory.

2.1.3 The purposes of theory in mathematics education

Approaches to developing theory have been part of the philosophical debate about the nature and purpose of theory across the last century (Carr, 2006). From these discussions, the main purpose of theory originates from societies' needs to contextualise, enclose and solve emerging problems. Research problems, when focused, become organised and approached within the context of a specific discipline (for example issues with teaching and learning mathematics). As constructs emerge for the research problems, theory explains the overlap between these constructs (for example the language of mathematics and content knowledge) (Oliver, 2000). Constructs such as metacognition, networking and language within mathematics education have been identified in Chapter 1 as examples of these emerging constructs. In addition, theory explains existing relationships between constructs; positions research problems within the scientific framework and creates new knowledge to revisit the needs of society.

Some purposes of educational theory are summarised in Table 2.1.

Table 2.1 Purposes of theory

Purpose of theory	Description of purpose	Source
Theory must explain the existing relationships between constructs	Theory serves as a system of tools. In this sense, theory explains and interprets classroom observations to justify the teacher's and the learners' roles.	Sriraman & English, 2010
Theory must position research problems in the scientific framework	Theory identifies the starting point of research through exploring the context of the problem statement and explaining how the problem can be interpreted. From this statement, theory has a particular characteristic that mirrors the properties of a model.	Nalzaro, 2012
Theory must cater for the needs and demands of society	Mainly, theory must improve teaching and learning experiences of mathematics across various contexts.	Sriraman & English, 2010
Theory must create new knowledge	Theory must explain the relationship between constructs that emerge into themes and rests on the needs of society. As these themes increase over time, new knowledge emerges and the theory will evoke new knowledge. This purpose of theory provides a theoretical lens through which constructs can be explored and knowledge created. Professional learning and development of practices to support students with problem solving in teaching mathematics education courses can produce different types of knowledge, including knowledge on subject didactics, subject content and social relationships.	Foster *et al.*, 2013
Theory impacts policy and practice	Theory must develop policy and have an impact on practice to explain its role in the scientific framework to implement and to explore (new) knowledge and skills. Furthermore, without this contribution a debate and consensus about theory is not possible and the bridge between theory and practice will collapse. Through combining observations, reviewing of scholarship across different disciplines and through inductive and deductive reasoning, this purpose of theory offers new ways of understanding and conceptualising everyday questions from social and natural worlds and how policy is implemented.	(Carr, 2006)
Education theory must improve teaching and learning experiences	When education theory is implemented from policy and practice, teaching-learning experiences can improve. For example, teaching through reflection on experiences and prior knowledge can be used and built upon to construct new knowledge Theory then provides a lens through which teaching and learning experiences can be reflected on to identify challenges and limitations and to explain these shortcomings (Foster *et* al., 2013). Theory must therefore allow for possible predictions and interpretations of learning trajectories and the quality that the teaching and learning experiences offer in this regard.	(Abrami *et al.*, 2011).

It seems from the overview on the different purposes of theory, that theory must first develop from practice and then, from this practice (new) theory can emerge. Therefore, the purposes outlined in Table 2.1 suggest that theory's intent is to elaborate on the relationship between knowledge and practice.

2.1.4 Praxeologies: the relationship between knowledge and practice

From the discussion on the purpose of theory it seems that theory explains the relationship between knowledge and practice. When knowledge emerges from practice, such as the case with constructive learning theory, knowledge and practice become inseparable (Bernath & Vidal, 2007). When knowledge is constructive through social interaction, the theory recognises and explains the underlying values and structures of these interactions and brings theory and practice together (Dean, 2013). Rodrigues, Bosch and Gascon (2008) refer to this knowledge and practice relationship as praxeologies.

Praxeologies explain the relationship between *praxis* (practice) and *logos* (knowledge). The most influential theories that associate knowledge construction and personal awareness in a society of co-constructors are those of socioconstructivism and metacognition (Valerie *et al.*, 2006). The researcher recognises that meaningful and collaborative learning experiences in mathematics education are the cause and result of various existing theories. Intertwined within the design of educational practice and theory processes, specifically in mathematics education, domain-specific theories can emerge as contributors to the understanding of how and why such educational praxeologies emerge. In the case of mathematics, praxeology's focus is on the specific local aspects of teaching-learning and doing mathematics. These praxeologies are aligned against the backdrop of the subject's didactics. Therefore, theories in specific instructional environments, or locale theories, have a specific nature when expressed as local instructional theories (Van Eerde, 2013). From these praxeologies, collaboration and reflection become a continuous activity (Rodriguez *et al.*, 2008).

Since theory can be seen as a summary of knowledge (Bernath & Vidal, 2007a), effective learning experiences require that knowledge must be clear and explicit (Alayont, 2014). Still, there is a scarcity of research on this topic (Kuhn, 2015), and research on constructing or exploring knowledge with theoretical and practical significance in teaching-learning and doing mathematics is at a standstill.

2.2 Theory and practice

Van Staden and Mentz (2014) propose that teachers' professional development and the reinforcement of teaching and learning practice strengthen the value of theory and highlight its implications in educational practice. While education theory influences teaching practice, it also reveals new knowledge about the teaching and learning experiences and put forward alternative practices and ideas that result from these experiences (Bernath & Vidal, 2007). This pragmatic view on the development of effective teaching and learning experiences relates to educational research design that merges education theory with the teaching and learning practice as noted by Barab and Squire (2004), Oliver (2000), Krishnaswamy and Chatur (2013), Cobb *et al.* (2013) and Goos (2014). Liden (2013) suggests that this theory and practice relationship must be revisited in empirical research, since the interpretation of the findings is dependent on theory and implicates new theory.

Liden (2013) claims that education theory can explain the relationship between constructs to apply, understand and explain knowledge in practice. Bernath and Vidal (2007) suggest that theories that explain the state of mind of students must move away from traditional educational pedagogy to create practically relevant educational theory.

2.2.2 New theory as a result of testing theory in practice

When mathematics teachers or mathematics education researchers test or apply their ideas or theories in practice in one context or another, new theories can emerge (Sriraman & English, 2010). The nature of this new theory about mathematics education poses questions to both teachers and researchers that require an interdisciplinary approach. In this sense, using theories from other fields can serve as a means to gain knowledge about specific aspects about the problems and questions that arise from practice within mathematics education. Silver and Herbst (2007) argue that through this borrowing of theory, teachers and researchers can select and arrange theories that will be helpful in the field. Even though theories of teaching-learning and doing mathematics follow in the footsteps of pioneers such as Steiner (1988), Sriraman and English (2010) argue that such universal and grand theories often overlook the context and embedded social-cultural settings of the mathematics classroom. For example, constructivism, according to Sriraman and English (2010), does not inform decision-making as part of problem-based learning experiences. Instead, its application in mathematics classrooms results in new knowledge about how students build new knowledge based on the architecture of their prior knowledge (Ernest, 2010). It appears from this argument that for theory to be implemented in practice, it must have a specific, meaningful and contextualised purpose to yield knowledge about teaching and learning experience and to enhance practice. The following section offers an overview of some theories in mathematics education.

2.3 Mathematics education theories

Lingard (2015) stresses the importance of mathematics education theories that filter the purposes, nature, and intent of the theory to model and define relationships between the teaching, learning and doing of mathematics. Sriraman and English (2010) claim that the variety of theories that aim to interpret the constructs' nature can exist underneath an overarching metatheory. The influences of philosophical ideas (Carr, 2006), experiences, beliefs, intentions and attitudes in mathematics teaching (Corneya *et al*, 2008) explain the nature of not only the constructs, but also the relationship between the theories, which act as standard theories underneath the overarching metatheory. Even though metatheory connects the underpinning relationships between constructs, it is the researcher's philosophical stance, or worldview, that contributes to the design of the theory (Kuhn, 2015; Schon, 1983; Carr, 2006). In turn, the theoretical contribution explains the greater phenomenon and brings together the standard theories with and by means of practice. Contextualising the educational theories that promote effective teaching and learning experiences integrates theory with practice and forms new local instructional theories, such as the type proposed by Silver and Herbst (2007).

2.3.1 Behaviourism and Cognitivism

For the purpose of this study, the researcher will not elaborate on the learning theories of behaviourism or cognitivism in great detail. Both behaviourism and cognitivism have accumulating principles that act within the social constructivist theory. According to Ernest (2010), cognition is an adaptive organisation of the experiential world. Wearne and Hiebert (1988) argue that theories that explain cognitive change are not entirely universal. Instead, cognitivism should be studied locally. A study done by de Jong *et al.* (2013b) indicates that cognitive change occurs in online environments such as group sites and online chats as well as in the natural and real-world classroom setting. One element of cognitivism that distinguishes it from constructivism is that it does not explain the search and discovery of knowledge.emphasised by Von Glasserfeld[7] (1989). When the individual adapts according to the experiences in the world, they do so largely in a pragmatic schema known as constructivism (Sriraman & English, 2010).

[7] Although some scholars might argue that Von Glasserfeld's view on cognition is not a recent source, his work is drawn upon in this study as he is considered the father of the movement between cognition and constructivism in educational theories (Sriraman & English, 2010)

2.3.2 Constructivism

Constructivism can be broadly defined as the architecture of knowledge (Ernest, 2010). The primary view on the constructivist approach to learning involves knowledge as a function of creating meaning from experiences (Ertmer & Newby, 2013). Constructivists do not share the views of cognitivists or behaviourists on reality. Rather, constructivist theorists emphasise the humanistic role in construction of knowledge and sense-making. Constructivism has its roots in the theories of behaviourism and cognitive science's learning theories. The student builds knowledge from personal interpretations, experiences and interactions into a unique, often shared, internal representation of knowledge. Ernest (2010) refers to the individual construction of knowledge as individual constructivism and the shared enactment of knowledge as social or shared constructivism. Knowledge is then embedded within a specific context and is framed as an interpretation or perceived form of the real world. Ertmer and Newby (2013) reflect on these lived experiences as construction of internal knowledge that can change through critical discourse and higher-order reasoning processes, such as metacognitive thinking. Through meaningful tasks and authentic experiences, students become aware of their critical understanding of the environment and ultimately structure their knowledge through facilitative thinking, regulation and application of their understanding of reality (Ertmer & Newby, 2013).

Constructivism as a learning theory can be viewed from three major philosophies of teaching and learning. These paradigmatic perspectives include radical, enactivism and social constructivism (Ernest, 2010)

2.3.2.1 Radical constructivism and enactivism

Radical constructivism refers to the function of cognition as an adaptive organisation of the experienced world and not only as self-discovery (Ernest, 2010). When compared to the cognitivist theory of learning, radical constructivism explains that cognitive structures are built to solve a particular problem as perceived necessary (Sriraman & English, 2010). Criticism against radical constructivism is concerned with the concept of the individual who seems separated from shareholders or co-constructors in real-world settings and experiences in terms of cognitive representations (Ernest, 2010). This in turn affects interpersonal communication, and as a result, a need exists to find a balance between the knowledge and application of knowledge in shared lived experiences. This need for a shared view on constructing knowledge from experience marks the starting point for social constructivism and creates the background for enactivism.

Enactivism explains that learning is shaped by both cognitive patterns and interaction with the world. Three main elements form the basis of enactivism: meaning, imagination and perception (Ernest, 2010). Although these elements compose the changing structure of actions, they are often referred to as abstract and generalised metaphors for thinking in the mind of the individual. The casual interactions that constructivists nowadays long for in a post-cognitivist epistemology or learning theory form the foundation for social constructivism.

2.3.2.2 Social constructivism

In a complex world where students and others interact with each other and the content of the mathematics curriculum, chains of social relationships emerge (Liden, 2013). Social constructivism originates from an explanation of the constructivist approaches to teaching and learning in these social settings to explore and explain these social chains. During social interactions, students that are teaching and learning within these experiences can construct new knowledge through their social interactions. The mechanisms of these individuals' conversations and collaborations include cell phones, interactive software packages and online social media sites such as Facebook and MySpace, as their individual and social cognitive and metacognitive functioning reflects across this social network (Simons, 2000). Students and teachers of mathematics can then, through such collaborative and shared experiences, build on previous knowledge and obtain new knowledge through constructive processes (Alayont, 2014). Learning is then not only an opportunity to construct new knowledge but also to be socially productive and collaborate in a meaningful experience (Nickerson & Whitacre, 2010). This socially constructive experience then focuses on the exchange between students' perspectives of the reality of their classroom teaching and learning practices and strengthens their own abilities to enhance the outcome of these experiences (Simons, 2000).

The above characteristics of social constructive learning theory seem to include self-directed learning and metacognitive knowledge and strategies as forms of awareness to regulate one's own teaching-learning and doing of mathematics. In addition, social constructivism explains the social networks that evolve as a result of these collaborative and constructive experiences. As a social theory, social constructivism expands across both macro- and micro-events in the classroom. Through this metatheoretical lens, social constructivism suggests that the theory not only explains universal relationships between teaching and learning constructs, but also expands on local instructional theories and practices. A relationship seems to exist between local theories and local instructional theories; although these two types of theories influence one another, they do not have the same purpose.

The key elements identified by Fung (2013) reflects not only the principles of social constructivism, but also that of a more recent learning theory that adapts these elements for online and web teaching environments. These principles include: engaging and participating in cultural awareness, create social meaning of and through the content, prior knowledge associates with current context by integrating student knowledge into the community, learning across multiple contexts, cooperatively and participatory. Chatti et al. (2011) refers to this learning theory as networked learning or connectivism. However, the scope of this research is limited to social constructivism, since it explores the networking environments and constructing of knowledge in online and offline communities of learning.

2.4 The relationship between local instructional theories and local theories

Approaches to developing theory rely on the perspectives that support the empirical research and the philosophy behind the theory (Liden, 2001). In addition, Wright, Brito and Cook (2011) claim that the purpose of a local theory is to explain emerging, tangible and actionable constructs in a local contextualised setting. In contrast, the purpose of local instructional theory is to design instructional guidelines that address issues emerging from classroom practice. According to Cradock *et al.* (2013), educational theory must reinforce professional practice, suggesting that the local instructional theory must contribute to education practice and the local theory must explain emerging constructs (Larsen, 2013 & Carr, 2006). As an argument for local instructional and local theories, Bernath and Vidal (2007) suggest that improving educational quality takes place when researchers bring forward new ideas and strategies to progress not only the teaching experience in the present but also to predict the quality of the learning experiences in the future. This heightened awareness of the implication(s) of educational theory for practice will require new forms of learning with their own, unique criteria and principles that will result in transformation of the teaching and learning enterprise. The researcher agrees with Bernath and Vidal (2007) that this transformation will not only enrich learning and the attitude towards learning, but also ultimately change deep-rooted learning behaviour and promote lifelong learning through reflective and researched teaching practice.

Local instructional theories offer teachers a structure for designing, engaging and refining such effective teaching and learning experiences (Gravemeijer, 2004; Nickerson & Whitacre, 2010). Figure 2.2 illustrates this relationship between local instructional theories and local theories, which revolve around emerging constructs.

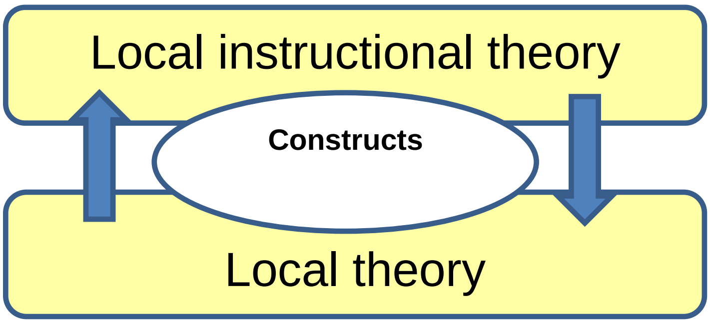

Figure 2.2 Relationship between local theories and local instructional theories

Source: Conceptualised from Gravemeijer (2004); Nickerson and Whitacre (2010)

Figure 2.2 shows the relationship between local instructional and local theories as being iterative in nature. It is conceptualised that the implementation of a local instructional theory can produce constructs which can then be theorised about in a local theory. On the other hand, the use of a local theory can result in an understanding of the relationship between the constructs and inform the development of local instructional theories.

2.4.1 Local instructional theories in mathematics education

Nickerson and Whitacre (2010) define a local instructional theory as a framework for design, engaging students in the development of effective mathematics lessons, whereas Gravemeijer (2004) refers to it as a description and rationale for the envisioned learning route. According to Sriraman and English (2010), theories have a structural and functional view, both of which organise the current study's attempt to design a local instructional theory, which through its function produces a local theory. The structural view refers to approaches or methods of teaching and learning that lead to the development of contextualised and localised instructional theories about mathematics education.

2.4.1.1 Structural view on local instructional theory design

Over the last five decades of mathematics education, research trends indicate that the formulation and structure of classroom experiences develop as restricted (or local) instructional theories. These local instructional theories appear to have a theoretical structure that contextualises the teaching and learning phenomenon and describes, interprets, explains and justifies the observations and reflections in mathematics classrooms. These local instructional theories also help to transform the practical issues in teaching and learning into research problems that can be studied in terms of mini-pedagogical experiences or research lessons (Cobb *et al*., 2013). These theories provide for the educational experiences, but also play a role in the function or purpose of the theory (Sriraman & English, 2010).

2.4.1.2 Functional view on local instructional theory design

When the local instructional theory is implemented, new ideas regarding best practices and effective teaching develop through observations of and reflections on the teaching and learning experiences. These ideas and new-found knowledge about teaching and learning can be researched in terms of emerging concepts. These concepts can then be explored, examined and understood within the contextual parameters of the local instructional theory and the relationship between these constructs can be explained in terms of the locale[8]. This leads to a view on the function of the concepts that emerge because of the instructional theory, as it opens up possibilities to explore constructs and develop new local theory. The local theory will, in turn, inform the local instructional theory, reform the design of the educational practice, and enhance the teaching and learning experience.

From the structural and functional view on designing new theory, it seems that the teaching perspective and teaching strategies can be theorised about in terms of a local instructional theory. Because of the implementation of the instructional theory in practice, new emerging constructs can be studied to determine the functioning of these constructs within the parameters of the instructional theory. The constructs can also be theorised about, result in a local theory and produce new concepts. A local theory of the teaching and learning phenomenon therefore seems to accommodate the local instructional theory, suggesting that these two types of theories are relational as illustrated in Figure 2.2.

2.4.2 Local and instructional theories in mathematics education

Examples of studies conducted to develop and design local theories in mathematics education are reviewed. The first case, where a local theory is produced from a local instructional

[8] The researcher reminds the reader that, for the purpose of this study, locale is a theoretical space or dimension identified to position the metacognitive knowledge and skills of individuals and groups (Ballarin, 2014).

theory, is borrowed from a study done by Belbase (2013). The second case reviews the local theory of Hiebert and Grouws (2007) as a result of an instructional theory.

2.4.2.1 Designing local instructional theory through local theory

In a self-reflective interview, Belbase (2013) reports on a reflection of beliefs about the use of Geometer's sketchpad as a medium to teach geometric transformations. Based on prior experiences and the pedagogical beliefs about teaching and learning, Belbase (2013) identifies four focal areas that hold the beliefs about what works in mathematics lessons and what does not. The focal areas include beliefs about the content, nature of teaching and learning, teaching and learning activities and concerns regarding the teaching and learning resources.

It is worth mentioning that a local instructional theory can also be a result of the local theory. The process of designing instructional theory and local theory is therefore seen as a relationship and not a hierarchical structure (Dai, 2012). However, in this study, instructional theory will be used to design a local theory and the following discussion therefore only reflects examples of this type of design.

2.4.2.2 Designing local theory through local instructional theory

In the current study, the teaching and learning experience is planned firstly by the design of an instructional theory, and lastly by the design of local theory about the experiences of students' metacognitive language and networking resulting from the local instructional theory. The scarcity of literature that includes examples of both local instructional theories and local theories in mathematics education, suggests further elaboration on the design of these theories (Nickerson & Whitacre, 2010). For this purpose, Table 2.2 lists and categorises some education theories, local instructional theories and local theories to illustrate the structure and function of these theories. Table 2.2 also includes some sources and the constructs that the studies deal with, as it highlights the scarcity among local theories in the fields of metacognition, networking and language in particular.

Gehlbach (1979) reported on the differences between instructional theory and learning theory. Gehlbach (1979) noted that teaching activities that are prescriptive are bound to be instructional in nature, suggesting that instructional theories are not entirely a new phenomenon. In comparison, a theory of learning considers the descriptive process whereas instructional theory is prescriptive.

Table 2.2 Some examples of local instructional theories, local theories and constructs in mathematics education research

Instructional theory	Local theory	Constructs	Source
(a) Connecting individual symbols with referents; (b) developing symbol-manipulation procedures; (c) elaborating and routinising the rules for symbols; and (d) using the symbols and rules as referents for a more abstract symbol system.	The theory explains how students become competent with the written symbols of the decimal fraction system.	Competence in decimal fractions Cognitive processes	Wearne & Hiebert, 1988
(a) Measuring; (b) reasoning about activity of pacing; (c) measuring with a "big step" of five = measuring by iterating a collection of paces; (d) measuring by creating a stack of Unifix cubes; (e) measuring by iterating a collection of 10 Unifix cubes, structuring distance into measures of 10s and 1s; (f) measuring by iterating the 10-strip, and using the strip as a ruler for the 1s; (g) measuring: strip alongside item; counting by 10s and 1s reading of endpoint, reasoning about spatial extensions; (h) means of scaffolding and means of communicating about reasoning about number relations.	The emphasis is on the possible instructional sequence – a theory of which offers an empirically grounded guidelines on how the instructional sequence can work.	Instructional sequences, addition and subtraction, small learning objectives	Gravemeijer, 2004
Sequence of key steps in the process of reinventing the quotient group concept.	Supports the guided reinvention of the quotient group concept.	(a) conjecture of the emerging symmetries of a square; (b) starting point context; (c) Identifying evens and odds in D8; (d) Viewing even/odd partitions as groups of subsets; (e) partitioning D8 to construct a four-element "metagroup"; (f) Identifying necessary conditions– Coset formation.	Larsen & Lockwood, 2013

There appears to be a trend in the metatheory applications in mathematics education and the years in which these contributions were published. Not only the metatheory but also the number of available examples of studies that report on instructional theories is a scarce topic in the 1980s to 1990s (Berliner, 2004). In their discussion of the development of theories and the changes in theoretical paradigms of mathematics education, Sriraman and English (2010:13) also refer to this trend: "We have witnessed, among others, shifts from behaviourism, through to stage and level theories, to various forms of constructivism, to situated and distributed cognitions, and more recently, to complexity theories ...".

From the review of the studies in Table 2.2, and in light of the changes in theoretical paradigms, four characteristics emerge from the comparison of local instructional theories and the design of local theories.

2.4.2.3 Comparing local instructional and local theories

First, local instructional theories and local theories do not appear to exist alone; instead, they underpin meta-or-standard theory, which supports and explains the instructional activities as well as the implications of these activities in practice. Furthermore, the instructional theories explain through a particular instructional sequence and conduct teaching and learning experiences. It appears that the instructional sequence serves as a guideline, providing a predetermined structure for the conjecture of the local instructional theory. The testing and refinement of this theory requires deep reflection (Choppin, 2011). Local theories that emerge from these instructional theories support and explain the impact or result of the local instructional theory's guidelines as implemented in practice. This notion of theory exemplifies the purposes of theory as discussed in paragraph 2.2.

Second, by implementing the instructional theory into practice, new knowledge emerges that supports the impact of the theory and results in a localised explanation of the instructional theory's structure and function. The examples of Table 2.2 illustrate characteristics of such education theories as behaviourism, cognitivism, constructivism and social constructivism, which serve as metatheories for the instructional theories. Third, the metatheories can act as standard theories in the above examples of instructional and local theories. In this role, the metatheories mainly focus on the effect(s) of the teaching practice and do not only explain the relationship between the constructs but also support the purpose of both the instructional and local theory.

Lastly, it seems that the constructs that emerge are predetermined and are purposively conveyed in the design of the local instructional theory. This predetermined or hypothetical conjecture of what might happen in class can be implemented as an instructional theory by following the guidelines. The local theory, then, predicts and explains the emerging constructs, which, in the examples of Table 2.2, do not necessarily form part of the steps or guidelines of the instructional theory. The local theory regards the constructs as the result or output of the instructional theory. Similarly, McKenney and Reeves (2013), Cobb *et al.* (2013) and Gravemeijer (2004) have reported on the input and output variables in the development of these local instructional theories as part of local theories' development. Figure 2.3 illustrates the design cycles of the development of local instructional and local theory.

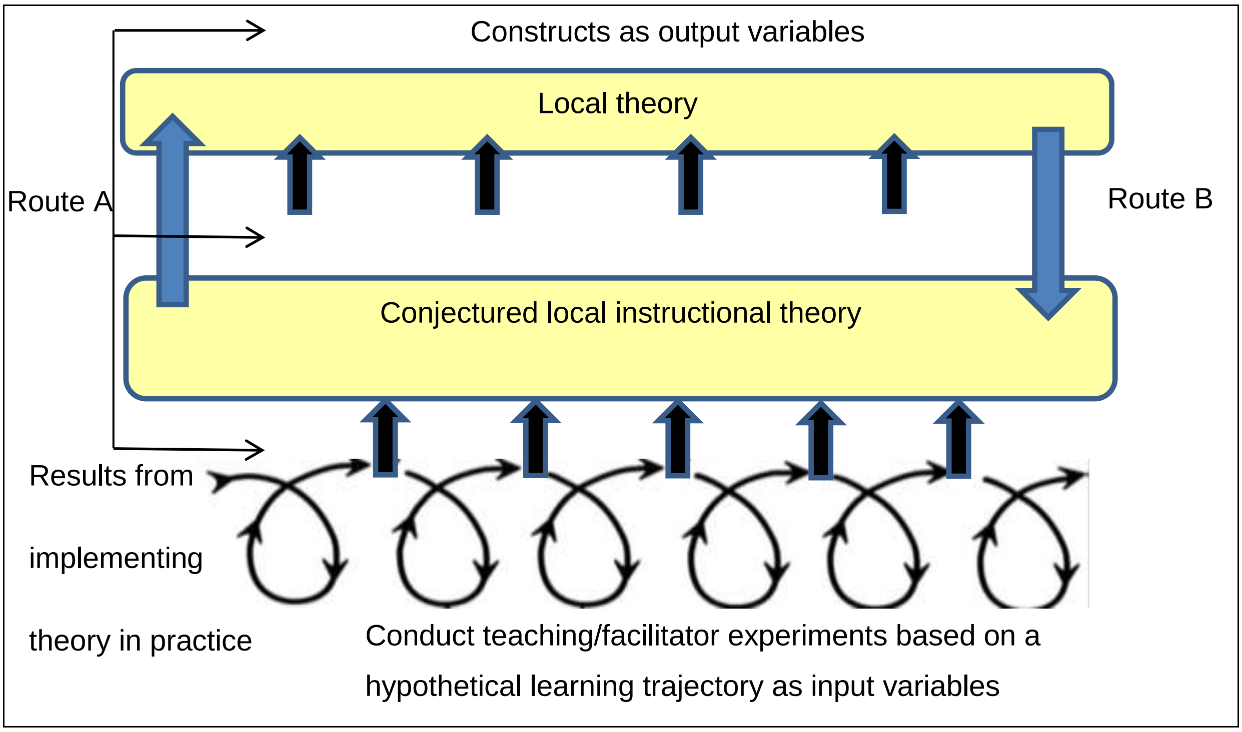

Figure 2.3 The design cycles of local instructional theory to produce local theory

Source: Adapted from Gravemeijer (2004); Van den Akker et al. (2006); Bustang et al. (2013)

Route A suggests that the local instructional theory informs the local theory as indicated by Larsen and Lockwood (2013). On the other hand, Route B indicates that the local theory can also inform the local instructional theory, as was the case with a study done by Nickerson and Whitacre (2010). The design cycle (also called a macrocycle) indicates similar continuous cycles (microcycles) that produce recurring or comparable results and enables the teacher as a researcher (Jagoda *et al.*, 2011) to formulate, test and refine the conjectured local instructional theory. In the macro- and microdesign cycles, the teacher as a researcher-facilitator (De Jong *et al*., 2014) makes use of a thought or teaching experiment. Cobb (2009) and Gravemeijer (2004) explain that the role of the teaching experiment is to test and improve the local instructional theory. The design of these teaching experiments forms the basis of a hypothetical learning trajectory as a set of classroom activities (Bustang *et al.*, 2013). Furthermore, from these emerge results; constructs can exist from which a local theory can be developed or designed. The first step towards this process is then to construct or develop an appropriate teaching or facilitator experiment.

2.4.2.4 Teaching experiments

Because of their familiarity in publication, teaching experiments will be referred to in this study and not facilitator experiments. However, Czarnocha (2008) distinguishes between teachers and facilitators' roles and suggests that the teaching experiments, through facilitation, are not directly instructed. For this purpose, the researcher takes on the role of lecturer and facilitator, but also acts as a researcher.

Teaching experiments are rooted in the attempt to understand students' thinking (Confrey, 2006) and collaboration. In order to characterise these mental and social practices, Piaget (1972), Dewey (1980) and Vygotsky (1987) argued that the process of students' thinking informs researchers about students' learning. Dewey (1980) held a pragmatic view on teaching and learning experiences that supports the use of teaching experiments and is reflected on by Confrey (2006), who quotes Dewey (1980:49): "A theory corresponds to the facts when it leads us to the facts, which are its consequences."

Based on these premises, teaching experiments should provide the opportunity to guide practice, explain the data and the evidence or constructs as they emerge as the consequences of implementing theory into practice. Confrey (2006) notes that there is a need to promote such a research culture in classroom practice. However, these teaching experiments are based on a hypothetical learning trajectory.

2.4.2.5 Hypothetical learning trajectory

Bustang *et al.* (2013) define the hypothetical learning trajectory (HLT) as a tool for planning students' learning experience. HLT comprises three distinct components, namely learning goals, descriptions of the learning activities and conjectures about students' thinking in the teaching experiments. At best, HLT prescribes the possible outcomes (Butang *et al*, 2013) connected to the theory (Confrey, 2006), associating the output constructs with the input teaching experiments (Van den Akker *et al.*, 2006).

2.4.3 Chapter synthesis

From the outline and discussion in this chapter, it appears from the review and examples of similar studies that the development of local instructional theories and local theories is a twofold process. First, by identifying a contextualised problem, a metatheory should be identified that will aid in explaining and predicting the research problem. Appropriate teaching and learning experiences should then be designed based on hypothetical learning trajectories. From these trajectories, teaching experiments can be designed and implemented to observe and reflect on emerging constructs or results.

Chapter 3

Mathematics education: problem-based learning and lesson study

The world [of teaching and learning] *is complicated, confused, impure* [and] *uncertain*

(Bourdieu, 1991:259)

3.1 Introduction

In Chapter 3, the teaching and learning of mathematics education[9] within the curriculum of higher education is described. This is followed by a discussion on some approaches to develop teaching and learning in undergraduate mathematics education courses. The discussion steers the focus towards problem-based learning (PBL) as an instructional approach to reaching the mathematics methodology module's outcomes, through adapted lesson study (LS) as one of the pedagogical mechanisms to facilitate the teaching, learning and doing of mathematics. By facilitating PBL in the context of LS, a local instructional theory (LIT) is proposed, based on the design principles discussed in Chapter 2. As argued in Chapter 2, the constructs of metacognitive thinking, its language and networking can emerge as students engage in the PBL task with LS, through LIT. Chapter 3 elaborates on the design and application of PBL and LS in theory and practice and suggests guidelines for the LIT.

3.1.1 Mathematics education

Mathematics education manifests as an educational field in the discipline of mathematics and its teaching and learning phenomenon (Fried, 2014). Mathematics education unites the theorems of mathematics and the theories of teaching and learning by explaining how mathematics is practised, understood and modelled in everyday experiences. From this view, Bernath and Vidal (2007) question the definition of mathematics and suggest that the different disciplines and scholarship must recognise the broad set of practices, knowledge and skills that stem from teaching, learning and doing mathematics. Limiting the definition of mathematics solely to its discipline will limit opportunities for lifelong learning locally and

[9] In this study, Mathematics education is the discipline uniting mathematics and its teaching and learning phenomena (Sriraman & English, 2013). It is an overarching term for the teaching and learning of mathematical content, related pedagogies and didactics.

globally (Bernath & Vidal, 2007). Instead, mathematics education is about fostering meaningful experiences in the mathematics classroom by joining realistic and research-based practice in the preparation for mathematics in everyday life (Fried, 2014). Higher education institutions must then promote the professional development of pre-service and in-service mathematics educators since they are expected to produce, to appease the demand of a society immersed in a mathematical world.

3.1.1.1 Mathematics education in higher education

With the focus on specialist fields such as science, technology, engineering and mathematics (STEM), higher education institutions agree that the fate of future societies' knowledge economy rests on "*commercially applicable knowledge*" (Temple, 2012:1). In addition, the competency to teach, learn and do mathematics is regarded as the cornerstone of mathematics education training programmes. At the heart of this teaching enterprise lie subject matter knowledge, pedagogical knowledge and skills that promote and foster lifelong learning as required for STEM-related specialist fields (Demir *et al*., 2012). Facilitating mathematics education as a subject in undergraduate courses includes the objectives of the course's curriculum, subject-related pedagogies, student assessment and evaluation of the effectiveness of the course and the quality of study materials. Devlin (2013) concurs with Demir *et al*. (2012) that fresh thinking about understanding teaching and learning experiences in university courses is required. Shulman and Shulman (2004) suggest that higher education institutions should find common ground on teaching and learning by using and adapting realistic problems anchored in theory and practice. When shifting from theory to practice, pre-service and in-service educators have to, first, understand learning experiences as a critical task in learning how to teach (Demir *et al*., 2012; Wirth & Perkins, 2012). This teaching-and-learning shift reshapes classroom experiences throughout the course and promotes lifelong learning.

Towards this shift in university classroom experiences, Goos (2014) claims that mathematics education should be taught and learnt from multiple theoretical perspectives, which include cognition, awareness of social and affective influences and curriculum design components contextualised against the political, economic and societal backdrop of the country.

3.1.1.2 Mathematics education in South African higher education

One of the 23 publicly funded universities in South Africa offers a four-year Bachelor's degree in Education (BEd), with specialisation in mathematics education. Students who enrol for the degree can qualify as educators in the junior primary, intermediate or senior and further education and training phases in the South African school system, with specialisation in two subjects or learning areas, including mathematics. These modules cover subject-specific knowledge and skills, the didactics and pedagogies of mathematics and its teaching and learning methodology. For the purpose of this study, one module has been identified as it prepares fourth-year education students, as pre-service mathematics educators, for specialisation in the teaching, learning and doing of mathematics in the intermediate phase. Successful completion of this module qualifies them as Grade[10] 4, 5 and 6 mathematics educators. For degree purposes, these pre-service educators are required to complete a practical component, during a three-week period, once in each semester. This allows them to visit a school and act as a pre-service and in-service mathematics teacher, giving them the responsibility of teaching mathematics in one or more of the grades they specialise in. The mathematics methodology module for intermediate phase mathematics educators (MMI 411)[11] is one example of a mathematics education module offered to fourth-year BEd students.

- **Mathematics methodology: intermediate phase (MMI 411)**

The MMI 411 module's content aspires to enable pre-service educators to become competent mathematics educators and aims to produce quality education to join the movement towards 21st century mathematics education (see 1.1.1) and to meet the demands of schooling in South Africa. Pre-service mathematics educators must be equipped with the knowledge and skills they need to teach in the diverse landscape of the South African curriculum. Some of the challenges that educators and learners might face include creating experiences where they can work collaboratively and gain knowledge and skills to facilitate change and development within teaching and learning (Posthuma, 2012).

As many of the schools in South Africa face complex barriers to teaching and learning (Wolhuter, 2014), the MMI 411 module's outcomes include such skills as planning, analysing, testing, reflecting and adapting mathematics lessons, during which students must demonstrate an appreciation for the value of mathematics and its teaching and learning towards preparing them for the education profession. These intentions highlight the

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[10] In the South African education system there are twelve grades offered across the twelve years of schooling. Learners' ages between these twelve years range from 6 to 18.

[11] For ethical reasons and anonymity, MMI 411 will be used as a pseudonym for the module identified in this study.

underlying values and ethos of the country's education within the framework of the Constitution (DOBE, 2012). As such, LS is identified as one of the mechanisms that can assist in preparing pre-service and in-service educators for the mathematics education profession (Young, 2013; Posthuma, 2012). Goos (2014) suggests that part of the development of pre-service and in-service educators' competency relates to the demand of a sociocultural perspective. By including sociocultural design elements in the curriculum, especially in the context of South Africa's diverse landscape, these pre-service educators are prepared for teaching and fostering learning in their own classrooms, as a product of higher education. Elements such as collaborative problem-solving opportunities must therefore be embedded in the curriculum.

3.1.1.3 The South African school mathematics curriculum

Even though higher education institutions collaborate on the status of the mathematics in South Africa (Tjoe *et al*.,2013), the media (Rademeyer, 2009) and international assessments (TIMSS, 2011) claim that quality of mathematics education is poor because the educators are unqualified to teach the subject(s) that they are expected to be specialists in. The lack of qualified educators has its roots partly in the previous education dispensation (where some educators had limited access to further education and training). However, Spady (2004) claims that there are other factors to consider, for instance, schools that continue to be under-resourced and do not have enough finances to provide stationery, textbooks and other basic teaching aids. The Centre for Development and Enterprise (CDE), as Simkins, Rule and Bernstein (2007) report, shows that lack of qualifications and poorly resourced schools make it difficult to meet the demand for mathematics for the future of the country and predicts a downfall in South Africa's mathematics education.

To address these issues, the CAPS curriculum (DoBE, 2012) offers specific aims and skills that include a critical awareness of the use of mathematical relationships in social, environmental, cultural and economic environments. The development of number vocabulary, number concept and calculation skills is promoted, teaching students to think and reason logically and to apply the mathematics knowledge by investigating and interpreting information in real-life situations. It is expected that subject didactics courses will empower pre-service educators to take cognisance of these aims and skills and realise them in their own teaching and learning.

To reach these aims and to foster the identified skills, the curriculum maintains five content areas[12] which include (1) numbers, operations and relationships, (2) patterns, functions and algebra, (3) space and shape (4) measurement and (5) data handling. Table 3.1 summarises these points for the discussion that follows.

Table 3.1 Aspects of place value in the Grade 6 mathematics curriculum

Content area	**Topic**	**Description relating to place value**	**Concept and skills**	**Term**
Numbers, Operations and relationships	Whole numbers	Number range for counting, ordering, comparing, representing and place value of digits	Recognising the place value of digits in whole numbers to at least nine-digit numbers	1
	Multiplication		Round off to the nearest 5, 10, 100 and 1000[13]	2
	Decimal fractions	Recognising, ordering and place value of decimal fractions	Place value of digits to at least two decimal places. Calculators can be useful tools for learners to learn about patterns when multiplying or dividing decimal fractions by 10, 100, etc. Understanding place value of digits in decimals will help learners when adding and subtracting. Learners can use the column method as they do with whole numbers. All problem types that are used for whole numbers can be used for decimal fractions.	
	Addition and subtraction	Recognising the place value of digits in whole numbers to at least nine-digit numbers	When learners can add and subtract six-digit numbers confidently, they may be asked to add or subtract very large numbers until more than six digits with or without using calculators.	3
	Division	Recognising the place value of digits in whole numbers to at least nine-digit numbers	Dividing whole numbers by 10, 100, 1 000, etc. helps to build learners' understanding of the place value of the digits in decimal fractions.	4

Source: Continuous Assessment Policy Statement (DoBE, 2012)

The DOBE (2012:10; 242) states: "Attention needs to be focused on understanding the concept of place value so that the learner develops a sense of large numbers and decimal fractions" and:

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[12] In this study however, reference will only be made to the content area of numbers, operations and relationships in Grade 6 with the specific content focus of place value as deemed necessary for the lesson plan design constructed from the scenario in the PBL task (see Addendum A).

[13] Notice that as numbers get larger learners will tend to use more than one calculating strategy at the same time (for example the multiplier is broken up into factors, but the multiplicand is broken down into place value parts) (DOBE, 2012:242).

> … as the number range for doing calculations increases up to Grade 6, learners should develop more efficient techniques for calculations, including using columns and learning how to use the calculator. These techniques however should only be introduced and encouraged once learners have an adequate sense of place value and understanding of the properties of numbers and operations.

Table 3.1 indicates that place value is a concept taught throughout the year in whole numbers, multiplication and decimal fractions, addition and subtraction and division respectively, across the four terms[14].

3.1.1.4 Some approaches to teaching and learning place value in Grade 6 Mathematics

Lai and Murray (2014) note that research on students' thinking about place value has identified three rules that explain their understanding and misconceptions. These are referred to as the whole number, fraction and zero rules (Lai & Murray, 2014).

The whole number rule involves the selection of the number with more decimal places as the larger of two decimals. The fraction rule involves the selection of the number with fewer decimal places as the larger of two decimals, whereas the zero rule is employed when students select the decimal with one or more zeroes to the immediate right of the decimal point as the smaller decimal. The misconceptions that students formulate regarding these rules, Lai and Murray (2014) explain, have their roots in a reliance on rote memorisation or syntactical rules, with little or no understanding of the underlying concepts (Lai & Murray, 2014; Hiebert & Grouws, 2007). When teaching and learning mathematics rely mostly on rote learning and memorising, education and its progress can suffer. Lai and Murray (2014) claim that this *syntactic* approach only creates a habit of merely applying rules to manipulate symbols and does not promote higher-order thinking.

This background suggests that South Africa's mathematics teaching and learning is not sufficient for the demands of the country. Recent amendments in the curriculum (e.g. a shift

[14] In the South African education system each school year is divided into four terms separated by a school holiday between terms. Each term is about three months long.

from OBE to CAPS) have produced a single comprehensive curriculum and assessment policy for each subject in Grade R–12 as a means of enhancing the quality of education.

Since the MMI411 module is designed for the preparation of intermediate phase mathematics educators, the following section only deals with a discussion of South Africa's Grade 4–6 mathematics curriculum.

3.1.1.5 Reshaping teaching and learning experiences in mathematics

Even though research in mathematics education is increasing (Sriraman & English, 2013), it is unlikely that mathematics educators are well prepared for the education challenges of the present and the unforeseen future (Demir *et al.*, 2012). In an attempt to reshape the teaching and learning experiences in mathematics classrooms, Dawson, Jaworski and Wood (2003) identified some concerns for the design and development of courses preparing students for their profession as mathematics educators. These challenges include mathematising, modelling and assessing mathematics, and its education, realistically and meaningfully. Dawson *et al.* (2003) note that in-service programmes through which middle school (intermediate phase) educators are introduced to new instructional approaches, lead to successful learning experiences. These experiences emerge when reshaping educators' beliefs about mathematics and their understanding of the teaching and learning implications, whether the approaches are old or new. Aiming towards new and improved research on mathematics education's teaching and learning, Van Eerde (2013) claims that innovative approaches should describe, compare, evaluate, explain, predict and advise on effective and appropriate classroom experiences. Mathematics education, as a subject, considers the content, knowledge, didactics and pedagogies of mathematics to create opportunities to enrich the teaching-learning and doing of mathematics.

Connecting teaching and learning experiences, calls for a need to theorise educational research and to analyse these understandings of teaching and learning (Lingard, 2015). Education theory must therefore guide the production of new knowledge about teaching and learning and promote an understanding of the experiences that they hold. By incorporating sociocultural perspectives in the teaching, learning and doing of mathematics, Gutierrez (2013:37) argues that an emphasis on social constructivism will rekindle mathematics education researchers' interest, to rethink what learning means. Developing learning theories must therefore contribute to the design of useful educational materials that will support effective learning through the intertwined nature of the theory and practice of mathematics education (Van Eerde, 2013). Further, Lingard (2015) highlights four pressing problems that frame the stance of mathematics education globally. Even though the problems were identified in a study done in the United States (US), each problem also reflects the teaching and learning conditions in South Africa (Leatham, 2013). Briefly, the problems describe that (i) differences in academic achievement among different social groups continue, and that (ii) socio-economic and cultural dissimilarities predict students' achievement, while (iii) students speak a language other than that in which they are taught, making communication of ideas culturally, linguistically and educationally more complex as (iv) additional demanding outcomes are expected to be reached.

Even though these problems cannot be directly controlled in the classroom, Hiebert (2013) explains that reflection on teaching and learning experiences can contribute to a change in teaching, which in turn will result in changed learning. Such reflective teaching practices are common in countries with high achievement rates in core subjects such as mathematics. For example, the Trends in International Mathematics and Science Study (Martin *et al*., 2012) identifies Taiwan, South Korea, Singapore, Hong Kong and Japan as superior countries when comparing top countries in terms of performance in mathematics education. According to Leatham (2013), these countries focus on developing students' inquiry, reasoning, conceptualising, communication and problem-solving skills. Hiebert (2013) explains that these high-performing countries have not contributed to pedagogical theory about improvement in mathematics education. Instead, their success in mathematics education seems to have cultural, economic and societal roots. However, these countries (e.g. the US and Japan) do contribute two essential mechanisms to improve the quality of teaching and learning in a country bound by diversity such as South Africa. For example PBL (see 1.3.4.2) from the Western education philosophy with its origins in the US (Clark, 2002) and LS (see 1.3.4.3) from the Eastern methodologies in Japan's professional development of educators (Demir *et al*., 2012) are considered the key regulators in the design and conduct of this study, as proposed in Chapter 1 (see 1.3.4 and 1.5).

3.1.1.6 Teaching and learning styles in mathematics education

Teaching and learning styles vary greatly in their nature and definitions (Sriraman & English, 2013). For this reason, Van Mierrënboer and De Bruin (2014) recommend that the researcher take a particular perspective on teaching and learning to limit the focus to factors necessary to identify the conditions for developing instructional theory. This suggestion seems appropriate, as LIT relates to instructional methods, learning processes and learning outcomes. In this study, learning is viewed in terms of gestalt theory's explanation by Van Merriënboer and De Bruin (2014:22), where the whole is greater than the sum of its parts.

It involves analysing one's own thinking processes and considering its implications for the larger group. For example, when students solve a problem collaboratively and the solution suddenly falls into place, it was solved through every individual's effort and input and not merely the contribution of a single group member. It symbolises social constructivist learning, yet the emphasis is on how every individual contributed to the group and what the group as a whole can learn from the experience. The students in turn produce a deep understanding of the problem and transfer their new knowledge and skills to new situations or new problems.

- **Teaching styles**

Young *et al.* (2010) have argued that educators who collaborate on their planning and teaching can realise and accommodate the needs of their students better than those who keep their lesson plans to themselves. Collaborative teaching acknowledges active involvement in student-centred learning (Chua, Morris & Mor 2012). This is especially true in the Asian setting, as demonstrated by Mokhtar and Majid (2006) who examined collaborative relationships between educators. Educators in secondary schools with more teaching experience were observed to be more likely to engage in collaboration with colleagues by sharing experiences, expertise and ideas. Furthermore, primary school educators tended to collaborate more because they organised more academic activities for constructivist teaching as opposed to secondary school educators who mostly employed a behaviourist or traditional mode of classroom practice.

- **Learning styles**

Vermunt, Vermetten and Lodewijks (1999) proposed an inventory of learning styles that integrates four components of active learning. These include cognitive and metacognitive strategies, students' mental models as their conceptions of learning and learning orientation (e.g. motives and beliefs). The cognitive and metacognitive learning strategies are associated with self-regulative components, whereas the mental models and learning orientation are associated with self-directive components (Vermunt *et al.*, 1999). These four learning styles, as Boyle, Duffy and Dunleavy (2003) explain, are meaning-directed, reproduction-directed, application-directed and undirected styles of learning. A meaning-directed style is refers to relating, structuring, and processing the subject matter critically, self-regulating learning processes and contents, constructing knowledge as learning conception, and personal interest as learning orientation. A reproduction-directed style refers to memorising, analysing, and self-test learning orientations, and a learning conception is one in which learning is viewed as the intake of existing knowledge. Concrete processing, a vocational learning orientation and a learning conception stress the use of knowledge and its applicability in real-world situations. Finally, an undirected learning style refers to a lack of regulation and unplanned learning in which great value is attached to cooperation with fellow students and the lecturer's constant appraisal. To change such classroom experiences, the teaching and learning styles must change and result in a change in pedagogical style.

Four overarching learning styles are known to accommodate students' experiences in mathematics education, as they contain components of student-centred learning, the telling-asking continuum, perception-based perspective and direct instruction teaching.

3.1.3.7 Student-centred learning

Van Staden (2011) claim that student-centred learning pedagogies focus attention on the learner's needs and abilities. They aim to help students achieve levels of engagement and thinking and oppose the traditional formats (where the teacher and the teacher's knowledge take centre stage) of learning. Approaches that include PBL as well as enquiry learning and project-based learning, discovery learning, case-based teaching and just-in-time teaching are examples of the teaching and learning styles that frame classroom experiences. Prince and Felder (2009) conducted a review of these learning and teaching approaches and conclude that student-centred learning (1) encourages deep learning, (2) improves critical thinking and self-directed learning, and (3) supports the theories of learning that explain how learning occurs socially.

Regarding the difficulties and challenges higher education institutions face in developing and presenting mathematics education courses, research suggests that little has changed. For instance, Hierbert (2013) argues that the way lecturers and pre-service educators understand, plan and apply course content does not necessarily improve the interaction and understanding among the individual and the content. In addition, this does not improve classroom experiences of mathematics educators, as Hiebert (2013) indicates in the discussion of three attempts highlighting the reform of teaching and learning styles in mathematics education.

3.1.3.8 Direct instruction teaching

Educators who employ the traditional direct instructional approach in their classrooms do not encourage students' interest in STEM-related fields (Voogt *et al.*, 2015) and do not facilitate students' critical thinking or problem-solving skills.

3.1.3.9 The telling-asking continuum

The telling-asking continuum explains that teaching can take place by telling students the mathematics that they learn using mathematical language (Papaja, 2012) while asking them for the mathematics that they learn in order to monitor their learning. When none of the students can answer the teacher's question, we often see the teacher move back towards the middle of the continuum, asking leading questions and supplying hints. Neither the teacher's use of leading questions and hints nor the reliance on an advanced student's telling the other students the mathematics to be learnt represents a theoretical alternative to teacher telling.

3.1.3.10 A perception-based perspective

Teaching that is based on a perception-based perspective involves creating situations that provide the students an opportunity for first-hand perception of the mathematics. It provides particular ways to give students access to the mathematics that they need to learn (perceive). Thus, teaching mathematics involves choosing representations and/or problem situations in which the mathematics is "transparent", so that students can perceive the mathematics. Students have more freedom than in direct instruction to think and communicate and thus more chances to engage their prior knowledge. Second, with the introduction of computer environments and manipulatives, students can engage with a richer set of mathematical representations on a regular basis. Third, intermittently and somewhat by chance, instructional tasks aimed at the perception of transparent relationships lead to cognitive processes that actually generate new conceptualisations. These principles of students' autonomous learning and their development of problem-solving skills for lifelong learning complement the aim of mathematics education. When individuals are confronted with simulated situations or contextualised scenarios and problems and encouraged to learn individually or collaboratively through self-study and research, such theories, models and practices are referred to as PBL (De Graaf & Kolmos, 2003). Based on its philosophy, PBL merits attention in this study.

3.1.3.11 Changing pedagogical styles: reshaping mathematics education experiences

If more lecturers can be found who are keen to change their pedagogical styles, namely the teaching and learning philosophies they hold, student participation will also increase and result in positive classroom experiences (Lee *et al*., 2013). Lecturers who rely only on a one-way philosophy of teaching and employ the traditional lecture or direct instruction teaching approach create a culture where students are unlikely to adopt a deep learning approach (Batdal, 2014). Lee *et al.* (2013) suggest that learning through problem-solving highlights the values and norms that are emphasised in student-centred classrooms and suggest that a PBL approach will encourage and motivate students to actively participate and self-direct their learning towards meaningful acquisition of the content, especially in higher education contexts.

3.2 PBL as an instructional philosophy

In the 19th century, universities were known as a place where an elite class of people spoke and debated about science in a *secret* language[15] (Davies & Lowe, 2009). Nowadays the debates are much more accessible and understood by the people immersed in the real-world problems that the universities converse about. With the rise of traditional curricula in the 1950s the gap between theory and practice has narrowed as new problems continue to emerge and new theory is brought forward, with new constructs that result in new knowledge. In education, perhaps more so in mathematics education, the historical split between theory and practice requires students to first master theory before trying it out or applying it in practice, as direct instruction or lecture method suggests. Yet, in the real world, familiar and unfamiliar problems continue to emerge (Proust, 2013) and consequently trigger students' theoretical knowledge in search of resolution. Students then communicate and collaborate as they hypothesise about the outcome of their planned activities and question possible implications of their actions in an attempt to solve these real-world problems. The kind of learning which result from the process of understanding and resolving such problems is phrased as PBL[16] and overarches the instructional philosophy[17] of this study. As Dewey

[15] The language Davies & Lowe (2009) refers to here is Latin, a language that was not accessible to the public. It has now been discarded in higher education and replaced by English as the modern academic language (Proust, 2013).

[16] For the purpose of this study PBL is reviewed in terms of lecturers and students, or pre-service educators, as an approach to facilitating course content in higher education.

[17] See 6.2.2 and § 3.2 for a review of the instructional philosophy of the researcher.

(1980) suggests, “The most powerful learning occurs when the student is dealing with uncertainty.”

The above discussion on emerging problems and their role in the theory-practice relationship positions PBL as an experience-centred teaching and learning approach. PBL is thus defined as an authentic, innovative approach captured within the principles of student-centeredness and contextualised by ill-structured real-world problems. PBL stimulates active learning and problem-solving through the integration and application of new knowledge and skills. In doing so, PBL motivates students’ engagement in coursework and encourages collaboration and communication through the promotion of metacognitive skills.

PBL has its origins in the 1960s faculty of medicine at McMaster University in Canada. Its philosophy of teaching and learning transformed the theory-practice relationship in higher education. Since its introduction, PBL developed through the 1970s and 1980s with modifications to cater for teaching and learning in a pragmatic sense (De Graaf & Kolmos, 2003). Faculty in other universities borrowed the philosophy and altered it with routines to suit the needs of the various disciplines it was applied in and adapted for. The disciplines that benefit from this philosophy include English (Leong, 2009), theology (Fung, 2013), data management (Chhabbra & Sharma, 2013) and more recently, mathematics education (Batdal, 2014). As PBL became an internationally known instructional method in the 1990s, its characteristics were used to facilitate problem-solving, experiential learning and reflection, all of which are necessary skills for the 21st century mathematics teacher (Fung, 2013).

PBL requires students to work together in groups as they engage in a series of tutorials and/or discussions regarding problems provided to them by the lecturer of their course. PBL has four interrelated dimensions, as noted by Barrett *et al.* (2012). First, PBL consists of an ill-structured, challenging problem presented to students, usually at the beginning of a learning cycle. Second, students work on the problem in small groups of 5 to 8 students. Resources are limited, as students must determine what resources they need, and obtain them on their own. Thirdly, compatible assessments must aim to ensure that authentic assessments are aligned with the learning outcomes of the course and the problem-based learning process. Finally, PBL is underpinned by a philosophy of higher education, focussing on students’ learning as opposed to educators’ teaching contextualised in a specific discipline or profession.

3.2.1 The educational rationale for PBL

The educational rationale for implementing PBL as a teaching and learning philosophy lies in its comparability with modern educational approaches. PBL's principles are founded on the application of knowledge, as it encourages self-directed lifelong learning through a scenario with hidden, well-structured or ill-defined problems in a particular context. By applying the educational principles of PBL, students link their prior knowledge to new knowledge (Gunawardena *et al.,* 2009). With the emphasis on active student-centred learning, PBL enables students to not only apply what they have learnt in context, but also to learn new knowledge and skills as they gather, organise and communicate about their knowledge and experiences. Through constructing such reflective thinking, students can perceive their future careers, fostering their intrinsic motivation to cultivate a habit of lifelong learning. The PBL task guides the process of discovery and application of knowledge, skills and attitudes. When the task is relevant, it builds on previous experiences by allowing them to participate and actively involving them with problems designed to create opportunities to take responsibility for their own learning. Their new knowledge and skills can immediately be applied in practice as part of cycles of action and reflection based on mutual trust, understanding and collaboration in a cooperative learning environment.

PBL differs fundamentally from traditional curricula approaches in the sense that PBL's educational objectives address the perceived problems in traditional curricula. Some advantages of PBL's approaches include its relevance and meaningful attributes to practice. Moreover, new knowledge and skills are acquired in context as it adds interest and motivation to allow students the opportunity to continually explore their knowledge. In addition, PBL offers students the opportunity to become aware of their own knowledge and that of their group members, and to accumulate an understanding of each other's learning needs and strategies. Such reflective practices not only enhance understanding and application of the course content (Temple, 2012) but also fosters students' metacognition, through communication and shared metacognitive thinking (Proust, 2013).

3.2.2 Principles of PBL tasks

PBL concepts relate to the variety of theoretical underpinnings associated with teaching and learning phenomena. The theoretical associations at the heart of PBL's philosophy include experiential learning (Cobb, 2013), reflection (Schön, 1983), constructivism (Piaget, 1980) and social learning (Vygotsky, 1987). Combining the principles of PBL tasks as stated by Mckee *et al*. (2013), De Jong *et al*. (2013) and Davies and Lowes (2009), the problem is seen as the starting point of the lesson and creates opportunities for self-directed learning, experience learning, activity-based learning, interdisciplinary learning, exemplary practice and group-based learning. Through working on such problems, students think and ask questions that activate prior knowledge and test their understanding about new knowledge. PBL reinforces their understanding by motivating further learning through practice and logical, analytical approaches to unfamiliar situations or problems that involve learning in context, and are integrative and collaborative.

When considering the nature of tasks that promote the principles of PBL, Dai *et al*. (2012) explain three aspects of educational situations that should be addressed in PBL. First, the task should be framed by the nature of the activity, tools and resources that guide the activities of the task. Second, the activities should be shaped by the norms and values recognised within a particular context. Third, the learning experiences and dispositions of the students who complete the task must be considered. It therefore seems that such educational tasks shape not only the activity but also the nature of the teaching and learning experience. Gresalfi, Barab and Sommerfeld (2012) call this reshaping *intelligent action,* suggesting that PBL tasks require prediction, reflection and evaluation. Moreover, in a socioconstructivist view, students complete the task collaboratively and can explain and support their own and other students' thinking as they become aware of the affordances of the task. Gresalfi *et al*. (2012) claim that the task targets prescribed conceptual ideas that do not only frame the activity, but also position students differently according to their potential to complete the task in the immediate local context (Dai *et al*., 2012). For example, if a task requires students to search for or compile specific criteria for designing a mathematics lesson, they might consider the implications for the steps they carry out in their search (procedurally) or focus on the implications of the criteria or components of lesson planning. As students engage with the activities required to complete the task, they might experience a change in their role and become hesitant or uncertain of what is expected from them. For this reason, and due to the expectations, goals and broader cultural values that enthral the task and the institution's vision, students' engagement with the task could differ across locale. The task therefore affects the curriculum directly and indirectly.

3.2.3 PBL and the mathematics education curriculum

PBL encompasses the various curriculum design components. These components include the objectives of the course, teaching and learning strategies, selection of appropriate content, information and communication technologies, depiction of educators and students' roles, organisation of learning content, classroom culture and applicable assessment strategies. According to De Graaf and Kolmos (2003), a change in one of these components involves a change in all components.

The Indonesian mathematics curriculum aims to teach and learn mathematics by including more than merely the content of mathematics. For example, understanding mathematical concepts and being able to explain and apply the relationships between them to solve real-world problems accurately and efficiently highlights the content of the curriculum. Furthermore, the development of critical thinking skills for analysing and identifying patterns and characteristics of mathematics, to manipulate these patterns and to generalise, to prove and to explain the ideas of mathematics, is highly regarded in the design of mathematics education.

This learning cycle is seen as an appropriate framework to consider in facilitating PBL since, for the purpose of the current study, it resembles the educational design research's macrodesign cycles (see Figure 1.1 and see 1.3.4.2) as well as the design cycles for local instruction theory and local theory development (see Figure 2.2 and see 2.4.2.2/3). Borhan (2012) briefly discuss the process of implementing PBL below, in relation to Figure 3.1 that illustrates the PBL learning cycle proposed as a framework for implementing PBL in this study.

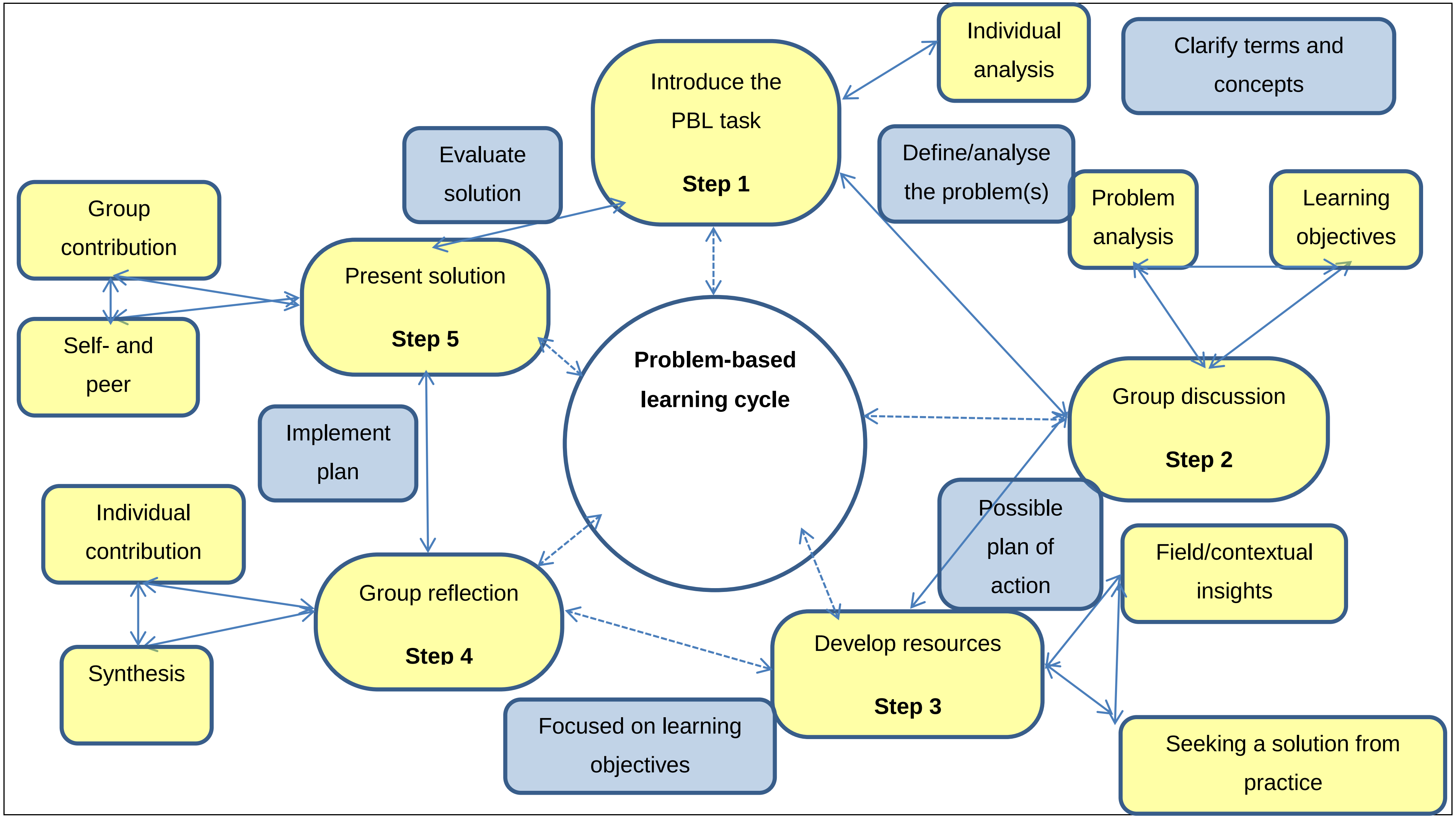

Figure 3.1 **Five steps of the PBL cycle for implementing PBL**

Source: *Adapted from Dai et al. (2012), Borhan (2012)*

3.2.3.1 Introducing the PBL task

Susilo (2013) introduced PBL to a group of secondary school science educators, with each teacher reading the problem and doing their own individual analysis. The educators had a chance to ask questions and clarify meanings as they familiarised themselves with the concepts in an attempt to define the essence of the problem. Each teacher then contributed an input to the group discussion. Each group member had to give an individual reflection on the analysis of the problem. Reflection can be by way of written documents such as notes or journal entries (Houghton, 2014).

3.2.3.2 Group Discussion

In a study by Rienties *et al*. (2014), students were nested in design groups. Students then planned, demonstrated, critiqued and communicated about their analysis to formulate an overarching goal for the group. During a group discussion, group members discuss their analysis of the problem and associate the underlying issues of the problem with particular roles that the group members may have. They also align the problems with the overarching goal and discuss possible learning objectives that accompany the problem and the anticipated learning experience. Students then develop an action plan where they consider what actions or challenges they should prepare for, who will carry out and implement these plans and by when they must take place, and for how long. Furthermore, students discuss what resources (e.g. money, materials etc.) are necessary to carry out the plan and communicate about what should be done and by whom.

3.2.3.3 Develop resources

Even though students' teaching and learning styles may differ (Ader, 2013), they have to develop resources that can be used to implement their plan of action. Houghton (2014) suggests that students should comment on the benefits of groupwork, since their inputs serve as additional learning resources. Resources will vary depending on the content and context of the PBL task. However, students must discuss which resources are needed and which are available and commit themselves to develop new resources, where necessary.

3.2.3.4 Group reflection

Students discuss issues they had with the underlying problems of the task; they collaborate on the difficulties with resource development and give reflective feedback regarding their experiences with the PBL task. During this group reflection stage, students are known to re-evaluate their progress and question their attempts, because one group member might present issues, concerns or comments that create uncertainty among group members. Johnson *et al*.

(1996) claims that reflection acts as a regulator in the dynamics of the group and serves as a mechanism to promote metacognitive thinking in the group's discourse. Reflection can call for responses to the context of the problem and can lead to change in a member's initial analysis and understanding of the problem (Lave & Wenger, 1991). In this regard, group reflection plays an essential role in the students' learning, since it presents them with an opportunity to become aware of the importance of collaboration and to understand the extent of the roles of the group's members. PBL can therefore develop reflection and thus promote metacognitive thinking about the learning processes, in order to elicit feedback about the analysis of the problem, the plan of action and the development of resources to execute the plan.

3.2.3.5 Present solution

After the group reflection, members of the group can be assessed individually (Brodie, 2008), in pairs or as a whole group (Fernandes, 2014). In this sense, the contribution(s) each member made to the group will play an important role as each aspect of the contribution should be reflected upon. Leong (2009) explains that it is possible that some students would be unwilling to share their ideas and might have trouble in communicating with others about what they were thinking or feeling. Furthermore, Hurme, Palonen and Jarvela (2006) explain that the solutions can also be presented in terms of written ideas, online blogs, printed statements or any form that presents a logical path of students' thinking. These solutions can also be monitored and evaluated by members of the group or by members of another group, to get feedback and an opportunity to explain what strategies were used and why the group members did what they had done.

3.3 Synthesis on PBL in higher education for mathematics education

According to Borhan (2012), reflecting on both problem-solving and the learning experience is critical in PBL. Malan *et al*. (2014) seem to agree that PBL is an impetus for promoting meaningful learning typified by students' self-directed learning and regulatory practices in a process of growth towards both dependent and independent lifelong learning.

As awareness of PBL's impact on teaching and learning expands, several studies reveal that PBL supports students' metacognitive competencies and provides them with realistic academic challenges that underlie mathematics education as a teaching and learning phenomenon. Because of PBL's constructivist nature, knowledge is not absolute, but is actively constructed. Therefore, PBL represents a shift from traditional approaches to the development of conceptual understanding and deep learning.

According to McDowell (2010), programmes in the US, where PBL was initiated, aim to prepare future mathematics educators by bearing in mind students' competency and preparedness to engage in the planning and implementing of mathematics lessons. Pre-service educators with little teaching experience can view lesson planning as time-consuming. They may then fall back on lower-level cognitive activities merely for the sake of producing a lesson plan relative to the assessment of the course (McDowell, 2010), rather than focussing on a creative, reflective and effective lesson planning that will facilitate meaningful teaching and learning experiences. In contrast, Japanese programmes suggest reflective coursework practices that commonly include what is known as LS.

3.4 The Japanese LS model

Lesson study[18] or 教研组 (jiaoyanzu/Jugyokenkyu) evolved from Japanese education practice as a supporting form of engagement in reflective teaching and professional development of pre-service and in-service educators (Choshi *et al*., 2014). Initially, participation in LS requires building a collaborative community of practitioners through critical reflection on teaching and learning practices, making use of locally available resources and shared didactics in teaching and learning-centred research groups. LS therefore stimulates both local and international debates about its effectiveness in designing, refining and presenting coursework. Through LS, Stigler and Hiebert (2009) assert that Japan has more successfully shifted their professional development programmes towards the ideals of National Research Council than the US itself has done. For example, Adams (2013) reports that teacher-initiated and publicly supported study groups frame Japan's educational landscape with an estimated 50 percent of all elementary school educators acting as registered members of Japan's research groups (Lewis, 2000). These groups have increased in popularity (McDowell, 2010) and in 2014 reached a point where week-long LS sessions were planned (Woodruff *et al*., 2013). For South Africa, LS seems like a promising approach for professional development of mathematics teaching and learning through collaboration about the design and delivery of, and reflection on, meaningful teaching and learning

[18] For the purpose of this study, LS will be reviewed in terms of pre-service and in-service educators' classroom experiences, where LS is introduced as a mechanism for facilitating mathematics education as a discipline. Note the shift from PBL as an instructional method to LS in the content of a mathematics education module (see 3.1.1.2 for a review on the MMI module).

experiences. In terms of South Africa's multicultural education landscape, the LS process can enable pre-service and in-service mathematics educators to develop innovative practices (Minar & Crown, 2001) to solve classroom problems (e.g. overcrowded and under-resourced classrooms) to improve the quality of the country's mathematics education. LS enables both pre-service and in-service educators to merge resources, knowledge and skills to collaboratively engage in and resolve both educational and curricular challenges (Tan, 2004). In doing so, LS promotes the systematic and collaborative efforts to conduct in-depth studies of the research lesson(s) to improve the quality of the teaching and learning experiences (de Oliviera & Cheng, 2011) of a learning community.

LS requires educators to perform various activities to improve the effectiveness of the lesson, including lesson planning, researching one's own lesson, debriefing the research lesson and reflecting on teaching practice (Young, 2010). Goos (2014) further affirms that collaboration in the planning and reflection of LS activities improves students/educators' problem-solving abilities. It therefore seems that more knowledge and skill can be developed through utilising a PBL philosophy to facilitate LS. From this argument, Lewis *et al*. (2006) suggest two common routes in the design and facilitation of LS. The routes include a general proof route and a local route (Lewis & Hurd, 2011). In this study, however, reference will only be made to the local route. As its nature suggests the local route allows for the design of a local theory that will reflect a set of local instructional guidelines (or LIT) aligned against the backdrop of a hypothetical learning trajectory (HLT) that will produce a local theory (LT) of the emerging constructs.

3.4.1 The local route of LS

As the local route suggests (Lewis *et al*., 2006), research on LS caters for instructional improvement. First, as Lewis *et al*. (2006) advise, the characteristics of the route of instructional improvement must be considered. The route characteristics of LS and the HLT allow the researcher to articulate the LS as a scientific enquiry and enable the researcher to organise the components of the LIT, LS and HLT in the framework of PBL as instructional approach. Table 3.2 therefore outlines how the route characteristics of LS and the HLT result in LIT with the philosophy of PBL as instructional approach.

Table 3.2 How LS and HLT result in LIT within a PBL approach

Philosophy of PBL as instructional approach		
Characteristics of the local route of lesson study (LS)	**Hypothetical learning trajectory of the module (HLT)**	**Guidelines for the local instructional theory (LIT) phases**
The lesson plan is developed locally and the local data obtained from presenting the lesson is used to improve instruction.	There is precision in the written and oral presentation.	Phase 1 – Investigation Long-term goals are considered for learning and development.
Once the lesson plan is presented, evidence of its effectiveness may be weak.	There are clear coherent arguments that reflect the style, sophistication and appropriateness of the presented lesson.	Existing curricula and policies are studied.
The lesson plan may depend on local materials and resources and these might change over time.	There is a clear understanding of the local needs in terms of resources and the necessary knowledge and skills to develop and design (new) resources.	Phase 2 – Planning Plan to conduct a research lesson and to collect data.
The lesson plan caters for continuous local adaptations including innovative design changes and changes in theory. Because of its flexibility, the lesson plan is expected to undergo improvement.	The presented lesson contributes to the needs and the level of conceptualising the mathematical texts and materials.	Phase 3 – Present and refine The data is then presented and its implications discussed.
The changes (or innovations) made to the lesson plan may adapt to each other. For example: if prior knowledge is assessed in the beginning of the lesson and it is clear that the route of the planned lesson cannot be followed, then possible structures should be in place to accommodate anticipated processes as early as possible.	Both basic and deep levels of reflection occurs at this stage as the needs, resources and classroom experiences are aligned with personal beliefs, values and expectations.	Learning resources, teacher resources and teacher knowledge are strengthened to improve the effectiveness of the lesson through refinement and reflection.
The knowledge that is produced from the locally presented lesson embodies an understanding of the knowledge and skills needed locally.	Observations from the classroom experiences can be reflected upon to accommodate changes in needs and instructional approach in order to improve the teaching and learning experiences.	Phase 4 – refine and represent The lesson plans reveal and promote students' thinking, whereas the resources support learning collegially during LS. The ideas are adapted and refined for re-presentation.
The local adaptions, improvements and diverse understandings of the product (or outcomes) of the lesson can be used to warrant its effectiveness locally.	Reflections on the meaningfulness of the presented lesson are measured against a specific prescribed set of criteria or outcomes	Motivation, improvement, collaboration and a sense of accountability are valued and associated with knowledge of the subject matter and knowledge of didactics, and emphasise long-term goals.

Source: Adapted from Lewis et al. (2006) and Schoenfeld (1992)

3.4.1.1 The kind of teacher involved in LS

LS allows educators to ask questions such as: How can we improve the way we teach X? (Minar & Crown, 2001:5), which create opportunities for educators to verbalise their thoughts and externalise their metacognitive thinking.

The increasing complexity of teaching mathematics then requires educators who:

- model lifelong learning
- cultivate 21st century competencies among their learners
- design and develop learner-centred classroom experiences and
- embed knowledge and technology resourcefully in lessons to meet the needs of their learners, including subject-specific software and social networking

Instead of focussing on group discussions regarding classroom practice, Young *et al.,* (2010) also emphasises that those who practise LS must systematically collect data and reflect deeply about their own experience and effectiveness in the classroom. To do so requires educators to collect data regarding learners' behaviour, emotions and cognition throughout the lesson. Following this route suggests a narrowing of the gap between theory and practice.

3.4.4.2 Locally authenticated materials, pedagogies and didactics

From the above discussion, LS appears to have a research nature about it, stimulating collaboration, networking, cognitive and metacognitive thinking and shared understanding and communication about the reflections before, during and after teaching. Moreover, LS in mathematics education promotes critical reflective thinking, problem-solving through revisiting the mathematical and didactical knowledge, the quality of teaching and learning experiences and an understanding of their theory and practice.

3.4.2 Collecting data in LS

The implementation of LS has the following characteristics:

- Collaboration, collegiality, and communication among educators and lecturers are fostered
- Implementation of research lesson is opened up to observation by others
- Mathematics lecturers become directly involved in mathematics instruction in school
- Mathematics educators become more empowered with curriculum development

These characteristics implicate the way in which LS in mathematics education is researched and suggest an overlap between the fields of metacognition and social network analysis.

3.4.3 Implications of LS on mathematics education research

Bustang (2013) identifies four forms of conceptual change that provide heightened metacognitive awareness manifesting through LS episodes. The types of change are conflict, intelligibility, plausibility and applicability of what is learnt throughout the LS cycles.

The first form, conflict, relates to manifesting differences through communication and collaboration. Second, intelligibility refers to the search for representations or examples to explain or express one's own understanding and moves away from internalised interpretations (or self-speech) towards experiences with others (who then have to interpret and understand what is said). Thirdly, plausibility refers to clarifying, elaborating and sharing the examples and experiences reflected upon, for greater clarification and collective understanding (by all LS group members) of the perspectives and experiences that LS initiates. Finally, the meaningfulness and applicability of the decisions made during LS design sessions are tested in terms of the appropriateness of the ideas or methods to resolve the problem(s) identified in the underlying theory and practice of LS. Based on the ideas embedded in LS, Table 3.3 presents an overview of the development of a LIT to facilitate LS.

Table 3.3 Overview of the development of LIT to facilitate LS

		Development of LIT		
Phase	**Characteristics of the local route of LS**	**Stages in LS**	**HLT of the module**	**Guidelines for the LIT**
1	The lesson plan is developed and presented locally and data obtained is used to improve instruction.	**Investigation**	There is precision in the written and oral presentation.	Long-term goals are considered for learning and development.
	Once the lesson plan is presented, evidence of its effectiveness may be weak.		There are clear, coherent arguments that reflect the style, sophistication and appropriateness of the presented lesson.	Existing curricula and policies are studied.
	The lesson plan may depend on local materials and resources and this might change over time.	**Planning**	There is a clear understanding of the local needs in terms of resources and the knowledge and skills required to develop and design (new) resources.	Plan to conduct a research lesson and to collect data through reflection and observation.
2	The lesson plan caters for continuous local adaptations including innovative design changes and changes in theory. Because of its flexibility, the lesson plan is expected to undergo improvement.	**Develop research lesson**	The presented lesson contributes to the needs and the level of conceptualising the mathematical texts and materials	The data are presented and discussed in terms of its implications.
3	The changes (or innovations) made to the lesson plan may adapt to each other. For example: if prior knowledge is assessed in the beginning of the lesson and it is clear that the route of the planned lesson cannot be followed, possible structures should be in place to accommodate anticipated processes as early as possible.	**Present research lesson**	Both basic and deep levels of reflection occur at this stage as the needs, resources and classroom experiences are aligned with personal beliefs, values and expectations.	Learning resources, teacher resources and teacher knowledge are strengthened to improve the effectiveness of the lesson through refinement and reflection.
4	The knowledge that is produced from the locally presented lesson is embodied in an understanding of the knowledge and skills needed locally.	**Reflection and refine research lesson**	Observations from the classroom experiences can be reflected upon to accommodate changes in needs and instructional approach in order to improve the teaching and learning experiences.	The lesson plan reveals and promotes students' thinking, whereas the resources support collegial learning during LS.
	The local adaptions, improvements and diverse understandings of the product (or outcomes) of the lesson can be used to warrant its effectiveness locally.		Reflections on the meaningfulness of the presented lesson are measured against a specific prescribed set of criteria or outcomes	Motivation, improvement, collaboration and a sense of accountability are valued and associated with knowledge of the subject, didactics and goals.

The following section elaborates on how the LIT develops and how it can be implemented.

3.5 A local instruction theory of PBL and LS

The ever-changing, complicated and challenging world requires students, our future educators, to go beyond merely building their knowledge capacity. They need to develop higher-order thinking skills (metacognition), critical thinking, decision-making and collaborative problem-solving behaviour (Miri, David & Uri, 2007:354). To reach these aims, the researcher, as facilitator, employed a PBL instructional philosophy through LIT and LS in the framework of mathematics education in one module at a higher education institution. The researcher therefore utilised the literature review on local instructional theories to plan and structure a proposed LIT for the purpose of this study (as conceptualised throughout Chapter 2). Consequently, employing a LIT allowed constructs to emerge that can be theorised about in a LT. The review of scholarship across Chapter 3 indicates that PBL and LS promote and foster constructs such as metacognitive thinking, metacognitive language and networking, among others. When these constructs are embedded within the design of LIT, they are expected to emerge when employing the LIT.

In order to facilitate the content of LS through a PBL instructional philosophy, the following theoretical guidelines steered the way in which the LIT as developed in Table 3.3, were implemented throughout this study. The first phase implicated the investigating and planning of the contextual, pedagogical and content-related issues regarding the PBL task. The second phase focused on how the lesson plan and resources should be developed and what teaching and learning strategies would be used. In Phase 3, the lesson was presented by a volunteer, reflected upon and refined. In Phase 4, the refined lesson was presented and a final reflection formulated. These four phases identified here reflect the LS cycles of Lewis *et al*. (2006) and are discussed below.

3.5.1 Phase 1 – Investigation and planning

Voogt *et al*. (2015) explain that, during LS, group members take on different roles as they contribute to and affect the design of the lesson. As group members reflect on each other's contributions, they act as co-designers and re-designers (Penuel *et al*., 2012). In the first meeting, group members negotiate the schedule for the sessions (Sun & Coulange, 2015), the curriculum goals and the background and needs of the school as emphasised in CAPS (DoBE, 2012) and the PBL task – Addendum A. The existing curricula and standards have to be investigated and group members must consider how they will collect data to determine the effectiveness of the lesson. An overarching goal has to be identified (Ertle, Chokshi & Fernandez, 2014) and must describe how group members plan to address the identified issues and reach the set goal(s) (Choshi, Yoshida & Fernandez, 2014).

3.5.2 Phase 2 – Development of the research lesson

The LS goal is revised in the second phase and the lesson plan can be developed. The processes and organisation of the content and the objectives of the lesson are considered. The main steps of the lesson plan, duration of activities, the nature of the teaching and learning activities and guidelines for instruction/facilitation are reflected upon (Choshi *et al*., 2014).

3.5.3 Phase 3 – Presentation and refinement of the research lesson

During this phase, one volunteer presents the planned lesson and the rest of the group observes. In this case, the guidelines by Choshi, Erle, Yoshida and Fernandez (2014) can be used to observe the presented lesson. It is cautioned that the observers should not intervene during the lesson's presentation as they should be positioned at the back of the class or the sides so that they do not interfere with the natural course of the lesson plan. Observing group members should note their observations and reflect on how the goals are reached through the planned activities. Afterwards, the group can schedule a feedback session to take turns and comment on what worked and what they think should change. It is also in this phase where the group members conduct research about the teaching and learning opportunities the lesson plan holds (Lewis *et al*., 2006).

3.5.4 Phase 4 – Representation of and reflection on the research lesson

According to Hobson and Vu (2015), after the lesson plan has been refined based on the observations and reflections in Phase 3, a volunteer again presents the lesson to a different class at the same or different school. Again, the rest of the group observes and have a reflection afterwards – repeating Phase 3. In this phase, what observers note about the lesson's functionality and its effectiveness is considered (Lewis *et al*., 2006). The reflections must inform the development of future lessons.

In conclusion, LS can be facilitated through a PBL instructional philosophy since the approaches of PBL and LS share similar principles, as outlined in this chapter. Once these principles are reflected in the HLT, a LIT with four cyclic phases is introduced. As argued in Chapter 2, the implementation of the LIT can produce constructs. For the purpose of this study, the body of scholarship concerning these constructs are explored and reviewed in Chapters 4 and 5.

Chapter 4

Metacognition, its language and reflections

> We must remember that [knowledge] by itself is literally meaningless; it has meaning only in so far as people know how to use it and how to create it and this involves language. Language, however has wider uses ... (Goldthorpe, 1968:9)

4.1 Introduction

The purpose of this chapter is to clarify and to discuss the concept of metacognition, its language and associations with reflection that are embedded in the local instructional theory proposed in Chapter 3. As emerging constructs, metacognition and metacognitive language are reviewed in both the individual and shared capacity of thinking about thinking. The chapter begins with an introduction to metacognition as the process of higher-order thinking and is outlined in three propositions derived from some of the definitions of metacognition. This is followed by a discussion on the metacognitive activity of reflection and its various facets, including affective issues in metacognition, which appears to have a close association with language. Then, distinguishing between past and current research, the focus moves to the way in which individuals communicate and express their metacognitive thinking. Particularly, the focus will be on metacognitive language as a language of learning that exhibits and promotes metacognitive thinking to foster metacognitive experiences through reflection.

4.1.1 Metacognition in the 21st century: lifelong

According to Bellanca et al. (2010), the opportunity is at hand for a 21st-century model for education that will equip students for the demands of society, study and careers in this millennium. With the vision of such 21st-century skills, Kayashima and Inaba (2003) claim that mathematics, communication as collaboration through literacy, flexibility and adaptability, initiative and self-direction, social and cultural skills, creativity, critical thinking and problem-solving are the underpinning themes for 21st-century learning. While some systems are in place for current curriculums, Wagner (2012) calls for all institutions to converse about the need for a curriculum that will scaffold these 21st-century skills.

Kayashima and Inaba (2003) argue that a lack of these skills will result in individuals who are relegated to low wages, low-skilled careers, and a lack of qualified engineers, medical doctors and an overall decline in the economy of a country. This is emphasised in the media (Rademeyer, 2009) and national publications (Siyepu, 2013).

It appears as though learners must adapt, or learn to adapt, to a changing world for securing their lives and the country's future economic competitiveness. Individuals who find it difficult to express their metacognitive thinking processes must therefore learn to associate their thinking with prior knowledge and experiences and externalise these internal processes. This externalisation can be encouraged in group settings where one participant may use a word or a phrase that another relates to and, as a result, the collaborative setting can include shared metacognition (Hurme *et al*., 2006; Wilson, 2001:5).

4.1.2 Metacognition, its nature and structure

In their *Tools for Learning Design Research* project, Stack and Bound (2012) focused on a model for integrating metacognition with practitioner-based research and concluded that a deepened understanding of the pedagogical knowledge and practices and innovative lessons were developed within various contexts. Mainly, their project aimed to overcome the barriers that are usually associated with a divide between the knowledge of theory (*logos*) and its application in practice (*praxis*). In particular, statements like "*learning is most often not embedded, learning is an isolated event and tight boundaries are set by the system"* are common concerns in curriculum development and evaluation (Stack & Bound, 2012:11). As an alternative, Woodruff *et al.* (2013) promotes a constructivist and humanistic philosophy into the design of curricula. Moreover, the philosophy of teaching and learning must support and embed a collaborative partnership between colleagues, teachers and students to foster a reflective atmosphere (Stack & Bound, 2012:16).

Schulman (1987:7) claims: *"*The teacher can transform understanding, performance skills or desired attitudes or values into pedagogical representations or actions … teaching must be understood to be more than the enhancement of understanding …"

In education, these opportunities allow students to collaborate and modify their learning and teaching strategies to include the learning of content and dealing with workload and course structure (Ozdamli, 2013:603).

The promotion of reflective thinking and reasoning can therefore be observed when individuals change their use of strategies during problem-solving. This awareness and acknowledgment of the choices made and how they affect future choices relates, first, to the nature of the experience and, second, to thinking about one's thinking in the experience (Proust, 2013; Hacker, Dunlosky & Graesser, 2009). Flavell (1979) calls this reflective process *metacognition.* Even though this concept has been researched in various contexts, its nature as regards individual networking and mediated networking capacity remains a scarce topic in the body of scholarship and deserves more attention.

Furthermore, how individuals express their metacognitive thinking and what words or phrases they use to mediate their own and others' thinking is an even scarcer topic (Proust, 2013; Whorf *et al*., 2012). Although well-crafted definitions for metacognition do exist to try and illuminate this fuzzy term (Murphy, 2012:4), it appears there is still a lack of clarity and agreement on what metacognition really is. This lacuna in the literature seems to hamper progress and understanding in the field of metacognition research, particularly its association with mathematics education and social network analysis. For example, Hurme *et al.* (2006) agree with Flavell (1979) that metacognition has its roots in the individual's knowledge of cognition and regulation. However, Schoenfeld (1992) suggests that metacognition is associated with problem-solving and mathematics learning, whereas Michalsky *et al*. (2009) acknowledge metacognition's role in peer questioning. Recently, Barzilai and Zohar (2014) explain metacognition as a complex and multifaceted concept that entails several types of knowledge.

These definitions infer that knowledge, skills and communication affect the metacognitive experiences as diverse constructs (Waters & Schneider, 2010) and suggest that the definition of metacognition in the literature remains vague. However, Proust (2013:13) offers a neutral explanation: *"Metacognition is the set of capacities through which an operating cognitive subsystem is evaluated or represented by another subsystem in a context sensitive way."*

For the purpose of this study, Proust's (2013) definition will suffice since it distinguishes between two levels of thinking. The first level refers to the operating nature of metacognition, which Brown (1987) and Efklides (2009) refer to as metacognitive and affective regulatory experiences. The second level refers to metacognition as a representative which informs about the knowledge – or lack thereof – before, during and after a task. Collectively, Waters and Scheider (2010), Lai (2011b) and Hacker *et al.* (1998) explain that metacognition emerges from teaching, learning and life experiences as a form of knowledge and action.

Based on metacognition's varying definitions, the researcher poses three propositions to outline the following discussion on metacognition. The three propositions derived from the above definitions explain that:

Metacognition is individual (Barrett *et al.*, 2012) and socially (Hurme *et al.*, 2006) mediated knowledge, which serves as a parallel system of knowing and awareness, instead of as a subsystem as claimed by Proust (2013). Metacognition is an individual and socially mediated construct of regulatory actions, including planning, monitoring and evaluating. Components of metacognitive processes that include metacognitive knowledge and regulation can be expressed verbally through a metacognitive language. The three abovementioned propositions will now be discussed in detail.

4.2 Proposition 1: Metacognition is a parallel system of knowing and is individually and socially mediated.

Metacognitive knowledge is regarded as a parallel system of knowledge since it refers to knowledge about oneself and knowledge about others. Within these two sets of metacognitive knowledge evolve three distinct types of knowledge that the researcher considers, for the purpose of this study, as levels of knowledge, borrowed from the work of Proust (2013), Hacker *et al*. (1998) and Flavell (1979). Each knowledge level is elaborated on below.

4.2.1 Three levels of metacognitive knowledge

Flavell (1979) distinguishes between three levels (or types) of metacognitive knowledge, namely knowledge of the person as self and others, task knowledge and knowledge of strategies. These knowledge types are considered as levels of knowing to position them in the conceptual framework of this study. The knowledge levels play a key role in reflection on the procedural, conditional or declarative knowledge types (Hurme *et al*., 2006).

The first level of metacognitive knowledge is discussed as knowledge of the person, where person refers to the self and others.

4.2.1.1 Knowledge of the person as self and others

Self-knowledge about a person's learning process can be correct or incorrect and can resist change (Veenman *et al*., 2006). For instance, a student may think or believe that enough time was spent on preparation for an exam, and then might blame the teacher if he/she failed the exam and claim that the exam was unfair or the teacher made it extra difficult (Veenman *et al.*, 2006). This example of misattribution to learning can create an experience where the student incorrectly assumes or interprets the knowledge of others, but in themselves lack the ability to amend their self-knowledge. It seems that students who reflect metacognitively have the ability to create a model of their own knowledge (Chapman & Inman, 2009) and compare this knowledge with an implicit knowledge of others. Moos (2014) claims that this awareness of self-and-other knowledge is not a personality trait, but can develop over time through interaction with others and their beliefs and by sharing their experiences. In this sense, metacognition is linked to theory of mind and to what Weil *et al*. (2013) refer to mentalising. It involves the ability to take into account another person's perspective. However, this process does not stop in the knowledge domain.

Weil *et al*. (2013) explain that these metacognitive experiences involve guiding and communicating about one's and others' thinking processes, suggesting that knowledge of the person, through reflection, can evolve into social interaction, exchanging and explaining of ideas and development of the overall betweenness of individuals. This awareness then calls for particular self-regulatory actions that, as Hurme *et al*. (2006) explain, control the knowledge of the person and frame metacognition in joint problem-solving situations as an affective metacognitive experience (Efklides, 2009). Such experiences can revolve around a specific task or problem situation during which students can obtain knowledge of the task as well (Flavell, 1979).

4.2.1.2 Knowledge of the task

Knowledge of tasks refers to a specific knowledge about the nature of experiences involving tasks or problems (Veenman *et al.*, 2006). It links the actions that were needed, the calculations carried out in mathematics operations and the information given or determined in a task with prior knowledge and knowledge of the self and others. During the interaction with tasks, students can engage with and regulate their approaches to the task according to what they hear or see while they perform the task. They can identify or recognise familiar task traits and the demands of the task. These traits can also take the form of problem-solving activities (Chapman & Inman, 2009), which foster metacognitive thinking and promote critical discourse. The task can be a practical problem, contextualised with specific knowledge of the subjects (e.g. mathematics) and didactics and pedagogies (Murata & Ohta, 2013). The task can also be associated with time restrictions and involves an understanding of what the task requires, what the goals are, what individual roles are and how to plan to reach the goals and to identify appropriate strategies to meet the demands of the task (Moos, 2014). Knowledge of the task develops self-awareness; knowledge of the person is therefore associated with awareness of strengths and weaknesses in one's own knowledge. Furthermore, the task requires strategies that map onto the task characteristics, which in itself can be a restriction if the knowledge of the strategies is not sufficient. Knowledge of the task involves knowing how and when to use different types of strategies.

4.2.1.3 Knowledge of the strategies

Knowledge of strategies includes using schema, drawing pictures or remembering something based on its characteristics, and is considered to be automatised, suggesting that strategies are actions that can be undertaken because they have become familiar routine procedures. However, skills refer to the intentional and goal-driven approaches which could be learnt from observation. The researcher agrees with Veenman et al. (2006:6), who state that clear and consistent conceptualisations of metacognitive skills and metacognitive strategy are still needed.

In this study, metacognitive knowledge of strategies then refers to the skills and strategies that are associated with the task demands and that can take the form of reading, writing, listening, drawing or calculating as it represents the student's enactments to reach the task's goal or solve a problem. It appears that knowledge of the person, task and strategies do not appear to be hierarchical as the individual can reflect on any one of these levels at any moment and exclude, or include, any of the other levels (Bormotova, 2010; Johns, 2009). However, Hacker (1998) organised the types of metacognitive knowledge in three domains to differentiate between the contexts and circumstances in which these knowledge types are reflected on and drawn from. Conditional knowledge involves specific circumstances from which strategies or skills are drawn from as procedural knowledge and declarative knowledge in the long-term memory. Because the person, task and strategic knowledge can exist as conditional, procedural or declarative knowledge (Proust, 2013; Murphy, 2012), the different domains of knowledge are interrelated. Each of these domains is discussed below.

4.2.2 Three domains of metacognitive knowledge

As mentioned in paragraph 4.2.1, the levels of metacognitive knowledge include knowledge of the person (KP), knowledge of the task (KT) and knowledge of strategies (KS). Figure 4.1

illustrates how these levels of knowledge are associated with each of the domains of metacognitive knowledge.

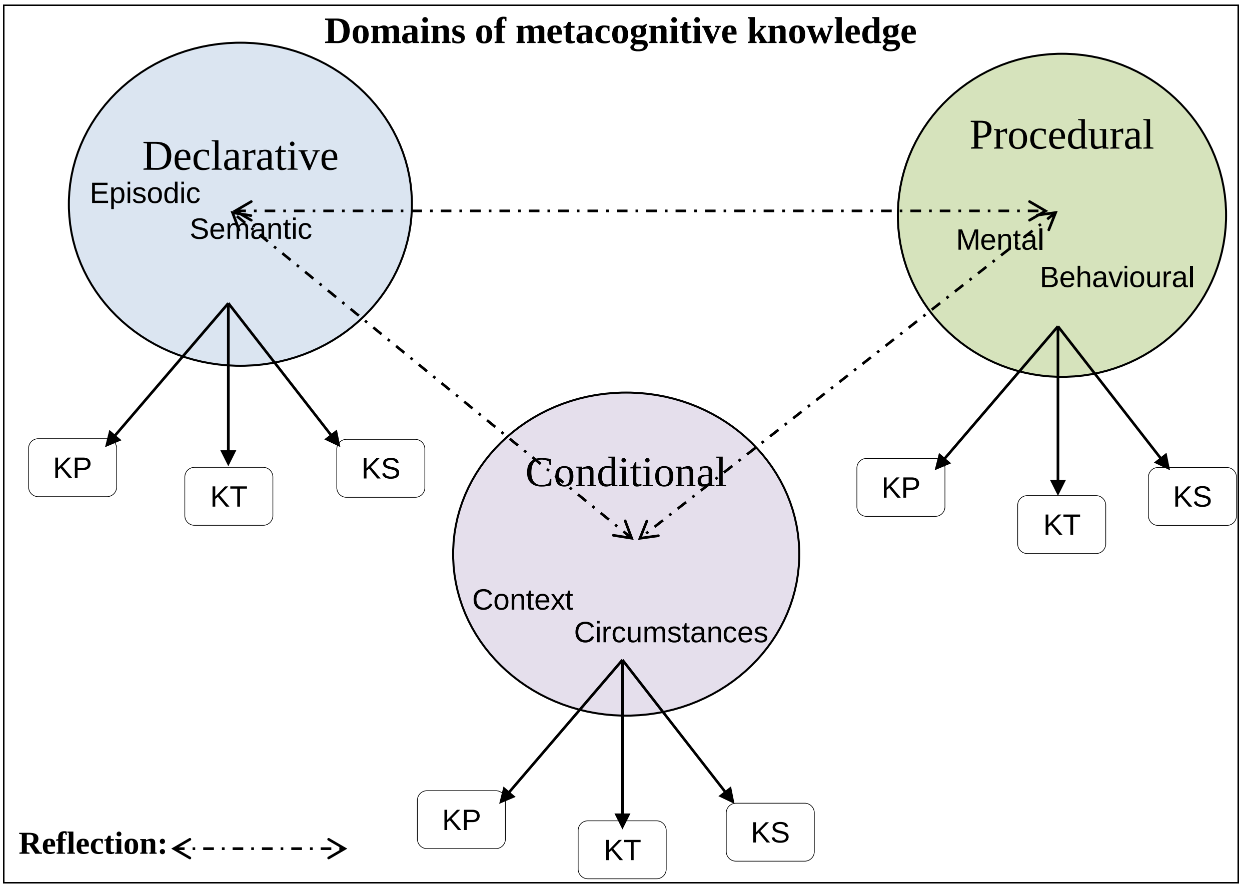

Figure 4.1 Domains of metacognitive knowledge

Source: Adapted from Hacker (1998) and Proust (2013)

The individual within the group is a carrier of many unique and shared knowledge types obtained from diverse metacognitive and affective experiences (Efklides, 2009). In the next section a discussion follows regarding the three overarching domains of metacognitive knowledge.

4.2.2.1 Declarative knowledge

Declarative knowledge is associated with the knowledge about oneself and the factors that affect cognition (Flavell, 1979 & Lai, 2011). Kuhn (2012) characterises declarative knowledge as the way in which the student understands his/her thinking, whereas Schubring *et al.* (2006) explain declarative knowledge as knowledge about oneself and the factors that affect performance. Declarative knowledge therefore consists of the factual statements of the episodic and semantic memory.

4.2.2.2 Procedural knowledge

Procedural knowledge refers to the awareness and management of cognition which includes knowledge about strategies (Lai, 2011). It can also refer to strategic knowledge (Flavell, 1979) and answers the question of how a particular strategy should be used. In essence, procedural knowledge gives directions for action which could steer mental activities and behavioural activities. Procedural knowledge therefore consists of the rules, skills and strategies that can be used to manipulate the declarative knowledge. Gobet *et al.* (2009) explains that procedural knowledge can be represented as a series of productions (or production systems) of the if-then format. For example: If you want to divide A by B, then count the number of times B can be subtracted by A (Ball & Wilson, 1990); and How do I challenge the idea without challenging the individual? (Murphy, 2012:16). Both the declarative and procedural knowledge types are interrelated with the context or conditions of the person, task and strategies.

4.2.2.3 Conditional knowledge

Conditional knowledge refers to the knowledge about when and why a strategy can be used (Schraw & Moshman., 1995 & Lai, 2011). Paris and Oka (1989) first coined the term conditional knowledge to refer to the individual's ability to judge, justify and explain the mental and behavioural actions they carry out. It also refers to the context or locale of the task (Waters & Schneider, 2010). Murphy (2012) explains that conditional knowledge denotes the knowledge of the conditions and circumstances under which some strategies will be more applicable than others. It can also include awareness of the knowledge of the conditions in which one would learn best or teach most effectively.

4.2.2.4 Synthesis of metacognitive knowledge (levels and types)

According to Proust (2013), the capacity to reflect on knowledge, strategies, representations and experiences has various changing possibilities, suggesting that reflection is a key role player in metacognition's regulatory domain. Metaphorically speaking, Jacobs and Gross (2014:2) state that metacognition allows us to change gears in our thinking. Proust (2013) explains further that the cognitive process is split up into two interrelated levels, known as the metalevel and object level. Efklides (2009) refers to the object level as cognition and the metalevel as metacognition. The reflection that occurs in the metalevel is a mental simulation of the strategies, representations and experiences of the object level. Thus, the metalevel serves as the higher-order cognitive reflections (metacognitive reflections) while the object level acts as the surface or platform that is being reflected on. There are thus two relationships that occur during reflection. First, it creates awareness of knowledge and, second, it regulates the knowledge. This construct of metacognitive regulation is discussed in the second proposition.

4.3 Proposition 2: Metacognition is individually and socially regulated

The regulatory actions can occur on a conscious, automatic response and non-conscious level, affecting the regulatory actions in different contexts, as the following sections explain.

4.3.1 Three types of metacognitively conscious experiences

Metacognition takes the form of conscious, automatic or nonconscious experiences, discussed below.

4.3.1.1 Metacognition and consciousness

In the literature, metacognition is generally presented as a conscious and deliberate activity (De La Feunte & Martinez, 2015; Hacker *et al*., 1998). Lau and Rosenthal (2011) argue that metacognitive knowledge depends on metacognitive representations that, through reflection, position the individual in a particular mental state. However, a conscious experience suggests that representations alone are not sufficient because some metacognitive knowledge can be automatically retrieved (for example, when driving a car becomes automatic), causing less conscious awareness of the procedural knowledge and more conditional awareness (for example, when looking at the scenery alongside the road). Cognition can therefore be carried out as an automatised process (Waters & Schneider, 2010) without much awareness (De La Fuente & Martinez, 2015). Even so, specific mental states require both the declarative and procedural knowledge types that exist in the conscious state. Motivating one to complete a task is the example used by Lau and Rosenthal (2011). The suggestion is that metacognitive knowledge, as the combined declarative, procedural and conditional knowledge, together with metacognitive experiences, is reflected upon during task activities (such as problem-solving during PBL).

On the other hand, Pasquali *et al*. (2010) claim that being consciously aware of one’s own metacognitive thinking suggests that the individual knows what they know and what they do not know. Proust (2013:67) argues that such distinctive evaluations of one’s thinking can interfere with the metacognitive representations obtained from reflecting on the declarative (I know that I don’t know), procedural (I know that I don’t know how) and conditional (I know that I don’t know how to know). Obtaining a clear picture of what one knows, or does not know, can result in higher-order cognitive functions (metacognition) that can be performed consciously, with or without knowing. Lau and Rosenthal (2011) explain that conscious awareness (of knowing and not knowing) affects performance of a task and the outcomes of the metacognitive experience.

The absence of metacognition in consciousness increases when the metacognition has become automatic (De La Fuente, 2015) and therefore can be regarded as cognition. On the other hand, the individual does not consciously seek to know in order to know, or the individual is not aware of his/her mental state and nonconsciously predicts what they know or do not know.

4.3.1.2 Metacognition and automatisation

According to De La Fuente and Martinez (2015), conscious processes and automatised processes complement each other. For example, when a student reads a text to understand what the task requires, the student has to be fluent in the language to understand the text. This reading exemplifies automatised cognition. One can also consider driving a car as an automatised process. However, when students analyse texts, connect main ideas and search deliberately for patterns to distinguish between relevant and irrelevant information, metacognition becomes a deliberate conscious higher-order cognitive process. Pasquali *et al*. (2010) explain that such metacognitive reflections on affective and metacognitive experiences emerge automatically when a person's declarative, procedural and conditional knowledge builds strong ties that associate new knowledge of the person, task and strategies with prior knowledge. This suggests that automatised metacognitive thinking processes include automated instructions to the self or others, as Hacker et al. (1998) explain, to check, judge or monitor.

4.3.1.3 Metacognition and nonconsciousness

When the controlling function directs the flow of the information obtained from the metacognitive refection, a mental picture or idea is constructed. This results in monitoring behaviour, which Nelson and Narens (1994) refer to as communication between the metacognitive knowledge and regulatory components. These two levels then form a metareflection. A series of these reflections can exist as a string of symbols, syntactical or semantic representations in the form of sentences, words, phrases or pictures that express the cognitive knowledge obtained through reflection. It therefore seems that metacognitive thinking denotes a language that is closely associated with the metacognitive knowledge domain and can be expressed in terms of the knowledge of the person, task and strategies (Whorf *et al*., 2012).

Conscious, automatic and nonconscious metacognitive thinking not only appear to produce mental images as a result of reflection, but also an understanding and/or regulation of the metacognitive knowledge (Hacker *et al*., 1998). Flavell (1979) and Brown (1987)[19] identified this cognitive controlling as self-regulation, a subcomponent of metacognition.

4.3.2 Domains of metacognitive regulation

In a study on problem-solving, Smith *et al.* (2009) note that students' ability to actively monitor and regulate their thinking processes relates to their prior knowledge. This suggests that the knowledge of the person, task and strategies, aligned within declarative, procedural and conditional knowledge domains, is a key in the metacognitive thinking process. However, if individuals are not aware (or do not know) that they lack the necessary knowledge, their performance can suffer (Wilson & Clarke, 2004).

According to Lai (2011), metacognitive awareness empowers students to regulate their cognitive processes to promote lifelong and self-directed learning. This empowerment involves planning, monitoring and evaluation (Atwater *et al.*, 2013). Such self-regulatory actions are strongly dependent on the constraints of the task and knowing one's strengths and weaknesses, and how these affect the metacognitive experiences that promote lifelong learning (Efklides, 2011). In addition, Akyol and Garrison (2011) refer to these knowledge processes as cognitive and affective states that enable the individual to monitor and regulate knowledge and to strengthen the capacity for new knowledge and strategies. Furthermore, Akyol and Garrison (2011) studied how these metacognitive aspects emerge from online forums and chat room discussions, and claim that such communities share and demand metacognitive knowledge to monitor and regulate one's own cognitive processes and those of others. In terms of this shared metacognition, self-knowing becomes shared and co-regulated.

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[19] Even though the work of Flavell and Brown is ground-breaking in metacognition research, Ertmer and Newby (1996) have introduced self-regulation as metacognitive regulation, because this term is more often used in the literature (Hacker *et al.*, 1998; Waters & Schenider, 2011; Proust, 2013). Self-regulation will then be discussed in terms of self-and-other control as metacognitive regulation, particularly because metacognitive awareness suggests that there is a social role in one's thinking (Liden, 2001).

Ader (2013) used a framework for metacognitive questioning and prompting to explore these regulatory processes and concluded that metacognitive regulation necessitates the value of feedback and self-talk as important aspects of language to regulate one's thinking. Moreover, Chua *et al.* (2012) identified cultural metacognition as a dimension of the metacognitive knowledge and regulatory domains, suggesting that metacognitive thinking is associated with language (Proust, 2013), as it contextualises the metacognitive experiences (Akyol & Garrison, 2011).

Hurme *et al.* (2006) explain that students who regulate their own thinking in collaborative learning environments also monitor and regulate their peers' thinking. Figure 4.2 illustrates the metacognitive regulatory domains of planning, monitoring and evaluation.

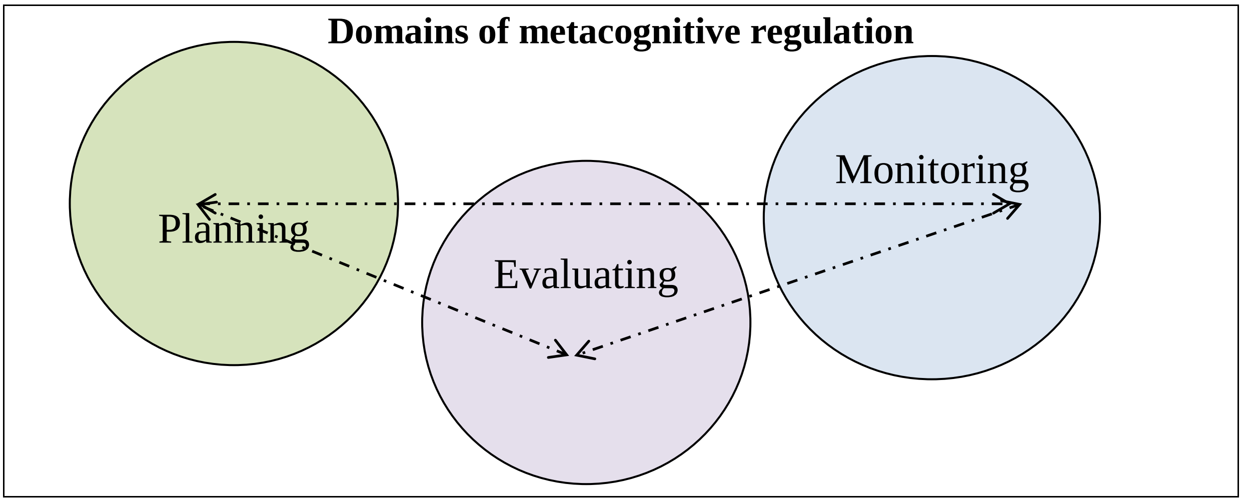

Figure 4.2 Domains of metacognitive regulation

Source: Adapted from Rodriguez and Bosch (2008)

The following section describes the metacognitive regulatory domains that control the metacognitive knowledge domains as illustrated in Figure 4.2.

4.3.3 Planning

Observing the use of metacognitive strategies in undergraduate students, Hurme *et al.* (2006) concluded that students involved in metacognitive activity did not consider aspects of planning. Instead, students engaged in problem-solving worked in groups and asked questions to explain their thinking and discuss their different views. Active participation in such networked learning environments allows students to construct meaning and to share their thinking, even though they do not plan deliberately. For Barzilai and Zohar (2014), planning involves setting goals, selecting strategies, predicting, organising strategies and sharing resources. Asking questions like: *Did I check alternative explanations?* creates awareness of the person and strategy knowledge and prompts students to engage in planning to regulate their cognition (Barzilai & Zohar, 2014:22). Veenman *et al.* (2006:27) make an important claim regarding planning when stating: "Either you are capable of planning your actions ahead and task performance progresses smoothly, or you don't and your actions go astray."

It therefore seems that a person cannot plan without reflection on and regulation of metacognitive knowledge. Veenman *et al.* (2006) further explain that planning involves self-instructions that usually go unnoticed if not explicitly verbalised. Proust (2013) and Hacker *et al.* (1998) suggest that language about one's own metacognitive thinking (metacognitive language) is associated with the metacognitive knowledge domain. However, from the explanation of Veenman *et al.* (2006), it appears as if such metacognitive language also relates to the metacognitive regulatory domain as an internal, self-talk and self-instructive language.

Murata and Ohta (2013) also established that poor performance during problem-solving is due to the lack of metacognitive planning, which corresponds with the monitoring of the problem-solving process and the understanding of the problem or task. Furthermore, Moos (2014) explains that planning is associated with the affective cognitive domain and metacognitive experiences when monitoring performance. This seems in accordance with Proust (2013), Hacker *et al.* (1998) and Flavell (1979), who claim that students who plan to solve a problem and who work collaboratively in doing so, will keep track of what they have already done, what they are currently doing and what they are going to do, and require monitoring throughout the problem-solving process.

4.3.4 Monitoring

A change in one's planning and approach to a problem or task requires monitoring and control of one's thinking processes (Markova & Legerstee, 2013). Lai (2011) claims that monitoring refers to attending to and an awareness of understanding and task performance, as it is associated with planning and evaluating (Schraw & Moshman, 1995). According to Akyol and Garrison (2011), monitoring coordinates the cognitive processes and engages recalling and rehearsing during reflection. It affects the cognitive state by making the individual aware of his/her willingness to change and regulate approaches and strategies during problem-solving. Ader (2013) claims that monitoring involves identifying patterns and judging mistakes as a conscious experience. Barzilai and Zohar (2014) therefore explain monitoring as seeking the truth of one's progress, suggesting alternative strategies and solutions, and creating a sense of certainty about one's knowing. Through monitoring one can therefore justify the declarative and procedural knowledge and judge the credibility of the conditional knowledge during problem-solving and task planning. In collaborative learning environments, students can monitor their own knowledge and that of their peers. While expressing their own and others' thinking, they can ask questions to monitor and communicate metacognitively about their thinking (Barzilai & Zohar, 2014). The group members can then adjust their thoughts and strategies as they share their metacognition and learn new strategies (Chua *et al.*, 2012), as it develops interpersonal and intergroup relations (Yzerbyt & Demoulin, 2011).

The monitoring of metacognitive experiences empowers students to formulate explanations of their own and others' thinking and allows them to reflect on, discuss and evaluate the planning and monitoring of these experiences.

4.3.5 Evaluation

In a dialogue, Hurme *et al*. (2006:187) gave an example of a metacognitive experience: "We changed the example, now it should be correct. Could someone check it? ... but it changes automatically … Why? ... Think!

According to Lai (2011) and Schraw and Mosham (1995), evaluation refers to appraisal of the effort, product and process of problem-solving and includes a reflection on and revision of task goals. Evaluation can then include an awareness of the discrepancies between the goal and the metacognitive knowledge by reflecting on the effectiveness of the strategies used (Moos, 2014) and the plans outlined (Ader, 2013). This, as the example of Hurme *et al*. (2006) suggests, can promote metacognitive questioning and foster critical reflection.

The need for additional resources can also be evaluated and creates awareness of what is known and not known (Darling-Hammond, 2009). McMahon and Luca (2007) explored metacognitive evaluations in a problem-based learning environment. They concluded that students evaluated their performance with the aim to improve their performance only when they knew what the criteria were for measuring the performance (by issuing a rubric for the course's outcomes). This awareness promoted questioning and motivated students to reflect on knowledge throughout the planning and monitoring phases to adapt their planning and use of strategies to elicit better performance.

Akyol and Garrison (2011) claim that evaluation improves critical thinking and making sense of one's ideas. Martinez (2006) suggests that evaluation should exist on a personal and social level. This implies that reflection first occurs on a personal level, in order to interpret one's own awareness, and then on a social level to express and share one's own understanding with peers. Through such collaboration, feedback from others can be used to enhance task performance, as group members form a network and the individuals in the network create ties between them as they communicate about their metacognitive thinking. This experience involves an analysis of performance, and questions whether an appropriate strategy was used. It also expresses the participants' metacognitive thinking in words (Hurme *et al*., 2006). Proust (2013) elaborates on such a metalanguage, which is discussed in terms of the third proposition.

4.4 Proposition 3: Metacognition can be expressed through a metacognitive language

When students communicate about their metacognition, they express their metacognitive thinking processes verbally. In this study, this verbal expression is referred to as metacognitive language. This concept is borrowed from the work of Frampton *et al*. (2009). Even though many scholars agree that the construct of language is situated in the metacognitive knowledge domain (Ducasse *et al.*, 2009; Chval *et al*., 2014; Hacker *et al*., 1998), Bermudez (2008) argues that assumptions drawn from reflections on the metacognitive knowledge occur as regulatory actions (for example, giving feedback after monitoring) and result in information obtained and used to make (new) decisions. Proust (2013:67) seems to agree with this argument in the following example:

> I may be cognitively impaired, and still be able to give you the correct answer to your query. In order to grasp my answer, you do not need to form a metalanguage about my language, translate my symbols into yours, represent my intention to address your questions in a relevant way ... You only need to use a certain interrogative syntax and associated intonation,

> and utter a sequence of symbols that my memory can associate with an answer.

The example suggests that, for metacognitive language to be enacted, there must be at least two speakers. One speaker recovers the information without having to interpret or uncover any intentions while the second speaker may question the intent of the information. In this case, no metacognitive language is necessary. However, in cases where communications are restricted by a lack of metacognitive awareness and where no metacognitive representations exist, metalanguage seems necessary.

Metacognitive language, from the arguments by Narens and Nelson (1994) and Proust (2013), is an expression of the deeper understanding when reflecting on metacognitive knowledge. In this case a feeling of knowing might be experienced (for example, I think the answer is ...). To ensure the answer, the idea or understanding must first be monitored. That means that different or more knowledge of an idea must be obtained (through metacognitive reflection) before it can be shared and agreed upon as fact. As suggested by Barzilai and Zohar (2014), this monitoring process will indicate whether the idea is true or false and if the information that is shared is understood socially.

4.4.1 Metacognitive language in teaching and learning

To create meaningful teaching and learning experiences, interactions and communications necessitate both knowledge and regulation of one's own cognitive processes and the sharing of these processes in a collaborative learning and problem-solving environment. Therefore, Hurme *et al*. (2006) argue that students should be closely involved with metacognitive thinking through engagement in collaborative learning environments in which the role of metacognition becomes transparent in their social interactions. In addition, instructional methods should be adapted to suit the cognitive and metacognitive needs of the students through questioning (Michalsky *et al.*, 2009), discussing (Choi, Land & Turgeon, 2005), planning and problem-solving (Hurme *et al*., 2006).

Communicating in mathematics allows one to develop new thinking about one's thinking, suggesting that such communications allow thinking to grow (Schoenfeld, 2013). Theoretical frameworks of mathematics and language have similar and distinct characteristics. Nachlieli and Tabach (2012) describe these characteristics as functions in terms of the knowledge domain of metacognition.

The foundation of metacognitive thinking, as part of shared and collaborative learning, is argued to lie in the theoretical framework of Vygotsky (1987)'s construct of ZPD to understand and explain the construct of metacognitive language. This framework suggests that learning requires a social presence, and the idea(s) or knowledge that accumulate from this learning is constructed through sense-making (Sfard, 2012), conceptualisation (Hurme *et al.*, 2006) and associations with and reflections on prior knowledge. The researcher views these concepts (metacognition, language and networking) in the light of the explanations and discussions that students exhibit when engaging in metacognitive activities. As such, students become aware of the gaps in their understanding and misconceptions when they realise *what,* if any, metacognitive expertise or competencies they lack. Students then communicate their ideas and learn to merge the internal metacognitive processes (of the individual) with the shared processes (of the group) by reflecting on their internal metacognitive knowledge. By doing so, they symbolise what they know and what they think or perceive their peers to know and understand. This metacognitive awareness occurs, as Hurme *et al*. (2006) claim, when students engage in PBL environments as they communicate their metacognitive thinking through metacognitive language to contribute to the group's overarching goal or shared purpose.

Semantics exists as a link between the different knowledge types (e.g. in the form of memory) and is responsible for creating links between prior and new learning. These links, when combined, exist as a semantic network in the metacognitive knowledge domain (Gobet, 2001).

4.4.2 Language and reflection

Bermudez (2008) claims that the absence of understanding one's thinking means that there was no language of thinking involved and that no communication thus exists between the metacognitive knowledge and regulatory domains. Chval *et al*. (2014) emphasise that environments and contexts that demand cognitive and metacognitive activities for all students must make use of multiple modes of communication (e.g. writing one's thoughts, verbal awareness, gestures and the use of pictures). This acts as reflections expressed in the form of language and can manipulate information before, during and after problem-solving

(Moshman, 2010). In addition, when students communicate reflectively they express both their metacognitive knowledge and their thinking about regulating their knowledge (e.g. monitoring, planning and evaluating their knowledge of the person, task and strategy). Ducasse (2009:82) proposes that when students reflect, a conflict of interest can emerge as this reflection becomes visible or shared with their peers. For example, Ducasse (2009) argues that students who reflect on their thinking express themselves (verbally and non-verbally) mainly through explicit composition and conflict resolution.

4.4.2.1 Explicit composition

Explicit composition refers to the components of students' reflections when identifying specific traits, factors or strategies during communication about problems, tasks and problem-solving and describing them in great detail. These could include descriptions of teaching and learning experiences, methods or strategies that students employ to succeed in mathematics. It also includes awareness of others as group members reflect on their own experiences while sharing them with the rest of the group. Group members can then reflect on specific traits, factors or strategies from their own metacognitive experience and communicate about those in a collaborative learning environment. It therefore seems that when students use a language of reflection during problem-solving, they can compose new ideas collaboratively by combining the different reflections drawn from their metacognitive knowledge and regulatory domains. In this sense, metacognitive language kindles reflection and fosters metacognitive thinking.

4.4.2.2 Conflict resolution

When students communicate their explicit compositions, conflict can arise and create an affective experience (Efklides, 2009). This occurs when one group member contributes a trait by sharing an idea or explaining a strategy, and another provides a different view or alternative strategy or predicts a problem with the implementation of the original idea (Ducasse, 2013). In this case, Proust (2013) suggests that a facilitator should pose an overall strategy to the group on which the group can reflect. However, Hacker *et al.* (1998) advise that facilitators should allow students to not merely exclude the idea, but to explain their thinking and to come to some agreement. Chval *et al*. (2014) explain that such collaborative teaching and learning environments should be guided by critical reflection, or, as Johns (2009) calls it, a deeper level of reflection.

4.4.3 Types of discourse

Nachlieli and Tabach (2012) identified three types of communication that take place during the teaching, learning and doing of mathematics, outlined in Table 4.1.

Table 4.1 Types and functions of discourse in mathematics education

Type of mathematics education discourse	The use of words and mental verbs	Visual or symbolic mediators	Observation of routines	Endorsing narratives
Ideational and mathematical[20]	Ideas emerge as students discuss mathematical concepts and the use of appropriate strategies	Algebraic symbols or expressions, graphs, tables in the context of real-world phenomena	Awareness of strategies and gradual increase in level of difficulty	Imperative description of mathematical ideas
Interpersonal and social	Informal and formal use of words. Using and/or sharing ideas	Refer to various scenarios, using gestures as a tool for thinking	Looking for ways to do things rather than asking for help	Translating and explaining for oneself and/or others
Textual and pedagogical	The use of terms or words in relation to one another	Teachers acknowledge learners' use of and readiness for understanding visual or symbolic representations	Forged linguistic ties between tasks and strategies	New information creates opportunities for change in the description and understanding of events

Source: Adapted from Nachlieli and Tabach (2012), Papaja (2012) and Vygotsky (1987)

It appears from the description of the functions of the types of mathematics education discourse that metacognitive language can be expressed in terms of information regarding knowledge of the person, task and strategies, and as declarative, procedural and conditional knowledge. Within explicit composition and conflict resolution, different types of knowledge of cognitive functions can be used to explain the choice and use of words, visual or symbolic mediators, observation and awareness of one's own and others' routines. This can then be interpreted as narratives of their thinking through the three functions of discourse in mathematics, derived from the work of Christie (2005) and Cradock *et al*. (2013). The authors explain that the use of metacognitive language allows the user to choose between a series of words, phrases and terminology to not only express their thinking and learning about mathematics, but also, to make interpretations when collaborating with others. In more detail,

[20] Even though Nachlieli and Tabach (2012) comment on mathematics as a school subject, the researcher concentrates on mathematics education as a module as discussed in paragraph 3.1.1.2 and therefore adapts the contents of Table 4.1 accordingly.

these words and phrases can be aligned to co-exist within the different components of the metacognitive knowledge domain. In addition, Papaja (2012) adopted Van Lier's (2000) interaction framework for communication. She decoded the interaction in terms of the functions the interactions have for teaching and learning based on communication, therefore language. In particular, both Papaja (2012:43) and Van Lier (2000) concluded that classroom discourse is not a random act. Instead, the "matter of who speaks and when is often governed by certain regulations", shaping the aim of communicating and the "processes and outcomes of individual development". Furthermore, Van Lier's (1994) framework for action research mirrors to some extent the proposed LIT phases outlined in Chapter 3. Following this process, Van Lier (1994:36) seems to emphasise regulatory behaviour in such interactive discourses:

> "We already think and talk ... but by making them [ideas] the object of systematic and sustained enquiry, we may ... become ... proactive".

This stresses metacognitive language's role in the metacognitive knowledge and regulatory domain. However, Nachlieli and Tabach (2012) explain that collaboration between students during such discourse caters for emerging thoughts that arise from the metacognitive experiences, which can be analysed according to the above functions of mathematics education discourse. Each of these discourse types are discussed below as proposed metacognitive language types.

4.2.3.1 Ideational language

Papaja (2012) explains ideational discourse as the expressing of ideas through the sharing of facts and experiences. The function (or purpose) of this discourse is to contribute to the discussion some ideas that might be worthwhile to take the group's progress further. Kuzle (2011) further elaborates that the expression of one's metacognitive awareness through self-reports during problem-solving situations in social contexts shows how metacognitive language can be used to describe awareness. Furthermore, evaluations and regulatory actions can enable group members to relate their metacognitive behaviours and improve their performance (Zawojewski *et al.*, 2013).

4.4.3.2 Mathematical discourse

Besides its use as a tool for social interaction (Ferreira *In* Maree *et al.*, 2012), metacognitive language also allows individuals to explicitly state their perceptual and decision-making processes (Frith, 2012). Moreover, Taylor and Cranton (2012) claim that knowledge of the person and of the world is largely mediated by the use of symbols that are conveyed through language (e.g. mathematical symbols). When these symbols are converted into ideas and verbally expressed, they can take on the form of a mathematical language. This type of metacognitive language can emerge when teachers or students explain what they see in a diagram (Papaja (2012)), explain a mathematical phenomenon or obtain clarity on mathematical concepts (Sfard, 2012). Sfard (2012) further explains that students who express their ideas in mathematical language provide a common ground on which other students can join the discussion and share their understandings. This suggests that metacognitive language is also associated with a social discourse (Papaja (2012)).

4.4.3.3 Social discourse

In order to provide an understanding of the specific elements that are referred to during discussions, Frampton *et al.* (2009:2) suggest the presence of a social language. In particular, students convey a "language of the mind" during conversations. Furthermore, through interaction individuals express different aspects of metacognitive language and Frampton *et al.* (2009:2) claim that "these aspects are important for one's own social understanding". For instance, Papaleontiou-Louca (2010:11) explains that the use of social language (e.g. we have or they think) offers a view on the mental states of others. Through such language, their understanding of intentions and a range of other mental phenomena can contribute to a "much more rounded understanding". This suggests that the use of metacognitive language can assist in understanding how others' mental worlds develop.

4.4.3.4 Pedagogical discourse

Stack and Bound (2012) are of the opinion that to foster a discourse on the use of pedagogy (e.g. lesson outcomes and teaching styles), teachers end up sharing stories about their experiences in classrooms and use pedagogical language to make sense of these stories. In terms of the questions students and teachers ask, Deed (2009) explains that the choices made and the strategies suggested serve as important pedagogical mechanisms. These allow the onlooker to theorise about their conceptualisation of what teaching and learning is and how it should be conducted. Pedagogical discourse as a metacognitive language therefore allows students and teachers to evaluate their learning experiences.

These verbal and non-verbal expressions of the self and mediated knowledge of others are contextualised (Proust, 2013) by a metacognitive language. This claim is discussed in the next section. Although the work of Nachlieli and Tabach (2012) and Papaja (2012) was used to theoretically identify the metacognitive language, the researcher acknowledges that these language types are referred to in the literature as types of collaborative discourses in mathematics. However, the researcher argues that when these language types are identified and aligned with emerging metacognitive components, the language then serves as a metacognitive language.

4.4.4 The use of mental verbs as indicators of metacognitive language

According to Waters and Schneider (2010) , declarative knowledge can be reflected upon by using mental verbs. These, as Grammer, Purtell, Coffman and Ornstein (2011) explain, are necessary to identify the language that individuals use to express their thinking. Some mental verbs include thinking, forgetting, wondering and knowing (Waters *et al*., 2010), and describe a sample of metacognitive language statements obtained from various authors. Also, Ruffman *et al*. (2002) and Frampton *et al*. (2009) offer summaries of the different mental state utterances.

It may seem that Table 4.2 presents a meta-analysis of the metacognitive language used in the literature. Although this is the approach the researcher used in the literature review, the intent is not to analyse or code statements obtained from the various sources for any data collection or analysis purposes. Instead, the lack of sufficient and appropriate examples of studies done on metacognitive language suggests that the researcher had to compile, at least, a table or a source to use as a guide or tool in preparation for the data collection and data analysis of this study. Table 4.2 therefore represents a tool, compiled from various sources, to identify or categorise metacognitive language.

Table 4.2 Sample indicators of metacognitive language obtained from the literature

Sample number	Description of a scenario where metacognitive language is used	Example of the metacognitive language used	Type of metacognitive language	Reflection on metacognitive knowledge domain	Level of knowledge	Reflection on metacognitive regulation domain	Source[21]
1	A teacher explains to students a strategy that they can use to check if they have used the correct steps in their calculations. The teacher explains that students must check to see if their answer makes sense.	*This is **how**[22] I want you to **check** if your steps are correct.* ***Look at your answer**.* *Does the [[23]answer] **make sense when you refer back to the question?***	Explanatory	Procedural knowledge Conditional knowledge	KS[24] KT	 Monitoring Evaluating Evaluating	Ornstein *et al.* (2011:40)
2	First, she explains that children need to reread their word problems if they are unsure about what to do.	***Remember how** we did this **last time**.* *We **went back** to our task and **reread** the [word problem] **out loud**.*	Mental verb Social Explanatory	Declarative knowledge Procedural knowledge Conditional knowledge Procedural knowledge	KP KS KT KS	 Monitoring Planning	Harris, Santangelo and Graham (2008:40)
3	The teacher gives the students some time to read and reread their word problems and asks one student to explain what he/she did, and to discuss why.	*Now, I have **noticed that you drew** three circles there, please explain what you were thinking.*	Mental verb	 Declarative knowledge	 KP	Monitored Planning	Kuzle (2011:19)

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[21] Even though these primary sources are referred to, the researcher acknowledges that the descriptions of the scenarios and examples in Table 4.2 are adapted to reflect similar situations in the context of mathematics education, PBL and LS. This is done because of the scarcity of literature in this regard.
[22] In Table 4.2 the researcher indicated some words in bold as examples of mental verbs (see 4.4.4) that are associated with metacognitive thinking, to substantiate how the examples of metacognitive language usage are categorised and aligned with the metacognitive knowledge and regulatory domains.
[23] Quotations are indicated between square brackets, or *[ ]*, as they represent a fictitious example of the use of metacognitive language, since the source only gives an example of the scenario with no accompanying quotations or excerpts.
[24] For a reminder about the use of this and similar abbreviations, please refer to paragraph 4.2.2.

Sample number	Description of a scenario where metacognitive language is used	Example of the metacognitive language used	Type of metacognitive language	Reflection on metacognitive knowledge domain	Level of knowledge	Reflection on metacognitive regulation domain	Source[21]
4	The student responded to the question by explaining that he/she was wondering what formula to use.	*Well, **I know that** I must be in a position **to know how** big the difference in the respective evidence in favour of A and B must be for **my decision to** be reliable in **this type of task.***	Mental verb Mathematical	Declarative knowledge Conditional knowledge Procedural knowledge Conditional knowledge	KP KT KS KT		Proust (2013:20)
5	While the student looks at his/her work again, the teacher notices another student throwing a piece of paper on the floor, and asks that students to explain how they solved the problem.	*You there!* ***What** did you **do**?*		Procedural knowledge			Wilson (2001:18)
5	The student explains that he/she rewrote the word problem in his/her own words and revised what has been written.	*[**I rewrote** the word problem in **a way** that it **makes sense to me** and **made sure** that what I had written down had **the same meaning** as the that of the word problem]*	Ideational Mental verb	Procedural knowledge Declarative knowledge Conditional knowledge	KS KP KT	Planning Monitoring	Wright *et al.* (2011:227)
6	The teacher moves on to another student who is looking at his/her work while frowning.	*How do you know you understand the work? ... Do you **realise when you make a mistake** or don't you?*	Pedagogical Mental verb	Declarative knowledge	KP	 Monitoring	Victori, Pinyana and Khan (2009:172)
7	The student explains.	*I **think** about it, but as I'm already busy with the other [sum]...Yes, **I am aware.***	Mental verb	Declarative knowledge Declarative knowledge	KT KP		Larson & Murtadha (2002:182)

Table 4.2 captures some examples of metacognitive language expressions. The table also indicates how the use of certain words or mental verbs (printed in *bold*) can ignite metacognitive thinking and produce new awareness about one's own and others' metacognitive thinking through reflection. It appears as though questioning and feedback play an important role in this reflection process. The examples suggest that there exists a socio-network between the teacher and the students (Dai, 2012; Borgati, 2012; Van Staden, 2011). Furthermore, these network characteristics are also noted as students reflect on particular aspects of the metacognition. For example, in Sample 4, the student reflects on declarative, procedural and conditional knowledge. However, this reflection seems to be driven by the teacher's comment in Sample 3. It therefore seems that reflection lies at the heart of the metacognitive process, as Dominowski (1998:201) states that reflection kindles metacognitive thinking.

4.5 Reflection and metacognition

Reflection as defined by Kelle, Klemke, Gruber and Specht (2011) is an active reasoning process that confirms experiences in problem-solving and involves social interaction. Reflection can be seen as a transformational process through which our experiences affect our objective way of thinking (Garcia, Sanchez & Escudero, 2007:1). According to Van der Walt (2008:79), all metacognitive knowledge is unified actively and collectively by reflection. It appears that reflection is a key component in the regulation of metacognitive knowledge and self-regulation. There are three stages of reflection – before, during and after action in the problem-solving process. After these three stages, the individual (as a reflective practitioner) may experience moments where synchronous reflection facilitates the so-called *aha* moment, which is a moment of acquired insight (Schon, 1983:3). In addition Waters *et al*. (2010:3) refer to metacognitive reflection and argues that a distinction can be made between cognitive reflection, metacognitive reflection and self-reflection.

4.5.1 Development of reflective thinking

Thinking about mathematics problems and reflecting on them is essential for interpreting the given problem. Provided details about what is needed are reflected upon to solve the problem (Zawojewski & Lesh, 2013:368). Schoenfeld (1992) mentions an *examining of special cases* for selecting appropriate strategies from a hierarchical description, but Zawojewski and Lesh (2013:369) argue that this will involve a list of prescribed strategies that is too long (prescriptive process) or too short (conventional process). Zawojewski and Lesh (2013:770) rather suggest a *descriptive process* to reflect on and develop sample experiences. The process should be focused on various facets of individual persona and differences, such as prior knowledge and experiences, which differ from one individual to the next.

4.5.2 Expansion models for reflective practice

According to Johns (2009), applying reflective practice is a powerful and effective way of learning. Three models for reflective practice feature in the next discussion. They include the reflective cycle of Gibbs (1988), Ertmer and Newby (2013), Johns (2009)′s model for structural reflection and the framework for reflective practice proposed by Rolfe (2011). The first model to be discussed is that of Gibbs (1988).

4.5.2.1 Gibbs’s (1988) model for reflection

Gibbs′s (1988) model is mostly applied during reflective writing. This model for reflection is adopted during problem-solving situations to assess first and second cognitive levels. For example, the student has to solve a problem. The situation is accompanied by feelings and emotions that will be remembered and reflected upon, obtained through prior experiences. A conscious cognitive decision must be made to determine whether the experience caused a positive (good) or negative (bad) emotion or feeling. By analysing the sense of the experience, other options are considered to reflect upon (Gibbs, 1988; Ertmer & Newby, 2013).

4.5.2.2 Johns’s (2009) model for structural and guided reflection

Johns′s (2009) model provides a framework for analysing and critically reflecting on a general problem or experience. Through scaffolding and guidance in more complex problems, students can reflect on their cognitive thinking in two phases.

One the one hand phase 1 refers to the recall of past memories and skills from previous experiences, where the learner identifies goals and achievements by reflecting on his/her past. A proposed way to do this is by using a videorecording of a situation where the learner solves a problem (Wilson, 2001). In this phase, learners take note of their emotions and what strategies are used or not. Phase 2, on the other hand, describes the feelings, emotions and surrounding thoughts that accompany their memories. A deeper clarification is given when learners have to motivate why certain steps were left out or why some strategies were used and others not. They have to explain how they feel and what makes them feel that way. In the end, the learners reflect on the inputs and outputs of factors that could have affected their emotions or thoughts in any way.

The third model that is proposed is the one by Rolfe (2011), known as a framework for reflexive practice.

4.5.2.3 Rolfe's (2011) model for reflexive practice

According to Rolfe (2011), the questions "what", "so what?" or "now what?" can stimulate reflective thinking. Table 4.3 demonstrates the models of Rolfe (2011), Gibbs (1988) and Johns (2009) as adapted by the researcher. It shows the movement of thoughts, actions and emotions between the different stages of reflection (before, during and after) in problem-solving.

Table 4.3 Integration of reflective stages and the models for reflective practice

Stage 1 **Reflection before action**	**Stage 2** **Reflection during action**	**Stage 3** **Reflection after action**
Descriptive level of reflection (planning and description phase)	Theory and knowledge building of reflection (decision-making phase)	Action-orientated level (reflecting on implemented strategy/action)
Identify the level of difficulty of the problem and possible reasons for feeling or not feeling "stuck", "bad" or unable to go to the next step. Pay attention to thoughts and emotions and identify them.	Describe negative attitude towards mathematics problems, if any.	Observe and notice expectations of self and others like parents, teachers or peers.
Evaluate the positive and negative experiences.	Analyse and understand the problem and plan the next step, approach or strategy.	Perform the planned action.
Create an awareness of ethics, beliefs, personal traits and motivations.	Recall strategies that worked in the past.	Reflect on the solution, reactions and attitudes.

Rolfe's (2011) model is simply descriptive of the cognitive levels and can be seen as a combination of Gibbs (1988) and Johns's (2009) models. The model explains that the student first reflects on a mathematics problem in order to describe it. In the second phase, the student constructs a personal theory and knowledge about the problem in order to learn from it. Lastly, the student reflects on the problem and considers different approaches or strategies to understand or make sense of the problem situation. By combining the three reflective models, it appears that reflection occurs on two levels. This is illustrated in Figure 4.3.

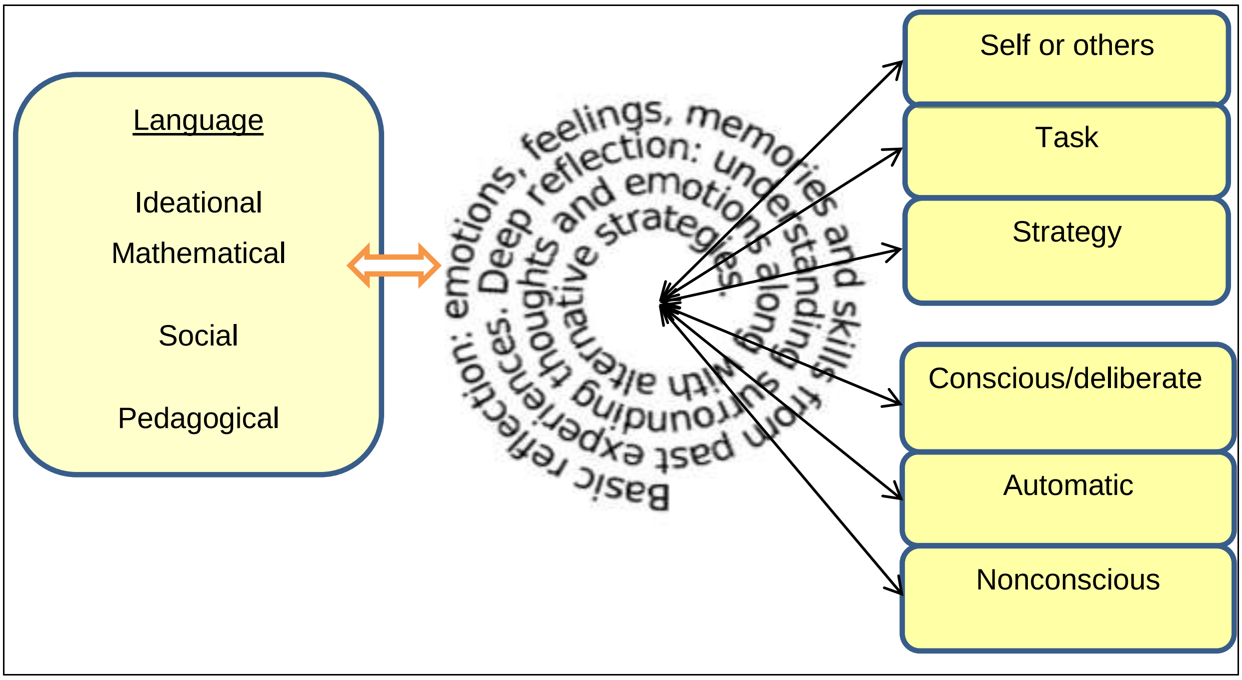


Figure 4.3 Levels of reflection

Source: Adapted from Johns (2009); Rolfe (2011); Papaja (2012) and Gibbs (1988)

First, it seems that reflection occurs on a basic level of awareness (which is mostly person- or task-orientated) and secondly a deeper reflection occurs in respect of reasons and sources for the metacognitive or affective experience (person- and strategy-orientated) (Efklides, 2009). The act of reflection seems to stimulate the level of awareness to plan and monitor tasks. It also seems that reflecting possibly manipulates and varies the knowledge and regulation associated with person, strategy and task characteristics during problem-solving. The researcher agrees with Efklides (2009:7) that metacognitive experiences relate to a process where progress, feelings and goalsettings are monitored and in turn produce a level of knowing of oneself and others.

4.6 Synthesis of Chapter 4 – building the conceptual foundation for a theory of metacognition

Chapter 4 reviewed the body of scholarship on metacognition, its associated language and reflections. Figure 4.4 offers a conceptual framework of this chapter to serve (in part)[25] as the conceptual-theoretical framework of this study.

[25] Based on the social nature of metacognition (Kalegh *et al.*, 2014), Chapter 5 reviews the literature to argue how metacognition, as a social construct, can be mediated in network environments. The conceptual framework illustrated in Figure 4.4 is therefore refined in Figure 5.7.

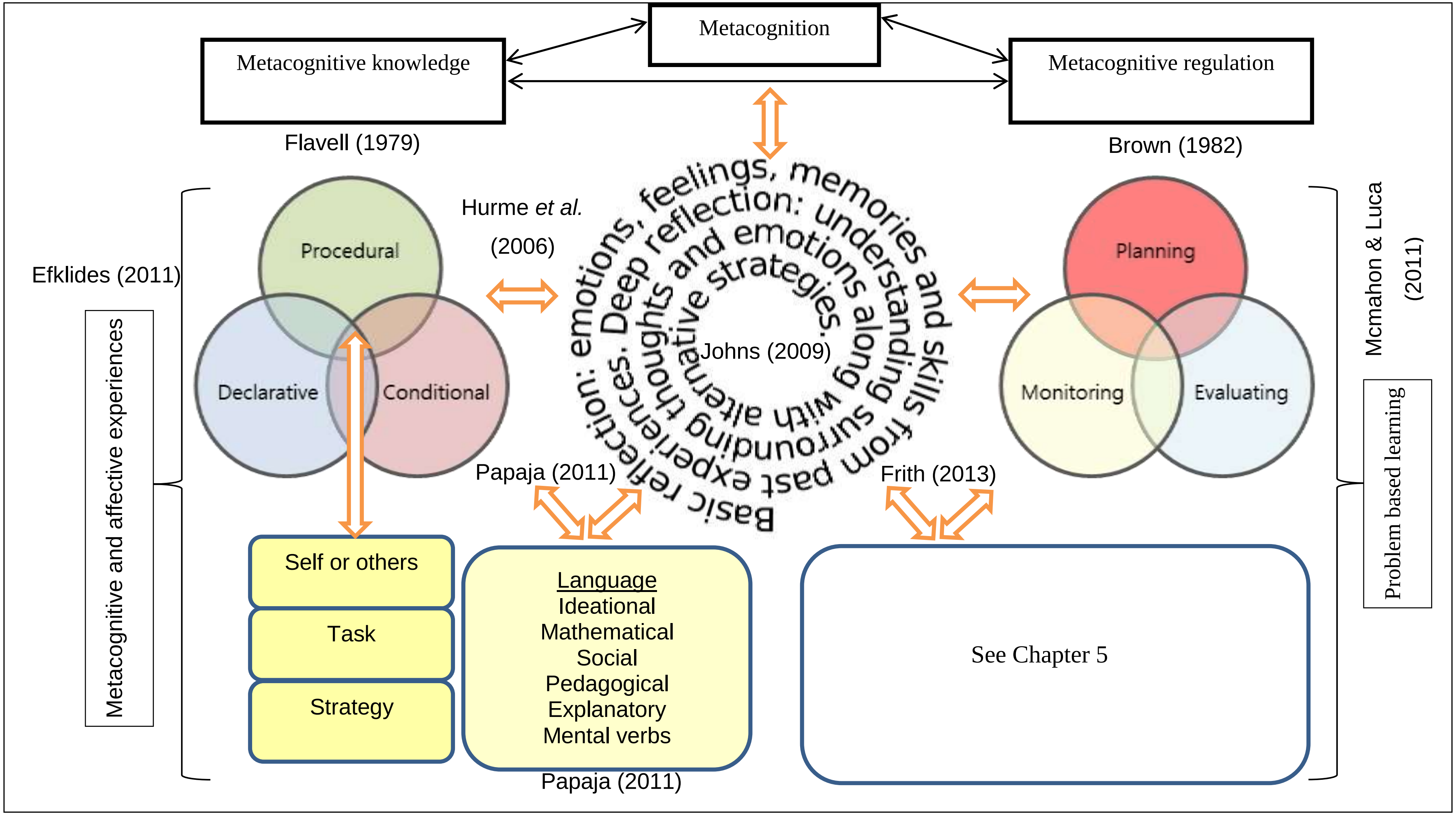


Figure 4.4 **Conceptual framework of metacognition, its language and reflection**

Based on the three propositions outlined in this study, the researcher conceptualises metacognition, its language and reflections as the projection of the individual (meta), and social dialogues with individuals can express their metacognitive knowledge and regulatory processes. The awareness of the subcomponents associated with these reflections occurs on a person, task or strategy level, bounded in a procedural, declarative or conditional knowledge domain. When we reflect on these metacognitive knowledge types through a basic or deep level of reflection, a conscious, deliberate, automatic or nonconscious form of regulatory planning, monitoring or evaluation exists which informs the reflection process and ignites the metacognitive knowledge. This knowledge is then objective to the embedded episodic, semantic, mental, behavioural, contextual and circumstantial cognition.

The discussion therefore suggests that metacognitive language and reflection is associated with the different components of metacognition. Furthermore, these associations appear to produce characteristics of network environments (Dai, 2012). Frith (2013) explains that these representations and expressions of thinking elicit explicit metacognition which enables individuals to share their experiences of action and thought with others. Rodriguez and Bosch (2008) describe such collaborative metacognitive experiences as the construction of complex knowledge networks and the enactment of this knowledge in a sociocultural and context-based environment. The researcher agrees with Hurme *et al.* (2006) and Pasquili (2010) that metacognitive thinking and reflection feature in these interactions and allow students to plan, monitor and evaluate their own and others' thinking, through communication and reflection in a *metacognitive network*. Chapter 5 therefore elaborates on this networking nature of metacognition.

Chapter 5

Metacognitive locale: networks and their conceptual-theoretical framework

The picture lapses into a still more confused indefiniteness, and this again increases a hundredfold when the subject of my thoughts changes. (Clarke, 1889:112)

5.1 Introduction

In Chapter 4 literature regarding metacognition, its language and reflection on thinking was reviewed, and it is argued that metacognitive language promotes metacognitive reflection in particular contexts, tasks and problems. Three propositions outline the literature regarding metacognition, its language and reflections. In Chapter 5 a fourth proposition is discussed, following a shift towards networks that represent the patterns of these metacognitive reflections. The proposition positions metacognition within network analysis as a means of examining the relationship that develops over time in a network in which metacognitive experiences are created and shared. The network exists then as a locale where individuals and groups mediate their metacognitive thinking during problem-solving situations. The researcher proposes a conceptual framework for the concept of metacognitive locale and concludes the chapter with a conceptual-theoretical framework of this study.

5.1.1 Proposition 4: The epistemic context-specific metacognition is a local construct that can be represented as a metacognitive network.

The individual and mediated nature of metacognitive knowledge and regulation exists in a metacognitive domain where language and context-specific knowledge acts as parameters of the individual and group's metacognitive stance. Dai (2012) explains that such collaboration occurs in a contextualised setting where participants engage productively through awareness of the interrelatedness between the overarching group goal (or problem), accountability of the group members, capacity for authority and creative use of resources. These four components suggest that metacognitive knowledge and regulation is not only communicated and negotiated about in social contexts, but also shared and distributed in a location where knowledge and the construction of new knowledge is a collective accomplishment of all the group members' inputs. Van Staden (2011) and McKetcher *et al.* (2009) explain that such collaboration can be represented by a sociogram, first introduced by Moreno (1956). The sociogram depicts the social networking of the group. In this study, however, this network is viewed from a metacognitive network perspective (Pasquali *et al.*, 2010) and serves as an expression of the collective representation of each group member's metacognition.

5.1.2 Metacognition as a construct of networks

Halgin *et al.* (2014) define a network as a set of actors that are linked together because of the type of relationship that exists between them. The actors could exist as nodes in the network whereas the links that connect the nodes act as ties that join and associate one node with another. In terms of metacognition, Rodríguez *et al.* (2008) emphasise that such network aspects can be considered when students collaborate during problem-solving, *and more so in mathematics problem-solving*.

Ballera *et al.* (2013) are of the opinion that students who solve problems collaboratively play an active role in their learning network. Furthermore, Barrett *et al.* (2012) argue that students' roles are on an individual, social and interactive level in the network. The researcher proposes in this study that the network also represents metacognitive thinking and reflection within the network on these levels. This argument seems to be supported by research conducted by Pasquali *et al*. (2010) on metacognitive networks[26].

5.2 The concepts of networks

As indicated in paragraph 5.1.2, networks consist of a collection of nodes and ties. This collective set can form new linear paths to other nodes within the network and, consequently, establish a location where these networks contribute to one another (Johnson & Johnson, 1996). The nodes can be tied directly or indirectly, via another node. As such, a pattern emerges that illustrates the position of the nodes and their ties in the network's structure. Table 5.1 presents an overview of how the social network concepts are aligned against the theoretical proposition of metacognitive networks and is explained afterwards in terms of Figure 5.1 and Figure 5.2.

[26] Even though the work of Pasquali *et al.* (2010) explains metacognitive networks as a neural construct, the researcher views Pasquali *et al.'s* (2010) contribution in light of social network analysis, as hinted on by research done by Hurme *et al.* (2006) and Van Staden (2011).

Table 5.1 **Aligning the concepts of social networks and metacognitive networks for use in this study**

Social networks			**Metacognitive networks**			**Section where the concepts of social networks and metacognitive networks are aligned**
Illustrated in Figure 5.1			Illustrated in Figure 5.2			
Network concepts (nodes and ties)		**Section where concept is discussed**	**Network concepts**	**Abbreviation/ representation**	**Description**	
Social roles (nodes)	Star	5.2.1.1	Declarative knowledge	DK	Acts as a node which resembles the metacognitive knowledge domain of declarative knowledge that the individual or group reflects on.	4.2.2.1, 5.3.1
	Bridge	5.2.1.2	Procedural knowledge	PK	Acts as a node which resembles the metacognitive knowledge domain of procedural knowledge that the individual or group reflects on.	4.2.2.2, 5.3.1
	Liaison	5.2.1.3	Conditional knowledge	CK	Acts as a node which resembles the metacognitive knowledge domain of conditional knowledge that the individual or group reflects on.	4.2.2.3, 5.3.1
	Gatekeeper	5.2.1.4	Knowledge of the person	KP	Acts as a subnode which resembles the level of the metacognitive knowledge domain of knowledge of the person that the individual or group reflects on.	4.2.1.1, 5.3.1
	Investigator	5.2.1.5	Knowledge of the task	KT	Acts as a subnode which resembles the level of the metacognitive knowledge level of knowledge of the task that the individual or group reflects on.	4.2.1.2, 5.3.1
	Coordinator	5.2.1.6	Knowledge of the strategy	KS	Acts as a subnode which resembles the level of the metacognitive knowledge level of knowledge of the strategies that the individual or group reflects on.	4.2.1.3, 5.3.1
	Socially weak member	5.2.1.7	Planning	P	Acts as a node which resembles the metacognitive regulatory domain of planning that the individual or group reflects on.	4.3.3, 5.3.1
Social relationships (ties)	Cognitive relationships	5.2.2.1	Monitoring	M	Acts as a node which resembles the metacognitive regulatory domain of monitoring that the individual or group reflects on.	4.3.4, 5.3.1
	Meta-affective Relationships	5.2.2.2	Evaluation	E	Acts as a node which resembles the metacognitive regulatory domain of evaluation that the individual or group reflects on.	4.3.5, 5.3.1
			Reflection	——	The route of metacognitive reflection that connects two or more metacognitive components.	4.5, 5.3.2,

In Figure 5.1 the concepts relating to social networks are briefly reviewed.

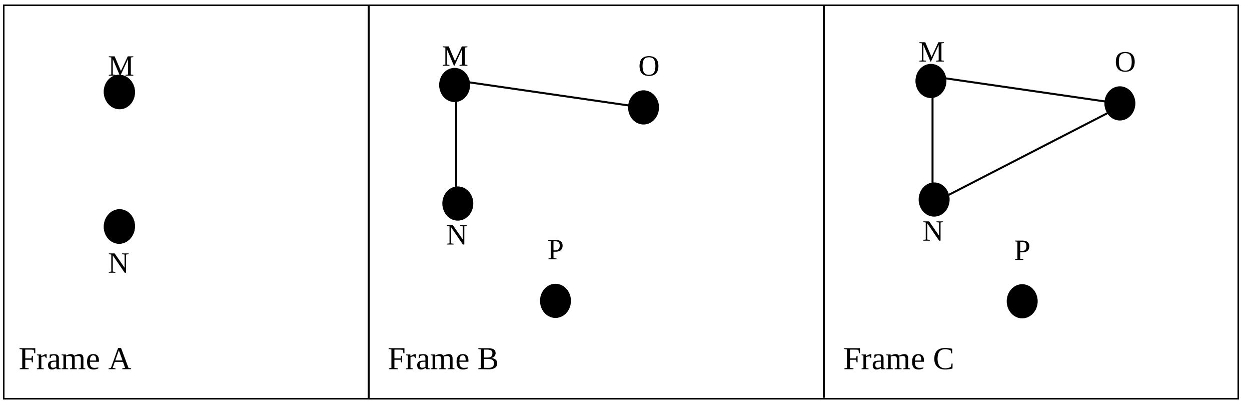


Figure 5.1 Representation of nodes and ties in a developing social network

Source: Adapted from Papacharissi (2011)

According to Papacharissi's (2011:8) explanation, Frame A represents two nodes (*M* and *N*). Each node is represented by a dot. In Frame B, *M* and *N* are connected by a tie. The tie (or relationship) between *M* and *N* can be represented by *MN* or *NM*. Also, two more nodes appear (*O* and *P*) and node *O* is connected to node *M*. The relationship between *O* and *M* can be presented by *OM*, or *MO*, depending on the type of relationship that the tie represents. Node *P* is represented as a node that is not associated with *M*, *N* or *O*. However, Borgatti and Halgin (2011) explain that *P* is still part of the network as networks do not have to exist through tied nodes alone. Frame C illustrates connected nodes *O* and *N* which creates a closed network between *M*, *N* and *O*. Collectively, Figure 5.1 shows that the network illustrated in frames A, B and C has grown in terms of its nodes and ties. This growth in the network can result in the formation of *strong links*, *weak links* and a *community* of nodes and links over time (Lazer *et al.*, 2009).

5.2.1 Social roles in the network

Pekericli *et al.* (2003) are of the opinion that individuals in a social network are considered to be on the lowest theoretical level. This suggests that the roles and relationships that are portrayed in social network analysis form the basis of the network on which other networks can develop as proposed by Elahi and Karlsen (2014). Pekericli *et al.* (2003) further explain that individuals in the network take on unique roles such as star, bridge, liaison and gatekeeper and socially weak member, and occupy these positions to allow the functioning of the network as a whole (Vogel, 2009). These roles are mainly determined by the nature of the interactions between group members (Chen, 2009). A discussion on these roles in the network now follows.

5.2.1.1 The star

The network's star, according to Long *et al.* (2012), refers to the individual who has the most social ties compared to other members in the group. They are also approached more often for advice and can generally be seen as the responsible person for the group's decisions (Van Staden & Mentz, 2014).

5.2.1.2 The bridge

Gray (2008) conceptualises the bridging role as the function of conflict resolution. The bridge manages the flow of information from one group member to another (Van Staden, 2011). The bridge often gains control over the other members in the group as the bridge can be seen as a source of inspiration and information (Chen, 2009). Ties or connections with a bridge can share knowledge with the bridge and become more empowered in the network (de Nooy *et al.*, 2011b).

5.2.1.3 The liaison

The liaison in the network is regarded as the group member involved with outside groups and has well-established relationships with members between two or more groups (Long *et al.*, 2012). The liaison often resembles the bridging role. Gray (2008) argues that liaisons are key regulators of the network, since they are responsible for extending the group's structure and opportunities by connecting their experiences with the experiences of other groups. Liaisons also facilitate collaboration (Long *et al.*, 2012) and control the circulation of ideas between group members (Vogel, 2009). The liaison can also be referred to as an itinerant (de Nooy *et al.*, 2011b).

5.2.1.4 The gatekeeper

Gatekeepers control the information that flows in and out of the group (Long *et al.*, 2012). Vogel (2009) explains that even though this control over the network's information-flow does not give the gatekeeper authority over the network, the gatekeeper can be regarded as a source of power over the control of the knowledge development in the group.

5.2.1.5 The investigator

The investigator in the group actively searches for information and allows access to this information, through the gatekeeper and coordinator, to other members (Van Staden, 2011).

5.2.1.6 The coordinator

The coordinator mediates the flow of information between group members and regulates their ideas and can often take on the role of a bridge as well (de Nooy *et al.*, 2011b). Furthermore, the coordinator does not have any ties with members outside the group that could benefit the group in any way. According to Kalish (2008), the coordinator receives information from other members in the group and, through negotiation, transmits this information to other less connected group members, for example the socially weak member.

5.2.1.7 The socially weak member

A group member with infrequent interaction with other group members can be regarded as a socially weak member (Chen, 2009). Chen (2009) explains that stronger members (such as stars and bridges) usually assist the socially weak member to contribute to the group. Long *et al.* (2012) elaborate that weak social ties are usually the cause of weak social relationships.

5.2.2 Social relationships in the network

Through reflection and collaboration, it is possible that group members in a social network can form relationships (Vogel, 2009). Similarly, Pekericli *et al.* (2003) argue that dependency relationships form over time between group members as they rely on each other to help them carry out their unique social roles. Such processes, Pekericli *et al.* (2003) claim, can be represented using social network analysis software (which was identified for the purpose of this study as NodeXL). Spiro *et al.* (2013) emphasise that within a social network framework, relationships exist either on an object level, suggesting cognitive relationships (Proust, 2013), or on an attributive level, suggesting meta-affective relationships (D'Mello *et al.*, 2013).

5.2.2.1 Cognitive relationships in the network

Panzarasa *et al.* (1999) note that cognitive relationships can emerge between group members throughout discussions. In addition, Elsbach (1999) explains that cognitive relationships exist because of the social roles individuals have. Eagle *et al.* (2009) elaborate that these relationships are intrinsic in nature (like metacognition) and that the context where this relationship emerges is a key indicator of the nature of the relationship (e.g. social or academic). A social cognitive relationship can then be referred to as friendship or caring for a loved one, whereas an academic cognitive relationship refers to the mutual understanding of workplace or problem-based tasks (Eagle *et al.*, 2009). Capaldi *et al.* (2014) explain that this nature can be externalised when individuals express their thinking through communicating with others and require attention and communication about feelings. These cognitive relations can be studied, as de Nooy *et al.* (2011b) suggest, through the use of social network analysis.

In particular, Gray (2008) claims that cognitive relationships are visible in the network characteristics (as relations between nodes) where leadership roles, decision-making and sense-making opportunities arise and when tasks are discussed. Furthermore, Long *et al.* (2012) elaborate that weak social ties are often the result of cognitive barriers between group members. As Long *et al.* (2012) explain, a need then exists to understand the translated thoughts of individuals in social contexts. Solem *et al.* (2015) suggest that an emphasis on cognitive processes in social network analysis allows researchers to understand how knowledge develops, how beliefs about one’s own strengths and weaknesses are formed and how strategies are monitored and controlled (regulated).

5.2.2.2 Meta-affective relationships in the network

Goldin (2002) claims that meta-affect is the affect of or the affect about the awareness of cognitive processes and can be externalised when monitoring the affect or viewing affect as a form of monitoring. It is argued in this chapter that these nodes can also represent the components of metacognition, discussed next.

5.3 The concepts of metacognition aligned with the concepts of networks

As the discussion in paragraphs 4.2, 4.3 and 4.4 indicates, metacognition is clearly multifaceted. For the purpose of this study, the researcher needs to elaborate on aligning the concepts of metacognition with those of networks. Borgatti and Halgin (2011:9-10) suggest that this is possible when “… we apply the same network theories to collective and/or nonhuman actors [nodes]…as we do to actors that are the individual persons”. In addition, McKetcher *et al.* (2009:116) claim: “Ties may be studied between individuals or groups of individuals, and may be used to show overlapping as well as non-overlapping networks.”

Therefore, in this study, the nodes will be used to represent the components of metacognition. The researcher acknowledges that different kinds of nodes have different capabilities and this will be considered when generating links (ties) between them and discussing these relationships. In order to align the concepts of metacognition with the concepts of networks, Figure 5.2 is used as an example and illustrates how, through metacognitive language and reflection, different components of metacognition can be tied[27].

[27] The researcher draws the reader’s attention to the similarities between Figure 5.1 and Figure 5.2 to illustrate how metacognitive components can be visualised as nodes to visualise a metacognitive network.

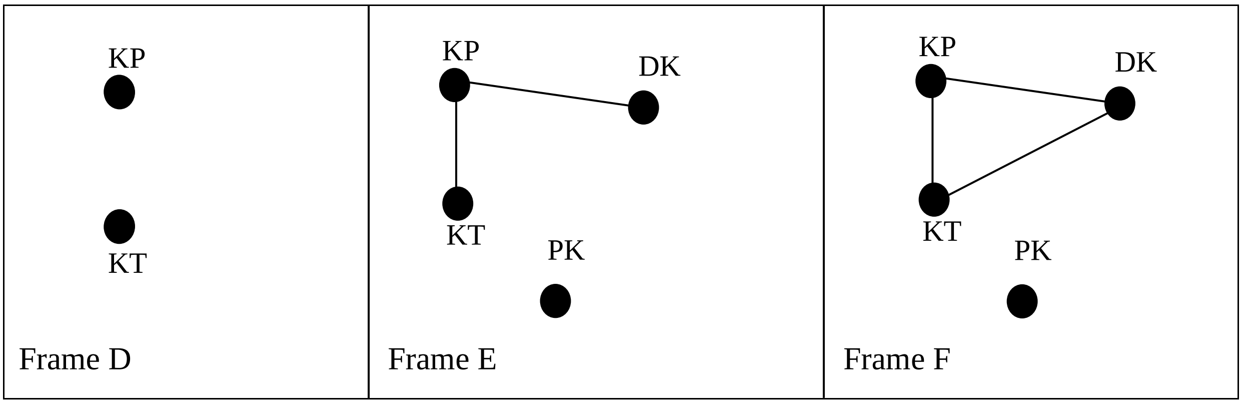


Figure 5.2 Representation of the components of metacognition and the reflection as nodes and links within a network

Analogous to Papacharissi's (2011:8) explanation of the concepts of networks (nodes and ties), in Figure 5.2 each metacognitive component is represented by a node. Frame D represents two metacognitive components represented as nodes (*KP* and *KT*). In Frame E, *KP* and *KT* are connected through reflection, illustrated as a tie. The tie (or reflection) between *KP* and *KT* can be written as *KPKT* and/or *KTKP*. Also, two more nodes appear (*DK* and *PK*) and node *DK* is connected to node *KP*. The reflection between *DK* and *KP* can be written as *DKKP*. Node *PK* is represented as a node that is not associated with *KP*, *KT* or *DK*. However, it seems that that *PK*, which represents the metacognitive procedural knowledge domain, is part of the metacognitive network although not reflected on. Frame F illustrates connected nodes *DK* and *KT*, which create a closed network between declarative knowledge, knowledge of the person and knowledge of the task. Collectively Figure 5.2 shows that the network illustrated in frames D, E and F has grown. This suggests, in this example, a growth in the metacognitive knowledge domain. This growth in the network can result in the formation of strong(er) links, weak(er) links and a collection of metacognitive and reflective components over time (Lazer *et al.*, 2009).

5.3.1 The concept of nodes

Ties and nodes differ, at first glance, in the way they are represented. The more ties attached to a node, the more complex the network becomes. Lazer *et al.* (2009) call this the *power law distribution*. Mainly, a node with the most ties connected to it can be seen as the most powerful node, or the node that has the greatest influence in the network. Where the node is located within the network is called the position of distribution, whereas the thickness, length and variety of ties are referred to as the degree of distribution. For example, two students may talk about their experiences with PBL and, in so doing, share their metacognitive thinking (Barrett *et al.*, 2012). In this case the students act as nodes and represent a social network

(Borgatti & Halgin, 2011). However, the information that the students share can also be considered nodes, as argued in paragraph 5.2.1. Nodes are the smallest units (Van Staden, 2011) in the network. They represent people, places or concepts (as is the case of metacognitive networks) and position the unit(s) within the network. Ties, on the other hand, act as pipes that allow the flow of information, ideas or knowledge from one node to another.

5.3.2 The concept of ties

Ties can be used to represent the relationship between nodes (McKetcher *et al.*, 2009). Borgatti and Halgin (2011:3) identify five types of ties in their review of network theories. These include: kinship, roles, cognitive, affective, interactions and transactions. Table 5.2 summarises these types of ties.

Table 5.2 Summary of ties within a network

Type of tie	Description	Example
Kinship	Kinship generally comprises of families, lineages, and clans where kinship is based on descent.	Brother, sister or other family member
Role-based	A formal social role that is usually voted on, applied for, administered to or takes the form of a responsibility taken on voluntarily or involuntarily.	Boss, teacher, facilitator, student, learner, group member
Cognitive	A perceptual relationship that acknowledges the existence of concepts, their relations, functions and associations.	Knowledge, skills
Affective	Life experiences and behavioural patterns that emerge from interpersonal and social relations.	Likes, dislikes, hates, loves and emotions
Interactive	Interactions can be modelled to represent the exchanging relationship between two or more people. These are usually within a particular context and consist of some linguistic form.	Giving advice, sending e-mail, chatting online, receiving notes
Transactional	This tie can represent itself more than once and usually occurs over a period of time. It creates stronger or weaker ties because it denotes the opportunity for one node to obtain more than another.	Signing a contract, coming to agreement, buying or selling

Source: Adapted from Borgatti and Halgin (2011),

For the purpose of this study, kinship, role-based, interactive and transactional ties will not be elaborated on. Instead, since the nature of metacognition is mainly metacognitive and meta-affective (see 5.2.2.1 & 5.2.2.2) the focus will be on the cognitive and meta-affective relationships. When students interact and share ideas or information at least once, a reciprocal relationship exists. For example, students can ask questions and obtain answers (Hurme *et al.*, 2006). The network can also be categorised either as a small-worldness network when the network has few links, or as a centralised network when most of the nodes interact with a central (powerful) node (Borgatti & Halgin, 2011).

5.3.3 Metacognitive networks

Metacognitive processes and the links between their reflections on metacognitive knowledge and regulatory domains can be represented as networks. The metacognitive networks can exist on an individual or social level (Rogan & Mors, 2014).

5.3.3.1 Metacognitive networks on the individual level

Metacognitive networks on the individual level represent the internal mental state of the individual's metacognition (Pasquali *et al.*, 2010). Such a metacognitive network can be described as an interpersonal metacognitive network, since it denotes the metacognitive thinking and metacognitive language of self-talk and expression of one's own metacognitive processes.

Each tie characterises the route of the metacognitive reflection and shows the route of reflection as it occurs on each of the components of metacognitive knowledge and regulation domains. Figure 5.3 illustrates a fictitious sociogram[28] to explain an individual's metacognitive network.

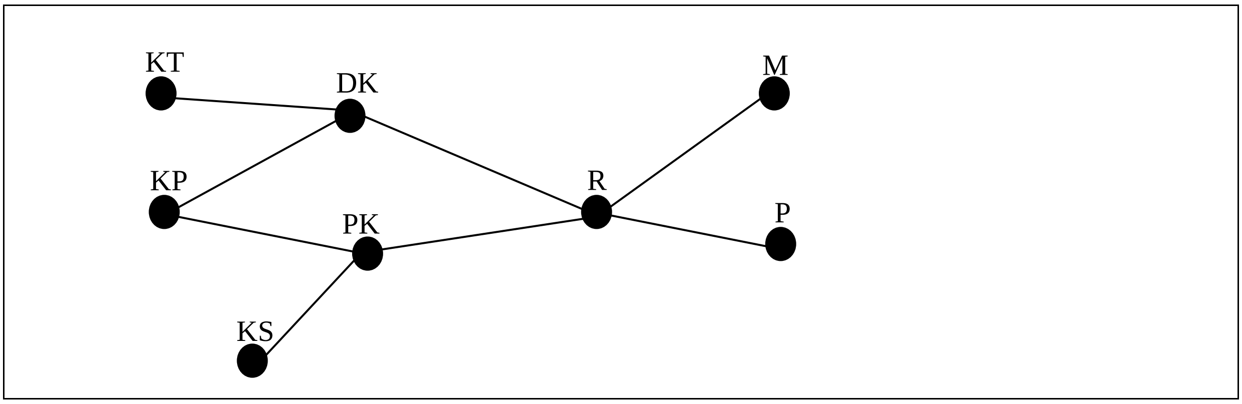


Figure 5.3 Metagram of a fabricated interpersonal metacognitive network

Source: *Adapted from Van Staden (2011), Pasquali et al. (2010) and McKetcher et al. (2009)*

[28] The concept of the *sociogram* is borrowed (Pekericli *et al.*, 2003; Gray, 2008; de Nooy *et al.*, 2011b)). Even though metacognition is considered to be a social construct (Hurme *et al.*, 2006), in this study the metacognitive network represents ties between the metacognitive components and its reflections that do not represent social ties. The term sociogram is respectfully adapted to suit the discussion. Therefore, sociograms of the interpersonal and social metacognitive networks are referred to as *metagrams* in this study when the network represents metacognitive components and not social interactions. In cases where social ties are discussed, the correct, inferred sociogram terminology will then be used.

The individual's metacognitive network is proposed as a theoretical construct in this study, illustrated as a metagram[29] (McKetcher *et al.*, 2009 & Moreno, 1956). Instead of persons representing the social reality, as sociograms are originally intended to do, the metacognitive network represented here illustrates the different components of metacognition and the route of reflection. As a result, the route and components (which act as ties and nodes) represent the network of the metacognitive knowledge (DK, PK and CK) and metacognitive regulation (P, M and E) as well as their underpinning levels. On the other hand, metacognitive networks can also represent a group's metacognition on a social network level.

5.3.3.2 Metacognitive networks on a social network level

On the social network level, a sociogram can be used to represent the patterns of social interaction. Hurme *et al.* (2006) explain that when students work together in joint problem-solving experiences, they form visible patterns that represent their interactions. These patterns can be used to visualise the type of interactions between students in a social network (see Figure 5.4)

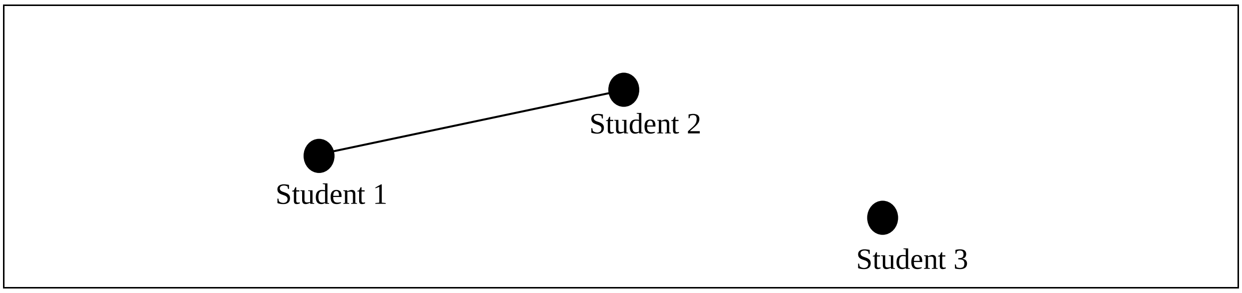


Figure 5.4 Sociogram of a fabricated network depicting the interaction of a student pair in joint problem-solving

Source: Adapted from Hurme et al. (2006)

Figure 5.4 illustrates two students that form a pair and a third student, not associated with the student pair. In this case, the two students illustrated as two connected nodes (Student 1 and Student 2) represent a pair. This relationship can be seen as a reciprocal relationship since each student shares their metacognitive thinking during joint problem-solving. However, the third node (Student 3) represents an isolated participant, one that does not share his/her metacognition with anyone else in the network. Such a relationship (not drawn) can be referred to as non-reciprocal. The distance between Student 1 and Student 2 can visualise the

[29] The abbreviations and symbols used are summarised in Table 5.1

nature of their interaction. This is referred to as the stress value and can be calculated using Chi-square statistics and stress-value to illustrate the goodness-of-fit of the network as a model (Hurme *et al.*, 2006). In their analysis of students' metacognition in joint problem-solving, Hurme *et al.* (2006) used a method referred to as multidimensional scaling (MDS), introduced by Vogel (2009). MDS can be used to graphically display the patterns of social interaction to clarify who the students interact with and if their interactions were reciprocal relationships. These relationships suggest that such networks represent active and not passive participation. It is proposed that that by exploring these relationships on the social and metalevel, one can reveal how metacognitive language, as a tool for stimulating interaction, can be used to foster metacognitive thinking through reflection and promote the development of such metacognitive networks. Hurme *et al.* (2006)'s findings, on the social-level, indicate that there are quite strong relations between supporting pairs' own thinking and social network analysis measures.

The interaction between the interpersonal metalevel and the social level are discussed in the following section by way of introducing the concepts of metacognitive locale as a network of metacognition.

5.3.3.3 Interaction between interpersonal and social metacognitive networks

The metacognitive network on the individual level can be seen as an interpersonal network that involves self-talk though a metacognitive language and reflection on the metacognitive knowledge and regulatory domains. The metacognitive network on the social level can be seen as a collection of the interpersonal metacognitive networks that share a particular setting or environment. When a network uses a particular language that is associated with the environment in which the network exists, a locale is formed (Grimm, 2012). Through shared metacognition, the interpersonal and social metacognitive networks can be tied to form a collective set of nodes and ties within the networks. When these nodes represent the metacognitive components and when the ties represent the metacognitive reflections, this collection of networks can be referred to as the metacognitive locale (Figure 5.5).

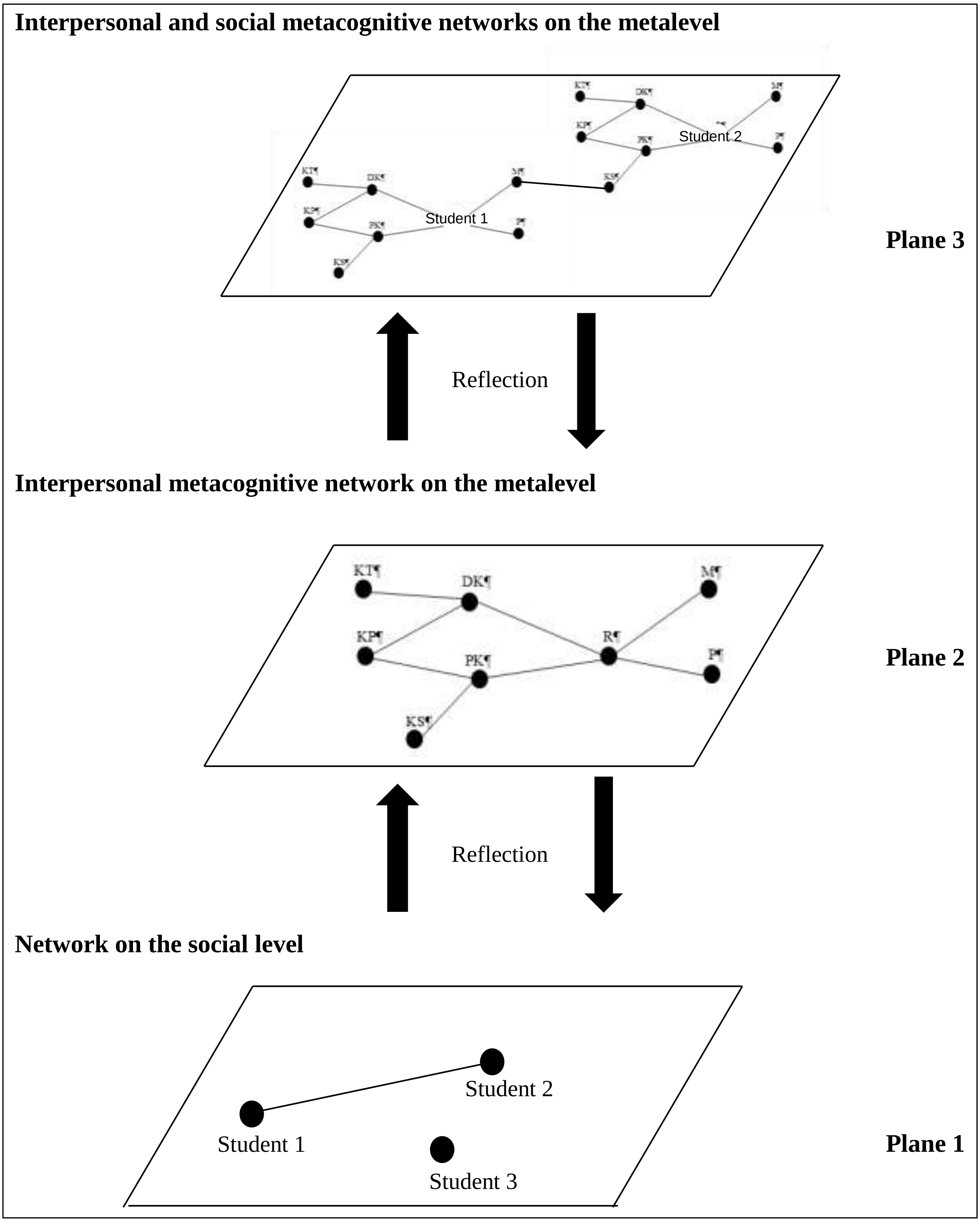


Figure 5.5 Three levels of the metacognitive locale

In Figure 5.5 three planes are aligned on which relationships between the nodes of the social, interpersonal and combined social-and-interpersonal networks are represented. On Plane 1 the social network is the first representation of the interaction between individuals. On Plane

2 an interpersonal metacognitive network is displayed, mapping the route of reflection between the components of metacognitive knowledge and regulation. On Plane 3 the interpersonal metacognitive networks of each of the nodes in Plane 1 are represented. The combined interpersonal metacognitive networks of the three students in Plane 1 are linked through ties that show the route of the reflection from one student's metacognition to another's via the integration of their metacognitive networks. Collectively, these planes represent the socio-and-metagrams of metacognitive thinking that originates from reflection. The linkages between the nodes (relationships between students and reflections between metacognitive components) suggest that there exists a community in the network that extends the geographic position of the students. The network displays students' metacognition (interpersonally and socially) within a specific circumscribed domain in which the three planes in Figure 5.5 are situated. Outside this network milieu, other networks can exist that may have stronger or weaker ties (Halgin *et al.*, 2014). However, in this study these external networks are overlooked as they do not form part of the scope of the study.

The architecture of metacognitive networks situated on the different planes rests on the principles of convergence (Papacharissi, 2011), which enables the reflections between the metacognitive components to multiply and overlap, showing a variety of distinct interpersonal and social levels of metacognition.

5.4 Towards a theoretical framework of the metacognitive locale

The discussion in this chapter suggests that metacognition is an individual and shared construct of knowing and cognitive control that promotes metacognitive thinking and development in collaborative learning environments. The awareness and control of one's own and others' knowing in these environments seems to develop within a network which is situated in the metacognitive locale. Borgatti and Halgin (2011) further suggest that the parameters of this locale are framed by the context of metacognitive knowledge (situated in conditional knowledge which is subdivided by declarative and procedural knowledge domains) and metacognitive regulation (situated in the planning, monitoring and evaluative regulatory domains). The metacognitive thinking in these domains is reflected upon and expressed, verbally[30], through metacognitive language which depicts the structure, type, route and strength of the metacognitive networks and collectively the metacognitive locale.

5.4.1 Standard theory 1 – metacognition (Flavell)

Chapter 4 dealt extensively with this theory. As an addition, metacognition is dealt with here as a theory, rather than a construct or concept. Borkowski (1996) argues that, across all reviews, metacognition consists of metacognitive knowledge, metacognitive judgements and monitoring and self-regulation. Furthermore, metacognition can be regarded as a theory about thinking (Kilpatrick *et al.*, 1997). As a standard theory, metacognition explains the relationship between what one is thinking and how this thinking occurs (Odafe, 2007). Moreover, Harkness (2015) explains that metacognition explains the processes involved during self-reflection and self-direction of one's learning. Metacognition is therefore a theory-level reasoning process (Herberg, 2011).

5.4.2 Standard theory 2 – the zone of proximal development (Vygotsy)

Siyepu (2013) used Vygotsky's ZPD as a theory to elaborate on what students can do with and without assistance from others. According to Vygotsky (2012:186), the ZPD is "the distance between the actual developmental level as determined by independent problem-solver and the level of potential development as determined through problem-solving under guidance, in collaboration with more capable peers".

This statement indicates that collaboration through a language of unity and understanding among peers is necessary and meaningful within problem-solving situations in mathematics education. It seems beneficial for the proposed study to view the assessment of students' abilities and to evaluate their instructional practices as important variables in understanding the language pre-service teachers use when expressing themselves in terms of teaching and learning experiences and in the design and development of such experiences (Siyepu, 2013). Vygotsky (1978) influenced a view on metacognitive theory through his discussion of transference from other-regulation to self-regulation, as social interaction plays a role in the development of higher-order (metacognitive) thinking (Papaleontiou-Louca, 2008:23). This is evident in Vygotsky's (1978:57) view on internal and external metacognition: "… first, between people (interpsychological) and then inside the individual (intrapsyhcological) ... All

[30] The verbal expression can occur internally (Van Lier, 2000) and externally (Papaja, 2012) in the form of text, sentences, pictures in both online (Garrison & Akyol, 2011) and offline (Hurme *et al.*, 2006) collaborative learning environments.

the higher functions originate as actual relationships between individuals." This statement suggests that metacognition is fostered and influenced by the active presence of and interactions with others. As a result of these interactions, Papaleontiou-Louca (2008:24) claims that an individual will develop and mature in forming their own assumptions and theories about themselves and others, suggesting that metacognition is not only associated with learning or development, but also the regulation of that learning or development in social contexts.

5.4.3 Standard theory 3 – social network analysis (Moreno)

Social network analysis is different from the traditional theories and methods used in educational research (Van Staden, 2011:29). For the current study, it provides the necessary techniques to understand social interactions in both real societies and online communities, which will be viewed in this study as the locale in which metacognition develops self-directed learning. It can be used to explore the invisible structures in students' mental states and the social structures that they exhibit when solving problems and communicating about those problems in an online forum. Social network analysis can also be used to elaborate on the theoretical, relevant and practical information regarding the users of a network, either in real societies or online communities. Since the 1930s, social network analysis has been used to explain and interpret social relationships to make predictions about those relationships. Only recently did researchers start using social network analysis to understand and explain educational phenomena (Papacharissi, 2011). However, because of its modern impact on understanding society, literature regarding the use of social network analysis to interpret and explain mathematics education students' metacognitive relations is still scarce. As a result of this lack, social network analysis will be employed as a theory to explain the network states of the participants in this study in a synthesised view with metacognition, ZPD and theory of mind under the overarching philosophy of social constructivism.

One approach Van Eerde (2013) claims to be beneficial for educational practice is that of educational design-based research. The aim is to develop instructional theories about teaching and learning and to develop materials that will improve classroom experiences. From this view it seems that educational design-based research has the capacity to narrow the gap between practice and theory. Intertwined within the design and theory progress, specifically in mathematics education, domain-specific theories can emerge as a contribution to understanding how and why educational praxeologies function. This function focusses mostly on specific local aspects against the backdrop of the education practice, including the understanding of teaching and learning phenomena. For this reason, such specific theories in specific instructional environments or domains are referred to as local instructional theories (Van Eerde, 2013).

5.4.4 Metatheory – social constructivism (Vygotsky)

To validate the use of LS, the research design should be based on a sound theoretical foundation. The theoretical principle of social constructivism promotes the social nature of knowledge, based on constructed interaction and engagement as a shared experience. Social constructivism is emphasised in this study as a metatheory that describes the construction of knowledge and experiences as a result of social interaction, collaboration and discussions (Van Staden, 2012). It promotes discourse, reflection and explanation, all of which support the principles underlying the LS cycles through engaging in and negotiating about activities by people from various backgrounds and various levels of experience and expertise. As a result of these engaging processes, the acquisition of knowledge and skills is in itself an adaptive function which organises one's experiences and mental processing in a social environment (Rock & Wilson, 2005).

5.5 Bringing it all together: the conceptual-theoretical framework

Chapters 4 and 5 review the scholarship regarding metacognition and conceptualise metacognition in terms of reflection, its knowledge and regulatory domains and the resulting networks that, when expressed through a metacognitive language, can be social or interpersonal in nature. Theoretically, these metacognitive constructs are argued to exist in a social space on three planes as illustrated in Figure 5.5. Furthermore, the relationship between these constructs can be explained through such standard theories as metacognition, social network analysis and ZPD. Also, these theories exist underneath an overarching metatheory of socioconstructivism. Chapter 5 elaborates on social network analysis, metacognition and the zone of proximal development as standard theories and socioconstructivism as metatheory in the theoretical framework of this study. The theoretical framework and its implications for this study are discussed as the framework serves to position the body of scholarship on LS, PBL, LIT and metacognition towards building LT within the scientific framework to deposit the ideas into the research design discussed in Chapter 6. Figure 5.6, therefore, serves as the conceptual framework underpinning this theoretical framework.

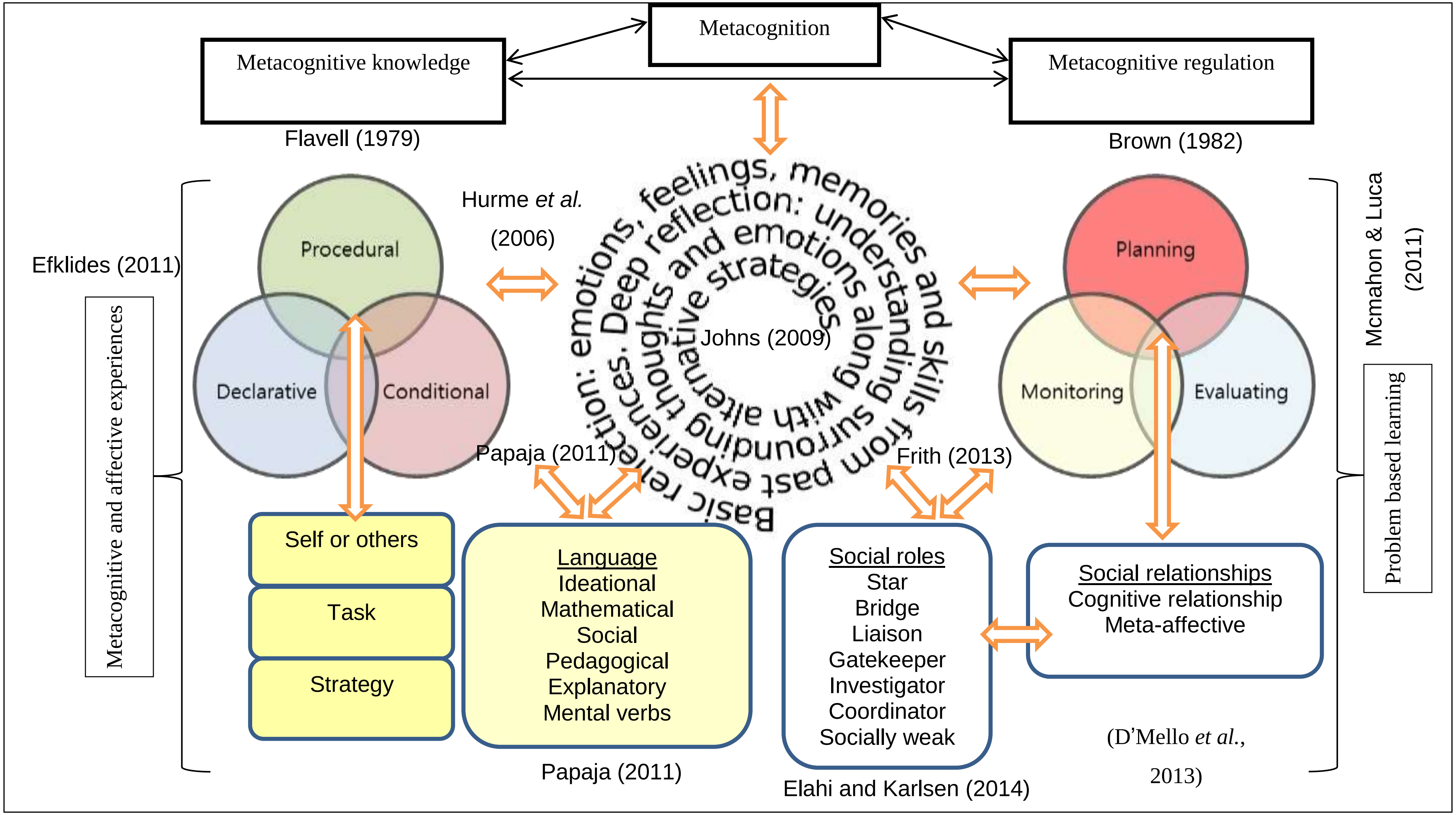


Figure 5.6 The conceptual framework of metacognition, its language, reflections and networking

In Chapter 4, the conceptual framework (Figure 4.4) was partially illustrated. By aligning the concept of metacognition with social network analysis, in Chapter 5, it seems that social roles (see 5.2.1) and social relationships (see 5.2.2) develop when individuals join in group discussions and form a social network. As such, these roles and relationships serve as the theoretical basis on which higher-order networks (metacognitive networks) are formed. Figure 5.6 illustrates the complete conceptual framework of metacognition together with its language, reflections and networking as conceptualised for the purpose of this study. The conceptual framework shows that when individuals contribute either metacognitive knowledge or regulatory components to a group discussion, they can take on certain social roles and develop social relationships. Based on these roles and relationships, they further engage in reflection on the contributions they and other group members make and, in doing so, express their metacognitive processes through a metacognitive language.

To answer the secondary and primary research questions of this study, the components conceptualised in Figure 5.6 are theorised about and explored to understand the relationship between these constructs that, as conceptualised from Chapter 2, emerge from implementing the LIT. The research design and methodology that were used to examine these constructs are described in Chapter 6.

Chapter 6

Educational design-based research methodology

6.1 Introduction to the research methodology

Chapter 1 orients the research and outlines the research problem. In Chapter 2, literature on theory and theory development was reviewed to gain insight into the nature of local instructional theory (LIT) and local theory (LT) development. It was conceptualised that the implementing of a LIT (read: local instructional theory) produces constructs that can be theorised about in a LT (read: local theory). Chapter 3 focused on problem-based learning (PBL) as an instructional philosophy and an approach to facilitate adapted lesson study (LS) as a pedagogical mechanism of the LIT within the context of mathematics education. Based on the principles of theory design, a hypothetical learning trajectory (HLT) and local instructional theory (LIT) was developed to foster metacognitively induced teaching and learning opportunities through which emerging constructs can be explored. In Chapter 4 and 5, these constructs were reviewed in terms of metacognition, metacognitive language and metacognitive networking (see Figure 2.1 & Figure 5.6)[31]. In Chapter 6 the emphasis is on the educational design-based research methodology used in this study.

6.2 Assumptions made by the researcher

The primary purpose of this study was to develop a LT of metacognition, metacognitive language and networking to explain the relationship between these constructs emerging from the LIT to improve and understand the teaching-learning and doing of mathematics. To this end, the researcher implemented an educational design-based research methodology (McKenney & Reeves, 2013) as it was seen fit for the development of theory (Dai, 2012) associated with LS cycles which form part of the module's content. The primary approach of data collection in design-based research offers an interpretive view and necessitates a predominantly qualitative approach (Creswell *et al.*, 2007). To interpret the emerging data, information provided throughout the study was viewed from a pragmatic perspective, that is, to inform the development of LT. Creswell and Clark (2007) explain that such a worldview

[31] The researcher reminds the reader that other constructs can also emerge from the LIT; however, for the purpose of this study, reference is only made to metacognition, metacognitive language and metacognitive networking.

can be used to theorise about how the research design can facilitate new theory, as it explains what works in practice and what not. In this study, to theorise about metacognitive locale, the LIT (see 3.5) produced different constructs as outlined below.

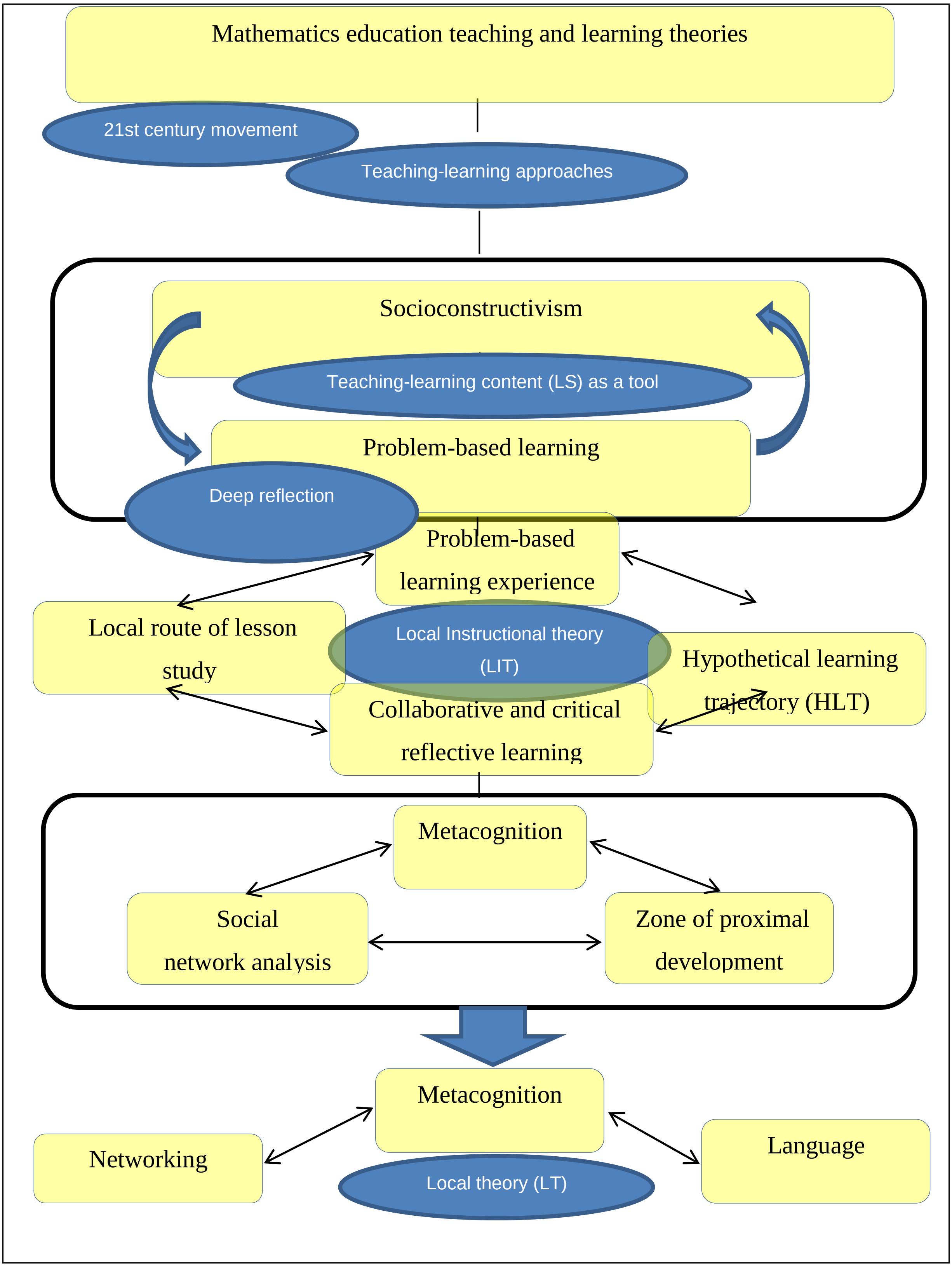


Figure 6.1 **The theoretical framework of the study**

Figure 6.1 shows that PBL and LS are captured in the LIT. Because this affects the research design of this study, the theoretical framework that unfolded and the methodological assumptions of the researcher are displayed here to show how the theoretical domain of metacognitive locale is the product of a range of ontological, epistemological and methodological underpinnings of the philosophy, theory and concepts that constitute this study. Based on these constructs' nature, the researcher formulated the conceptual, theoretical and philosophical assumptions that outline the research design in this study.

6.2.1 Conceptual-theoretical assumptions

Since metacognition, social learning and communication are conceptually part of the principles of PBL and LS, it was anticipated that constructs of metacognition, metacognitive language and metacognitive networking emerge when following the five PBL steps, six LS phases and the four EDR stages as accumulated in the LIT. These methodological contexts provide the researcher the opportunity to explore and understand the relationship between the constructs that emerge from theory.

Following the guidelines of the LIT, the constructs appear to exist in three different, yet supporting layers or strata as collections of networks, illustrated in Figure 5.5. The social, interpersonal and socially shared metacognitive networks across the three strata reflect the intricacy of the philosophical, theoretical and conceptual nature of the study as well as the interrelatedness between the concepts and constructs that accumulated in the conceptual framework.

6.2.2 Philosophical-theoretical assumptions

The teaching-learning and doing of mathematics is grounded on the philosophical and theoretical underpinnings of socioconstructivism. Frith (2012) explains that social constructivism is one of the many teaching-learning paradigms associated with mathematics education. Since PBL shares these underpinnings to prepare pre-service teachers for their future careers within the context of the module, the content of LS serves as a mechanism to introduce the theory and practice of mathematics education to pre-service teachers by following the guidelines of a LIT (see 3.5).

In reality, change is constantly taking place and the individual is an active, reflecting agent of this change, conducting and/or transforming reality by thought and consequently action (Taaitila & Raij, 2012). For this reason interpretivism and hermeneutics were identified as suitable philosophical lenses for the data analysis process and interpretations to generate LT to explain the role that metacognitive language and metacognitive networks play in individuals' metacognitive locale and to understand how these networks develop, thus answering the research question.

6.3 Paradigmatic assumptions and perspectives

Interpretivism adds potential to the research design by offering a view on the technical and practical change of the world in which the data exists. It relates to the recollection, thinking and understanding rather than creating (Reid *et al.*, 2013). For a design-based research approach, this seems appropriate since it offers an interpretation and reinterpretation of what is envisaged and reflects the philosophical assumptions of this study.

Mayoh and Onwuegbuzie (2013) point out that hermeneutics, on the other hand, is a superior choice when content about realities and context requires inductive analysis. Isoda (2011) seems to agree and contends that the application of hermeneutics rejects positivism and postmodernism in qualitative research. This suggests, as Friedrichs and Kratochwil (2009) describe, that hermeneutics in social studies tends towards a pragmatic philosophy that leads to the analysis being framed in such a way that it allows for (more) progressive understanding throughout the study. This developing nature of hermeneutics to understand the underlying meanings (or parts thereof) that (co)construct the framework of the research design, can reveal various interpretations.

6.3.1 The pragmatic paradigm necessity: aspiring to use an educational design-based research approach with an embedded multi method design

The philosophical stance that informs the educational design-based research design in this study is embedded in a pragmatic paradigm. The purpose of this perspective is to provide the research methodology with the necessary logic and structure to guide the process of the research (Sarantakos, 2010).

6.3.2 The Pragmatic paradigm

The focus of pragmatism as a worldview is on the research outcome and informs the actions, situations, and consequences of inquiry, rather than predetermined conditions (as is the case with for example post-positivism). The emphasis is mainly on "what works" and how it works (Bignall *et al.*, 2014)). Creswell and Clark (2007) therefore offer the following thoughts on pragmatism:

- Pragmatism is not limited to a particular system of philosophy or reality and can guide, for example, the research design of EDR.

- Since pragmatism is not bound, the research problem and research purpose drives the pragmatic outlook on the data.
- Many approaches to collecting and analysing data are encouraged.
- Pragmatist research occurs in a variety of research contexts, including social sciences.
- Pragmatism is applicable for studies in an external reality (visible world) and a world lodged in the mind.

6.3.2.1 The logic and uses of pragmatism

Friedrichs and Kratochwil (2009) argue that pragmatism is necessary to acquire reliable and useful knowledge for both theory and practice applications. In particular, pragmatism allows for the production of inductive theoretical propositions as the result of empirical observations. Pragmatism therefore appears to serve as an instrument for developing theory with epistemological and methodological awareness.

6.3.2.2 The structure of pragmatism

The structure of pragmatism seems to exist in research in which theory development is conducted by continuously moving backwards and forward between abstract and empirical levels of inquiry up to the point where the theory feeds the research and vice versa (Friedrichs & Kratochwil, 2009).

6.3.2.3 How pragmatism guides the research process

Johnson and Onwuegbuzie (2004) explain that pragmatism places a high regard on experiences in reality as the external world and how these experiences influence the inner world. Also, the use of different theories can be instrumental for developing new theory. Pragmatism acknowledges that truth and knowledge can change over time. It allows for the testing and validating or refuting of theories. After the researcher reviewed a list of strengths and weaknesses of pragmatism (tabled in Johnson & Onwuegbuzie, 2004), it seems as if pragmatism has received preference over other worldviews and is deemed fit for the current study's purpose. In the next section, emphasis is on the design-based research approach suitable for research in a pragmatic paradigm.

6.4 Design based research as an emerging paradigm

Educational science draws on multiple theoretical and research perspectives to explain the conditions through which teaching and learning develop (Barab & Squire, 2004). Design-based research (DBR) is grounded in the belief that the nature and conditions of teaching and learning experiences are engineered to sustain learning through complex interventions (Bell, 2004), as is the case with this study. This is done to better promote metacognitive thinking as an educational goal, to overcome the issues in mathematics education and to acknowledge the need for development of novel pedagogies. For this purpose, the current study draws on the principles of Anderson and Shattuck (2012) of design-based research as an emerging methodology. Anderson and Shattuck (2012) define design-based research by real educational contexts amidst the design and testing of appropriate interventions. These interventions occur through multiple methods of data collecting and analysis, which involves multiple iterations and improvements. This is ensured by promoting a collaborative partnership between participants to negotiate the design principles and to meet local needs that will not only advance theoretical agenda but also uncover, explore and confirm theoretical relationships that share the epistemology, ontology and methodology that underpin a PBL philosophy (Anderson & Shattuck, 2012:16-17).

Against this background, a research design was needed to develop theory in an educational setting directed at LS towards examining the relationships between the, produced constructs of metacognition, its language and networking. The current study proposes that these constructs together foster students' metacognitive locale, a proposed theoretical domain where metacognition develops, in the higher education environment. Norwich and Ylonen (2013) confirm that DBR allows for the systematic monitoring, refining, goal attainment and evaluation that (similarly to LS) include the importance of context, which is a factor in local theory development. For the purpose of this study, educational design-based research (EDR) was regarded as instrumental for the research design as discussed below.

In the context of mathematics education, an educational design-based research approach was incorporated to develop a local theory. Figure 6.2 illustrates a summary of the educational design-based research methodology used in this study.

Summary of the educational design-based research methodology

Primary research question

In what way(s) do(es) understanding of the identified metacognitive language and metacognitive networks contribute to theory building concerning students' metacognitive locale in teaching-learning and doing mathematics?

Secondary research questions

Secondary research question 1	Secondary research question 2	Secondary research question 3	Purpose
What do students' metacognitive language entail when teaching-learning and doing mathematics?	What do students' metacognitive networks entail when teaching-learning and doing mathematics?	How can metacognitive language and metacognitive networking foster shared metacognitive experiences?	To produce constructs from the LIT to develop LT

PBL steps (Dai, 2012 - § 3.2.2)

1. Introduce the PBL task
2. Group discussion
3. Develop resources
4. Present solution
5. Group reflection

LIT phases following the local route of LS (Gravemeijer, 2004 – see 3.4.1)

1. Investigation and planning
2. Develop research lesson
3. Present research lesson and reflect
4. Refinement of research lesson and represent

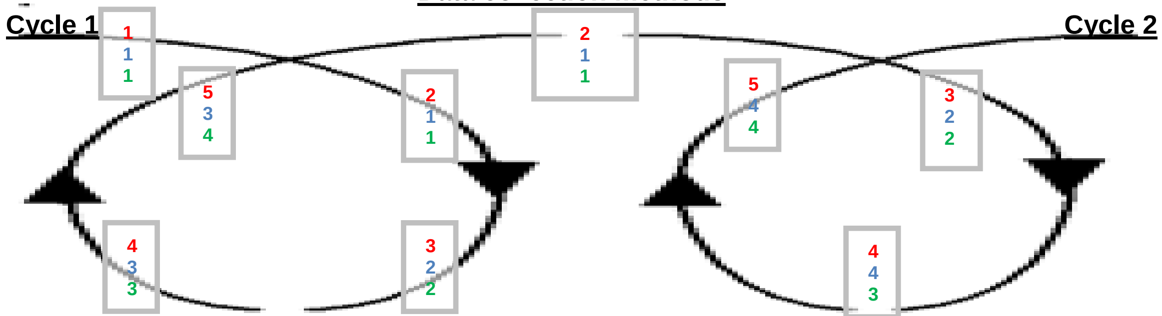
EDR stages (Amiel & Reeves, 2008 – see 6.4 & see 1.3.4.1)

1. Analyse practical problem in collaboration
2. Develop solution
3. Test and refine solution
4. Reflect to enhance solution

Population: (N=60), **Sample**: (Group A, n = 6; Group B, n = 5)

Data collection methods

Based on the **HLT,** guidelines of the **LIT** were followed in each design cycle to produce emerging constructs.



Emerging constructs:	Metacognition	Metacognitive language	Metacognitive networking	Data analysis
Stratum 1	X	X	Social roles and relationships	Interpretivism Hermeneutics
Stratum 2	X	X	Social roles	
Stratum 3	X	X	Social relationships	
Theoretical framework:	**Metacognition**	**ZPD**	**Social network analysis**	Socioconstructivism

Figure 6.2 Summary of the educational design-based research methodology

The following sections elaborate on the educational design-based research methodology portrayed in Figure 6.2.

6.5 Population and sampling of participants

Creswell and Clark (2007) explain that the sample refers to a subset of the target population and any assumption or generalisation of the sample reflects on the population. In the current study, all fourth-year students (N = 60) who registered for the mathematics methodology module were invited to take part in the research. These students served as the population of the study from which the sample is drawn.

6.5.1 Inviting students after the whole class meeting

As part of the outcomes of the module, students were required to design and present mathematics lessons for work-integrated learning[32] (WIL) as part of their undergraduate coursework in the intermediate phase. To contextualise the lessons, a PBL task was designed for a whole class activity, which outlined the expectations of the students throughout the module as well as the philosophical principles of this study that underpinned the PBL task (see 1.3.4.2 & Addendum A). The whole class was introduced to the PBL task by means of a Powerpoint presentation of the printed out version of the PBL task (see Addendum A). Students had to read through the task followed by a class discussion regarding the expectations of the task to get clarity and to ensure that everyone understood what was expected of them. It was then explained that the task was based on a real school with real problems and that students who wished to participate in the study could prepare their lessons for the school as a component of service learning and that these lessons could be used, refined and adapted for WIL. Based on the principles of LS, students had to form groups consisting of five to eight members. They were also encouraged to keep journals for reflections throughout the study and work together in a series of design sessions. The journals were used to aid in their reflections (Bormotova, 2010) and to provide the researcher with some biographical information of the participants, outlined in Table 6.1.

- [32] Work Integrated Learning refers to practical teaching or in-service training, as it may be known in other countries,

Table 6.1 Summary of the two groups' participants and their biographical information

Biographical information[33]	Group A						Group B				
	S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**	**S10**	**S11**
Aware of the requirements/norms for work in group settings	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Creates mind maps/concept maps	✓				✓	✓					✓
Reads the study material more than once	✓		✓	✓				✓	✓		✓
Only reads study material once[34]						✓			✓		
Identifies/highlights important points	✓	✓	✓	✓	✓			✓			✓
Identifies what is not important	✓			✓				✓			✓
Summarises the important points in own words	✓		✓		✓	✓		✓		✓	✓
Understand what they have read	✓		✓	✓	✓			✓		✓	✓
Feel that they do not understand what they have read		✓				✓			✓		
Thinks that the design sessions are useful for learning the module contents	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
Uses the internet to get extra relevant information in addition to the study material	✓		✓			✓		✓			✓
Communicates with other group members about the work via mobile cell phone, e-mail or online forum	✓	✓	✓	✓	✓			✓	✓	✓	✓

[34] No participant expressed that they did not read the study material at all. This was perhaps due to the reason that tests, assignments and exams were used to assess all registered students for the module MMI 411 – for course purposes.

The biographical information identified in Table 6.1 was obtained by identifying themes of the similarities and differences between participants, based on the readings of their reflection journals, taken from the idea by Bormotova (2010). In particular, the journals were used as a means for participants to reflect before and after the design sessions. After the design sessions, participants were required to write down what they did in the design sessions and how they prepared for the design sessions and the module. They also had to write down what challenges they encountered with the research lesson. Based on those readings, Table 6.1 was compiled and is briefly discussed below.

The two groups, Group A and Group B, had six and five members respectively. All eleven participants were registered for the fourth-year mathematics education module for intermediate phase mathematics at one rural university in the North West province. Table 6.1 shows which components the participants expressed in their reflective journals. The symbol ✓ indicates that the participants have somehow mentioned this component in their journal. If the space is left open, it does not necessarily mean that the participant does not (for example) read the material more than once. However, participants did receive an individualised table during the last session where only their biographical information was noted and were asked to see if they agree with where the ticks were made. They were also informed that this table was compiled from reading their journals. No participant made or suggested any changes.

6.5.2 Inviting students to form LS groups for the design sessions

Since participation was voluntary, no limitation was put on the number of students or groups who wished to take part in this study. At first, three groups of students (n = 15) volunteered, but within the first month one group withdrew as its members felt that they had too many other responsibilities[35]. Two groups of students remained and participated in both design cycles throughout the year.

All participants were full-time fourth-year intermediate phase mathematics education students enrolled for a Bachelor of Education (B.Ed.) degree. They have had mathematics as a major for two years and had taught and observed mathematics lessons for the intermediate phase (Grade 4 to 6) on six occasions as part of WIL since their registration for the degree in their first year.

[35] See journal extract in Addendum G

The criteria for selecting these participants included convenience, purposiveness, access and willingness to participate, as suggested by Wellman *et al.* (2009). The participants' enrolment for the module, the preparation they received and notes (which covered theory on LS and metacognition) made these students suitable candidates to design a local theory during the adapted LS approach.

6.6 Research site(s)

The university classroom, library and school were three research sites identified to conduct the study, elaborated on in this section.

6.6.1 The university classroom

The first research site involved the university classroom, which provided the opportunities for sampling, sharing the experiences within the design cycles with the rest of the class and facilitating LS and metacognition as part of the mathematics methodology module. These opportunities were scheduled in students' timetable for the classes and did not form part of the design group sessions or LS cycles. Therefore, no data were collected during class time.

6.6.2 The university library

Since participants volunteered to put aside time for participating in this study, the design sessions were planned and arranged to be conducted outside of class time in an environment where participants had access to resources, in a safe place to comfortably discuss and design the research lessons. The university library offers several rooms for groups of students to meet and work together. These rooms include appropriate lighting, desks and chairs, computer facilities, airconditioning and internet access. During these sessions, videorecordings were made of participants' discussions, planning and design of the research lessons. A schedule (see Addendum B) was used to plan these meetings.

6.6.3 The school classroom

To teach, observe and reteach the research lesson, on which the context and problems of the PBL task were based, two local township schools were identified as part of a service learning opportunity. The two groups visited the two schools after arrangements were made with the principal and Grade 6 mathematics teachers who kindly agreed to assist during classes and to make their classrooms and learners available. The arrangements were such that no school activities were hindered during the visit and since teachers would teach *place value* on the day of the visit, no teaching or learning opportunities were lost. Teachers of these classes were present during the teaching of the research lesson to help in instances where translation from English (the teaching medium) to Setswana (mother tongue of learners) was necessary.

Learners and teachers did not form part of this study; instead, they served as parameters of the context of teaching in South Africa and thus no data were collected about them. The students, however, who presented and observed the lessons, were videorecorded.

6.7 Data collection instruments

The data were qualitative in nature. A PBL task was designed to contextualise the phases of LS and the activities in the design cycles.

6.7.1 Data collection instruments: The PBL task

To facilitate metacognitive experiences, a PBL task was designed and issued to the participants (de Jong, Verstegen, Tan, & O'Connor, 2013). The guidelines provided by Lee *et al.* (2013) were used for the PBL task's design. This PBL task, a fictitious scenario, was based upon various problem cases, which participants had to identify and relate to the evidence in the PBL task. The PBL task provided the participants with a contextual problem, which they had to solve as a group, using adapted LS as an approach to design and re-design a research lesson. The main role of the PBL task was to kindle participants' metacognitive thinking to explore, examine and understand the phenomenon of metacognition within mathematics education practice. The research lesson served as a key to unlock their metacognition, to externalise and explore it for the purpose of this research. Participants then solved and discussed the PBL task in a series of design group sessions.

6.7.2 The discussions as a research instrument

Before and after the design group sessions, the participants were encouraged to meet and reflect about or comment on their ideas and experiences with regard to the PBL task. These discussions were also planned as starting points for the design group sessions.

6.7.3 The design group sessions as a research instrument

A series of three design sessions were conducted through which participants solved a PBL task while being videorecorded. The number of sessions depended on the number of LS phases and whether participants felt the need for further discussions. Data were collected and analysed in accordance with the previous sessions' approach; that is, the discussion in the previous session(s) served as a starting point for further discussion and design. The Freinet-PBL learning cycle by Borhan (2012) was employed as an outline of the design group sessions' processes. The PBL cycle includes five stages: (1) introductory phase; (2) a group discussion about the PBL task and clarifying any concerns or questions participants may have about the goals for the session(s); (3) resource development for the research lesson; (4) presentation; and (5) group reflection. The main goal was to design a research lesson to observe and analyse participants' metacognitive networking and metacognitive language when they designed, presented and re-designed a mathematics lesson on place value for a Grade 6 class situated in a rural area.

On completion of the first draft of the research lesson, a volunteer was asked to present the lesson and make a videorecording of this presentation at a school where other members of the group could reflect on the lesson and how it was received. After the presentation, they had to give feedback through generated notes, observations and recommendations for refinement of the lesson based upon their own reflections and experiences. This added validity and trustworthiness to the data as it represented multiple perspectives on the same phenomenon.

6.7.4 The videorecording of the presented lessons as a research instrument

All participants reflected and commented on possible changes to the lesson. This presented lesson was videorecorded for thorough feedback on how the lesson worked in practice. The videorecording was given to participants before the re-design phase, after the presentation of their lessons. The aim was to promote a discussion about the videorecording to foster their metacognitive thinking about the presentation. This re-designed lesson was presented again, reflections were done and the group members were asked to share their experiences about the design sessions with the rest of the class.

6.7.5 The researcher as a research instrument

The researcher acted as a facilitator who objectively ensured that the design group sessions were organised and contributed meaningfully to the solving of the PBL task. He also ensured that the participants could negotiate freely and responsibly and that the group (network) reached its goal, namely to design, implement and reflect upon a research lesson as stipulated in the PBL task. The researcher had to prepare resources such as those listed in section 1.6.2.3. He also took on the role of network administrator. The researcher evaluated and monitored the participants' progress and discussions. Care was taken that group members only gave appropriate feedback in their discussions and that the discussions were monitored for both quality and ethical reasons.

6.8 Data analysis

Two types of data analysis were used, namely conversation analysis and coding of the verbatim transcriptions[36].

6.8.1 Network analysis software

When researchers analyse qualitative data, the findings typically suggest association between themes, categories and the relation between the data and theory (McKetcher *et al.*, 2009). Practitioners and non-practitioners of network data analysis such as social network analysis need appropriate software packages that effectively analyse and visualise these networks for design, tutorial or research purposes (Bonsignore *et al.*, 2009). However, such analysis requires domain experts that can use adequate programming language associated with the software package for manipulation and visualisation of these network types (Smith *et al.*, 2009).

For the purpose of this study, the researcher trialled network analysis software packages suggested by McKetcher *et al.* (2009) for use in a Windows 8.1 operating system. The aim was to discover, explore and visualise embedded network data that is both social and conceptual in nature, within transcribed narrative data of focus group interviews. Programs such as Multinet, Pajek and UNICINET were mainly considered, as numerous sources (Bonsignore, *et al.*, 2009; McKetcher *et al.*, 2009; Van Staden, 2011; Mrvar & Ljubljana, 2014) report on successful implementation of network analysis using these and similar programs representing social networks. Bonsignore *et al.* (2009) explain that visualised networks such as the image files produced by network and SNA software must meet the following conditions:

- Nodes must be clearly visible.
- Nodes' degrees must be countable.

[36] Participants were Afrikaans- and/or English-speaking students. For this reason the data collected were in some cases Afrikaans transcriptions and in other cases English. The researcher acknowledges that because this study is intended to contribute to teaching-learning scholarship regarding metacognition, the presentation of the findings were written in English in chapters 7 and 8. The original transcriptions can be found on the accompanying CD (for Group A and B). The sessions were conducted mainly in Afrikaans, but students were allowed to ask questions if they were unsure and they were allowed to respond in English, which the English-speaking students did. The researcher, however, did take great care in accordance with the study leader and the transcriptionist to analyse the transcriptions and present the findings in English, when spoken in Afrikaans, to ensure that the data were transcribed, translated and interpreted correctly.

- Links between nodes must exist as a relationship between one source node and a destination node.

- Groups of nodes, or clusters, and outliers must be identifiable.

For this reason, a SNA package was sought that met the conditions stipulated by Bonsignore *et al.* (2009) and at the same time offered sophisticated and fairly manageable network analysis without the obstacles of technical skills and experience that experts in the field are familiar with.

To draw metagrams to illustrate the metacognitive networks of the participants that developed through their collaborations and discourses during the design group sessions, the transcribed interviews first had to be converted into social network analysis data. To do so, McKetcher *et al.'s* (2009) five-path analysis was adapted to a three-step path, refined and followed. McKetcher *et al*. (2009) explains that software programs such as Microsoft Office Word, Excel 2003, ATLAS.ti 5.0, SPSS version 13, and Multinet version 4.44 can be used to convert the transcribed interviews into social network data.

The researcher decided to use NodeXL instead of Multinet 4.44 or Pajek 4.01, since it is compatible with Windows 8 and 8.1. Other social network analysis software packages were also considered, including Netminer, StOCNET, Structure and UCINET. However, for the purpose of this study, NodeXL provided the tools for analysis and visualisation of collaborative networks, innovations networks, data-mining and many other features related to social and natural sciences (Mrvar & Ljubljana, 2014). Each package considered offered some difficulties that were expected to hinder the data analysis process and that required knowledge and vocabulary associated with SNA, which pose obstacles for those who are not familiar with network metrics or lack the technical skills and experience associated with the programs. The researcher as a proletarian of SNA software was of the opinion that time-consuming analyses could affect the network analysis process.

Although there are many SNA software packages, NodeXL was identified as a suitable tool to use for the purpose of this study.

6.8.2 NodeXL as suitable SNA tool

Node XL, an open-source add-in toolkit for networks within Microsoft Excel that can be used to discover, explore and visualise network data (Smith *et al.*, 2009), was identified as an appropriate tool for network analysis. It offers a flexible, interactive and effective exploratory interface for network analysis, particularly for complex systems such as those that underpin focus group interviews or social groups (Smith *et al.*, 2009). NodeXL adds network metrics to Microsoft Excel, for example degree, centrality measures, clustering and network visualisation (Bonsignore *et al.*, 2009) and promises a familiar environment to work with to those who already have experience with Excel, even though for the purpose of this study these features and network characteristics were overlooked.

NodeXL also offered the researcher the option to import files into other social network analysis programs' format to conduct further exploration of the networks. As this was not necessary to answer the research questions, it is anticipated for inclusion in future research projects. In addition, NodeXL's output image files can be printed in colour with clear distinction between the nodes and links, whereas the researcher argues that MultiNet offered a less attractive version of the same data.

To transcribe and code the design sessions and to search for embedded network data, the process described below typically involves importing, processing, calculating and refining the data, resulting in a series of network graphs that tells a useful story (Smith *et al.*, 2009:3). Microsoft Office Word, Atlals.ti 7.0 and NodeXL were the three software packages used throughout this process. First, data obtained from the interviews were transcribed verbatim using Microsoft Office Word, then the transcriptions were coded in Atlas.ti 7.0 and finally imported in NodeXL to explore and visualise the embedded networks.

Analysis took the form of a series of three analysis events (or strata), with the main purpose to analyse and code the data, determine the nature of the data through inductive analysis and visualise the metacognitive network structures that the data hold.

The process of data analysis follows below.

6.9 *A priori* codes in the network analysis

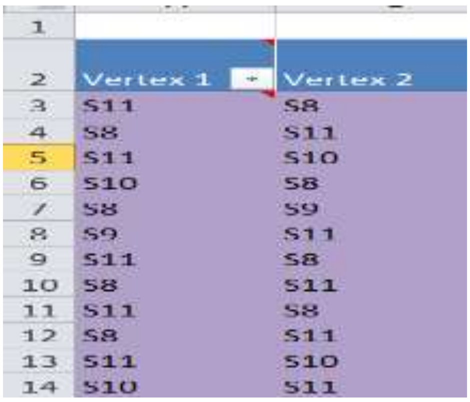
Analysis started immediately after each LIT phase was completed and prepared for coding. Based on the conceptual-theoretical framework of this study (see Figure 5.6 & Figure 6.1), the following *a priori* codes were identified (Table 6.2).

Table 6.2 ***A priori*** **codes for the social, interpersonal and socially shared metacognitive networks**

Theme	Category	Code	Description	Codes for social (stratum 1), interpersonal (stratum 2) and socially shared metacognitive networks (stratum 3)										
				S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**	**S10**	**S11**
Metacognitive knowledge (MCK)	Declarative knowledge	DK	Knowledge about oneself and the factors that affect performance	S1DK	S2DK	S3DK	S4DK	S5DK	S6DK	S7DK	S8DK	S9DK	S10DK	S11DK
	Conditional knowledge	CK	Knowledge about when and why a strategy should be used	S1CK	S2CK	S3CK	S4CK	S5CK	S6CK	S7CK	S8CK	S9CK	S10CK	S11CK
	Procedural knowledge	PK	Knowledge about how a strategy should be used	S1PK	S2PK	S3PK	S4PK	S5PK	S6PK	S7PK	S8PK	S9PK	S10PK	S11PK
Level of metacognitive knowledge (LMCK)	Knowledge of the person	KP	Knowledge about the strengths and weaknesses of oneself or others	S1KP	S2KP	S3KP	S4KP	S5KP	S6KP	S7KP	S8KP	S9KP	S10KP	S11KP
	Knowledge of the task	KT	Knowledge about the nature of the experience regarding the task	S1KT	S2KT	S3KT	S4KT	S5KT	S6KT	S7KT	S8KT	S9KT	S10KT	S11KT
	Knowledge of strategies	KS	Knowledge about the types of strategies that could be used	S1KS	S2KS	S3KS	S4KS	S5KS	S6KS	S7KS	S8KS	S9KS	S10KS	S11KS
Metacognitive regulation (MCR)	Planning	P	Setting goals, selecting appropriate strategies, predicting the outcome of one's actions and organising strategies and resources to align with planned experiences	S1P	S2P	S3P	S4P	S5P	S6P	S7P	S8P	S9P	S10P	S11P
	Monitoring	M	Engage in recalling and reflecting to judge consciously the ideas, actions and contributions of one self and others	S1M	S2M	S3M	S4M	S5M	S6M	S7M	S8M	S9M	S10M	S11M
	Evaluating	E	Appraisal of the efforts, processes and products of the planned experiences and the revision of goals	S1E	S2E	S3E	S4E	S5E	S6E	S7E	S8E	S9E	S10E	S11E

The design group sessions were videorecorded for analysis. Each session was transcribed in and saved as a separate Microsoft Word File (e.g. Group A – session 15 February) which indicates the group's name and the date the session took place (see the schedule in Addendum B for a complete list). The design group sessions served as a type of focus group interview during which each participant was assigned a unique pseudonym (e.g. S1) incorporated in a dialogue format when transcribing. The pseudonyms also acted as codes for each stratum and were useful for identifying the nodes of the networks. This served a dual purpose as pseudonyms of the participants were used to identify them in all the transcriptions and to make the data obtained from the participants accessible and identifiable in NodeXL, particularly in the output image files of the networks. After transcribing the interviews in the design group sessions, a thematic method of analysis was employed in which each of the three strata was analysed using the codes in Table 6.2 to exploit network data embedded within each stratum. Here it ought to be noted that each transcription was analysed according to the particular network data embedded in the strata. For example, in stratum 1 social network data were analysed through interpretivism, in stratum 2 interpersonal metacognitive network data were analysed through hermeneutics and in stratum 3 socially shared metacognitive network data were analysed through both interpretivism and hermeneutics. Each transcription was thus analysed three times, each time with either a social, interpersonal or socially shared metacognitive network that had to be understood and interpreted. Collectively, to answer the primary research question, the networks in strata 1, 2 and 3 form the domain of the metacognitive locale theory. A NodeXL template was created and the contents of Vertex 1 and Vertex 2's columns were copied from Excel into NodeXL's Edge sheet. For example:

Sample Edge sheet



	Vertex 1	Vertex 2
3	S11	S8
4	S8	S11
5	S11	S10
6	S10	S8
7	S8	S9
8	S9	S11
9	S11	S8
10	S8	S11
11	S11	S8
12	S8	S11
13	S11	S10
14	S10	S11

Sample Vertex sheet

	Vertex	Color	Shape	Size	Opacity	Image File	Visibility	Label
3	S11							S11
4	S8							S8
5	S10							S10
6	S9							S9
7	S7							S7
8	S7KP							S7KP
9	S7E							S7E
10	S7KT							S7KT
11	S8E							S8E
12	S8P							S8P
13	S9KP							S9KP
14	S9P							S9P
15	S10KP							S10KP
16	S11KP							S11KP

For analysis, the path outlined in Table 6.3 was followed.

Table 6.3 Path of data analysis – Three steps in Word, Atlas.ti and NodeXL for the three strata

<table>
<tr><th colspan="2">Strata[37]</th><th>Stratum 1</th><th>Stratum 2</th><th>Stratum 3</th></tr>
<tr><td rowspan="6">Path and software package or tool</td><td>Word</td><td colspan="3">1. Verbatim transcribing of the videorecordings while, in dialogue format, clearly indicating S1, S2 etc. and separating the identified participant from the quotation using a colon (e.g. S1: this is easy).
2. Also indicate in brackets to which participant the response or reply is (e.g. S1 to S4).</td></tr>
<tr><td rowspan="3">Atlas.ti</td><td colspan="3">3. Import the verbatim transcription as a primary document
4. Create new or import codes by using Table 6.2
5. Create families using S1, S2, S3 etc.</td></tr>
<tr><td>6. Autocode all S1, S2, S3 etc.
7. Manually code all participants' responses using Table 6.2 according to who said what in reply or response to a question or statement made by another group member.</td><td colspan="2">6. Create families for MCK, MCR, LMCK and S1, S2 etc.
7. Manually code according to Table 6.2 regarding the codes for strata 2 and 3</td></tr>
<tr><td colspan="3">8. Save file as (data analysis – transcript file name)
9. Open quotation manager and select output to open in Web browser
10. Copy the entire table.</td></tr>
<tr><td>Excel</td><td colspan="3">11. In a new Excel document, paste the table copied in step 10.
12. Delete unnecessary columns (e.g. ID, size, density, author, created & modified)
13. A, B, C and D are remaining columns. Clear the contents of column B
14. Rename column A (Vertex 1), B (Quotation), C (Vertex 2) and D (Line number)
15. Select column A, Data, text-to-column, Delimited, next and choose a colon (:) as the delimiting symbol
16. Save file as coded data table (strata 1, 2 or 3)</td></tr>
<tr><td>NodeXL</td><td colspan="3">17. Copy contents of Vertex 1 and 2 from the file saved in step 16 into NodeXL's Edge sheet.
18. Copy the contents of the line numbers column.
19. Refresh the graph</td></tr>
<tr><td colspan="2">Data output type or format</td><td>Social network data</td><td>Interpersonal network data</td><td>Sociometacognitive network data</td></tr>
<tr><td colspan="2">Theoretical associations to interpret findings</td><td>Social network analysis</td><td>Metacognition
ZPD</td><td>ZPD
Metacognition
Social constructivism</td></tr>
</table>

[37] For details about strata 1, 2 and 3 please see paragraph 5.3 and Figure 5.5.

The discussion below elucidates the data analysis procedures for each stratum.

6.9.1 Analysing stratum 1 through interpretivism and social analysis

In stratum 1, the social network data were analysed by interpreting the social roles and relationships between participants. As Sarantakos (2010) suggests, interpretivism was used as a framework to understand the social world of the participants within the two groups. The emphasis was on interpreting the meaning of the participants' personal views, opinions and their observations during the group discussions and the presentation of the research lesson. This was done, as Sarantakos (2010) explains, to interpret the social relations and their local meanings.

In the analysis of stratum 1, the verbatim transcriptions were first read through and coded. Since the focus was on social network analysis, care was taken to code participants' responses in two ways. For example, each participant was assigned a unique code (e.g. S1) automatically to identify who was speaking at the time. When a group member commented on what was said or asked, the comment is either part of a current conversation or the start of a new conversation or idea being shared. The automated coded quotations were then reread to determine which participant(s) related to the first code. It was expected that, on the social level, there existed some social network ties between these participants (Bonsignore *et al.*, 2009). Because of the interactivity in the group and the collaborative nature of the PBL task, a social network was formed, serving as the basis of the metacognitive locale as it provides a framework around which strata 2 and 3 developed. Therefore, only the social roles and relationships were considered and any other network metrics were ignored.

6.9.2 Analysing stratum 2 through hermeneutics and conversation analysis

In stratum 2 the emphasis was on the interpersonal metacognitive networks, including the metacognitive knowledge and regulatory domains that had become explicit in the conversations during the design group sessions. Following the guidelines for hermeneutic analysis by Sarantakos (2010), coding was conducted by eliciting an understanding of each participant's individual contribution to the group. Then the transcription was coded according to the categories associated with metacognitive knowledge and regulatory domains for each individual. The result was an interpersonal metacognitive network view of each participant's metacognitive knowledge and regulatory components. In this sense, Table 5.1's network concepts were adhered to.

6.9.3 Analysing stratum 3 through interpretivism and hermeneutics

In stratum 3 the social roles and relationships were sought to understand the ties between participants' social and metacognitive networks to interpret how their interpersonal metacognitive network contributed, or not, to the socially shared metacognition of the group as a whole. Here the emphasis was on shared metacognition. In particular, the metacognitive language participants used to express their metacognitive processes was acknowledged and, through this language, the shared metacognitive networks were identified.

An example of how the resulting network maps for each stratum looks like and how the networks are interpreted follows in the next section.

6.10 Interpreting the findings and network maps of the three strata

The following discussion explains how the strata displayed in Figure 6.3 should be interpreted. It is believed that it would be helpful to the reader if the networks are interpreted alongside a copy of the strata's colour separated networks[38], or part thereof, highlighting the particular stratum as a separated network.

[38] Due to the limited amount of space, it is not possible to include these separate illustrations further in the findings sections for all networks; however they are available in the Addendum H. It is therefore anticipated that the reader will follow the guidelines in the following example to interpret the networks presented in Chapter 7 and Chapter 8.

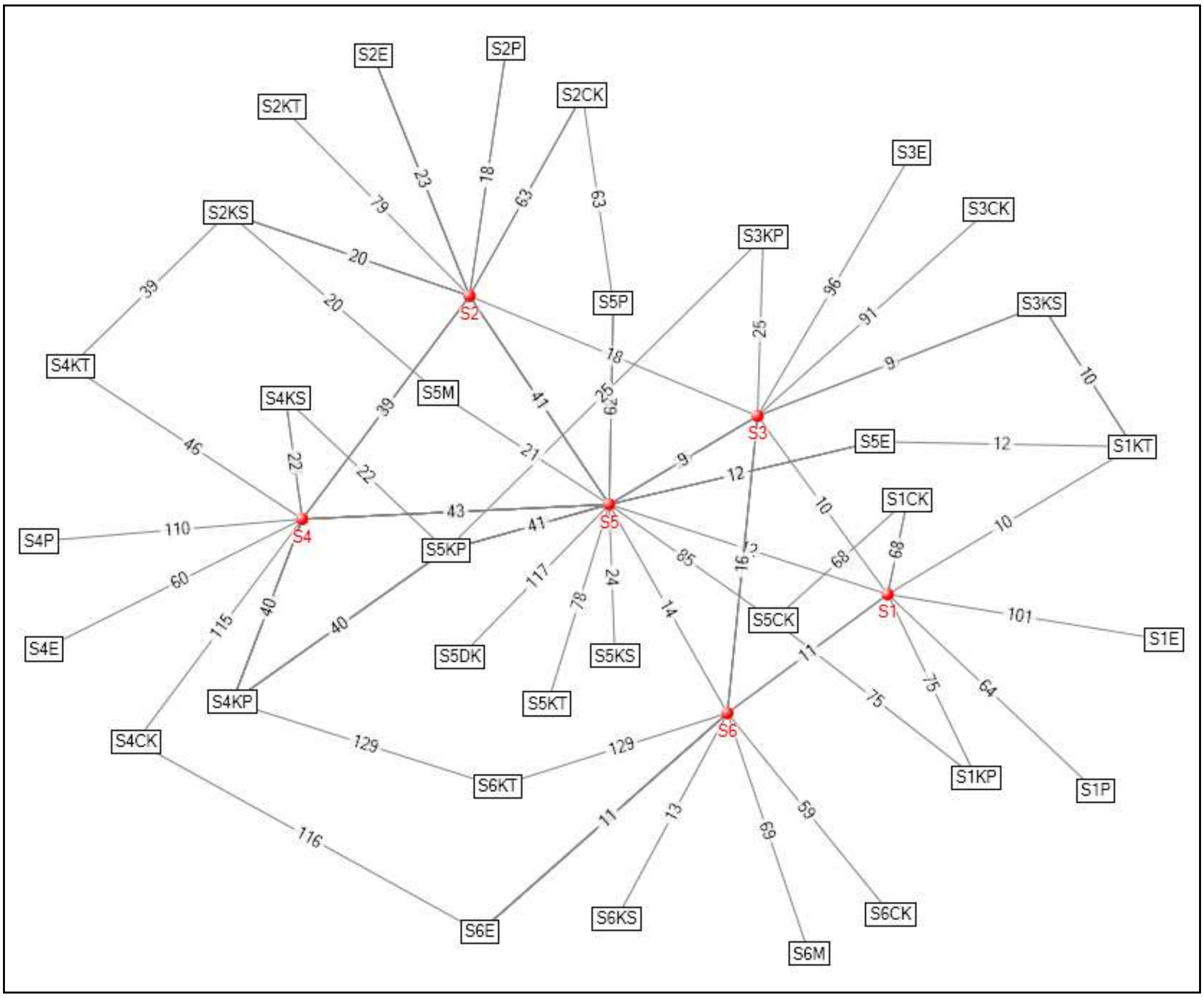


Figure 6.3[39] **Sample network map/illustration of the social-metacognitive network of the findings in stratum 3**

Figure 6.3 shows a sample of the network illustrations of Group A's socially shared metacognitive network that is representational of strata 1, 2 and 3 during the first LIT phase.

[39] The reader is reminded to look at the colour-separated view of the embedded strata in Figure 6.4

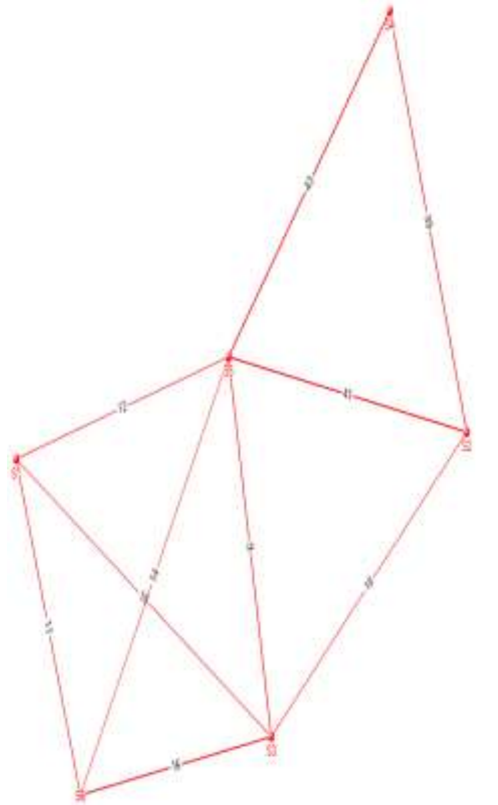
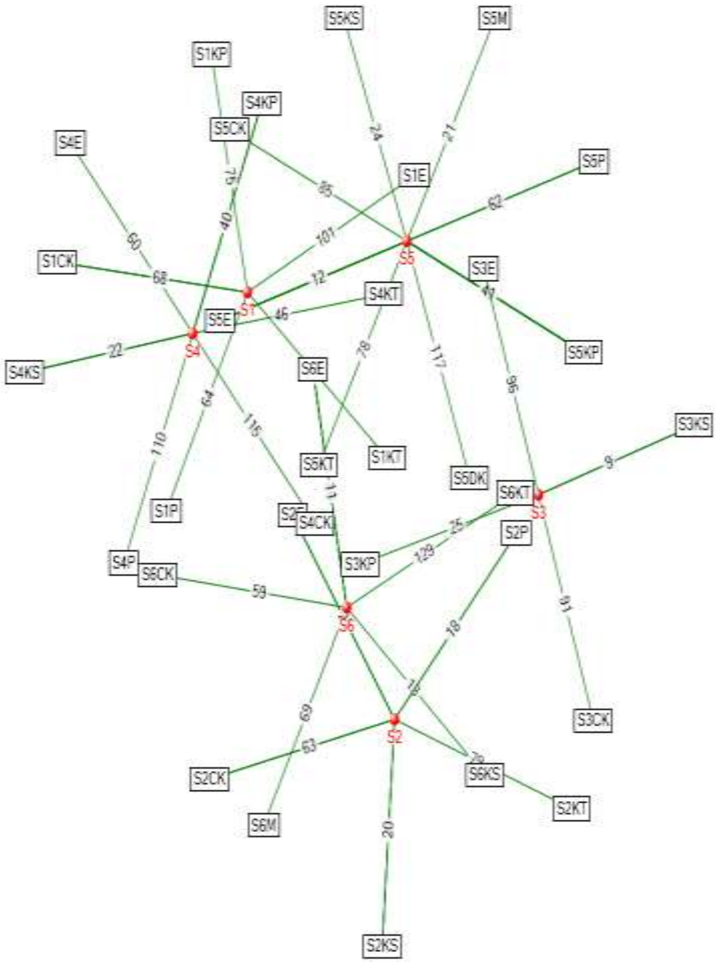
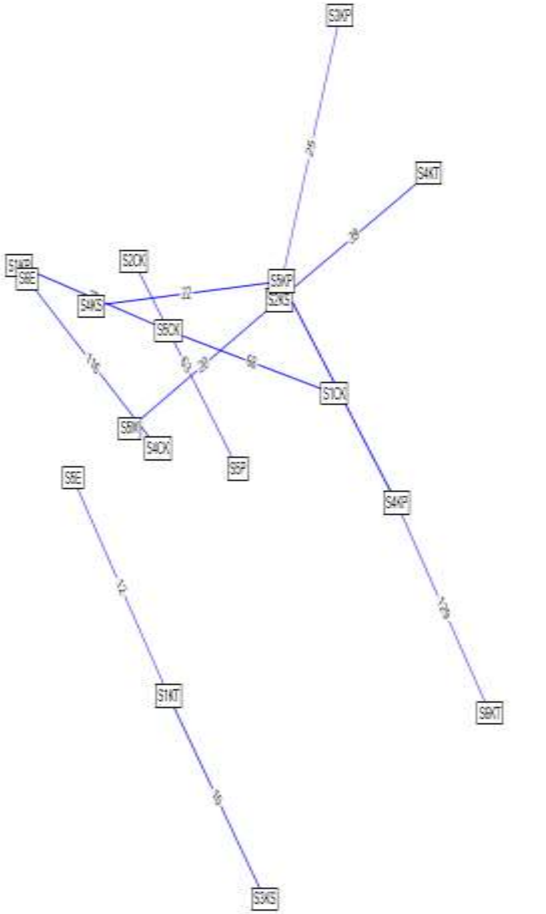
Social network	Interpersonal metacognitive network	Socially shared metacognitive network
		

Figure 6.4 Sample extracts of colour-separated networks in strata 1, 2 and 3 as viewed collectively in Figure 6.3

Stratum 1 is illustrated by the spheres labelled S1, S2, S3, S4, S5 and S6, and the ties between these nodes comprise the social network of the group. Each sphere symbolises a particular student who acts as a node in the social network. The ties between the students that connect two or more nodes, represent the social relationships among group members. To guide the reader through the network, line numbers were used representing a quotation line of the appropriate transcript coded to that particular student, serving as evidence of the relationship between two nodes. For example, *S1* and *S6* are connected by a line numbered *11*. The quotation reads: *Like an activity* [S1$^{1:11}$]. In this quotation, S1 showed her *knowledge of the task* by giving an example of a type of task that she thought would be appropriate to use, while S6 replied: *Yes* [S6$^{1:12}$], suggesting that she agreed that this type of task would be appropriate. S6 therefore agreed and *evaluated* S1's contribution in such a way that the social relationship was interactional in nature. In this sense, S6 had as a coordinating role whereas S1 served as a source of information.

Stratum 2 represents the interpersonal metacognitive network of each student. Again, line numbers are used to show the path of the metacognitive knowledge (CK, PK, DK), metacognitive knowledge level (KT, KS, KP) and metacognitive regulatory domains (P, M, E) indicated as nodes in the text boxes. The lines in this network are all connected to a student holding that particular metacognitive component. This constitutes the individual's metacognitive network. For example, S3's interpersonal metacognitive network (see lines 25, 96, 91, 9, 18) comprises KP, E and CK. Furthermore, these metacognitive components are either connected to other group members' networks (e.g. S3KP:S5:KP through line 25) or they remain disconnected (e.g. S3CK in line 91) but are still part of the student's network. It is important to note that the nodes in stratum 2 are individual in theory nature; however, when a node of one student's metacognitive domain is connected to that of another student, the result is a social-metacognitive network, represented in stratum 3.

The social tie between S3 and S1 in stratum 1 is indicated by line 10. A possible explanation of this tie lies in S3' knowledge of strategy (KS) in line 9 as it is connected to S1's knowledge of the task in line 10. These lines represent parts of S3 and S1's socially shared metacognitive network in Stratum 3. Collectively the networks in stratum 1, 2 and 3 represent Group A's metacognitive locale, a theoretical dimension consisting of three interrelated networks that are social, interpersonal and social-metacognitive in nature and which explains the relationship between students' metacognition, metacognitive language and networking.

6.11 Ethical issues of the research

According to Resnik (2014), ethical issues are the norms applied throughout research. Cohen *et al.* (2004) explain that ethical issues protect from harm, give informed consent, ensure privacy and honesty. Each student signed a letter of informed consent which permitted the researcher to include the student in the research. The letter of consent also informed students about voluntary participation and the use of pseudonyms and that all information would be treated confidentially. The students were allowed to withdraw from the study whenever they felt like it, as members from the third group did.

The following ethical issues (see Table 6.4) are derived from Creswell *et al.* (2007); Maree *et al.* (2012) and Welman *et al.*(2009) were also considered.

Table 6.4 Ethical issues considered

Ethical issue	**Approach followed in this study**
Anonymity	Students' right to privacy was protected by using pseudonyms, thus keeping them anonymous in the social, interpersonal and socially shared metacognitive networks.
Confidentiality	All information regarding schools and students was kept confidential.
Biased usage of language	Language and/or words in the study were not biased against any person, group or organisation.
Emotional/physical risk	No participant was exposed to any physical or emotional harm.
Rights of participants	Participants were not disempowered or marginalised. They were free to withdraw from the study at any time.
Generalisation of findings	The findings and discussions were generalised to the sample within the population and great care was taken to limit the research to the students' locale.
Research instruments	The research instruments were suitable for answering the research questions.

The ethical issues outlined in Table 6.4 ensured professional research practice and affected the trustworthiness of the research, discussed next.

6.12 Trustworthiness and issues of validity

Validity scrutinises effective research. As such, the following principles of validity for qualitative research proposed by Cohen *et al.* (2007) are considered.

6.12.1 The natural setting as a principal source of data

The university classroom where the participants received their training, the school classroom in which participants presented the research lesson and the venues where participants communicated about their ideas, were available natural settings familiar to the participants.

6.12.2 Socially situated data

The main enterprise of the education profession (collaborative teaching and learning opportunities) in South Africa was embraced by providing participants the opportunity to communicate freely and responsibly about their ideas. They shared their experiences and opinions about the necessary adjustments for the research lesson in a social setting.

6.12.3 The researcher is part of the researched world

Regarding the researcher as part of the researched world enhanced validity by allowing the researcher to be part of and familiarise himself with the research site.

6.12.4 There is holism in the research

Constructs of this study were explored to determine, explain and visualise the underpinning relationships between the participants and their use of metacognition within LS design, teaching and learning. This allowed the researcher to follow the participants throughout the whole process (holistic) of data collection and reflection.

6.12.5 The researcher is a key instrument in data collection

The researcher was not only an observer but also a facilitator who aimed not to teach the participants, but rather to ask questions that helped them reflect, in a sense, and unlocked their metacognition, expressing it for the purpose of this research.

6.12.6 There is a concern for process rather than outcomes

It was important that this study contribute to the educational development of the participants. Hence, the process of LS and design was incorporated in their existing mathematics methodology module. This allowed them the opportunity to use and share the skills, knowledge and attitudes that were fostered throughout the design sessions.

6.12.7 Seeing through the eyes of the participant and not the researcher

Since the participants’ views and interpretations of the PBL task, the LS approach, the design of the research lesson and its presentation were at the centre of this research, specifically their contributions to the design sessions led to the understanding of their metacognition and overall, their metacognitive locale. The researcher aimed to reveal the participants' expressions in their own words (through verbatim transcriptions), reflecting and letting them reflect upon the value it had for the PBL task, which in itself served as a tool for the research.

6.12.8 Participants' validation is important

Because the participants' expressions were videorecorded, observed and reflected upon in the discussions, the researcher interpreted their expressions and not his own.

The data collection methods and analysis were as detailed as possible, emphasising their descriptive validity. The interpretations were not those of the researcher, but the accumulated views of the participants' expressions about their experiences. This ensured the interpretive validity. The theoretical validity of this study therefore lies in the explanations of the constructs scrutinised for each participant in terms of all the variables to answer the research question(s). Since this study aimed to contribute a local theory, the generalising validity was ensured by only generalising the findings within the sample of this study. Evaluative validity was improved by making use of the videorecorded lesson presentation, as it served as a means of validating what the participants interpreted from their experiences and reflections. Validity was further ensured by prolonged engagement in the field as the educational design process consisted of many opportunities such as design group sessions and videorecorded lesson presentations to collect multiple data types (Cohen *et al.*, 2011).

6.12.9 Inter-coder reliability

In addition to the trustworthiness issues outlined above, the researcher also made use of inter-coder reliability (Nieuwenhuis, 2010). Inter-coder reliability refers to the extent to which the same codes were assigned to transcribed text by different coders. Intra-coder reliability therefore refers to the consistency of coding the same idea. Throughout the current study, the researcher and supervisor collaborated on the consistency of the codes and the ideas that were coded to ensure that the findings presented reflect this consistency and can therefore be considered trustworthy and reliable.

6.12.10 Crystallisation

Besides the abovementioned issues of validity, crystallisation was also employed as a validation strategy (Creswell & Plano Clark, 2011). To enhance the paradigmatic interpretations in this study, crystallisation was used because it relates to multiple data sources and theoretical lenses (Ellingson, 2008). The aim was not to assume a more valid truth, as is the case with triangulation, but to open up a more complex and thorough understanding of the issues explored. In addition, multi-vocality ensured that the participants' views were recognised along with the members' reflections (Tracy, 2010). Crystallisation validates the findings through multiple methods of data collection and analysis. In this study, crystallisation provided a "deepened, complex and thoroughly partial understanding of the topic" (Maree *et al.*, 2012:41).

In addition to the issues of validity and trustworthiness discussed above, researcher bias was removed by clarifying the researcher's conceptual-theoretical, theoretical and philosophical assumptions (see 6.2) before starting the research whose findings are presented in chapters 7 and 8.

6.13 Chapter summary

Chapter 6 positions the research methodology of this study in terms of three overarching methodological approaches. First, Figure 6.1 shows the concepts and constructs in terms of the theoretical framework of this study. It was conceptualised that the implementation of the LIT produce constructs to theorise about in a LT. Figure 6.2 shows how the PBL steps, the LIT phases and the EDR cycles interact and explains the data collection and analysis procedures. In particular, a unique three-step process of data analysis is offered in Table 6.3 to show the path of data analysis with such software packages as Word, Atlas.ti and NodeXL. Chapter 6 also presents a way in which the findings obtained from this process can be interpreted. The chapter closed with a discussion on the trustworthiness, reliability and ethics issues. The findings obtained from implementing the research design outlined in Chapter 6 are presented in Chapter 7 (for Group A) and Chapter 8 (for Group B).

Chapter 7

Presentation of the findings of Group A

7.1 Introduction

Chapter 6 described the research design and methodology of the study. The data collection procedures and analysis plan were emphasised to understand and explore the constructs of metacognition, metacognitive language and metacognitive networking as they emerge from the LIT. The LIT consists of guidelines that reflect the phases of LS with the embedded principles of PBL and the HLT. Chapter 7 presents the findings that were obtained in the first and second educational design-based research cycles of Group A.

The analysis of the data were done with the conceptual-theoretical framework (see Figure 5.6 & Figure 6.1) in mind. The findings were obtained by implementing the LIT based on a HLT of LS within the context of mathematics education through a PBL instructional philosophy to improve the quality of teaching-learning and doing mathematics. The design sessions focused on the analysis of a PBL task and the planning, design, presenting, refinement and re-presentation of a research lesson based on the concept of place value for a Grade 6 intermediate phase mathematics classroom situated in a rural area, as explained in Chapter 3. Conceptually, metacognition was analysed according to the themes of metacognition, metacognitive knowledge and the metacognitive regulation, and through interpretivism and the hermeneutics of the metacognitive language participants used to express their opinions, ideas, views and observations during the LIT phases.

7.1.1 Research focus during analysis

The primary research question to answer was:

How can metacognitive language and metacognitive networks foster group members' metacognitive locale in teaching-learning and doing mathematics?

To answer this question, three secondary research questions were identified and explored.

In order to answer the research question, to understand group members' metacognitive language and networking, and to develop a theory of group members' metacognitive locale, a series of design sessions were conducted over a period of two semesters with two groups of final-year mathematics education group members (Group A, n = 6; Group B, n = 5). After analysing the transcripts of the design sessions, three constructs emerged that constituted group members' metacognitive locale in the form of a social, interpersonal metacognitive and socially shared metacognitive network. The results of the networks were further classified under the following sections:

- Stratum 1 – social network illustrated with a sociogram
- Stratum 2 – interpersonal metacognitive network illustrated with a metagram
- Stratum 3 – socially shared metacognitive network illustrated with a sociometagram.

For analysis and crystallisation purposes, the research design and presentation of the findings are limited to the conceptual-theoretical framework of this study (Leshem, 2011). In this study, the researcher found other themes as emerging constructs after implementing the LIT relating to LS and PBL. However, themes such as affective issues, linguistics, beliefs and values did not form part of the scope of this research or the conceptual-theoretical framework and were excluded from analysis. The analysed findings presented here only pertain to the constructs of metacognition, metacognitive language and networking across the three strata. Since the analysis of the social network was done solely for merging the group members' position and relationship in the social network with their interpersonal metacognitive networks, further analysis on the network metrics was also omitted. The initial goal, then, of stratum 1 is to comprehend the basic structure of the social network around which the group members' metacognition developed. An outline of the presented findings is discussed in the following section.

7.1.2 Preliminary discussion on the presentation of the results

In Chapter 8, the findings are presented in a similar fashion as of those in Chapter 7. The ways in which the results are presented are the same. This preliminary discussion therefore refers to the findings of Group A and Group B. First, a network map of each stratum's illustrated networks is provided for each of the two groups. The findings of the illustrated networks are then presented and, where necessary, group members' views, observations, reflections or responses in the design group sessions are presented to support the information embedded in the network illustrations.

In stratum 1, the social network's findings are given in terms of the role(s) group members play in the network, with the accompanying relationships (ties) between members of each group. This served as the framework or basis on which the next strata developed. In stratum

2, findings of each group member's interpersonal metacognitive networks are presented in terms of the metacognitive knowledge and regulatory domains of the network and the metacognitive language used to express these networks. The interpersonal metacognitive networks are then aligned against the role of the group member in the social network. In stratum 3, findings of the group's socially shared metacognitive network are presented by merging the social network and the interpersonal metacognitive networks, focussing on the relationships between group members. Hence, each stratum reflects components of metacognition, metacognitive language and networking as emerging constructs of the implemented LIT. The findings are then interpreted and discussed in Chapter 9 where the crystallisation of the findings aids in building the theory of metacognitive locale. Even through the LIT's phases structured the research design, its purpose was solely to create the locality of the study, limiting the study from other content of the module and focussing only on the PBL philosophy that underpins the study. The presentation of the findings for strata 1, 2 and 3 are then offered here, hinting of the LIT phases but not encapsulated by their unique qualities. Therefore, the LIT phases were not refined, as the purpose was to explain the constructs and their relationship, not to refine the LIT. This was conceptualised in Chapter 2. It is possible that other routes could be taken for developing LT, refining LIT and/or predicting constructs. However, for the purpose of this study only route A (see Figure 2.3) was followed.

7.2. Metacognition, metacognitive language and networking as emerging constructs of the LIT

Group members offered up extra time to devote to the design sessions:

> *We have a group assembly [17:S3:247][40] every week Tuesday half past seven we get together [17: S2:247]...for two hours [18: S1:247]*

Based on the philosophical principles of PBL and LS, it was anticipated that implementing the LIT phases would allow the constructs of metacognition, metacognitive language and networking to emerge. The design sessions allowed the researcher to explore the constructs as components and to theorise about them to produce a LT. The findings of the verbatim transcribed responses, analysed and coded, were arranged in three themes that confirm the emerging constructs of the LIT.

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[40] The numbers here indicate where the quotation comes from in the original transcription. The number indicates what document the quote is taken from (17), the second identifies the participant (Student 3) and the third shows the line number (247) in the transcription where the quotation lies.

7.2.1 The construct of metacognition

Group members from both groups showed growth in their understanding of LS and its value for practice as they learnt to think about their thinking and to manage their ideas, a metacognitive conduct. At first they were unaware of knowing how to plan and set realistic goals for LS. They had diverse ideas that were "scattered" as they were unsure about what activities to include in their lesson and how to choose the most suitable ideas. They did not know what to focus on, when to focus on the content and/or when to focus on the methodology of the lesson:

> *I would say that we were not sure what to expect… We have to start all over… We felt that our ideas were not important as some ideas will work and others' not… Now we are much more confident…*

Their growth includes relating the theory of LS with the practice of LS:

> *There are many ideas which we can do better…it was very insightful…we looked at how the learners did the work…they all seem to understand the work, but they made the same small errors… If we had to do it again, we will be better prepared…*

Group members reflected on the design of the lesson and on its presentation, thus reflecting on the teaching and learning experience. In the second and third session, they divided the work amongst themselves and arranged additional sessions. They worked together, collaborated and made decisions about their ideas, specifying what to include and exclude in the lesson. Even though they managed their resources (e.g. time) better, they still shared an important concern:

> *We talk past each other and do not understand what the other one means.*

This suggests that they became aware of a metacognitive language necessary to succeed in their attempts.

7.2.2 The construct of metacognitive language

Not being able to express their reasoning clearly, and lacking the vocabulary of their thinking about their thinking, group members explained that even though they worked together, they often became frustrated when their ideas were not understood or recognised:

> *But we do not understand what each other mean. I don't know what they don't know and what someone else is thinking about my idea [1-5A:19-21]…we had too many good activities and we do not know how to say that we do not want to use this or that idea…*

Group members also seemed to have evolved in their awareness of this language in the sense that:

> *We now know each other and we can say this or that won't work. At first, we were afraid of what someone else might say…we didn't want to demotivate anyone… and felt that we need some way of saying what we think…*

In order to get focus, group members discussed what their lesson outcomes should be. This presented a problem, not only for interpreting the curriculum documents, but also to put down in words a clear and explicit lesson plan and to communicate about what they expected from their learners at the end of the lesson:

> *We have an outcome, we want them to be able to identify and apply…The outcomes must be smart. It must be specific, measurable, attainable, relevant and traceable…so we have to look at it again…*

The nature of networking between group members also developed along with their metacognition and metacognitive language.

7.2.3 The construct of networking

Group members later decided to work with each other's ideas, and not necessarily to come up with new ones all the time:

> *We are all here, we know what to do. We have to take everyone's ideas into consideration…We started with an idea and talked about it and made a combination of all the ideas. It helped us to understand the concept better… Lesson study is difficult if one student feels separated from another… When we get together we can say, this stuff works great and then we take other ideas and we build on that.*

Group members had realised what their strengths and weaknesses were, and that the nature of their networking was likely to be the key to overcome the barriers group members had during LS.

Metacognition, metacognitive language and networking appear to relate to the conceptual framework of the experiences with LS when aligned against the LIT, offering a view on some implication of LS, the LIT and the development of LT. Even so, the three constructs exist on three levels, or strata, which explains the emergence of each construct either on a social or interpersonal level. The first level's findings are presented in terms of stratum 1, the social network.

7.3 Stratum 1: Analysis of Group A's social network

Two themes that act as functions of the social network of Group A frame the first stratum. The themes of *social role* and *social relationship* (Table 5.1) were identified for this purpose. Even though this was not the aim of the study, the roles and relationships of the social network facilitated an understanding of the social network that served as the framework on and around which the theory of metacognitive locale was built. Figure 7.1 shows the social network of Group A as it emerged throughout the design sessions.

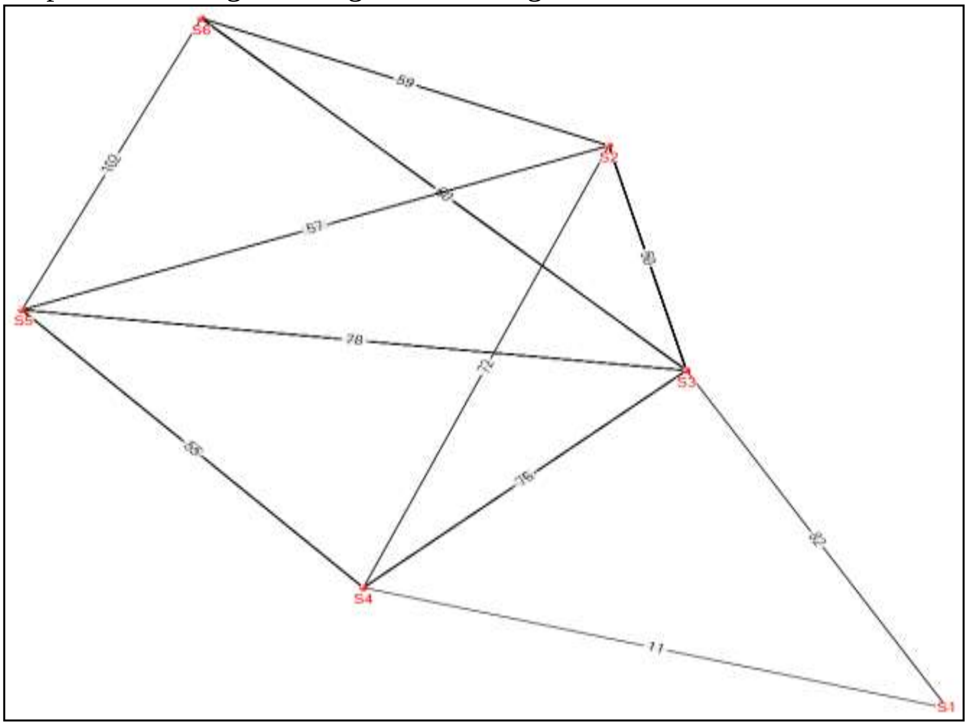


Figure 7.1 The social network map of Group A across the design group sessions, illustrated using NodeXL

From the analysis of the transcriptions of the design sessions that were coded as social network data (e.g. who responded to whose ideas) 11 social network ties (or lines) were revealed. Each tie in Figure 7.1 represents a group of lines of communications via the social relationship(s) between group members in the network. Therefore, each line number in stratum 1 consists of a group of coded quotations between group members and depicts the relationships between them and their roles and contributions during the LIT phases. This was done because, as Butts (2008) suggests, the analysis of the roles in the social network is a means to reduce the complexity of the network architecture.

7.3.1 Social roles in the network

The social roles in Table 5.1 were used to identify group members to determine their role in the first stratum. Since the focus is not on the social network metrics, the direction of the network nodes was omitted, with roles such as representatives and gatekeepers not identified (de Nooy, Mrvar, & Batagelij, 2011). A number of roles were present and are discussed here.

7.3.1.1 The star

The strongest node in stratum 1 appears to be S3 as he has connections with all group members (S1, S2, S4, S5 & S6) and can be referred to as the star of the social network in Group A. S3 has many social ties with the rest of the group and seems to play a central role by acting positive about the decisions made and accessing and managing information:

> *I think we now have an idea for a lesson. Let's begin and expand…*
>
> *[1:S3:49]*

His role of "star" seems to be associated with asking questions and gathering information regarding the main points of the lesson plan. This includes the outcomes, content, teaching strategies and assessment. He does this by asking such questions as:

> *Why don't we test their prior knowledge before the lesson? [S3:246:70]…why not do it a day before …? [1:S3:76]…what is our outcomes again? [1:S3:99]…can you bring it in with the base ten blocks? [1:S3:173]…Do they have to represent it? [2:S3:124]…*

7.3.1.2 The liaison

The liaison seems to be S1. She asked if they could suggest a school at which they wanted to present their lesson. This is based on her previous experience at a school with, according to her, the same contextual issues as highlighted in the PBL task.

> *Can we choose the school? [6:S1:284]…their [spoken] language is very good, but do not let them write. Maybe it is because I taught English there [19:S1:269]…it's about 5 kilos on [19:S1:294]…some sit three at a desk [19:S1:461].*

7.3.1.3 The bridge

In Figure 7.1 it appears as if S3 and S4 act as bridges between S1 and the rest of the network. Van Staden (2011) explains that the "bridge" permits weaker linked individuals in the network to stay connected with the rest of the group by allowing the information to flow between group members. Throughout the LIT phases, S3 and S4 seem to provide information to S1 and appear to progress S1's ideas by translating them for the group, for example:

> *Look, you can still give the groups names and tell them but there are now hundreds… [2:S3:220]; or you have to represent it physically [2:S1:222]; this is where the base ten blocks come in [2:S4:226]*

When another group member (e.g. S5) responds with a counter-example or similar idea, S3 and S4 support S1:

> *But we are going to…use matches, rocks and paper [2:S5:228]; Matches are very cheap [2:S1:229]; Yes [2:S3:230]; Just don't let them [the learners] walk away with it [2:S4:231]*

S2 formed social relations with four of the group members (S6, S5, S4 & S3). Her role as a bridge in the network seems based on the questions she asked:

> *But how are we going to test their prior knowledge? [1:S2:52]…How can we can do revision? [1:S6:59]…Can we give an activity? [1:S6:68]*

7.3.1.4 The investigator

In a way S4 expresses an investigative role when S1 presents a template to the group that they can use for the lesson plan, and considers some suggestions:

> *Here is a lesson plan template that I have made. It asks for learning outcomes, skills, assessment standard and question… [5:S1:71]. Just read the learning outcomes again [5:S4:80]… We have the lesson outcomes, we just have to adapt from there [5:S4:8]… I like the blocks they have, we can print those blocks out [5:S4:15].*

After the research lesson, S4 asked for the videorecording to help her reflect on the lesson.

7.3.1.5 The socially weak member

One characteristic of a socially weak member is that, according to Van Staden (2011), the individual prefers communicating over social media instead of elaborating on ideas in face-to-face conditions. When the researcher asked the group members to discuss the things they wished to change in the lesson plan S2 asked S1:

> *You are very quiet, so why? [18:S2:193]*

Directly afterwards, S4 also stated:

> *Yes [S1] you have not said anything so far [18:S4:195]*

The researcher, as facilitator, wanted to abide by the ethical issues and did not want S1 to feel uncomfortable. He suggested an alternative through which all group members could share what they thought, and in doing so steered attention away from S1. The researcher then proposed that group members write their ideas in a reflective journal and submit them individually. This alternative was met with enthusiasm as S1 replied:

Sharp…I like that idea [S1:411] …We all like it… [S1:415]

The role of a socially weak member seems to fit S1 when reflecting on another statement that refers to the completion of the lesson study template:

I completed my own form [S1:68]

Furthermore, the brokerage roles that were identified are coordinator and liaison.

7.3.1.6 The coordinator(s)

Often group members found themselves getting distracted from the general aim, the overarching lesson study goal and the planning of the research lesson. In such cases, it seemed as if S4 steered the group back to focus, providing reasons along the way:

> *Now we are thinking too far into the future [1:S411]...then we begin thinking of something else [1:S4:36]...just build a foundation of place value first [2:S4:32]...*

Since no other social ties existed to suggest that S1 was related to S5, S6 or S2; only S4 and S3 served as coordinators in the group. This happened when S1 raised a concern or issue and S4 and/or S3 coordinated the issue in line with what other group members said.

The analysis of the patterns of the individual roles in stratum 1 was required to make sense of the kinds of interactions in the group. In effect, Gee (2014) explains that such domain-specific patterns can be used to transform theory by explaining and modelling the structures in the network. For this purpose, the social relationships between the group members were scrutinised in the next section.

7.3.2 Social relationships

Mainly social roles exhibited two types of social relationships: meta-affective relationships and cognitive relationships, represented in Figure 7.2.

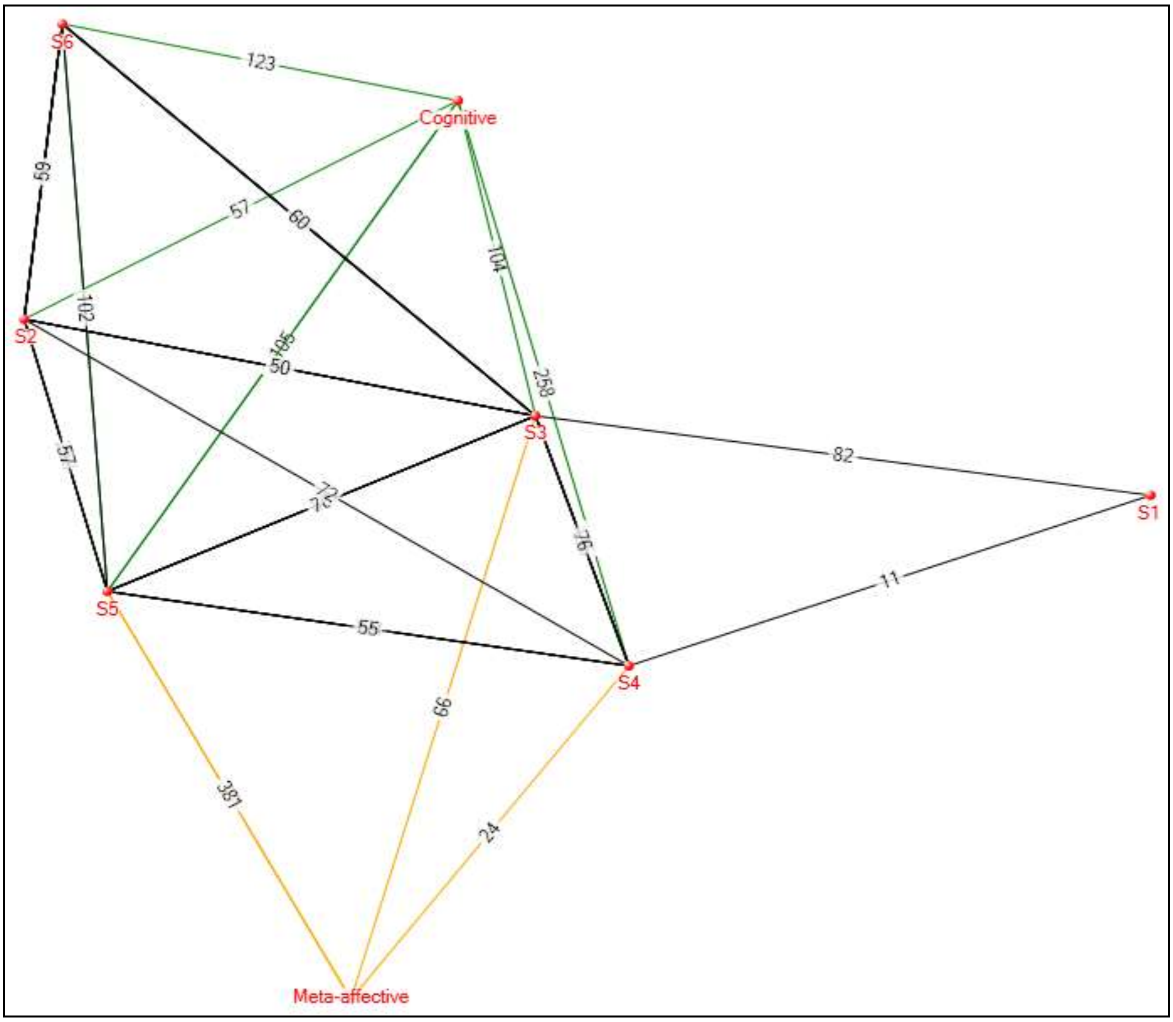


Figure 7.2 The presence of cognitive and meta-affective relationships in the social network

Group members' meta-affective and cognitive relationships are embedded in the social network. These relationships seem to exist between S3, S4 and S5 (line 24, 66 & 381) for the meta-affective social relationship and between S2, S3, S4, S5 and S6 (line 123, 258, 57, 104 and 105) for the cognitive relationship. Note that S1 was not connected cognitively or meta-affectively by any member in the group.

7.3.2.1 Meta-affective relationships in the network

Even though the aim of the study does not lean towards an impression of affective issues, the findings did offer a view on group members' *feeling of knowing.* In order to elucidate the metacognitive networks, such metalevel feelings are associated with metacognitive experiences (Efklides, 2011), and thus the presence of a meta-affective relationship was considered to be instrumental in understanding metacognition on the social level. For example, S2 seems to speak for the whole group when she claims that they *feel* that the use of larger numbers in the Grade 6 class's place value task helps the learners understand better and that:

> *We feel that the learners know colours better and they will visualise it better [3:S2:41]…I feel that it helped in the one group [6:S2:306]*

The presence of this *feeling* was also experienced and acknowledged by others in the group:

> *I feel that there is not enough emphasis on the pronunciation of the words [6:S5:60]; …we don't feel that its right [2:S4:1]… it feels as though we need a model [2:S3:3]*

These noetic feelings show the meta-affective relationships in the social network and can be seen as a form of monitoring (Lai, 2011). Besides these meta-affective relationships the social roles also seem to relate to the cognitive relationships embedded in the network.

7.3.2.2 Cognitive relationships in the network

Cognitive relationships exist when group members communicate about their understanding of the concepts involved (e.g. in LS and the components of the research lesson's plan). These relationships were identified when cues were used where group members specifically focussed their attention on aspects such as activities, tasks, lesson plans, outcomes, goals, assessment and strategies.

Thinking about and knowing the lesson plan's outcomes seemed central as a cognitive relationship where group members collaborated about the activities:

> *Learners must be able to represent place value using base ten blocks [1:S5:100], we can then quickly break... [1:S3:103]…our outcomes [1:S5:104]…yes, our outcomes… [1:S2:105]*

The analysis of the social roles and social relationships in stratum 1 revealed distinctive characteristics of each group member in the group that are helpful for facilitating an understanding of their metacognitive networks. Table 7.1 summarises the roles and relationships of Group A.

Table 7.1 Summary of the roles and relationships in Group A's stratum 1

Social role	Meta-affective relationship	Cognitive relationship
Star	S3	S3
Bridge	S2, S3, S4	S2, S3
Coordinator	S3, S4	S3
Socially weak member	*	
Investigator	S4	
Liaison	*	
Group member	S5	S5

*Even though S1 was identified as the socially weak member and the liaison of the group, she did not have a meta-affective or cognitive relationship with any of the group members.

The findings also indicate that a particular social role associates with the relationships that formed between group members. In order to explore these individual and shared qualities in terms of metacognition, stratum 2 shows the findings to understand and interpret the interpersonal metacognitive networks that have emerged from implementing the LIT.

7.4 Stratum 2: Analysis of Group A's interpersonal metacognitive networks

In stratum 2, an account is given of the interpersonal metacognitive network for each of the six members of Group A. These accounts are given according to the categories in Table 6.2 (DK, PK, CK, KT, KP, KS, P, M, E) and for each group member individually (S1, S2, S3, S4, S5, S6). The interrelatedness and overlapping nature of group members' networks are synthesised and presented in this chapter and discussed in more detail when aligned in Chapter 8. The analyses of the group members' networks are presented consecutively, starting with Student 1 (S1) and ending with Student 6 (S6).

Figure 7.3 shows an overview of how Group A's interpersonal metacognitive networks are connected via the social network.

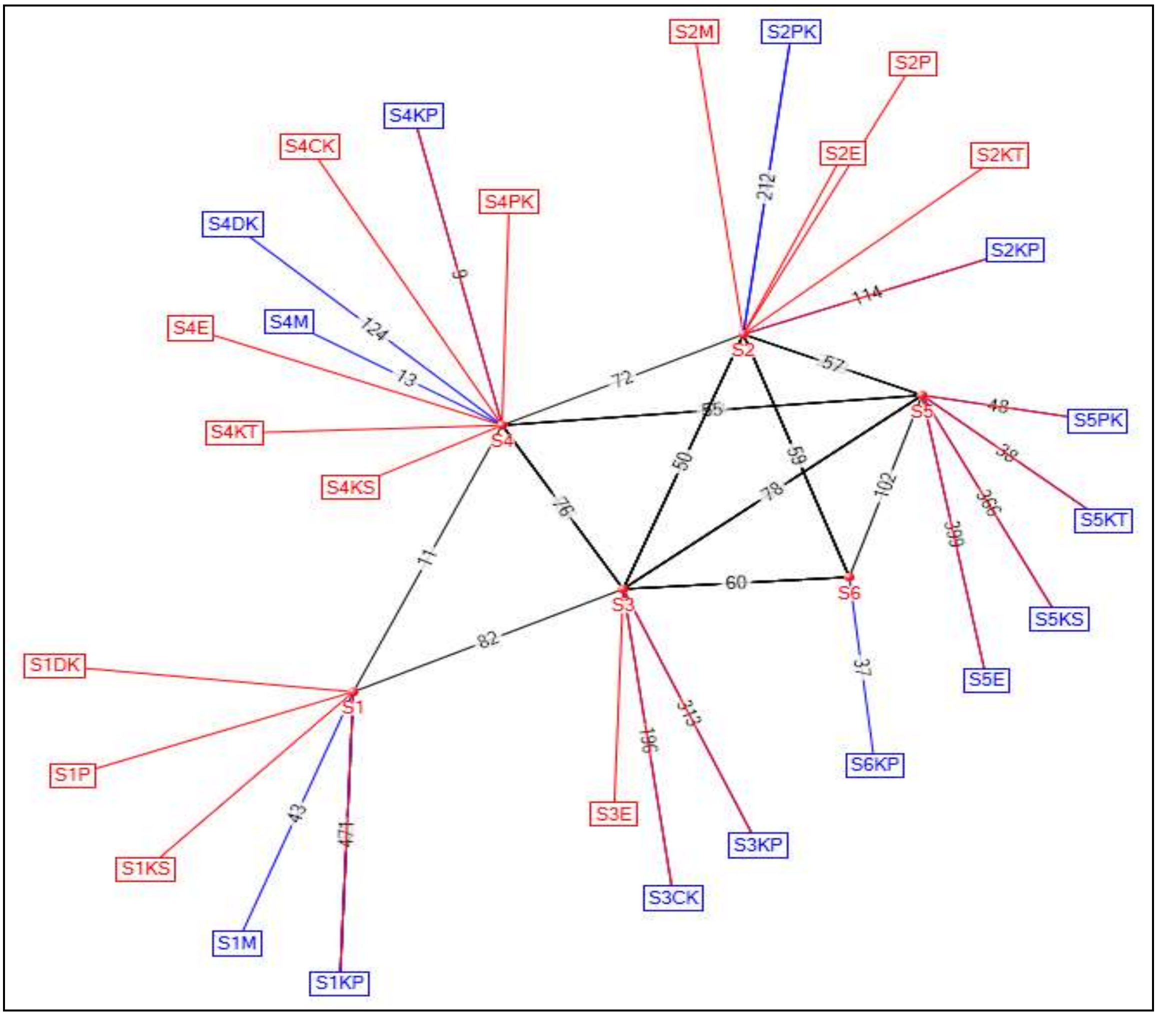


Figure 7.3 Group A's interpersonal metacognitive networks

In Figure 7.3 group members differed in terms of their interpersonal metacognitive networks. These networks reveal what metacognitive knowledge or regulatory processes each group member expressed during the LIT phases. The weakest network seems to lie with S6 (line 37), who also does not have any social relations with the rest of the group (see Table 7.1), whereas the strongest network belongs to S4 (adjacent to line 72 and 76).

The account of each group member's interpersonal metacognitive networks in stratum 2 will now follow.

7.4.1 Account of Student 1's interpersonal metacognitive network

Suggesting that the outcomes are the starting point of the lesson plan showed that S1 was strategically ***planning*** her approach:

> *I think we must first start with the outcomes because the outcome is going to be our basic guideline [5:S1:29]*

While judging the quality of the teaching experience they are planning, S1 compares what they are doing against what happens in the school who they are preparing the lesson for, showing that she is ***monitoring*** their efforts:

> *I think not at all, because what we learn here and what they are doing there is entirely different [19:S1:34]*

Even though the interpersonal metacognitive networks were identified using the mental verbs as key indicators (e.g. *think(ing) or know(ing))* there were also components of group members' metacognitive knowledge and regulatory domains that were not directly associated (or coded) with the mental verbs. For example:

S1 expressed in terms of her ***declarative knowledge***:

> *First we are going to write the numbers on the board, then we underline like a certain place value and then they have to say… [6:S1:142]…we then like give learners a number and they have to stick it in the right column [6:S1:159]*

By reflecting back to the first presentation of the lesson, S1's ***knowledge of the strategies*** regarding teaching became evident:

> *I told him* [a learner] *that if you have ten of them it makes this, then I said that the hundred can be represented in that big square, because then, look, it fits in there. [20:S1:108]*

S1' ***knowledge of the person*** appears to be based on an assumption of Grade 6 learners' strengths and weakness, who she taught before, when she states:

> *They are kids. They don't even know what volume is [1:S1:472]...the kids don't get these things right: adding and subtracting [2:S1:18]*

7.4.2 Account of Student 2's interpersonal metacognitive network

In terms of ***planning*** S2 suggested:

> *…write what we are going to do in the introduction, what we are going to teach and learn and what we are going to do at the end and the activities also [6:S2:11]*

When ***monitoring*** their decision about using a terminology poster in the classroom, S2 said:

> *But look, last time we said that there is a problem that not all the kids are English, can read English [1:S2:69]*

It seemed important to S2 that the lesson plan accommodate learners with a visual preference for learning, showing that she has some ***knowledge of the person***(s) for whom she is preparing the lesson:

Use the blocks...then it is visual for them [1:S2:152]

Reflection on the presented lesson gave this group member the opportunity to ***evaluate*** their efforts:

Yes, I think our lesson worked. I think it worked very well [20:S2:33]

When asked how they planned to assess learners' prior knowledge about place value, S2 expressed her ***procedural knowledge*** in this regard:

Say for example a kid does not understand…you then not only work with one kid. Everybody must be able to understand. So then you explain using the columns, saying exactly the same as you would for the individual. But let us look at how they write it in expanded notation. So if the kid does not understand the one way, we can show them the other. [2:S2:240]

S2's ***knowledge of the task*** was focused on the lesson plan as the task itself:

Yes, and from there you can approach your teaching and learning phase [5:S2:6]

7.4.3 Account of Student 3's interpersonal metacognitive network

Giving praise for a suggestion, thus ***evaluating***, was one of S3's *star* qualities:

Even if they don't get around to it, they can still tell the teacher…and she can still use the idea. It's a good idea [4:S3:64]

When it was suggested that they use a PowerPoint presentation as part of their teaching approach, S3 was hesitant to accept and responded by highlighting the conditions under which such technology will not fit:

Okay, I do not think they have technology and stuff in those schools, so you cannot use PowerPoint there you are going to use the black board… [1:S3:183]

When reflecting on the ***knowledge of the person,*** S3 cautioned:

But listen here, the kids do not get these things right [2:S3:19]

This student also reflects on the PBL task, highlighting his ***conditional knowledge:***

OK, I don't there is so much technology in that school. So you cannot use the PowerPoint there [1:S3:183]

7.4.4 Account of Student 4's interpersonal metacognitive network

When reflecting on what group members should do in circumstances where learners are struggling with distinguishing between the ascending and descending values, S4's ***conditional knowledge*** seems to be general, claiming that circumstances determine what you do:

We feel that you do as is necessary… [6:S4:276]

S4's ***knowledge of strategies*** about teaching was associated with a feeling of knowing:

If you perhaps see that the kids are struggling …then you ask…and we just let them turn the question around… [6:S4:278]

The ***procedural knowledge*** of S4 relates to how a concept will be taught and the ***knowledge of the person***:

Then we begin thinking about something else, we then have half a concept of, okay, they know what it is. Then we can go on to adding and subtracting and the smaller stuff [2:S4:36]

When explaining what the introduction of the lesson entails. S4's ***declarative knowledge*** emerged:

Then we are going to use our worksheets for expanded notation that we also use for assessing the prior knowledge, because it comes from Grade 5. [7:S4:124]

Reflecting on the value of the lesson plan, S4 showed that she could ***monitor*** the progress of the learners and has ***knowledge of task*** of lesson planning:

I just feel that it is not a rigid thing that you can use. If, say, there is something we feel is necessary or that we want to do but unforeseen circumstances…then we need to spend more time on that [6:S4:226]

Upon ***evaluating*** the use of the base ten blocks, S4 said:

Yes, yes but we also say the big block and the small blocks...but then volume comes in again. [1:S4:141]

7.4.5 Account of Student 5's interpersonal metacognitive network

At some point, group members discussed the importance of allowing learners to use calculators and other tools in their teaching. When S5 suggested that they should give the learners something to work with, rather than work on, she explained her ***knowledge of strategies*** that would help them:

So that you can encourage them and tell them its fine, go back, take out the paper, put it next to you and work from there. [4:S5:18],

She explains this further when expressing her ***procedural knowledge*** and her ***knowledge of the task:***

I think also that…we also talk to them about going from the more basic to harder, to move from there to what they understand where they can apply what they just learnt in the lessons. [4:S5:48]

Sounding excited about what they have planned, S5 ***evaluated*** their efforts critically:

I don't think this is part of their prior knowledge…but I like the idea, this is going to work [1:S5:399]

7.4.6 Account of Student 6's interpersonal metacognitive network

This student seemed a bit distracted from the sessions while group members gave their ideas and shared their experience. She expressed herself vaguely when saying:

Yes, no, I don't know [1:S6:120].

She was not as quiet and withdrawn as S1, but her expressions made it difficult to code her metacognitive network, giving the impression that she was either waiting for the bigger picture:

So where are we going from here? [1:S6:175]

or looking for someone to conceptualise and explain it all to her differently:

So, what you saying? [1:S6:101]

She did, however, express a single cue to show her ***knowledge of the person***, hinting that she knew what amount of work these learners could deal with in one period:

For me, the thing is, this whole of the decimal story is just too much for the kids in one lesson [2:S6:37].

Table 7.2 provides a summary of the interpersonal metacognitive networks of the group members in Group A. Afterwards, the findings of stratum 1 and stratum 2 were aligned to reveal stratum 3, which develops alongside the metacognitive knowledge and regulatory processes expressed by group members' metacognitive language.

Table 7.2 Summary of Group A's interpersonal metacognitive networks

Group member	Metacognitive knowledge	Level of metacognitive knowledge	Metacognitive regulation
S1	DK	KS	P, M
S2	PK	KP, KT	P, M, E
S3	CK	KP	E
S4	CK, PK, DK	KS, KT	M, E
S5	PK	KS, KT	E
S6		KP	

Through their expressions of their metacognitive processes, group members provided a profile of their unique contributions to the group. In stratum 3, these contributions are looked upon in terms of how the interpersonal metacognitive networks and the social networks (in strata 2 and 1) respectively surface towards an overarching network reflecting group members' socially shared metacognitive processes.

7.5 Stratum 3: Analysis of Group A's socially shared metacognitive networks

Metacognitive language appeared to play an important role when group members monitored whether they were understood:

> *Do you understand what I mean? [1:S3:126],*

or when they wanted to find out if they understood correctly:

> *…do you get what I am saying? [S6:246:131]...I get what you are saying [1:S3:132]…Okay, so now we have two things...what is place value? ... and what is base ten blocks? [1:S3:157]*

Group members evaluated not only their own contributions to the design sessions, but more so the contributions of their peers. Particularly, group members evaluated each other's ideas by either agreeing with "*yes, hmmm or because...*" or when in disagreement they gave an explanation instead of saying no:

> *I don't think this is part of their prior knowledge [7:S5:247]...they will not know with their prior knowledge what this number is [2:S5:262] We shouldn't think too much about making it interesting for them...,we must just stick with the columns [2:S2:212]*

The findings of stratum 3 are outlined in terms of the conceptual framework, that is, where each individual's interpersonal metacognitive network was seen as the result of or the catalyst for the other group members' networks. For example, one group member may pose a question or idea to the group, expressed through a form of metacognitive language. Another group member responds to this after reflecting. The response is a component of either metacognitive knowledge or regulation expressed through a similar or different metacognitive language.

In each of the LIT phases, group members collaborated about the decisions regarding the research lesson. In the following section, Figure 7.4 shows the findings of each of the LIT phases and what metacognitive language group members used to express their metacognitive processes. Aligning this language with the metacognitive components was seen as necessary to link group members' social and interpersonal metacognitive networks to identify and illustrate the embedded socially shared metacognitive networks in stratum 3.

Even though the focus is not on the LIT phases, using the phases as an outline for presenting the results of stratum 3 served as a means to reduce the amount of data to provide an in-depth analysis of the results. The accumulated socially shared metacognitive networks as they have emerged are illustrated in Figure 7.4.

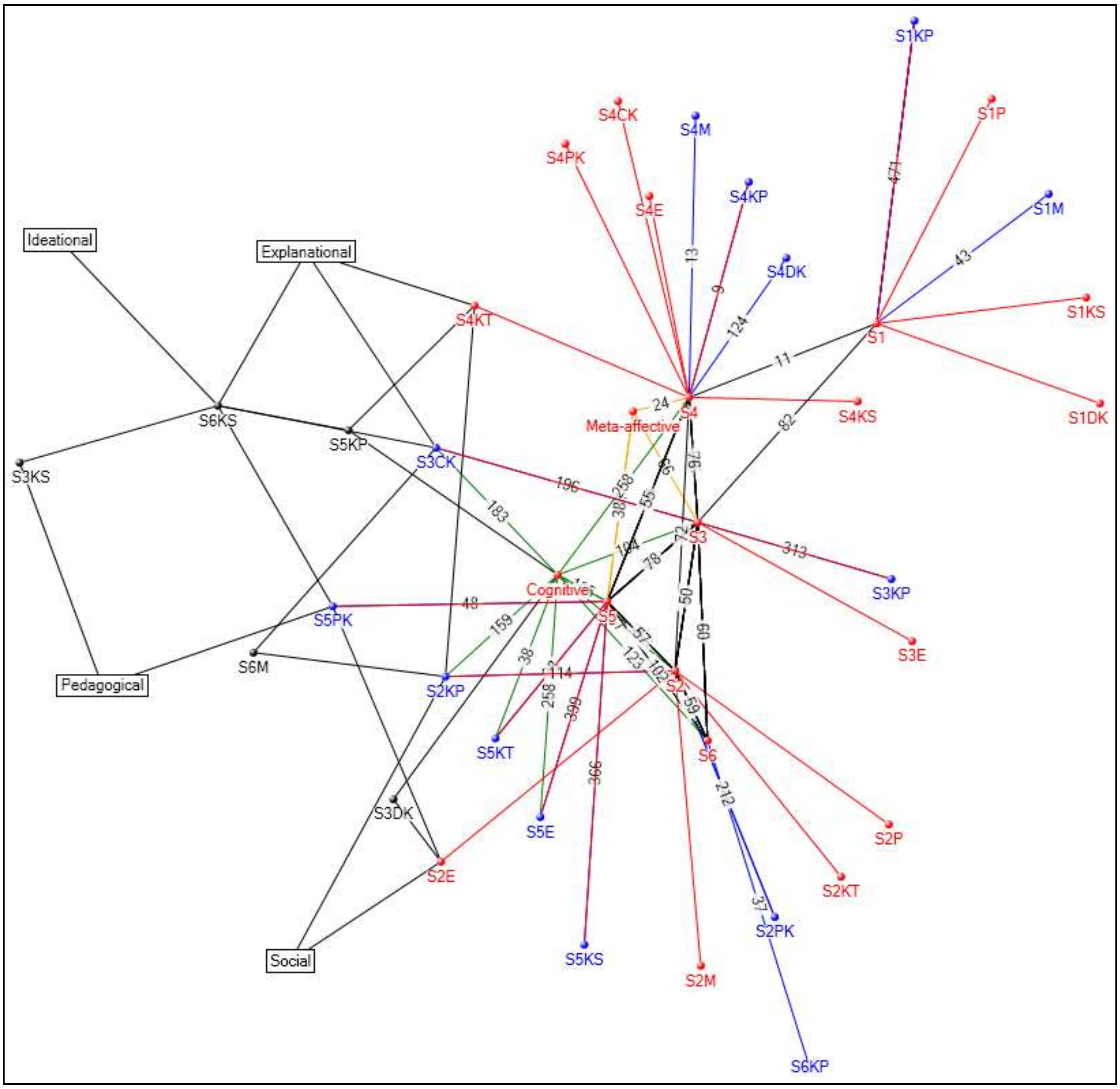


Figure 7.4 Group A's expression of metacognition via metacognitive language[41]

[41] Please refer to Addendum H for the colour separated networks

At this point, the researcher cautions that the metagrams in stratum 3 should not be misinterpreted. If, for example, the metagram shows S4KT, S2KP and S5M linked to ideational language, it does not necessarily suggest that these three group members are linked socially, or that they share their metacognitive processes. Instead, this only shows what group members have expressed these metacognitive processes in that particular metacognitive language. In this sense, the language serves as a theme under which specific metacognitive components are categorised. To be more explicit in the findings and to avoid such confusion, the findings of stratum 3 are presented using separate tables for each LIT phase, allowing the researcher to give a more in-depth account of how the identified metacognitive components of each group member were expressed with the related metacognitive language. These tables also show the social ties between group members, suggesting that the metacognitive processes of one member follow those of another. The result is a socially shared metacognitive network in which group members reflect on each other's contributions during the design sessions. These reflections then produce ideas, solutions, problems, probing questions and strategies that enable each member to contribute their respective interpersonal metacognitive networks to meet the overall goal of the group. When one group member, for example, expresses metacognitive monitoring of another member's knowledge of the task through explanatory language, then these two members share their metacognition by means of that language to such an extent that they, through social interaction, regulate one another's metacognition and/or create awareness of each other's metacognition. One member who does not express any knowledge of the task, for example, can hear about another member's expression and, through reflection, monitor that member's contribution, forming a socially shared metacognitive network between them.

The findings of the socially shared metacognitive networks in Figure 7.4 are presented in the following sections in terms of the metacognitive language group members used to express their metacognitive networks throughout the LIT phases. Also, since group members worked together and discussed their thinking in a group, they are referred to as group members in these phases.

7.5.1 Findings of phase 1: Investigation and planning

After investigating the information provided by the PBL task, group members collaborated and formulated the long-term goal(s) which directed their efforts in the design sessions:

> *We want to equip learners with problem solving-strategies, metacognitive thoughts and to be able to work successfully and independently. We want*

our learners to use their environments as resources. In this way we hope to enrich the learners' knowledge and impact on the teachers' attitudes.

The group members decided on the topic and formulated an idea on which they built the rest of their efforts.

During planning, emphasis was on testing the learners' prior knowledge with regard to the concept of place value and outcomes. Table 7.3 summarises the relationships between group members as expressed by their use of metacognitive language.

Table 7.3 Interaction flowchart of the investigation and planning phase – testing prior knowledge

Metacognitive language	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**
Ideational						KS[10]
Social (we)		E[2] KP[7]				
Pedagogical			KS[11]		PK[3]	
Explanation			CK[5]	KT[8]		KS[4]
Mental verb			DK[1*]		KP[9]	

It was S3 who urged the group in the beginning of the session to expand on their idea for a lesson. He did this by using a mental verb:

I think we can perhaps begin by expanding... [1:S3:49]

In this statement S3 expressed his DK, declaring *what* they had to do. This was followed by S2's E as she talked about *us* and used this social language to highlight what they knew and do not knew:

We said in the introduction that we will test their prior knowledge. But how are we going to test their prior knowledge when we are there? [1:S2:52]

Her question was almost immediately answered by S5, expressing her PK using pedagogical language:

We have to look at what level they are on [1:S5:55]

Further, S6 used explanation language to explain that they would then know if they had to do revision, reflecting her KS. In addition, S3 explained that this would help them to know what level the learners were on and if they understood, reflecting the conditions of the task.

Table 7.4 Interaction flowchart of the investigation and planning phase – outcomes

Metacognitive language	S1	S2	S3	S4	S5	S6
Mathematical		DK[2]				
Social (we)			KT[1/3]			
Pedagogical						KS[4]
Mental verb					DK[5]	

At times, some group members were uncertain what their outcomes for the lesson plan were, then others reminded them:

> *What are our outcomes again? [1:S3:100]...Learners must be able to represent place value through the base-ten blocks. [3:S5:101]*

S3 guided the group to go on, now discussing the outcomes of the lesson plan:

> *Okay...are we quickly going to...Outcomes...that's first [1:S3:106]*

At first it seems as if S3 and S2 drive the discussion regarding the outcomes with mathematical and social language:

> *Yes, what place values do you get? [S2], Uh, What are place values [S3] Can we not ask them like a question to let them think... [S3]...How about: representing a certain number with base-ten blocks? [S2]*

Then S6 abruptly expressed her KS through pedagogical language:

> *Then perhaps you can ask them: what is the difference between this block and the smaller block? [1:S6:122],*

to which S5 replied with a mental verb:

> *I think we must let them understand ...how to use it [1:S5:123]*

Up to this point S1 did not say anything, to who S5 said:

> *...I don't know what you are thinking [1:S5:261]...*

The remainder of the session was devoted to ideas on how to teach the concept of place value. Group members ended up discussing topics that, afterwards, they realised were not part of their outcomes:

> *Okay, so we are only going to focus on place value [2:S5:308]...nothing with decimals, adding or subtracting [2:S6:310]... Now it is a bit more chilled down [2:S2:313]...remember that when you work with volume, you work with cubic units, not with place value [2:S3:19]*

These were important starting points in the next session where group members developed the research lesson.

7.5.2 Findings of phase 2: Developing the research lesson

The development phase mainly revolved around groundwork for the lesson's activities and required two sessions. Group members seemed to answer the question of *how* to teach place value (teaching strategies) and with *what* (teaching resources) by collaborating.

This time S5 proposed that everybody give their ideas, they listen to it and combine, or as S4 puts it:

> *Take six people's lessons and make it one [3:S4:22]*

Table 7.5 Interaction flowchart of the developing phase – *teaching resources (place value chart, worksheet and arrow cards)*

Metacognitive language	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**
Mathematical						KS[3]
Social (we)						KT[5]
Pedagogical		KT[6]	KT[4]	KT[7]		
Explanatory					KP[1]	KT[2]

S5 seemed concerned about the contents of the worksheet, explaining:

> *They get a worksheet where the base 10 blocks are in but they don't know what base 10 blocks are or how to use it to represent or what it represents [5:S5:171]*

Then S6 explained the value of the resources:

> *I was thinking that I can make a place value chart. A big one, almost a big as a TV and then learners can use the arrow cards as well. [5:S6:114]*

She further elaborates using mathematical language:

> *Different numbers can be made from the arrow cards but if you put them, the cards, together then it gives you a number. If they put that number on the zeros then they get a number but if they take it apart, they still see here is 2000, here's 200. [5:S6:126]*

Previously, S1 suggests that they should not start with the introduction, explaining they must:

> *Begin with the outcome [5:S1:52]...If you start with the introduction, you box yourself in [5:S1:101],*

with which S3 seemed to agree:

> *First get your relevant information and all your knowledge [5:S3:105]*

After discussing the aim and focus of the lesson in the first session of this phase, group members were better prepared for the second one:

We are finished brainstorming [7:S6:104]...but we do not have a pertinent question [7:S6:110]

Thereafter group members developed the tasks and strategies they thought necessary in the teaching-learning phase.

S2 suggested that they make posters with the terminology, followed by S4's expression of her understanding through pedagogical language:

So the introduction will be the terminology that we explain on the poster. Then we will show them...and she also brought nice worksheets of expanded notation that they can use for testing their prior knowledge [7:S4:124]

S5 reminded the group that they have planned to make a wordlist for the words in English and Setswana, whereupon S4 commented:

But we are going to give them the definition in English [7:S4:133]

Table 7.6 Interaction flowchart of the developing phase – *teaching strategies*

Metacognitive language	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**
Ideational		DK^3			DK^1	
Pedagogical			KT^2			
Explanatory		PK^4				

Regarding teaching strategies, S5 explained:

If you explain place value, then you say place value is this and this and this. Look, it is number one on the word list. [7:S5:155]

S3 seems to agree and explains further what he thinks S5 means:

So that you don't do everything at once [7:S3:157]

S2 then recommended a tool they could give the learners:

We are going to make big posters to put up in the classroom. Then we can make a copy, like a small page that the kid can stick in his book. Then if he gets homework that he can still have the stuff in his book. And then go back and say, this is what we did today... [7:S2:159]

The responsibility of group members also became clear as S2 explained:

Then there will be assistants. Say for instance the assistant goes and help to quickly see if the kids understand [7:S2:176]

7.5.3 Findings of phase 3: Presenting and reflection of the research lesson

As mentioned, one student volunteered to present the lesson to a Grade 6 mathematics class in a rural school, while the others observed and could assist where necessary. In the case of Group A, S4 volunteered. Although the researcher did not collect any data of the learners or teachers in the school, the group members' observations and reflections were considered in terms of the metacognitive language they used to express their metacognitive processes. The mathematics teacher was also present in the class to assist with language switching between English and Setswana, if necessary; however, no learner asked this teacher any questions.

During phase 3, only S4's voice was recorded as she presented the lesson. If another group member helped out some of the learners, it was regarded individual in nature as they did not share what they were doing with the rest of the class. These individual cases were reflected upon in the refinement phase. Some of the learner activities required that learners had to select a card, volunteer to go to the board, put the number on the board and say the number out loud to the rest of the class, while S4 assisted. The lesson was presented over two periods for which group members prepared and that the non-presenters had to reflect on.

The account that follows presents the findings of S4's metacognitive language during the presenting of the lesson in terms of the different lesson components indicated in Table 7.7.

Table 7.7 Interaction flowchart of the presenting the lesson phase – *the lesson plan in practice*

Metacognitive language	Volunteer	Testing prior knowledge	Introduction	Teaching and learning phase
Mathematical		KT[3], KS[5]		KT[6]
Social (we)	S4	KP[2], E[4]	?	M[7]
Explanatory		M[1]		

When testing learners' prior knowledge, S4 started the lesson by explaining the task to the learners out loud, telling them what was expected of them (refer to Addendum A for the task). In doing so, she was **monitoring** their strengths and weaknesses by asking them:

> *You guys can do that? [14:S4:9]…You can multiply right? [14:S4:13]*

Her **knowledge of the person** attributes of her group members became evident as she referred to them socially when telling the learners:

> *If one of you struggle, put up your hand and one of us will come and help you guys [14:S4:17]*

During the time when learners completed the prior knowledge activity, the group members helped out by putting posters up on the board and walking around, monitoring what the learners did. In some cases, they stopped, pointed with their finger at a learner's work, bent over and talked to the learner, appearing to explain something to them. After about fifteen minutes S4 seemed eager to go on with the lesson and started asking the learners what answers they got. She wrote these on the board, going through each question as she showed her **knowledge of the task** by expressing it in mathematical language, and **evaluating** learners' responses and calling them *you*, not referring to any one individual as learners called out the answers as a group:

> *Well done. So if you write it like this. If you write it like four times ten plus eight times one, it's correct [15:S4:12]*

She explained another way to write it, showing her **knowledge of strategies** through mathematical language. The lesson carried on in this manner, with S4 reading part of the task and thinking out loud. Every time she wrote on the board, she used mathematical language. By the time that she got to question number five, she invited a volunteer from the learners to write the answer on the board, asking the rest of the class:

> *Is that everybody's answer? [15:S4:46]*

She carried on from there without waiting for the learners' response, appearing to assume that everybody had got it right. She then referred to the posters they put up, and explained how the place value chart works, using numbers provided by the learners:

> *We say one thousand three hundred and thirty six…we are going to give you an exercise …you have to fill in this place value chart… you have to fill those numbers into the column that is given to you… [15:S4:145]*

She explained the value of various base ten blocks using the posters. She then referred to the exercise and the posters (see Addendum C). It did not seem that S4 introduced the topic, as she only said after testing the learners' prior knowledge:

> *Okay, moving on. You guys, place value. On the board, can everybody see the million? [15:S4:46]…*

Reading the numbers on the posters, telling the learners that they have another activity to do, seem part of the teaching and learning phase:

> *Let's quickly do an example [15:S4:84]*

Using the base ten blocks, she exhibited her own **knowledge of the task** and **monitored** learners' understanding that:

If I gave you ten small blocks, are you going to keep them like this or are you going to exchange it for a whole ten? [15:S4:68]

While busy explaining what to do with the tool that S6 suggested, S4 realised that:

We don't have enough time left [15:S4:86]

and asked learners to paste everything in their books. They were instructed to carry on with the worksheet for homework.

7.5.4 Findings of phase 4: Refining and re-presenting the research lesson

Group members were asked how they felt about the research lesson, to which S3 and S4 replied:

The time was limited [17:S3:14], It was very nice, the kids work together nicely and they are very sharp. They picked up on something just like that (snaps her finger)*. They liked the colour posters…Well done! [17:S4:21]*

In contrast, S2 commented:

We spent too much time on the lesson; we drifted from the point instead of focusing… [18:S2:40]

Reflections on the research lesson's effectiveness and group members' experiences during the presentation of the lesson showed their awareness of the use of the resources and strategies they had worked on. It seemed as though group members became aware of their own and others' concerns, strengths and weaknesses, as Table 7.8 shows.

Table 7.8 Interaction flowchart of the reflection and refinement phase – *awareness of self and others*

Metacognitive language	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**
Social (we)			KP^4			
Explanatory		KP^3		KP^1		
Mental verb				KP^2		

It appears that the same problems they had in the planning phase kept them from feeling successful in their attempts, as S4 explains:

But we were unsure what to do, I mean, if we have to carry on now, if we have to plan another lesson we will know exactly what to do because we saw what we did [18:S4:44]

She carries on, using an important mental verb:

…and what we did not know [18:S4:45]

Although LS requires collaboration, it seemed that group members did not experience the planning or implementing of the research lesson in the same way. Regarding the conditions of the experiences, S2 said:

> *We had too many activities…and nobody wants to say this or that won't work…we cannot hurt someone else [18:S2:62]*

and S3 agreed:

> *Yes, from now on we will be straightforward with each other [18:S3:72]*

In the follow-up session, the emphasis was on *what* to improve on and *how* to improve the research lesson towards preparation and refinement for re-presenting the lesson.

Group members decided that the testing of prior knowledge would be a groupwork activity where learners can answer the questions in groups of three to four. Each learner would receive the worksheet (see Addendum C) and complete it in the group to save time. In the teaching and learning phase, the terminology poster and the base ten blocks poster was converted into an electronic version using Microsoft PowerPoint. During re- presentation, the group went to another school with similar conditions to both that of the PBL task and the first school. S2 volunteered to present the lesson. Table 7.9 shows how this language relates to specific metacognitive components.

Table 7.9 Summary of the metacognitive language and metacognitive components in stratum 3

Metacognitive language	**S1**	**S2**[42]	**S3**	**S4**	**S5**	**S6**
Ideational		DK			PK	KS
Mathematical		DK		*KS, KT*		KS
Social (we)		E, KP	KT	*E, KP, M*		
Pedagogical		KT	KS, KT	KT	PK	KS
Explanatory		PK, KP	CK	KT, *M*	KP	KS
Mental verb			DK		KP, DK	

[42] The researcher reminds the reader that S2 and S4 both volunteered to present the research lessons. Therefore, the findings of these participants may seem denser than those of the other group members. For this reason, the metacognitive processes of S2 and S4 expressed during presentation of the research lesson are indicated in italics.

Table 7.9 shows the summative findings of Group A in stratum 3 in terms of metacognitive language and the metacognitive components expressed through each language type across the LIT phases. The components indicate which metacognitive processes were involved before, during and after the expression of metacognitive language. Overall, S1's socially weak relationship with the rest of the group appears to relate to the use of metacognitive language, as no socially shared metacognitive network emerged from this participant's expressions. However, the rest of the group had expressed their metacognitive processes in such a way that it seems as if they shared the regulatory responsibilities by reflecting on each other's metacognitive knowledge, as presented in the findings across Tables 7.3 to 7.9. It seems that, for these group members, the metacognitive components expressed can be categorised according to the distinctive metacognitive language used. For example, ideational language was mainly used to express DK and PK domains of metacognitive knowledge, whereas KS was the level of metacognitive knowledge and this language was also used to monitor group members' processes.

Mathematical language mainly expressed DK, KS and KT with no metacognitive regulatory processes. Social language related to both the expression of metacognitive knowledge (e.g. KP and KT) and metacognitive regulation (M and E). Pedagogical language was associated with KT and KS, whereas explanatory language gave insight into group members' PK, KP, KS and KT, with the exception of evaluating and planning. It also seems that planning (P) was not expressed verbally at the forming of these socially shared metacognitive networks. It is mainly seen as an overarching process to which each individual contributed metacognitive knowledge or regulatory components by planning, as it were, the lesson. As such, ideational language was used to contribute ideas to the group. Whether the ideas involve time management or the use of an alternative strategy, they remain ideational. Even though some group members did express planning as their interpersonal metacognitive networks, they did not share this component socially (see 7.5.4) with the rest of the group. However, subcomponents of planning were also not expressed metacognitively, including prediction and organising. The mental verbs used to express metacognitive processes were related to the cognitive relationships between group members (compare Figure 7.2 with Table 7.9). Mainly, *think(ing)* and *know(ing)* were keywords used to express their metacognitive processes, particularly for metacognitive knowledge (e.g. DK and PK).

The construct of metacognitive language, analysed in stratum 3, was intended to identify and represent the socially shared metacognitive networks between group members. Overall, the metacognitive language was situated in the two domains of metacognitive knowledge and regulation; however, it appears as though more emphasis was placed on expressing knowledge of strategies, person and task than on planning, evaluation and monitoring, making metacognitive language dominant in the metacognitive knowledge domain, particularly with the levels of knowledge (e.g. KP, KT & KS). Metacognitive language then served as the product of the social and interpersonal metacognitive networks and appears to have acted as a catalyst for the socially shared metacognitive network.

In Chapter 8, the findings of Group B are presented in the same manner as in Chapter 7.

Chapter 8

Presentation of the findings of Group B

[The star] like the Chesire cat, fades from view. One leaves behind only its grin, the other, only its ...attraction (Seife, 2005)

8.1 Introduction

Chapter 8 presents the results that were obtained in the first and second educational design-based research cycle of Group B in a similar fashion to that of Chapter 7 for Group A. The results of Group B are also presented across stratum 1, stratum 2 and stratum 3. The focus was on the constructs of metacognition, metacognitive language and metacognitive networking, emerging after implementing the LIT. In stratum 1, the results of the social network were presented in terms of the social roles and relationships between members of the group. In stratum 2, each group member's expression of his or her metacognitive processes was analysed in terms of the interpersonal metacognitive networks. In stratum 3, the metacognitive language that group members used to express their metacognitive processes was analysed in terms of the metacognitive knowledge and regulatory components, in order to show how the metacognitive components emerge from the discussion, discourses, ideas and views that group members hold.

8.1.1 Preliminary discussion on the presentation of the results

In Chapter 8, the same outline used in Chapter 7 was employed to present the results of Group B. Although the two groups' results are presented separately, it is by no means the purpose of this study to compare them. Instead, the results of Group A and Group B will serve to accumulate, in Chapter 9, towards crystallisation of the study, aligning the results of both groups to discuss and interpret the results and to theorise about the students' metacognitive locale.

8.2. Metacognition, metacognitive language and networking emerging constructs of the LIT

Similarly to the constructs that emerged throughout the design sessions of Group A (see 7.2), Group B also, as expected, produced statements of such a nature that the constructs underpinning the LIT could be highlighted. The findings of these constructs for Group B are presented below.

8.2.1 The construct of metacognition

Throughout the sessions, participants expressed their metacognitive processes. Similarly to Group A (see 7.2), Group B's members ideas were distributed across teaching and learning strategies, module content, dates for the sessions and the overarching LS goal, and they made similar statements to those of Group A:

> *We didn't know what to do…When we walked out of the first class, we did not think that we were ready to teach…now we are more confident...we learned a lot from each other.*

Participants of Group B also expressed their metacognitive processes through a metacognitive language.

8.2.2 Metacognitive language

At one point early in the first LIT phase, S7 explained that she felt that the group was not making any progress. To do so, a metacognitive language was used as group members (particularly S10) highlighted communication as an important aspect of LS, indicating that what they said and how they understood things seemed related:

> *I suggest that we talk with each other. As a group we have to come together and do research about a specific subject to expand our knowledge so that we can develop an effective lesson [1:S10:43]*

This statement suggests that group members realised that they would need to work together if they wanted to succeed in solving the PBL task.

8.2.3 Networking

Group members also explained how working together helped them to collaboratively design a research lesson based on the PBL task.

> *Working together helped a lot…the work was not as complex as the articles, because we could explain what we have read…We did our planning to meet at different dates and planned accordingly…each group member got certain tasks to do and we did it, all of us. We knew what was expected from us and we supported each other.*

The findings of these constructs as they emerged by implementing the LIT are presented below in terms of how the constructs emerge on the three strata.

8.3 Stratum 1: Analysis of Group B's social network

Two themes that act as functions of the social network of Group B structured the first stratum. The themes of *social role* and *social relationship* (see Table 5.1) were identified for this purpose. The researcher reminds the reader that the roles and relationships of the social network were intended to facilitate an understanding of the network as a framework on and around which the theory of metacognitive locale is built. Figure 8.1 shows the social network of Group B as it emerged through the design sessions.

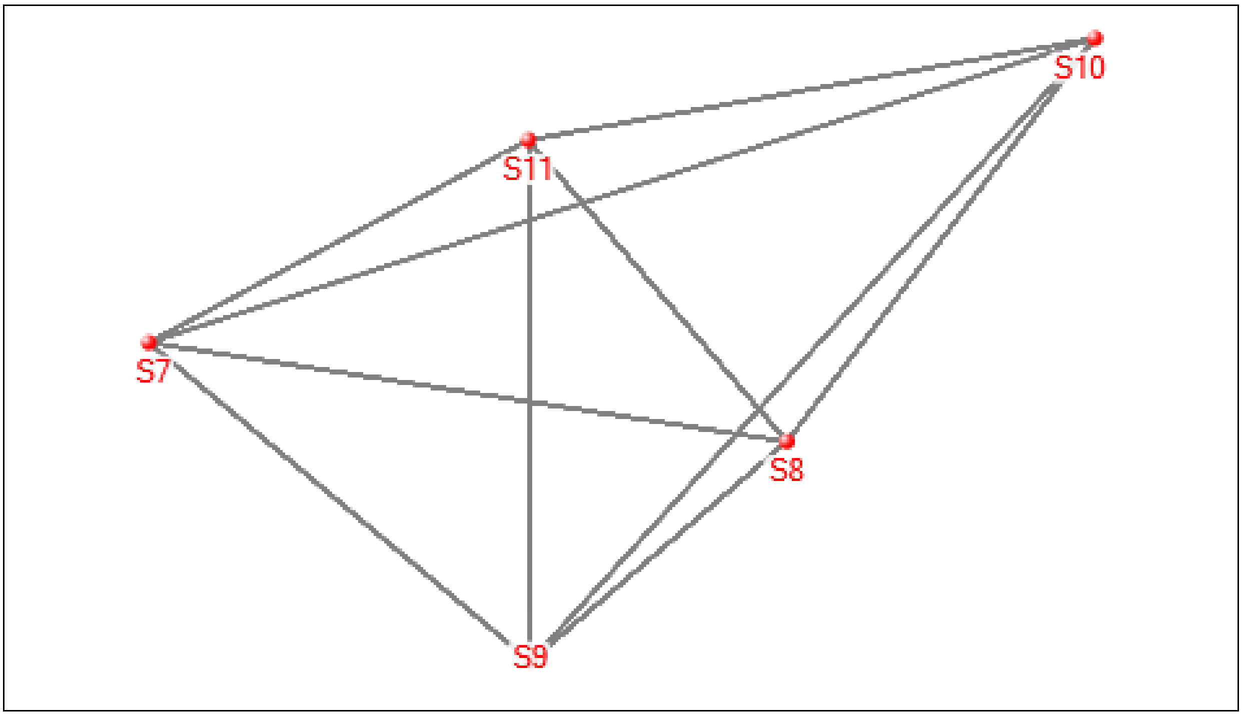


Figure 8.1 The social network map of the participants across the design group sessions, illustrated using NodeXL

The social network roles and relationships for Group B are presented below.

8.3.1 Social roles in the network

Similarly to Chapter 7, the social roles in Table 5.1 were used for social network analysis in the first stratum. Again, the focus was not on the social network metrics, omitting representatives and gatekeepers (de Nooy *et al.*, 2011a), but on the roles and relationships within the social network. In contrast with Figure 7.1, the results of Figure 8.1 show that all group members in Group B contributed in the design session in such a way that they had an equal number of ties (four) with other group members. Even so, they differed in terms of how they expressed themselves and how they portrayed the different social network roles.

8.3.1.1 The star

The star in the network was identified as S8. This student showed regulatory behaviour through which he evaluated, made decisions, accessed and managed information and expressed this role when he stated:

> *I think our lesson worked, it worked very well [5:S8:45]*

He also arranged when the group should meet again and took on the role of a leader across the sessions:

> *I think we must put everything on our memory sticks. Then we come together one day and then copy it... [6:S8:7]*

He also seemed passionate about their group's overarching goal as he explains:

> *Let's say, we go to the school, present this wonderful lesson and the kids all understand. Then you take your materials again, you end up with the same problem you had before you got there [3:S8:132] ...that's why I suggest we give the lesson plans to them* (the teachers at the school)*, so that they can have something to work on in the future [3:S8:150]*

8.3.1.2 The liaison

The liaison, who allows group members to access resources beyond the scope of the network, seemed to be S11. This group member focused more on creating and sharing connections with others outside the group. However, these connections were regarded as useful since they would help towards preparing for future lessons:

> *I think it would be better if we had a teacher in our class, because then maybe she could have said to us what she thinks was really good or maybe she said Ok, but she does not usually do that in her class...[4:S11:452]...because I know my mother works at the ...school...she's a Grade R teacher...and she has an assistant in her classroom...perhaps we could do something like that? [4:S11:397]*

8.3.1.3 The bridge

Two of the members in Group B acted as a bridge to allow the flow of information between S10 and the rest of the group. These members were S11 and S9. Whenever S10 contributed to the group, as a socially weak member, S9 or S11's statements always followed. For example, when S9 said:

> *The handing out* (of worksheets) *would be difficult [4:S9:102]*

S10 explained that they should

> *Print it out and give it to them a day or two before. Then they can cut it out and they have it ready when we start teaching [4:S10:119]*

This was followed by S11 who explained that this was a useful idea, and that different grades or classes could be reached if they did this:

> *So I had this project where I taught ...The grade eights had to collect specific lengths of sticks...they had to be calculate the amount needed...each grade*

had a different task to do...at the end we could make blinds for the classroom...[4:S11:121]

8.3.1.4 The investigator

Even before the researcher gave details as to what the participants could expect in the class in terms of the research, S9 said:

I would like it if we can get a camera and record the session. We can put it on a tripod and everything [6:S9:253]

S9 also gave a detailed reflection about a learner's understanding after observing the research lesson:

I think at the end they were just looking and rewriting the number...but you could see that they were copying the number into columns, so I don't think they had the concept of that and I think if you gave them the number in words, I don't know if they would be able to do it still [6:S9:325]

8.3.1.5 The socially weak member

In Group B, the socially weak member was S10. Her contributions to the design sessions were in general reflections of her own observations. For example, when asked how she experienced the schools they visited, she said:

It's the same, they just wear different uniforms [4:S10:27]

This was regarded as a blunt statement, since S10 volunteered to present the research lesson in the first school. She did, however, acknowledge the role the other group members played during the presentation of the lesson:

These are things you wouldn't have noticed if you were teaching the class on your own [6:S10:44]

After having considered the idea about the individual journals (see 7.3.1.5) the researcher also mentioned this to Group B:

I thought about letting you write your ideas in a journal because if you do it in a chat room online, you can go back to see what somebody else had written. If you write it in your journals, it is a bit more personal.

To which S10 responded:

Yes, that's much better [6:S10:222]

8.3.1.6 The coordinator

S8 was regarded as the coordinator in the network. Besides his role of "star", S8 also steered the group to focus:

Yes, but when we look at the work they did... [5:S8:29]

He also evaluated the effectiveness of the lesson plan:

I think it worked. I think it worked very well [5:S8:46]

8.3.2 Social relationships in the network

From the beginning, it was as though the members of Group B knew what they wanted to do and they worked towards the group's goal. On reflection, the researcher found this group to be hard-working. This was evident in the cognitive relationships, yet the group had a less familiar attitude with each other than those members in Group A, which is reflected in their weak meta-affective relationship displayed in Figure 8.2.

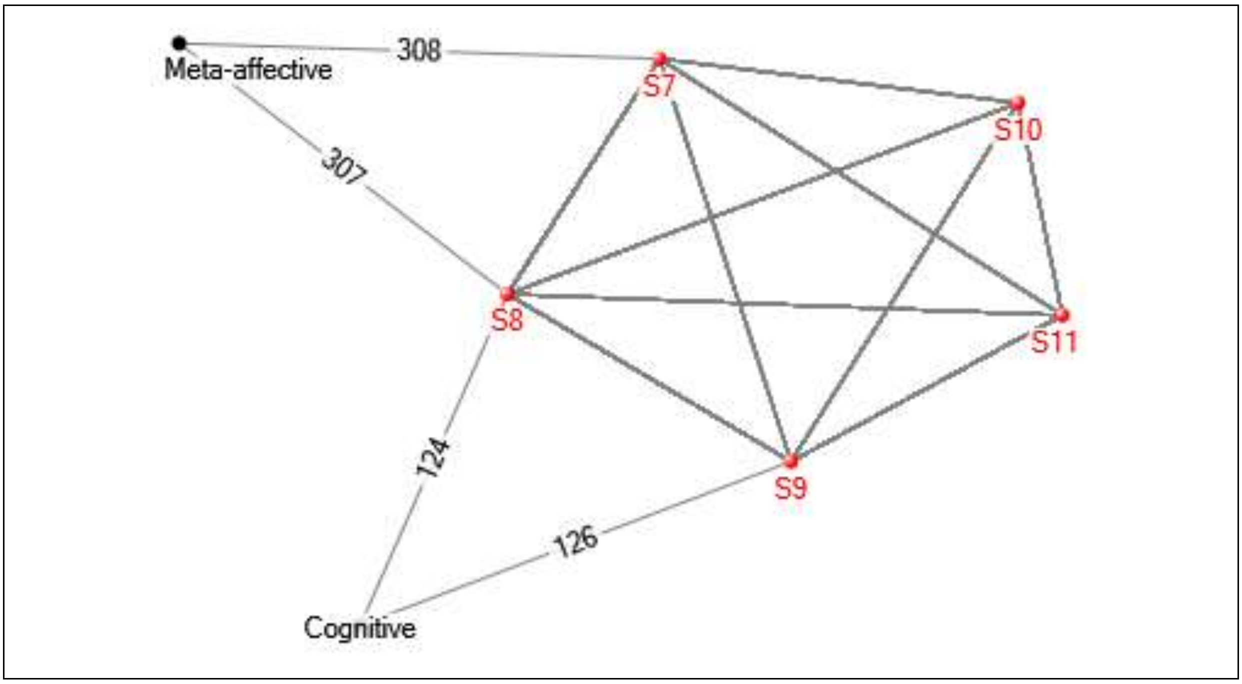


Figure 8.2 The presence of cognitive and meta-affective relationships in the social network

Figure 8.2 shows how these two relationships emerged from the design group sessions of Group B.

First, the findings of the meta-affective relationships are presented.

8.3.2.1 Meta-affective relationships in the network

In Figure 8.1 it appears as though each member equally contributed to the group discussions and equally communicated with each other. In Group A, it seemed that group members' ideas were scattered. However, the members of Group B seemed close in terms of their sharing of ideas and feelings. They also exhibited cognitive and meta-affective relationships as illustrated in Figure 8.2. Mainly, there seems to be an overlap between S8 and S7's meta-affective and cognitive relationships when they discussed the effectiveness of ideas incorporated to the lesson plan:

> *It feels as if it helped in the one group [4:S7:307]. I don't know how it worked for the others. [4:S8:308]*

This overlap marks two distinctive qualities of the meta-affective relationship: being aware of the result of their collaborations and knowing what they do not know. Furthermore, the relationship between the meta-affective and cognitive associations was also marked by several other statements of affect:

> *It felt as if it was an ordinary school [5:S8:22]. I feel there was not enough...time planned to do this [5:S8:88]. Look at it, feel how you feel about it and then they can still look at their own stuff [3:S8:149]*

Notably, only S8 and S7 expressed this *feeling of knowing* throughout the sessions, as none of the other group members used the phrase *I feel* or *we feel* to refer to their meta-affective relationships. This suggests that there was a weak meta-affective relationship between other group members, as hinted upon in the statement by S7:

> *I feel that there is a gap. I am not saying that there is not enough work done, I just think it feels as if our production has a problem [1:S7:39]*

8.3.2.2 Cognitive relationships in the network

Cognitive relationships were identified by looking for key indicating phrases such as *I think* where group members expressed their metacognitive processes with mental verbs. The cognitive relationships between group members were mainly expressed between S8 and S9, particularly when these group members reflected before and after action:

> *I think that will help [3:S9:162]... If we make it* (the poster) *we tell the teacher that they must throw it away, because we are coming back to discuss it at a later stage [3:S8:124]. I think if he sees it, maybe he will understand [3:S9:156]*

The social roles of and relationships between Group B's members show that each member related to either of these relationships or both, even though they did not directly use such phrases as *I think* or *I feel*. Instead, group members expressed agreement with either *feeling* or *thinking* (for example) when they said "*hmm*," "*yes*" or "*no*" and "*huh-uh*". Table 8.1 summarises the roles and relationships between group members of Group B.

Table 8.1 Summary of the roles and relationship in Group B's stratum 1

Social role	**Meta-affective relationship**	**Cognitive relationship**
Star	S8	S8
Bridge	**	S9
Coordinator	S8	S8
Socially weak member	*	*
Investigator		S9
Liaison	**	**
Group member	S7	

***** S10 was identified as the socially weak member and did not show any meta-affective or cognitive relationships with the rest of the group.

** S11 was identified as a bridge and liaison, yet she did not show any meta-affective or cognitive relationships with the rest of the group members.

8.4 Stratum 2: Analysis of Group B's interpersonal metacognitive networks

In stratum 2, the interpersonal metacognitive network for each of the five group members in Group B is presented. The analyses of the group members' networks are presented consecutively, starting with Student 7 (S7) and ending with Student 11 (S11). Figure 8.3 shows an overview of how Group B's interpersonal metacognitive networks are connected via the social network.

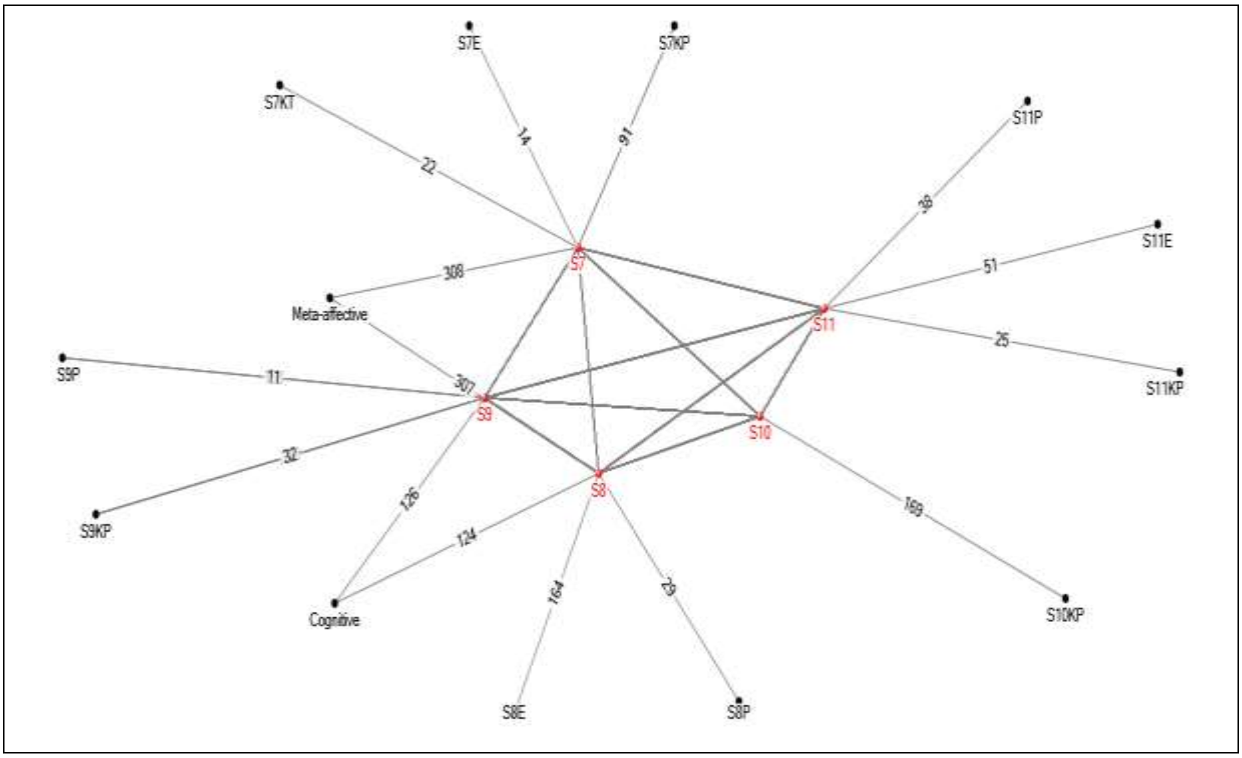


Figure 8.3 Group B's interpersonal metacognitive networks

Differences between group members' interpersonal metacognitive networks are visible in Figure 8.3 and discussed in the following account of each participant's interpersonal metacognitive networks in stratum 2.

8.4.1 Account of S7's interpersonal metacognitive network

This student was generally quiet, and when she spoke, she spoke mostly when agreeing with other group members, suggesting she waited for others to present ideas to the group. Throughout the sessions she expressed her ***knowledge of the person***, for example when reflecting on the learners' experience in the classroom:

> *Well, they did grasp it quickly [5:S7:91]...And they have good manners [5:S7:34]*

Other metacognitive components such as ***monitoring*** and ***knowledge of the task*** were expressed during the presentation of the research lesson.

8.4.2 Account of Student 8's interpersonal metacognitive network

Student 8 had many inputs throughout the sessions and seemed to have played an important role when decisions had to be made. He confirmed whether the ideas of others would work, thinking critically about their contributions as he ***evaluated***:

Yes, that is a good idea. It can work. To give a template and then an example of what the lesson might look like [3:S8:164]

This student seemed to have some prediction about what the school would be like and what learners' performance entailed when ***planning***, being surprised afterwards:

I really expected something different. For me it felt like any ordinary school, but without grass [4:S8:14]. If we look at the work, they knew more than we expected they would [4:S8:29]

8.4.3 Account of Student 9's interpersonal metacognitive network

S9's interpersonal metacognitive network mainly consisted of ***knowledge of the person***, as she reflected on the learners' strengths and weaknesses before and after the presentation of the research lesson.

Yes, they are all pretty good, actually [5:S9:32]...You could see in their faces that they understand [4:S9:340]

Even when reflecting upon the observation of the lesson, S9 could indicate that some learners struggled with some of the concepts, yet she did not specify what exactly the learners struggled with:

...she was the clever one, and she answered wrong [4:S9:2]

In terms of ***planning***, S9 claimed that she had some expectations about the school and that these were unanticipated:

The school environment was a lot better that I thought [5:S9:11]

and predicted that the teacher would find the lesson they prepared useful the next year:

Then next year he'll think: oh wait, this section. I have a lesson plan for it. I have the resources ready for it. I can use it... [3:S9:158]

8.4.4 Account of Student 10's interpersonal metacognitive network

So far, it seems that group members could remember some characteristics about the learners in their classrooms. Student 10, for example, recalled two learners at the back of the class that:

...really struggled [5:S10:169],

expressing ***knowledge of the person*** and suggested that they should pay more attention to those concepts. Even though this student shared this quality with the rest of the group, she did not express other metacognitive components that could have been identified.

8.4.5 Account of Student 11's interpersonal metacognitive network

During preparation of resources, S11 expressed her ***knowledge of the person*** by making a general statement:

> *...sometimes the learners are afraid to go back, either because they think the teacher might be upset or they feel that they might be stupid or things like that [4:S11:25]*

After the presentation of the research lesson, S11 showed that she can ***evaluate:***

> *So the fact that we got most, everything, done in the short time we planned for, I think that was good [5:S11:51]*

She also expressed her metacognitive ***planning*** with regard to the poster the group decided to make. She also considers leaving room for improvement:

> *I will have the outline with words and everything. So if someone wants to add something, they can [3:S11:38]*

Table 8.2 presents a summary of the metacognitive components of Group B's interpersonal metacognitive networks. In comparison to Group A, Group B showed less components in the metacognitive knowledge domain (e.g. DK, PK, CK). This does not suggest that these components do not exist, since group members did express components on the person, task and strategy level of metacognitive knowledge which, as argued (see 4.2.2), also infers declarative, procedural and conditional knowledge and suggests that Group B' members expressed themselves only on a basic level of reflection (see 4.5).

Table 8.2 Summary of Group B's interpersonal metacognitive networks

Participant	**Metacognitive** knowledge	**Level of metacognitive knowledge**	**Metacognitive regulation**
S7		KP, KT	E
S8	PK		P, E
S9	CK	KP	P
S10		KP	
S11		KP	P, E

Through their expressions of their metacognitive processes, participants provided a profile of their unique contributions to the group. In stratum 3, these contributions are looked upon in terms of how the interpersonal metacognitive networks and the social networks (in stratum 2 and 1), respectively, surface towards an overarching network which explains their socially shared metacognitive processes.

8.5 Stratum 3: Analysis of Group B's socially shared metacognitive networks

On different occasions, group members expressed their contributions to the group using language typically associated with groupwork, collaborative problem-solving and discussion contexts. In the following sections, only that language type that associates with the expression of metacognitive processes (for example, ideational language used to express knowledge of the task) is reported on (see 4.4.3). The outline of the presentation of the findings below reports on the expression of metacognitive components through metacognitive language as expressed in the different LIT phases.

Figure 8.4 shows the socially shared metacognitive networks linked via metacognitive language.

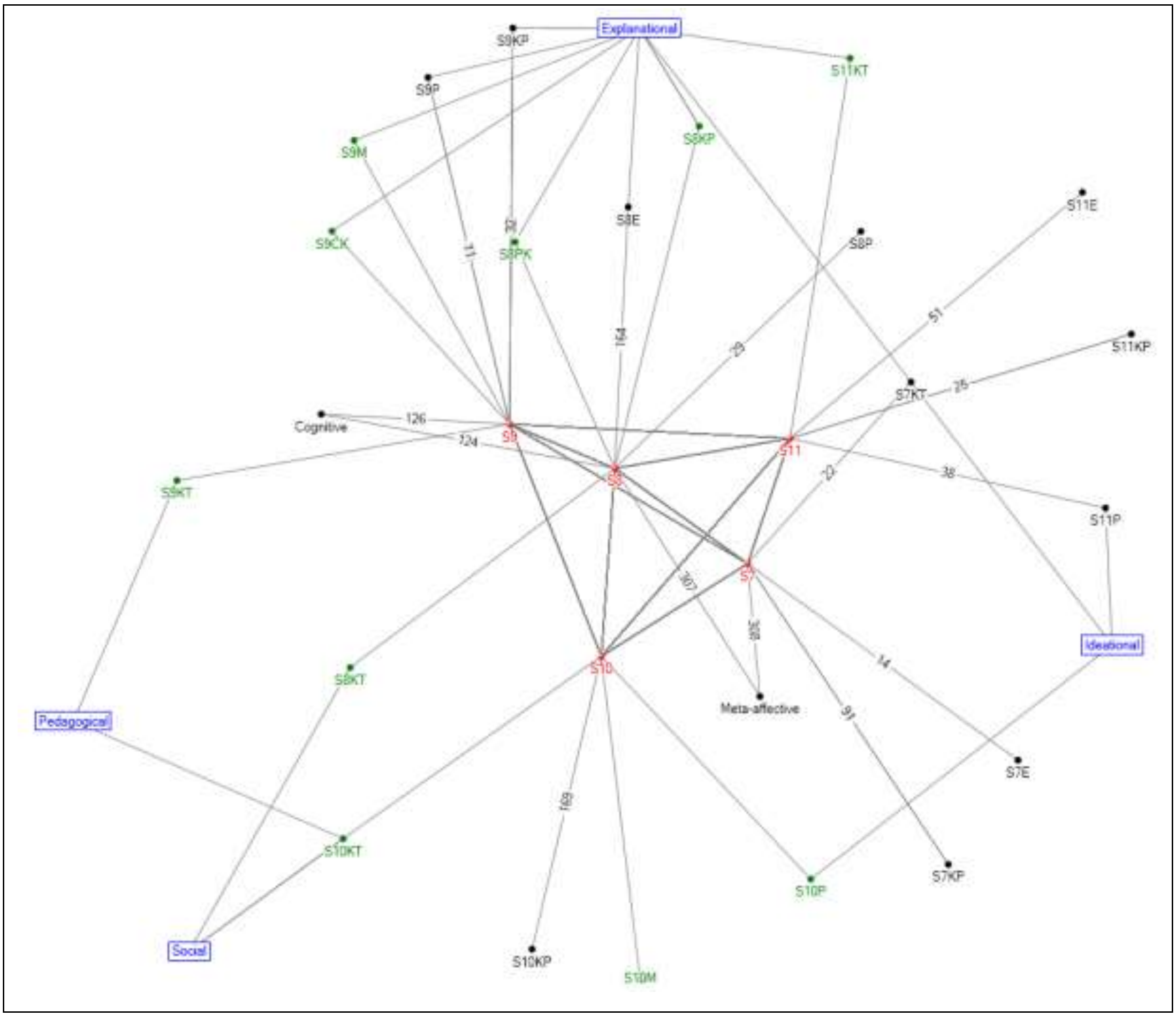


Figure 8.4 Group B's socially shared metacognitive networks linked via metacognitive language

A discussion regarding Figure 8.4 now follows in terms of the metacognitive language used in every phase

8.5.1 Results of phase 1: Investigation and planning

For Group B, the discussions in LIT phase 1 revolved around four main ideas. These include: (1) getting clarity on the concept of place value and determining the outcomes for the lesson; (2) discussing the use of a worksheet; (3) developing a poster to assist learners with the language barrier between English and Setswana; and (4) identifying key aspects they needed to address in the introduction. At this point, Group B appeared more focused on the task of developing the research lesson when compared to Group A (see 7.5.1) since they had in the investigation and planning phase already highlighted the point of communication and the focus of the lesson they needed to prepare for (see 8.5.1). Following is a presentation of the findings of the first LIT phase. Tables 8.3 up to 8.7 present the interaction among group members in terms of the metacognitive components expressed through metacognitive language throughout the phases.

Table 8.3 Interaction flowchart of the investigation and planning phase – outcomes and concept clarification

Metacognitive language	**S7**	**S8**	**S9**	**S10**	**S11**
Ideational	KT[1]				
Social (we)		KT[4]		KT[2]	
Explanation	KT[6]	KP[5]	KP[3]		

Early in the session, group members explained that:

> *This part of the CAPS document says that we must round off only from zero to one and fractions come in later... [1:S7:8]...So we have decided, in the first lesson, we are not yet going to talk about fractions and rounding off. We stick with the meaning of place value and we have decided to use cans which we have painted different colours [1:S10:14]...if it is colours, we know they will recognise it, for place value [1:S9:20]. That's why we will also paint it with colours [1:S8:21].*

Towards considering the outcomes, group members explained:

> *Before I present a lesson, I first want to know the basics....it doesn't help you present this wonderful big lesson and...the learners don't understand [254:S8:256]...we can use columns, then the learners gets a chance to see if they can do place value [3:S7:99]..That's good [3:S11:101]*

In planning, the discussion regarding the worksheet's relevance and content revealed group members' application of what they have learned in the module, as S10 explained:

> *We then use a worksheet [3:S10:68]… but only later so that we can formulate effective questions [3:S10:71]*

Yet, in developing the research lesson, nothing was said about effective questioning.

8.5.2 Results of phase 2: Developing the research lesson

In LIT phase 2, group members focused on developing the research lesson's accompanying teaching resources and the teaching strategies.

Mainly, this phase was devoted to discussing the teaching strategies, the resources and the procedures for teaching the concept of place value as captured in Table 8.4.

Table 8.4 Interaction flowchart of the developing phase – *teaching resources (place value chart, worksheet and arrow cards)*

Metacognitive language	**S7**	**S8**	**S9**	**S10**	**S11**
Social (we)				KT^{5}	
Pedagogical			KT^{4}	KT^{2}	
Explanatory		PK^{3}			KT^{1}

Instead of providing numerous ideas, members of Group B expressed their knowledge of the task by expressing their ideas through social and/or pedagogical language. S11 reflected on the lesson study goal by referring to their overarching LS goal, yet she claimed they had a problem reaching it:

> *We want to improve their attitude [4:S11:215] I think I have a reason for why we struggle so much, it is because if a teacher presents one lesson, then you have time to see what the learners can do…[4:S11:235]*

and S10 used social language to generalise a misconception of the information as she neglected to refer back to the PBL task (see Addendum A):

> *and we don't actually know anything about the learners [4:S10:241]*

S10 showed her knowledge of the task when she expressed herself through pedagogical and explanatory language:

> *We said they write any number on the board, a big number, on the board. You can ask the learners, five learners each give a number. Then you combine these numbers. We use then use different place values and… underline for example four, then the kids have to like think what is the place value of the underlined digit. Our aim is to work up to a million, according to the caps document [5:S10:18]*

It appeared that everyone was in agreement as they answered simultaneously "yes" or "no" to the researcher's questions when he repeated their statements, making sure that he understood. When asked if they were still going to use the coloured cans, the group members responded "no" and explained that they would instead use a place value template table that every learner would receive (see Addendum D and Addendum F). How this was to be used was explained by S8:

This is what we are going to use. The ones we are going to make red, the hundreds we are going to make blue. Then we said, okay, we associate the number on the board with , for example, 7134, then the four in this place we associate with red to help the kid to visualise that specific number is ones. We first had it on cans, and we feel that they know colours better and they will be able to visualise it better [5:S8:29]

At this stage S9 steered the group back to focus when she expressed herself through pedagogical language:

I think we have too many activities. For a lesson to work your learners must have the necessary prior knowledge of the lesson. There must be one activity that is theoretically founded. Otherwise the lesson won't work without the necessary background and the time necessary to complete the lesson. [5:S9:15]

In the third phase, these considerations were observed as S7 and S8 presented the research lesson.

8.5.3 Results of phase 3: Presenting and reflection of the research lesson

In the third phase, S7 and S8 volunteered to present the research lesson together. During the lesson's presentation S8 and S7 supported each other, although from the observation in the classroom, it appeared as if S7 was acting as the teacher and S8 only supported S7, not teaching anything. S8 therefore assisted S7. Table 8.5 presents of the interactions among the group members. Note that S7 and S8 presented the lesson whereas S9, S10 and S11 reflected on it. S7 and S8 did not make any attempt to reflect with the rest of the group.

Table 8.5 Interaction flowchart of the reflection on the presented lesson – classroom management and the effectiveness the lesson plan and resources

Metacognitive language	**S7**	**S8**[43]	**S9**	**S10**	**S11**
Ideational				P[3]	P[2]
Mathematical	KT, E, M, PK,	KT			
Explanatory			P[1], KP[4]		
Mental verb				M[5]	

[43] The researcher reminds the reader that S8 and S10 both volunteered to present the research lessons. Therefore, the results of these participants may seem denser than that of the other group members. * is used here to identify the metacognitive components the volunteers expressed during the presented research lesson.

The lesson was mainly presented in a traditional fashion, that is, the teacher (S7/S8) told the learners what to do, asking questions and giving feedback through closed or convergent questions:

> *No, no, no wait, what is the value of the six?... Huh-uh…six hundred thousand. Now try again… ok, carry on [4:S7:295]…*

When the learners answered (five thousand), S7 asked a closed question again, seeming to get confused, at which point S8 helped out:

> *But what is that? Is it the value of the place value or the place value? [4:S7:234]… of the digit? [4:S8:236]… the digit's place value [4:S7:238].*

As the learners responded, S7 evaluated:

> *Well done. It is the digit's place value. Ok… what's the place value that is underlined?*

In terms of expressing her declarative knowledge and procedural knowledge S7 did one of two things, depending on if she was teaching the whole class or if she was helping one learner while the others were busy with the worksheets.

On the one hand she showed the learners what to do, expressing her declarative knowledge through mathematical language, yet she did not explain it in words:

> *If we got ten of these little squares, we pack them together like this* (arranges them on the board), *we get the whole [4:S7:22]*

On the other hand, she explained clearly that:

> *You see, ten of these can fit in one of these squares. OK now I want you to use that base 10 blocks and present this number, 67, with the base 10 blocks for me… use your base 10 blocks, keep your numbers separately but use the base 10 blocks [4:S7:41]*

After the presentation of the research lesson, group members had an opportunity to reflect on what worked for them and what did not work for them, to encourage them to adapt and refine the lesson in the next design session. These reflections mainly revolved around such issues as classroom management (e.g. handing out of worksheets), the effectiveness of the lesson plan and the usefulness of the resources that they had developed.

It is worth noticing that even though S7 and S8 presented the lesson, they did not make use of metacognitive language to express their metacognitive networks during the reflection session.

Instead, the findings in Table 8.5 only show the components expressed during the presentation of the lesson. This also shows that these two group members did not contribute to the discussion afterwards, suggesting that the interaction was not between S7 and/or S8 and the rest of the group.

S9 first mentioned the issue of classroom management. This was followed by S10 and S11's statements indicating their explanatory and ideational language:

> *…just the handing out will be difficult, because it is a lot of handing out [6:S9:102]…it could be difficult, but maybe just put it on a desk so long [6:S11:106]…or in their desks then you can tell them it's there. Surprise! [5:S10:108].*

In terms of evaluating the effectiveness of the lesson and how the learners experienced it, S9 used explanatory and mathematical language:

> *They were very smart. They got the base 10 blocks and everything, and when we did it in class they really seem to understand the place value… [7:S9:164]*

This was also followed up with a contrasting evaluative statement and S10's monitoring:

> *I saw the one guy, he did something wrong on his worksheet, and you can see either he hasn't read the question or he didn't understand it [7:S9:166]…I also noticed it [7:S10:169]*

Group members were then in agreement that they could only judge the effectiveness of the lesson if they could get the worksheets back after the learners had finished with them. They also considered the value and purpose of the poster they had created (see Addendum E) and refined in the next phase.

8.5.4 Results of phase 4: Refining and re-presenting the research lesson

After reflection, group members refined the research lesson towards representing it in the LIT phase 4. The refinement is based on the discussions in the previous session's reflections.

Table 8.6 Interaction flowchart of the reflection and refinement phase – *awareness of self and others*

Metacognitive language	S7	S8[44]	S9	S10	S11
Mathematical				*M, E*	
Explanatory		E, KP	M, CK	*PK*	
Mental verb				KT	

This refinement of the research lesson resulted in a re-presentation of the lesson at another school with similar conditions. For reasons of convenience, the second school visited was the same one as Group A visited in LIT phase 4 and care was taken not to present the lesson to the same Grade 6 class that Group A had. Upon reflection, group members did not suggest major changes for the second round, as S8 explained through social language:

> *I would not have changed anything. We saw what worked and what did not, and we worked on that. It [the lesson] worked then [8:S8:22]*

When considering the development of their ideas and their contributions, it seemed that the context of the schools for which the lesson had been prepared, played an important role in their critical thinking:

> *We had a lot of ideas, and we could never really come to a consensus. I think what happened is, when we decided on an idea, there were always problems with the idea we had, for instance when we wanted to work with the cans we were wondering if they would know the cans [8:S9:26]*

They then tried to make it easier for themselves by moving away from the obstacles, which were mainly contextual:

> *We then tried to make it easier for ourselves and for the kids [8:S8:115]. After every session, we realised what we did wrong and we thought about what we had done [8:S10:153]*

S10 volunteered to re-present the lesson to conclude the LS cycle. During presentation, she seemed rushed to get the work finished and, similarly to S7, asked many closed questions. Because she was eager to finish on time (as the periods lasted 30 minutes), she did not give

[44] The researcher reminds the reader that S8 and S10 both volunteered to present the research lessons. Therefore, the results of these participants may seem more dense than that of the other group members. * is used here to identify the metacognitive components the volunteers expressed during the presented research lesson.

learners a proper chance to answer the questions. She did, however, offer them her support when they felt they did not understand:

> *OK, and number two, write expanded notation as a number. Okay, let's go, can you do that? You can multiply? You're Grade 6 right, you can multiply, right…if one of you struggle, put up your hand and one of us will come and help you guys, OK? [7:S10:14]*

Throughout the presentation of the lesson, S10 taught in a similar fashion as S7 as she evaluated learners' understanding through mathematical language and read out loud:

> *Well done, OK, so if you write it like this: four times ten plus eight times one, it's correct. [8:S10:19]*

Using mathematical language and the place value poster she guides the learners:

> *Huh-uh. OK. Why did you say 50? Who said 50? Don't worry; don't worry it's not wrong. Why did you say 50? Okay you guys listen, look, look closely, alright. We just put this number out of expanded notation, so if I ask you what's the value of the 5, in what column is it? [8:S10:185]*

Table 8.7 offers a summary of the different metacognitive components expressed through metacognitive language. The researcher reminds the reader that Table 6.2 can be referred to in order to revise the components and their abbreviations used in Table 8.7.

Table 8.7 Summary of the results in stratum 3

Metacognitive language	**S7**	**S8**[45]	**S9**	**S10**	**S11**
Ideational	KT			P	P
Mathematical	*KT, E, M, PK*	KT		*M, E*	
Social (we)		KT		KT	
Pedagogical			KT	KT	
Explanatory	KT	KP, PK, E	KP, P, M, CK	*PK*	KT
Mental verb				M, *KT*	

[45] The researcher reminds the reader that S7 and S10 both volunteered to present the research lessons. Therefore, the results of these participants may seem more dense than that of the other group members. * is used here to identify the metacognitive components the volunteers expressed during the presented research lesson.

Table 8.7 shows a summary of Group B's use of metacognitive language in stratum 3 in terms of the metacognitive processes expressed across the LIT phases.

Ideational language was mainly used to express KT and P whereas mathematical language expressed KT, E, M and PK. Social and pedagogical language were used to express KT, with no association with any regulatory process. Explanatory language was used to express KT, KP, PK, E, P, M and CK, showing its wide use throughout the sessions. Mental verbs were mainly used to express the cognitive thinking processes (e.g. I think or I remember) and was used when monitoring and expressing KT.

When metacognitive language is used to express the metacognitive processes, the interactive flow charts (in Tables 8.3 to 8.7) show that group members contributed to the group's discussions and regulated each other's contributions when planning, monitoring and/or evaluating.

In Chapter 9, the findings of Group A and Group B as presented in chapters 7 and 8 are discussed towards constructing the theory of metacognitive locale.

Chapter 9

A narrative of the theory of metacognitive locale: discussions, limitations and recommendations

Sometimes when I consider what tremendous consequences came from little things ... I am tempted to think ... there are no little things. (Covey, 1992:287)

9.1 Introduction

In this educational design-based research study the data for two groups of fourth-year intermediate phase mathematics education students were collected, analysed and presented separately. Chapter 7 presents the findings of Group A and Chapter 8 presents the findings of Group B. Chapter 9 discusses the findings and merges Group A and Group B's data sets to answer the secondary research questions. After contextualising the findings in the conceptual-theoretical framework, possible explanations, limitations and recommendations follow in an attempt to answer the primary research question.

9.1.1 A brief overview of the study – a summary of previous chapters

In this study, metacognitive locale is a proposed LT to explain the relationship between metacognition, metacognitive language and metacognitive networks, contextualised in a fourth-year mathematics education methodology module for the intermediate phase. To do so, a LIT was first developed and implemented through a PBL instructional philosophy (see 3.2) and an HLT (see Table 3.2) of LS (see 3.4) over a series of iterative phases as discussed in chapters 3 and 6. By following the guidelines of the LIT (see 3.5), metacognition, metacognitive language and metacognitive networks emerged as constructs embedded in the theoretical foundations of HLT, LS and PBL. These constructs are conceptualised in the conceptual-theoretical framework of this study as explained in chapters 4 and 5. In Chapter 6 the educational design-based research methodology, theoretical stances and paradigmatic perspectives were discussed. Chapters 7 and 8 presented the findings of the study stretching across three strata that form the infrastructure of the LT for Group A and Group B. In Chapter 9 the key findings are discussed in terms of the standard theories, the relationship between the constructs are emphasised and a narrative account of the theory of metacognitive locale is presented. In closing, Chapter 9 formulates guidelines for the operationalisation of the theory of metacognitive locale to answer the secondary and primary research questions. Limitations and recommendations are also elaborated on before concluding the chapter with a personal reflection.

9.1.2 How the theory of metacognitive locale was constructed: structure of the discussion towards building theory

In terms of their function, the propositions are used here to outline the discussion sections in Chapter 9 to show how these functions, as guiding theoretical assumptions, relate to each stratum. In doing so, the key findings are conceptualised in accordance with these theoretical assumptions (as the propositions suggest) through standard theory (e.g. social network analysis, metacognition and ZPD). Thereafter, in building the LT alongside this conceptual-theoretical framework, encapsulated by social constructivism as metatheory, the narrative of the theory of metacognitive locale unfolds.

9.1.3 Propositions

This study dealt with the development of a LT to explain the relationship between emerging constructs after implementing a LIT of PBL and LS. As a reminder, chapters 4 and 5 together offered four propositions intended to address the conceptual and theoretical nature of these constructs. The propositions are briefly revisited in Table 9.1

Table 9.1 Summary of the theoretical propositions

Proposition	Function in study	Sections describing the concepts related to the propositions
(1) Metacognition is individually and socially mediated knowledge.	Interpret and understand metacognitive knowledge components	4.2.1, 4.2. 2
(2) Metacognition is individually and socially regulated.	Interpret and understand metacognitive regulatory components	4.3.1, 4.3.2, 4.3.3, 4.3.4, 4.3.5
(3) Metacognitive processes can be expressed verbally by means of a metacognitive language.	Interpret and understand metacognitive language	4.4.1, 4.4.2, 4.4.3
(4) The epistemic context-specific metacognition is a local construct that can be represented as a metacognitive network.	Interpret and understand metacognitive networks and their representations as metagrams	5.1.1, 5.1.2, 5.2.3

Table 9.2 represents the extent to which these propositions have been met in the current study, discussed afterwards.

Table 9.2 Overview of the findings in terms of the four propositions for Group A and Group B

Group A				Group B		
Literature supporting the findings	Section of the analysis and presentation of the findings	Domains/levels of the emerging constructs as underlying concepts	Propositions	Domains/levels of the emerging constructs as underlying concepts	Section of the analysis and presentation of the findings	Literature supporting the findings
Bormotova (2010) Frith (2012) Kuzle (2011:16) Rodriguez and Bosch (2008) Hurme *et al*. (2006) Parviz *et al*. (2014)	7.4.1, 7.4.4 7.4.1, 7.4.4, 7.4.5 7.4.1, 7.4.2, 7.4.4, 7.4.6 7.4.2, 7.4.4, 7.4.5 7.4.2, 7.4.4, 7.4.5 7.4.3, 7.4.4	Declarative knowledge Knowledge of strategies Knowledge of the person Procedural knowledge Knowledge of the task Conditional knowledge	Metacognition is individually and socially mediated knowledge MCK	Knowledge of the person Knowledge of the task	8.4.1, 8.4.3, 8.4.4, 8.4.5 8.4.1	Kuzle (2011:16) Rodriguez and Bosch (2008) Hurme *et al*. (2006) Parviz *et al*. (2014) Efklides (2011)
Hurme *et al*. (2006) Rodriguez and Bosch (2008:289)	7.4.1, 7.4.2 7.4.1, 7.4.2, 7.4.3 7.4.2, 7.4.3, 7.4.5	Planning Monitoring Evaluation	Metacognition is individually and social regulated MCR	Monitoring Evaluation Planning	8.4.1 8.4.2, 8.4.5 8.4.2, 8.4.3, 8.4.5	Barrett *et al*. (2012) Goos *et al*. (2004)
Bormotova (2011) Bermudez (2007:285) Tabach and Nachlieli (2012) Papaja (2012) Van Lier (2000)	7.5.1, 7.5.2 7.5.2, 7.5.3, 7.5.4 7.5.1, 7.5.2, 7.5.3, 7.5.4 7.5.1, 7.5.2 7.5.1, 7.5.2, 7.5.3, 7.5.4 7.5.1, 7.5.4	Ideational Mathematical Social Pedagogical Explanatory Mental verbs	Metacognitive processes can be expressed verbally by means of a metacognitive language MCL	Ideational Social Explanatory Pedagogical Mental verb Mathematical	8.5.1, 8.5.3 8.5.1, 8.5.2, 8.5.1, 8.5.2, 8.5.3, 8.5.4 8.5.2 8.5.2, 8.5.3, 8.5.4 8.5.3, 8.5.4	Bermudez (2008:288) Hurme *et al*. (2006:196) Papaja (2012) Van Lier (2000)
Jimoyiannis, Tsiotakis and Roussinos (2013) Voogt *et al*. (2015) Hurme *et al*. (2006). Tjoe *et al/* (2013)	7.3.1 7.3.2 7.4 7.5	Social networks Social relationships Interpersonal metacognitive networks Socially shared metacognitive networks	The epistemic context-specific metacognition is a local construct that can be represented as a metacognitive network MCN	Social networks Social relationships Interpersonal metacognitive networks Socially shared metacognitive networks	8.3.1 8.3.2 8.4 8.5	Jimoyiannis, Tsiotakis and Roussinos (2013) Voogt *et al*. (2015) Hurme *et al*. (2006).

9.2 Contextualising the findings within the literature (Discussing Table 9.2)

According to Papacharissi (2011), the architecture of a network rests upon the principle of convergence and the nature of the overlapping connections between the nodes. In the current study the constructs were explored across three strata that symbolise the network platforms as social, interpersonal metacognitive and socially shared metacognitive. The researcher is of the opinions that social networks can represent *higher-order properties* that cannot be explained merely by the nodes or the ties between them. As such, the nodes and ties in the social network (stratum 1) form the basis for the metacognitive locale's development across the LIT phases. This *social base* is then the foundation on which the other strata develop.

First, a LIT was developed and implemented from which three constructs emerged that were embedded in the LIT's underpinning hierarchy of PBL, LS and HLT. The constructs were explored in terms of their nature and relation to one another. In Chapter 9, this relationship between and the nature of the constructs are theorised about, highlighting the theoretical architecture which structures the theory of metacognitive locale.

Papacharissi (2011) explains that networks exist because of the growth in knowledge, ideas and influence. In the current study, strong ties exist in strata 1 and 2, yet the weaker ties mainly appear to connect some of the different interpersonal metacognitive networks of the participants. As a result of such social interaction, the interpersonal networks are merged and a third stratum is formed, reflecting the socially shared metacognitive network of the group. Furthermore, the social links between participants (in stratum 1) can be explained by interpreting the interpersonal networks in stratum 2. This seems to be a result of the metacognitive language expressed by the students. Papacharissi (2011:13) explains this by stating, "information doesn't come from either the weak or the strong ties; it arrives through intermediate ties. People rarely use their weak ties because they very rarely communicate through them."

A discussion regarding the key findings of this study now follows.

9.2.1 Proposition One

Metacognition is individually and socially mediated knowledge

Students' contributions to the group were directed at reaching the groups' overarching LS goals. Throughout their attempts, the students expressed different metacognitive knowledge components and, by so doing, contributed various forms of procedural, declarative and conditional knowledge on either a person, task or strategy level. In addition, the students revealed different social roles and relationships that developed throughout the sessions. Serving as a basis for collaboration regarding the PBL task, these roles and relationships provide a social network analysis perspective on how metacognitive knowledge is associated with the individually and socially mediated knowledge in the group. For example, the students reflected on their own and each other's strengths and weaknesses – which correspond with findings in a study conducted by Bormotova (2010). In such network contexts, Hurme *et al*. (2006) explain, the student asks questions or seeks help from others. During these interactions, individuals contributed their knowledge about unique experiences and what they knew and provided different viewpoints which then led to the construction of new knowledge in the group, as also noted by Frith (2012). This, as Rodriguez and Bosch (2008) explain, does not happen automatically. Instead, the awareness of the cognitive processes of self and others brings about metacognitive experiences. Students in this study also elaborated on such experiences by reflecting on their feelings about what they knew or did not know (as emphasised by S2 and S7 – see 7.4.2, § 7.5.2 & § 8.4.2, § 8.5.1). Also, Kuzle (2011:16) reports that *these feelings of suddenly knowing* open up possibilities for more metacognitive experiences to construct mediated knowledge individually and socially as a result of metacognitive reflection. Although not all group members contributed equally (S1 and S11 were socially weak – see 7.4.1 & § 8.4.1), their contributions were valued theoretically and practically. For instance, Parviz *et al*. (2014) explain that a limited knowledge of strategies is the cause of poor metacognitive experiences (reflecting the socially weak members). This suggests that, as Efklides (2011) claims, metacognitive experiences associate with meta-affect and can affect the course of the outcome of cognition. This may explain why socially weak members have weak and/or no meta-affective relationships and consequently weak and/or no cognitive relationships (Papaleontiou-Louca, 2008).

On the contrary, the current study showed that participants who expressed and reflected upon person, task and strategy knowledge made more rigorous and comprehensive decisions, as was the case with Group B (see 8.5.1 and Table 8.3). This finding is supported by Parviz *et al*. (2014:39), who claim that effective writers (who) used the person, task, and strategy knowledge more appropriately [have] the ability to make sound decisions.

Furthermore, the constructed and mediated metacognitive knowledge was also regulated.

9.2.2 Proposition Two

Metacognition is individually and socially regulated

In addition to the metacognitive experience, the stars (S3 & S8– see 7.4.1& 8.4.1) and bridges (S3 & S4 – see 7.4.1; S9 and S11 – see 8.4.1) in the networks directed and supported the social interactions between group members. This quality of the star role was also coupled to regulatory behaviour, as noted by Hurme *et al*. (2006), and supports the second proposition.

Through planning, monitoring and evaluating, participants reflected on the contributions they and the rest of the group made. In terms of planning, the setting of goals and the contribution of ideas necessitated a deeper level of reflection. For example, group members reflected on the ideas and monitored and judged each other's contributions. Group A, at first, seemed distracted from their LS goal and expressed this throughout the meta-affective relationships. This, as Rodriguez and Bosch (2008:289) contemplate, suggests that individuals who experience difficulty in problem-solving have an *inability* to monitor and regulate their thinking. This explains why socially weak members also have weak cognitive and meta-affective relationships. Furthermore, the interpersonal metacognitive networks that students had, suggested their potential for metacognitive regulation. However, they did not always regulate their ideas and explained that they were cautious not to hurt someone's feelings and at first found it difficult to express how they monitored others' ideas. When they did monitor or evaluate each other's ideas or the presented lesson, the students gave explanations for their judgements and in doing so assisted with and scaffolded the construction of new and refined ideas, suggesting co-constructed knowledge through self-and-other regulation. Goos *et al*. (2004) explain that those students who monitor, judge and evaluate their own and/or others' contributions scaffold each other's understandings and contributions to the problem-solving process. This scaffolding nature of self- and other's regulation mirrors Vygotsky's (1987) ZPD in terms of the cognitive relationship between participants. When students work together on the PBL task, they portray different social roles and act, in the case of a star for example, above their own level of metacognitive affordances (Barrett *et al*., 2012). This, for example, explains that through metacognitive regulation (e.g. planning, monitoring and evaluating) students can create opportunities for themselves to acquire knowledge and resources and, through regulating others' contributions, acquire new knowledge. Therefore, individual and social regulation can facilitate the (co)construction of (new) knowledge.

9.2.3 Proposition Three

Metacognitive processes can be expressed verbally by means of a metacognitive language

In total, six types of metacognitive language emerged from the analysis of the collaborations in the design sessions. The researcher reminds the reader that these types of language are derived from the work of Nachlieli and Tabach (2012) and Papaja (2012) – see Table 4.1.

Participants' metacognitive knowledge and metacognitive regulatory processes were verbally expressed in terms of these six metacognitive language types, each posing a unique function of the discourse in the design sessions.

The researcher therefore believes, pragmatically, that participants' awareness of these metacognitive faculties assisted in the coordination of the social network, namely the roles and relationships between group members, even though Bermudez (2008:285) is of the opinion that social activities do not necessitate *the intervention of language*. Furthermore, Bormotova (2011) explains that through such narratives, participants engage in deepened metacognitive reflections and in a sense ignite their socially shared metacognitive faculties. As each metacognitive language type was used to express either metacognitive knowledge or regulatory components, it offers a view on participants' interpersonal metacognitive networks – which is the metacognitive contributions they make to the group. When these metacognitive contributions (e.g. knowledge of the person, task or strategy) are regulated, and therefore acted upon metacognitively, the group seemed to acquire a meta-metacognitive contribution as group members (co)constructed and (co)regulated their knowledge of the ideas, mathematics, pedagogies and an understanding of their own and others' explanations through social interaction (see Table 7.8 and Table 8.7), encapsulated by the social language through which they expressed their orientation towards each other's metacognition, namely the location of their metacognitive stance. Furthermore, the use of mathematical, social and pedagogical language allowed the students to reflect on, understand and reconceptualise their ideas and ultimately produce a lesson plan that they felt was effective in the context for which it was designed. Tabach and Nachlieli (2012) elaborate that during processes where individuals talk about and share their ideas, new learning is enabled and enacted. In this study, metacognitive language was an introduced theoretical construct used to identify and understand the way in which students express their interpersonal metacognitive processes. From understanding these processes, the researcher attempted to theorise about the social interactions, moving towards a local theory of students' metacognition.

For the purpose of developing LT, the complementary relationship between metacognitive language and metacognitive processes that are reflected upon and regulated seems interrelated, suggesting that metacognitive language kindles the reflective process and brings about metacognitive knowledge through regulation. This seems to illustrate what Bermudez (2008:288) calls the "coordination in resources allocation and communal activity at the linguistic level".

Returning to the co-construction and co-regulation of (metacognitive) knowledge, Hurme *et al* (2006) aver that there is an association between metacognition and the features of communication. This seems to be supported in the study by Ballera *et al*. (2013), which revealed that deepened learning increases communication and socialisation, possibly at the outflow of what Hurme *et al*. (2006:196) refers to as the "relations between supporting pairs' own thinking and social network analysis".

9.2.4 Proposition Four

The epistemic context-specific metacognition is a local construct that can be represented as a metacognitive network

As discussed above, knowledge can be constructed, reflected upon, reconceptualised and reconstructed in order to collaboratively solve a problem (PBL). The findings of this study therefore propose that these emerging constructed levels of knowing are the result of several conditions and processes, contextualised by the implementation of a LIT. In joint problem-solving, each group member's contribution led to the flow of ideas that when reflected upon, exposed students' metacognitive knowledge and regulatory processes. Through shared content, shared ideas, a shared development environment and shared metacognitive language, participants of this study verified the fourth proposition. Jimoyiannis, Tsiotakis and Roussinos (2013) illuminate these expressions as the exchanging of ideas to develop critical and social reflective thinking, rooted in the concepts of social network analysis. As such, this study echoes the level on which students contribute, access, manage, and therefore, network knowledge and regulatory processes, similar to the results of Voogt *et al*. (2015) and Hurme *et al*. (2006). The fostering of these locally induced principles of network analysis (e.g. language and context) precedes collaborative knowledge building (Tjoe *et al*., 2013). Therefore, the fourth proposition's notion of metacognitive networks explains that these collaborations evolve from and into interpersonal metacognitive networks (see Figure 7.3 and Figure 8.3) as well as socially shared metacognitive networks (refer to sections 7.5 and 8.5) as a product of the metacognitive language.

The findings of the four propositions outlined above can then be positioned on three cohesive theoretical networks, identified in this study as strata 1, 2 and 3, discussed next in an attempt to outline and construct the local theory.

9.3 Theorising about the findings through the four propositions

As conceptualised, the constructs that emerged by implementing the LIT are reflected in the four propositions outlined to explore and understand these constructs. Figure 9.1[46] illustrates the three strata on which the findings emerge (as captured across Chapter 7 and Chapter 8 and summarised in Table 9.2). Together, the three strata scrutinise the architecture of the theory of metacognitive locale.

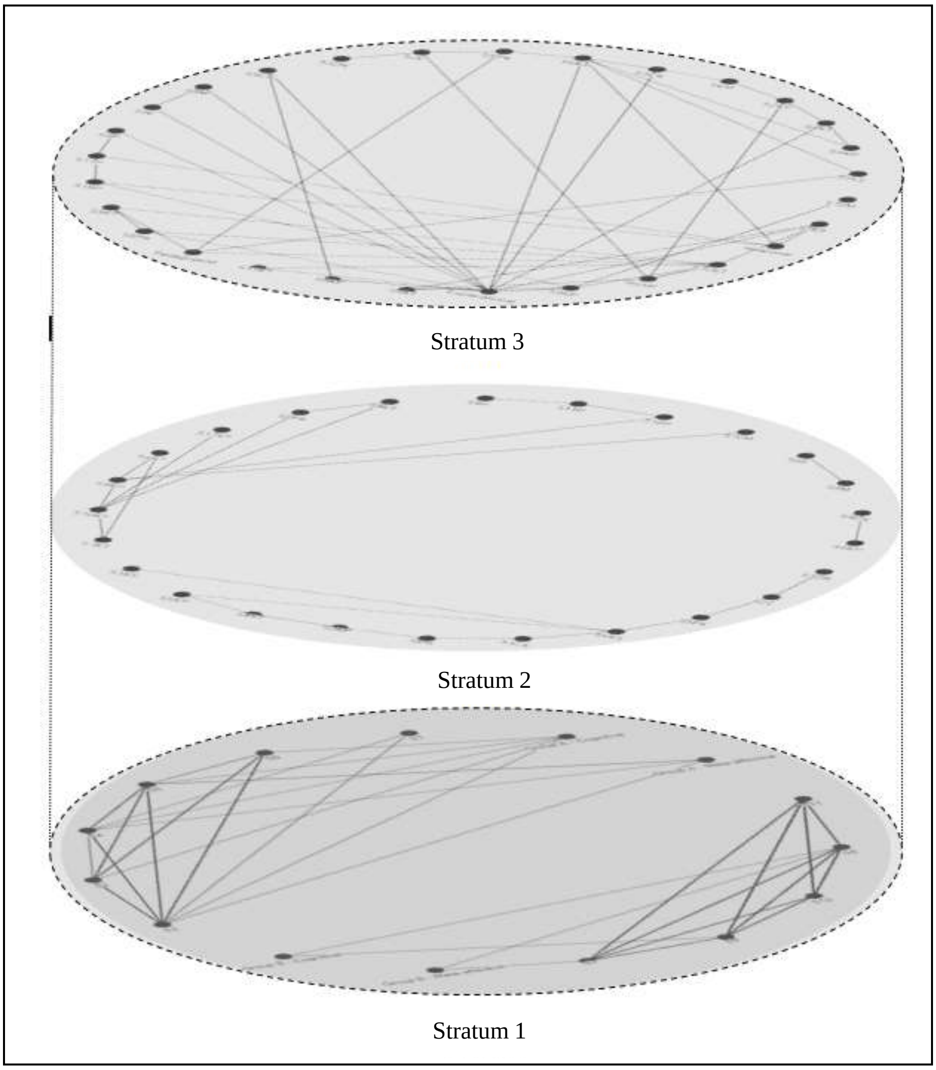


Figure 9.1 Illustration of the metacognitive locale

[46] Refer to Addendum I for a more detailed portrayal of each of the strata illustrated in Figure 9.1.

Figure 9.1 shows how the findings obtained from the networks drawn in NodeXL can be situated on three strata. The theory of metacognitive locale therefore explains the interrelated nature of the relationship between the constructs of metacognition, metacognitive language and networking that exist on across these strata.

9.3.1 Discussion of the findings of stratum 1

Stratum 1 represents the social network which serves as the basis from which other networks evolve. In particular, this network consists of the social roles of group members and the social relationships between them. Although the social network is not intended to answer the research question(s), it does provide a context and framework for doing so.

Even though the social roles and relationships are drawn upon in this study to explore the individual and shared network characteristics of the participants, the discussion of the findings does not reflect these roles and/or relationships. Instead, the findings of stratum 1 can help explain the nature of some of the data through an understanding of the theory of social network analysis. Stratum 1 therefore enables the researcher to interpret the findings as the roles and relationships that act as explanations in interpreting the findings of strata 2 and 3 through the theory of social network analysis, in accordance with the theory of metacognition (stratum 2) and the theory of the ZPD (stratum 3).

9.3.1.1 Interpreting stratum 1 through SNA

The dynamics between the members in the two groups were constantly changing throughout the sessions as members became more interconnected and communicated to solve the PBL task's problem. Similarly, Ballera *et al*. (2013) explain that this tendency to act as a group and become familiar with one another is characteristic of SNA studies. Even though the stars in the networks were identified, other group members' participation and the roles they portrayed suggest that the knowledge and regulatory actions occurred from, with and via all group members. Even the socially weak members' meek contributions were transformed through discussions as all members' contributions were considered collectively.

In the following discussion of stratum 2, these roles and relationships are drawn upon, to understand the context, and/or to explain the nature of the interpersonal metacognitive networks and socially shared metacognitive networks.

9.3.2 Discussion of the findings of stratum 2

Barett *et al*. (2012) explain that individuals bring unique characteristics to group settings, which Efklides (2011) refers to as levels of knowledge of person, task and strategies. The current study shows that during each of the LIT phases and through participants' collaborations, the construct of metacognitive knowledge emerged either as a knowledge domain (e.g. procedural, declarative or conditional) or knowledge level (e.g. person, task or strategy). It appears from Table 9.2 as if Group A expressed more metacognitive knowledge domains and level characteristics than Group B. In particular, the scarcity of metacognitive knowledge of strategies, procedural and conditional knowledge suggests why Group B felt that they did not have to change their initial lesson plan. It was only after a follow-up session that participants of Group B expressed their awareness of the issues with their lesson plan. Kuhn (2012) explains that individuals can gain knowledge from others, even though this knowledge is not consistent, in the sense that not all group members gain equally. To elaborate, Kuhn (2012) clarifies that cognitive collaboration does not always yield benefits for greater cognitive development. In terms of Group B, it seemed that some students (e.g. S10) did not benefit or develop new knowledge after their collaborations. Wilson and Clarke (2004) explain that those who do not develop new knowledge are likely to be unfamiliar with the language that other members use to express their reasoning. This suggests that merely interacting does not promote or foster metacognitive knowledge; it necessitates a shared understanding of what this knowledge entails and seems to depend on the metacognitive nature of the interaction. Not constructing new knowledge does not necessarily indicate poor performance.

In addition, since group members were expected to solve the PBL task, the opportunities to plan the task and execute these plans might have been uncommon, as De Backer *et al*. (2015) explain. Frith (2012) claims that knowledge of others (person knowledge) and knowing the intentions of others can hinder performance. It is therefore important that participants focus on an overarching goal that shares their intentions, rather than focussing on individual needs. The aim of collaborative opportunities should be that the whole group benefit by the individual contributions.

Both Group A and Group B reflected in terms of their metacognitive regulation as they planned together, monitored and evaluated each other's contributions. Hurme *et al*. (2006) explain that the individual characteristics depend on the individual's beliefs with related social dispositions (roles). Such interpersonal networks therefore provide information about how metacognition is configured in problem-solving. For example, the expressed metacognitive knowledge and regulatory components not only show what individuals' metacognitive skills are, but also infer what metacognitive skills have not been expressed or developed. This also reflects students' metacognitive needs (Norwich & Ylonen, 2013) Stratum 2 therefore exhibits the nature of the interpersonal metacognitive networks as expressed throughout the sessions. Chatzipanteli *et al*. (2014) further elaborate that students who use metacognitive strategies (such as planning and evaluating) to diagnose problems and solve them, remember and learn more than others. Students can then adapt their ideas by becoming aware of their own metacognitive skills. Mavarech and Kramarski (2014) propose several strategies to aid in developing this awareness, one of which suggests providing opportunities for groupwork. Chatzipanteli *et al*. (2014) agree that it is through cooperative structures and social learning that new metacognitive networks can be built.

In the following discussion, these interpersonal metacognitive networks are drawn on to identify the metacognitive language used to express these networks and to elaborate on how the metacognitive language and the interpersonal metacognitive networks promote socially shared metacognitive networks.

9.3.2.1 Interpreting stratum 2 through metacognition

Garrison and Akyol (2011) are of the opinion that the increase in the use of social networking is the reason why individual metacognition is a key to understanding metacognition as a social construct. Furthermore, to develop metacognitively, the individual requires various types of metacognitive knowledge and regulatory skills, which can be developed through teaching and learning. Hence, the clear expressions of individual reflections can be used to direct individual awareness and allow others to share in the regulation of their metacognition. Interpersonal metacognitive networks, then, refer to the monitoring and evaluating of one's own strengths and weaknesses (Chen *et al*., 2012). Chen *et al*. (2012) emphasise that such individual characteristics can influence greater social learning through a common understanding of the expressions. Barzilai and Zohar (2014) therefore claim that individual metacognition relates to the nature of the individual's knowledge and of the processes of knowledge. These metacognitive skills can then inform the individual's knowledge and contribute to the development of others' knowledge. The findings suggest that when students use their weak ties, such as those reflected on in the interpersonal metacognitive network, and express their thinking about this network through metacognitive language, a strong tie can be formed socially. The result is a socially shared metacognitive network that underscores an existing interpersonal metacognitive and social network.

9.3.3 Discussion of the findings of stratum 3

The idea of collaboration, as Frith (2012) explains, is that individuals must work together to achieve a greater goal. Deed (2009) explains that to reach this shared goal, group members must use their unique metacognitive processes (knowledge and regulation) and allow others to examine their learning experience. Deed (2009) adds that the exact decision-making processes of the individual remain unknown. However, when individuals come together and share their ideas, they express these internal metacognitive networks and develop their learning by building on these complex inner structures. Therefore, an understanding of the metacognitive language used to express the metacognitive thinking processes seems vital for understanding how social interactions and group members' contributions pave the way towards developing socially shared metacognitive networks.

The present study was grounded in metacognition, social network analysis and the ZPD as theories of social constructivist learning. Its roots therefore extended from an individualist and constructivist approach to learning, through metacognition; towards a self-directed and collaborative dialogue between group members. Goos *et al*. (2004) explain that such a design promises overt social interdependence for success in problem-solving. With respect to the socially shared metacognitive processes, individuals' awareness of their one knowledge triggered selective knowledge and regulatory actions and expressed their metacognitive processes through a metacognitive language. Students' thinking aided better planning for effective solution approaches, and regulation of each other's thinking facilitated the development of new ideas to monitor and select. These metacognitive skills therefore proved to be important for productive problem-based learning, also noted by Kuzle (2011).

Metacognitive experiences therefore appear to improve the capacity to engage and succeed in cooperative learning environments (Deed, 2009).

9.3.3.1 Interpreting stratum 3 through the ZPD

In terms of metacognition, the researcher modifies Vygotsky's (1987) definition of the ZPD to infer the metacognitive development of the participants and adapts Ramos *et al* (2013)'s view on ZPD and metacognition. In this regard, ZPD refers to the theoretical distance between one individual's interpersonal metacognitive network and the potential development for that network as determined by what they can achieve through collaboration with others who possess, at that time, a co-metacognitive component. The co-metacognitive component refers to the component(s) necessary to complete a metacognitive experience within the network. For example, when one student expresses metacognitive knowledge about the task and offers an idea, then another group member reflects on and monitors the idea, perhaps asks a question and evaluates the contribution. Together the metacognitive knowledge and regulatory components are contributed to the group and if accepted and collaborated upon, the reflections are likely to foster a shared metacognitive experience.

Gunawardena *et al*. (2014) explain that collaborative problem-solving group thinking processes move from a given individual context through discourse, action, reflection, and reorganisation towards a socially shared metacognition. ZPD implicates social constructivism as the transition from one student's metacognitive thinking to another's and can be viewed as a shift from the one with less metacognitive skills to one with a more sophisticated and complex network of metacognitive thinking (Belbase, 2013). In doing so, students can scaffold each other's progress (Gunawardena *et al*., 2009) by managing their knowledge and the knowledge of their peers at their own pace in order to regulate their thinking.

Within the context of a mathematics education module for intermediate phase fourth-year pre-service teachers at a rural university in North West, South Africa, a local theory about students' metacognition was developed. Collectively, the propositions and the findings that reflect the proposition across strata 1, 2 and 3 enable the researcher to answer the following secondary research questions. First, discussing secondary research questions 1 and 2 offers a challenge since they seem to have an interrelated nature. The challenge then was to decide which secondary research question (question 1 or 2) could be answered first. For example, students expressed their interpersonal metacognitive networks through a metacognitive language. Their metacognitive networks can be socially shared when understanding and reflecting on the expressed metacognitive language. Therefore, the researcher argues that, in order to answer the first research question, an understanding of the second one is necessary. With this in mind, the following discussion answers the research questions, based on the exposition of the propositions and their relation to the three strata.

9.4 Answering the secondary research questions

In the current study, the constructs that were explored emerged across three strata that symbolise the network platforms as social, interpersonal metacognitive and socially shared metacognitive networks. Towards building theory, the aim of this study has been to identify these metacognitive networks and their associating language, to illustrate and to understand them through a theory of the metacognitive locale that the networks represent. The findings of the four interrelated propositions were arranged, theoretically, on strata 1, 2 and 3.

The propositions (9.2) and strata (9.3) together disclose the secondary research questions and inform the primary research question.

9.4.1 Question 1: What does students' metacognitive language entail when teaching-learning and doing mathematics?

From the findings throughout the design sessions and the discussions above, it appears as if students who participated in this study expressed their individual and shared metacognitive processes through metacognitive language. In particular, students expressed metacognitive knowledge and regulatory components using phrases associated with ideational, mathematical, social, pedagogical and explanatory language. These phrases often included the use of mental verbs to hint on the expressing of their thinking.

9.4.2 Question 2: What do students' metacognitive networks entail when teaching-learning and doing mathematics?

Throughout the study, the students expressed their metacognitive processes, which through social network analysis revealed metacognitive networks embedded in their use of metacognitive language. Students expressed their underlying metacognitive knowledge domains (e.g. declarative, procedural and conditional), levels (person, task and strategy) and metacognitive regulatory domains (e.g. planning, monitoring and evaluation). These metacognitive networks revealed interpersonal metacognitive networks which showed an individual's metacognitive contributions to the group and socially shared metacognitive networks, revealing how the interpersonal metacognitive networks are shared and merged.

9.4.3 Question 3: How can metacognitive language and metacognitive networking foster socially shared metacognitive experiences?

When a student expressed his/her metacognitive processes through metacognitive language, other group members could reflect, on a basic or deep level of reflection, on their contributions. From these reflections, other group members also expressed their metacognitive processes (as indicated in tables 7.3 to 7.9 and tables 8.3 to 8.10). As such, group members then contributed their metacognitive components (e.g. knowledge of the task) and regulated each other's contributions by monitoring, planning with and evaluating others' contributions. Based on the conceptual-theoretical framework in Figure 5.5 and Figure 6.1, these contributions led to the development of new ideas and knowledge which are in turn regulated and co-regulated. During this co-construction of (new) knowledge, group members share their experiences, ideas and contribute, metacognitively, to the group's overarching LS goal. It is through these contributions, it seems, that participants of this study shared their metacognitive processes with their group members through social interaction (e.g. social roles and relationships) and through this interaction either contributed to another member's metacognitive knowledge or regulated their metacognitive knowledge. Since these metacognitive experiences are social in nature, they require the use and understanding of metacognitive language. It appears from the findings of this study (Table 7.10 and Table 8.10) that no one particular metacognitive language type can be associated with specific metacognitive knowledge or regulatory components. Instead, through expressing the metacognitive components, participants externalise their (interpersonal) metacognitive networks that consist collectively of the individuals' metacognitive skills and knowledge. Through metacognitive reflection, these skills (e.g. planning and monitoring) and knowledge (e.g. of the person and strategies) instil an awareness of one's own and others' metacognition. On a theoretical level, these functions of metacognitive language and metacognitive networking are therefore argued to co-exist in students' metacognitive locale, where they form social, interpersonal and socially shared metacognitive networks through the expression of and reflection on each other's metacognitive contributions.

Collectively, the secondary research questions above aimed to address the primary research question, discussed next.

9.5 Putting it all together: answering the primary research question

The following discussion serves to answer the primary research question:

In what way(s) do(es) understanding of the identified metacognitive language and metacognitive networks contribute to theory building concerning students' metacognitive locale?

First, the researcher is of the opinion that the theory of metacognitive locale is a theory of social practice. To elucidate this statement, Figure 9.2 shows how the components of the theory of metacognitive locale can be organised.

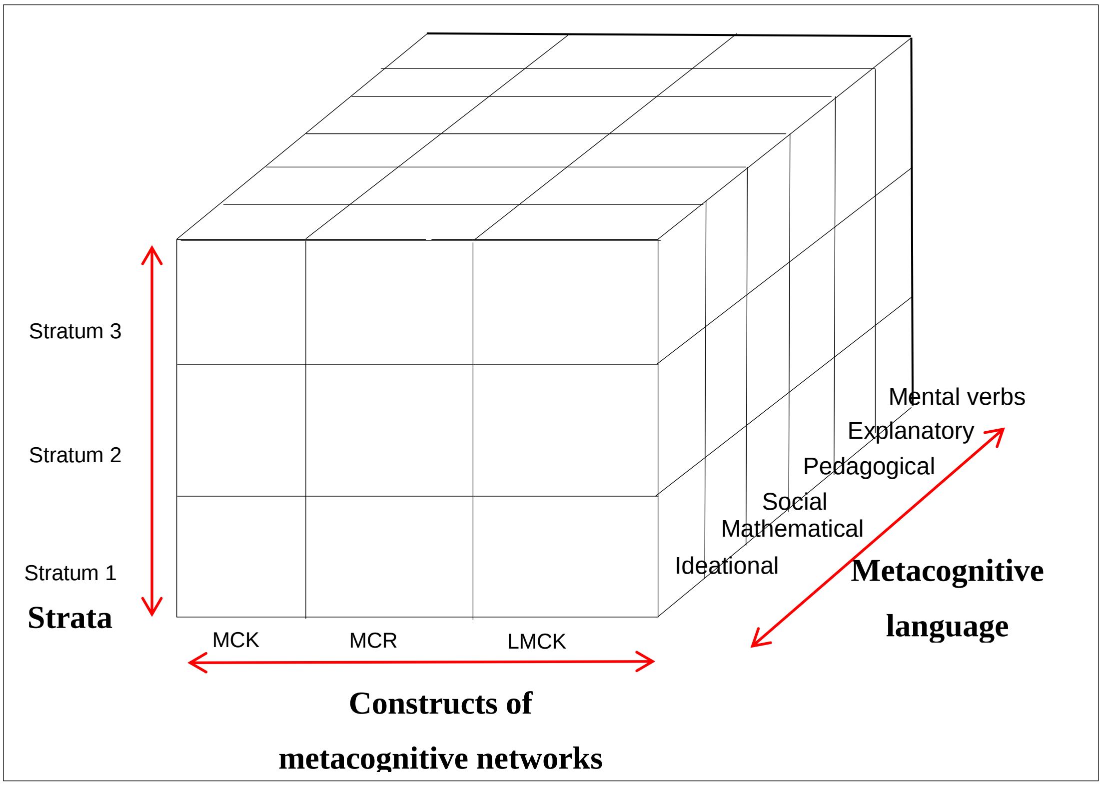


Figure 9.2 Taxonomy of the components of the theory of metacognitive locale

In Figure 9.2 the organisation of the constructs of metacognitive language and metacognitive networks is displayed in relation to each stratum. In order to translate emerging theory from the findings to develop LT, the architecture of the network rests upon the nature of the overlapping connections between the dimensions of the locale reflected in the networks (Papacharissi, 2011). Furthermore, stratum 1 serves as the foundation for the evolving metacognitive networks (also refer to 7.1.2).

In stratum 1, participants communicated with each other about their contributions towards solving the problems highlighted in the PBL task. In doing so, they took on different social roles and formed cognitive and meta-affective relationships. Based on their expressions of their thinking while acting out these roles, metacognitive networks emerged throughout strata 2 and 3. In stratum 2 the interpersonal metacognitive networks were identified. These include networks of the participants' metacognitive knowledge and regulatory components on a person, task or strategy level. When communicating about their thinking, participants used a metacognitive language to express these inner workings of their mind to other group members. They could then reflect on each other's individual contributions and regulate each other's metacognitive knowledge. In doing so, they shared their metacognitive knowledge and regulatory skills in the third stratum. The third stratum shows their combined metacognitive knowledge and regulatory components as expressed through a metacognitive language.

Together, the three strata show that a LT of students' metacognitive locale can be developed through what Haftman and Wenzel (2013) refer to as a context with fixed parameters and assumptions. In the current study, the parameters consist of the three constructs embedded in the LIT, whereas the assumptions comprise the four propositions, suggesting a way in which a LT of metacognition can be developed.

9.6 The narrative of the theory of metacognitive locale

A local theory provides an abstract boundary or set of dimensions that explains the definitions and theorems that relate to the context with its parameters and assumptions. In the current study, after careful exposition of the literature and the findings, the local theory's architecture was introduced across three strata together with four interrelated propositions, reflected in the findings of chapters 7 and 8. Therefore, grounding the findings of this study in the theories of SNA, metacognition and the ZPD, the constructs' relationship appears to have a localising nature, integrated by three levels of strata and contextualised in LS and PBL by the LIT. The strata comprise the dimensionality of metacognitive development across the LIT phases and suggest that LT can be developed about students' metacognitive language and networking.

9.6.1 The (first) definition of metacognitive locale

Since a locale is restricted by the network and language of the locale, it is suggested the dimensions of metacognitive locale exist collectively because of the three strata in which the network develops. These include social, interpersonal and socially shared metacognitive networks, each contributing to the larger locale. As such, the network comprises of a series of smaller networks, each stratum consisting of one or more strata. In theory, metacognitive locale then constitutes a network of thought that is in a state of constant evolution and cannot be obtained in isolation, as the complexity and volume of the locale appears to develop alongside the complexity of the social roles and relationships, which coincide with the changing interpersonal metacognitive networks. Once these networks are linked through a common understanding of the metacognitive language used to express the flow of metacognition within the network, the network evolves as both a singular (interpersonal) and collective (socially shared metacognitive) network, permitting the network to exist in a social reality where the LIT is implemented.

The interdependence of the network's language and the experience of metacognitive growth (individually or socially) seem to precede a shared level of metacognition. The findings in this study suggest that contextual variables (such as metacognitive language) of the network in a social reality consist of the language (itself) expressing the metacognitive processes and the accumulation of these processes, resulting in a socially shared metacognitive network. The network's structure and development therefore relies on the locality of group members' metacognition, or their theoretical position in the locale. The analysis of the locale's social strata made it possible to isolate fundamental cues, subnetworks and metalinguistic phrases relating the metacognitive networks to their social dispositions and the infrastructure of the metacognitive locale.

As research progresses on metacognition and, in particular metacognitive locale, its definition is open for development and refinement, leaving the following (first) proposed definition parsimonious for such purposes:

Metacognitive locale can be defined as a theoretical dimension (or set) consisting of three cohesive networks that are social, interpersonal and socially shared metacognitive in nature and can be used to explain the relationships between metacognition, metacognitive language and networking as emerging constructs of a LIT on PBL and LS.

The definition of metacognitive locale explains that when students are immersed in a social reality (e.g. problem-solving in a group setting), they take on social roles and develop social relationships. These relationships are cognitive and meta-affective in nature as they express students' *thinking* and *feeling* with regard to their ideas and views about problem-solving. These relationships seem to be embedded within the metacognitive language students use to express their metacognitive processes, suggesting that students hold characteristic social roles related to their use of mental verbs (such as thinking and feeling). When students express a similar metacognitive language (or mental verbs), their cognitive and/or (meta-) affective relationship(s) develop(s) into a socially shared metacognitive network. Students then reflect on their own and each other's metacognitive processes and share interpersonal metacognitive ties. As a result of such metacognitive reflections, it is theorised that students share their metacognitive processes in a theoretical dimension (or locale) in which their metacognition develops and/or contributes to the group's goal.

9.6.2 Application of the theory of metacognitive locale

In order to apply the theory of metacognitive locale, the researcher suggests that the theory of metacognitive locale is developed in and is thus suitable for use in the conditions where:

- Theory is to be developed about metacognition
- Methodological contributions are desired to study metacognition and its development
- Social contexts and the roles they play on individual and socially shared metacognition are under scrutiny
- Networks come into play in the conceptual understanding of metacognition in teaching-learning and doing mathematics

The theory of metacognitive locale is therefore contextual and limited by the didactical environment or the theoretical variables (e.g. instructional philosophies) in that environment. The theory of metacognitive locale differs from similar theories (see Table 2.2 & Mosham, 2010) in the sense that it is contextualised, first, in the teaching and learning of module content towards facilitating LS. Second, it describes the relationships between constructs, of which one (metacognitive language) is in itself theoretical in nature as the researcher borrowed its components from the field of social linguistics and metalinguistics for collaborative learning in similar social spaces. The researcher reminds the reader that locale theories are limited to a particular context. Therefore, when applying the theory of metacognitive locale, the context should reflect the conceptual, theoretical and philosophical assumptions held in that study; otherwise, as Ballarin (2014) explains, the local theory cannot be reproduced. The theory of metacognitive locale therefore contributes to the list of metacognitive theories which include tacit, incremental, informal and formal theories (Mosham, 2010). The way in which this study underwrites this contribution is discussed next.

9.7 Contributions of this study

The researcher proposes a design-based theory of mathematics education students' metacognitive locale to explain the relationship between constructs that emerge from implementing a LIT. The contribution of this study is threefold, contextually, theoretically and methodologically manifested in the implications of the theory.

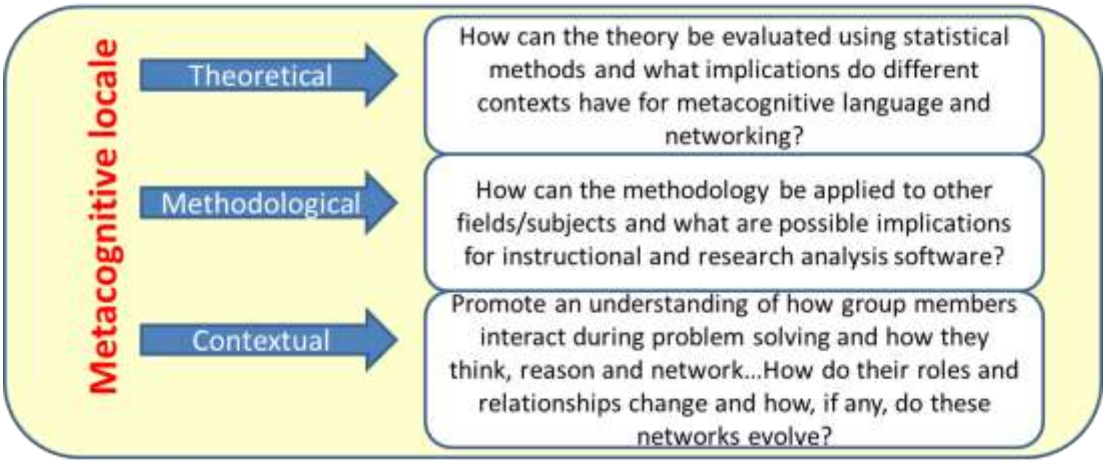
Contextually, the participants who volunteered to take part in the design sessions contributed to the schools, teachers and the learning they were involved with. The participants (students) and the teachers received material they could reflect on and refine and became aware of the advantages of LS, especially in the South African context in a country where LS has not been fully recognised and implemented in all rural areas (Posthuma, 2012). The students themselves have experienced what benefits LS have for professional development and service learning.

Theoretically, the study offers a local theory with constructs otherwise unknown in metacognition research (metacognitive language and metacognitive networks) to explain the relationship between these constructs and their value for developing socially shared metacognitive networks.

Methodologically, the study offers three distinct practices for research in metacognition theory. First, metacognition is in itself a challenging topic for data collection and analysis in qualitative studies. By combining interpretivism and hermeneutics as lenses for understanding the data, the constructs of metacognition, metacognitive language and metacognitive networks were explored in an educational design-based methodology. Second, before data collection, a LIT was developed as a theoretical guide towards facilitating the design sessions. The result was localised emerging constructs, narrowed down to only those that emphasised the study's purpose. Third, there is a paucity of literature offering guidelines for data analysis and presentation of the findings of socially shared metacognition in qualitative studies. By importing the data into NodeXL and conducting social network analysis, the researcher managed to propose an original data analysis plan, particularly for researchers interested in social network analysis and metacognition.

9.8 Recommendations for further research

Parallel to the anticipated contributions of this study, the following contextual, theoretical and methodological recommendations are proposed.



Contextually, a PBL task could also be developed in the context of urban schools. As such, refinement of the LIT can inform decisions about the content of the module where, for example, LS can be replaced or positioned with teaching and learning theories suitable for a course in the didactics of mathematics, or tried out with any other course or content.

Theoretically, the relationship between the constructs across strata 1, 2 and 3 can be used to inform the development and refinement of the LIT, which then impacts on the HLT for evaluation of the theory or the quality of teaching and/or learning in terms of the content of LS.

Methodologically, the use of semiotics as a lens is suggested as the understandings and interpretations of the current study might also develop through textual analysis. Semiotics involves Vygotsky’s (1987) theory on social-cultural thinking and explains how various resources can support and guide the learning activity, choice of pedagogy and the learning tasks. In this sense, semiotics could provide a means of exploring the language used to express metacognitive thinking and to uncover the process whereby this metacognitive language evolves in itself and the network. Correspondingly, the results can be interpreted from a structuralist or post-structuralist perspective, focusing mainly on the taxonomy of different possible metacognitive network structures and the metalinguistic phrases used to express them. In addition, data metrics, which were omitted in this study, could offer a statistical view on the metacognitive networks towards refining the taxonomy to generate a statistical model of the metacognitive locale in a quantitative study.

I also recommend that this proposed contribution should be studied further to develop and refine its structure within different theoretical views and contexts. Perhaps this can be done by conducting statistical analysis of the network data.

In terms of the methodology, future research could focus on how this theory can be applied in other disciplines to develop informed instructional practices and even consider development of software for analysing teaching and learning experiences to determine further implications of the theory.

In close, teacher educators who familiarise themselves with this theory and its underlying assumptions and who apply its principles in teaching and learning environments can make more informed choices regarding curriculum development with a PBL instructional philosophy to engage with their students' metacognitive locale.

9.9 Limitations of the study

Throughout the study some limitations emerged that were conceptually and theoretically unpredicted. These could either have been results of the implemented LIT, in which case it is not a limitation but emerging constructs that require further investigation, or contextual parameters that have somehow left their footprint in the LT. For example, the study did not focus on other emerging constructs besides metacognition, metacognitive language and networking. These *hidden constructs'* roles do not appear to have had an influence on the findings nor the interpretation of the theory of metacognitive locale. When the researcher planned the research design, he did not anticipate limited interest in online discussions, contrary to Garrison and Akyol (2011), and so the social network data were overlooked in the online discourses of a community of inquiry. Furthermore, the withdrawal of Group C early in the study permitted the researcher to carry out the research with a smaller number of participants. This study's design did provide for in-depth exploration and analysis of Groups A and B's participation, resulting in the LT. Commitment to the design sessions and the LIT's guidelines was crucial for participation in the study, especially because of its social nature. Fortunately, the participants in Group A and B stayed committed across the two research cycles during the course of the design sessions.

9.10 In closing, a personal reflection and overture into the thesis

The teaching, learning and doing of mathematics have always been three contradictory themes of my life. A life in which the nature and structure of reality was mainly constructed as an ontology of knowing and applying mathematics. Until now.

The experience of searching for and uncovering the infrastructure of metacognitive locale and theorising about its constructs and taxonomy within this reality has expanded and created awareness of my own social, interpersonal and socially shared metacognitive networks. This now marks only the start of a lifelong journey of (self) discovery and the realisation of the impact of and on this ontology of metacognition, to help perceive and, perhaps locate, our purpose in this reality. In doing so, our understanding of the reality of teaching and learning steers the worldview we have formed about it. We, then, have to acknowledge that a world of teaching and learning cannot be constructed on its own, but only by the co-existence of what we think and feel as reflective practitioners in this world. What we need now is an understanding of this` world, not merely of the different teaching and learning styles and how to accommodate them or the strategies imposed to solve mathematical problems, but to understand how such styles evolve and how the strategies are reflected upon. We need to reflect on *how to know* and on the metastructures that exist, to reinforce *a way of* knowing *what we know* and *what we do not know*.

We, also, need to enforce this knowledge in our practice when teaching, learning or doing mathematics. Nothing less than this will enable us to prepare students for lifelong learning, for knowing their knowing and the knowing of others. This study offers, towards this ideal, a local theory that could reveal at least a glimpse of what these ideas might embrace. Yet, in proposing the theory of metacognitive locale, I cannot help but wonder about the greater intervention and integration with the even more complex system(s) in which we define ourselves as individuals and groups, and how our own contributions unfold and touch the lives of so many others socially, interpersonally and metacognitively.

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Addendum

Addendum A

The PBL task

Date: 05 February 2014

From: The independent teacher

To: mathematics education students

Hello all,

I am a recently appointed teacher at a rural South African school in the North West province near my old hometown. I just love my job. It is the perfect environment to make a difference, particularly in the lives of my students. My colleagues don't seem to share my enthusiasm. The more senior teachers tell me that this aspiration will soon pass. They also don't really take part in coaching sports in the afternoon. In fact, when the final school bell rings, most of them are already on their way, while their learners have to lock the classrooms on their way out. I don't really know what their hurry is, it's not that there is a lot of traffic, since most of the kids walk home or hitch a ride on a passing tractor. The kids do like to play soccer after school though.

I'm writing because I need your help. My younger nephew told me about all the big ideas he has for next year when he starts as a teacher in the city. While he has a BEd degree and majored in Mathematics, I don't know if he is going to make it. The thing is, I have been teaching for 13 years and got a diploma in education back in 2000. Al lot has changed in education since then. I remember back in the days when I was in my final year in the Mathematics didactics class, at that time I did not go through too much effort in reading up or talking to my friends about the work. It felt as though it will just take too much effort anyway. We didn't really think that teachers will need all that theory. For us it was just another big word, like pedagogy and constructivism, especially for me. I am more of a do-it-yourself person. I see myself as an independent teacher. I like to follow my own way, express my own creativity and do things, the way I want to, when I want to. The problem is that, with so many changes and challenges, I don't think I'm fit to still be a teacher, let alone starting a new teaching career at a rural school. My experience as a teacher has always taught me one thing: everybody is different. At my new school, the more senior teachers insist that I do things their way, but I feel that I have more experience than they do since I taught at a top school in the city for most of my teaching career. For example, they emphasise drill and practice methods where rote memorization and fast recall seem to be all that matters to them. Statements like, *I've been doing it for years, and it works* usually accompanies their disagreement with what my nephew is suggesting. Its times like these that I don't know if I should agree with them and do as they do, or if there is some alternative out there that I, or they, are overlooking. It seems like every teacher teaches his/her own way.

Another problem is that the teachers, who make themselves available for help to mentor newcomers like me do not even have a degree. They were available, had some experience in similar schools, or have taught in the FET colleges. I sometimes think they don't even know how to plan a proper lesson. In this case, I feel I am more qualified and more experienced than they are but I struggle to get better marks form my math students. I read up on the matter of education and found that today's younger teachers are more prepared for all of this. For instance, they now talk about metacognition, where I am still clinging to behaviourist teaching and learning. They also know how to help their learners become better at problem solving, something I try to capture in word sums. I want my students to succeed. I also want to teach these professed metacognitive skills so that they can become better learners and I can grow as a teacher. Above all, I want to feel that they are well prepared for the next grade, or topic, but it feels as though we are stuck and I am repeating every lesson with little or no success. My nephew suggests that we should look at the learning trajectory of mathematics to determine where the learners are at the moment, then we must work together, all the grades and develop some kind of a lesson study group.

What makes things difficult is that resources are limited. To put matters into perspective, kids do not have free access to water. Instead, learners and teachers must all share the same tap in the school, which is the one in the principals' office. We do get some subsidy from the department and recently bought four computers for the newly build staffroom. However, some of the locals have stolen the copper cables and so we experience frequent power cuts. In the nearby town, so load shedding is also a problem and there are speculations that this frequent shutdown will get worse in the future and spread to nearby communities. Luckily, my class faces north so we have enough natural light in the summer. I wish I can take my class to the library, but the school did not receive any books so far, so we changed the library into another classroom, since most of the class sizes range between 40 to 65 learners in a class.

I e-mailed my nephew about this and asked whether he has any suggestions for me to improve my teaching. Now my nephew claims that he uses computers and power points for his teaching. He also claims that, at his school, teachers come together once a week or so and discuss issues they have when implementing their planned lessons. The focus is not on discipline, but rather on reflecting on what they planned to do, and how it worked, or not, in practice. They also discuss new or alternative ways of getting better results. I wish we could do it here at Requiro Primary.

I have heard that the Department will give the school money, and other necessary resources, if they manage to get learners to excel, especially in English and Mathematics. My Grade 6 class recently had the highest average in the school (38%). I considered this as an accomplishment, keeping in mind that I teach for the first time in a rural environment with little or no support and a general lack of resources. Also, did I mention that most of the learners speak Setswana at home? Although I do teach in English, there are still certain words that I know they do not understand.

Do you think it is possible to get these learners to do better? If so, what can be suggested to, not only uplift their marks across in grade 6 but also uplift their spirits and make them want to learn more and become equipped for the 21st century. I want them to not only pass, but also to master the content and solve problems, think critically. I want them to become lifelong learners. I really do believe that if they can do well in school, they will have a better and brighter future. If their marks do go up, and the school shows improvement, we can get more support, grow as a school and I suppose there would be more respect from the senior teachers towards me.

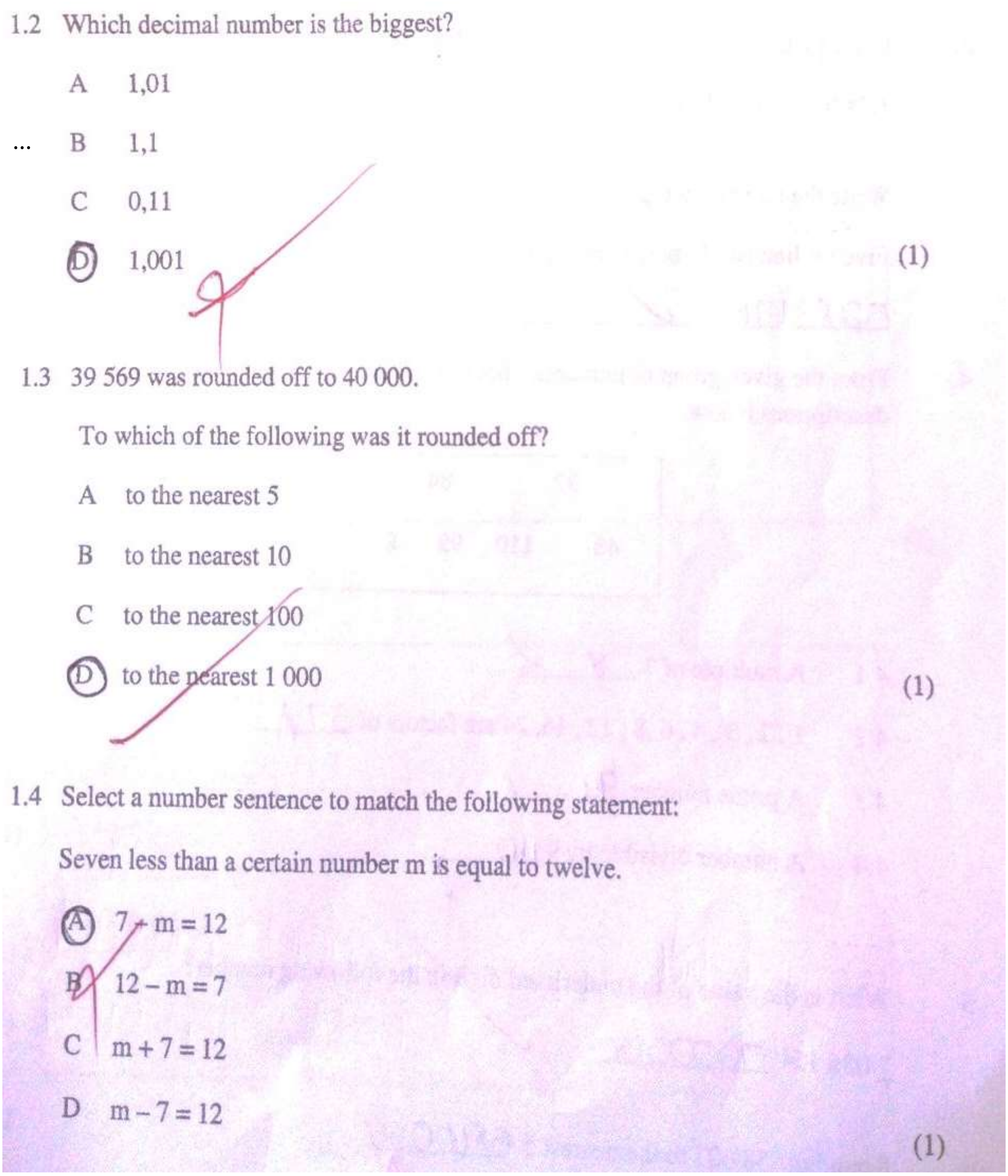
Here are some documents I added to give a clearer picture of where I am teaching.

I first add a satellite image of where the school is situated (I got this from Google).

Here is a photo from the nearby neighborhood, close to Requiro

Most of the learners walk to school. Those who live in the township nearby

Learners seem to struggle mostly with place value. Here is a copy from one learners' ANA test which I have marked:



1.2 Which decimal number is the biggest?

A 1,01

... B 1,1

C 0,11

D 1,001 (1)

1.3 39 569 was rounded off to 40 000.

To which of the following was it rounded off?

A to the nearest 5

B to the nearest 10

C to the nearest 100

D to the nearest 1 000 (1)

1.4 Select a number sentence to match the following statement:

Seven less than a certain number m is equal to twelve.

A 7 – m = 12

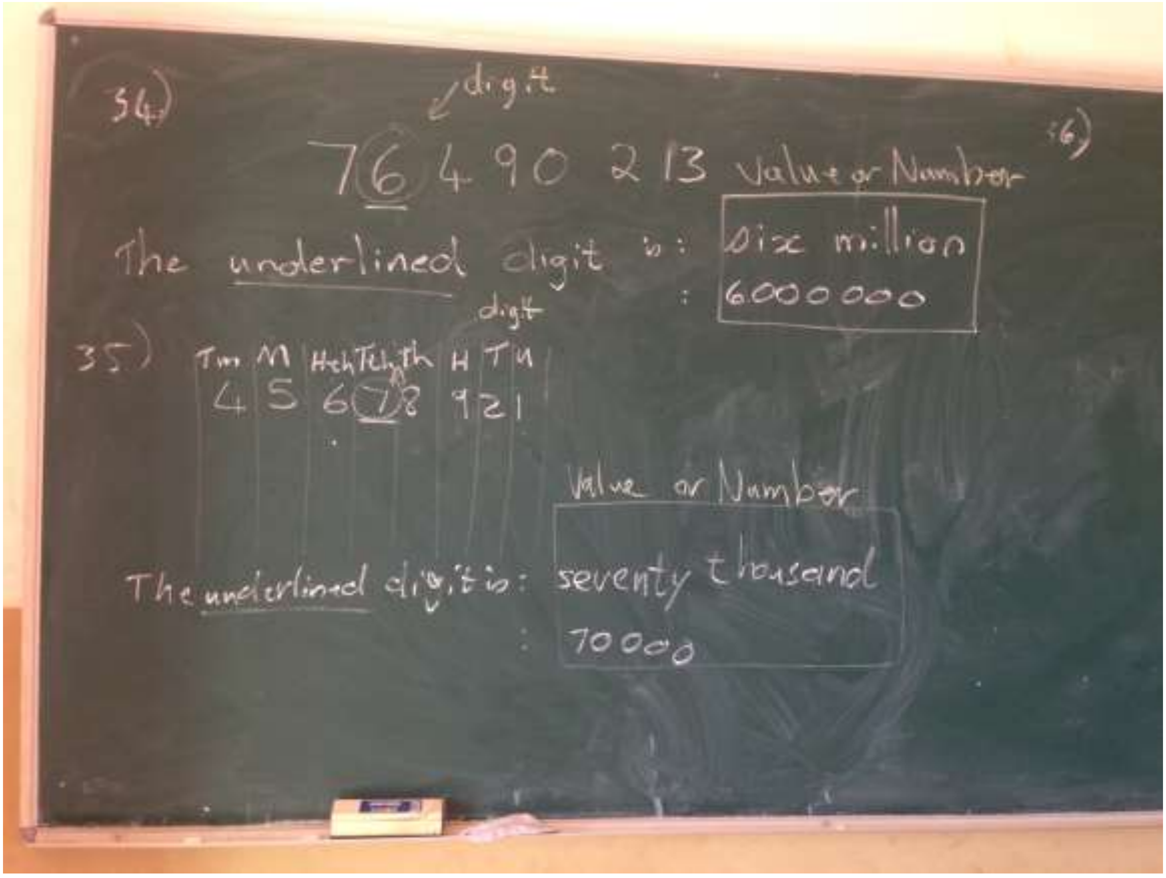
B 12 – m = 7

C m + 7 = 12

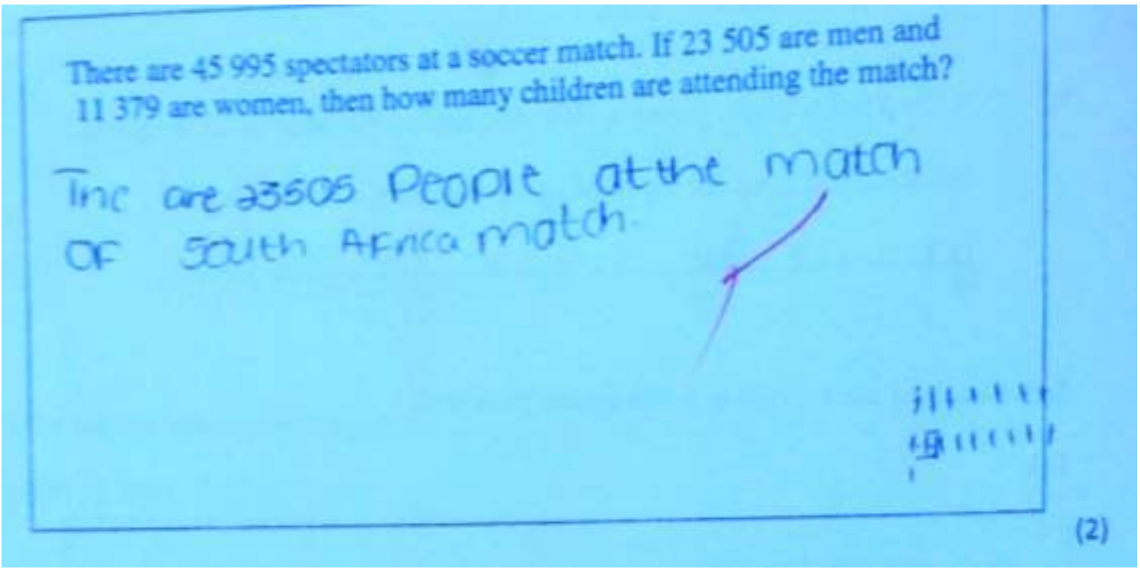
D m – 7 = 12 (1)

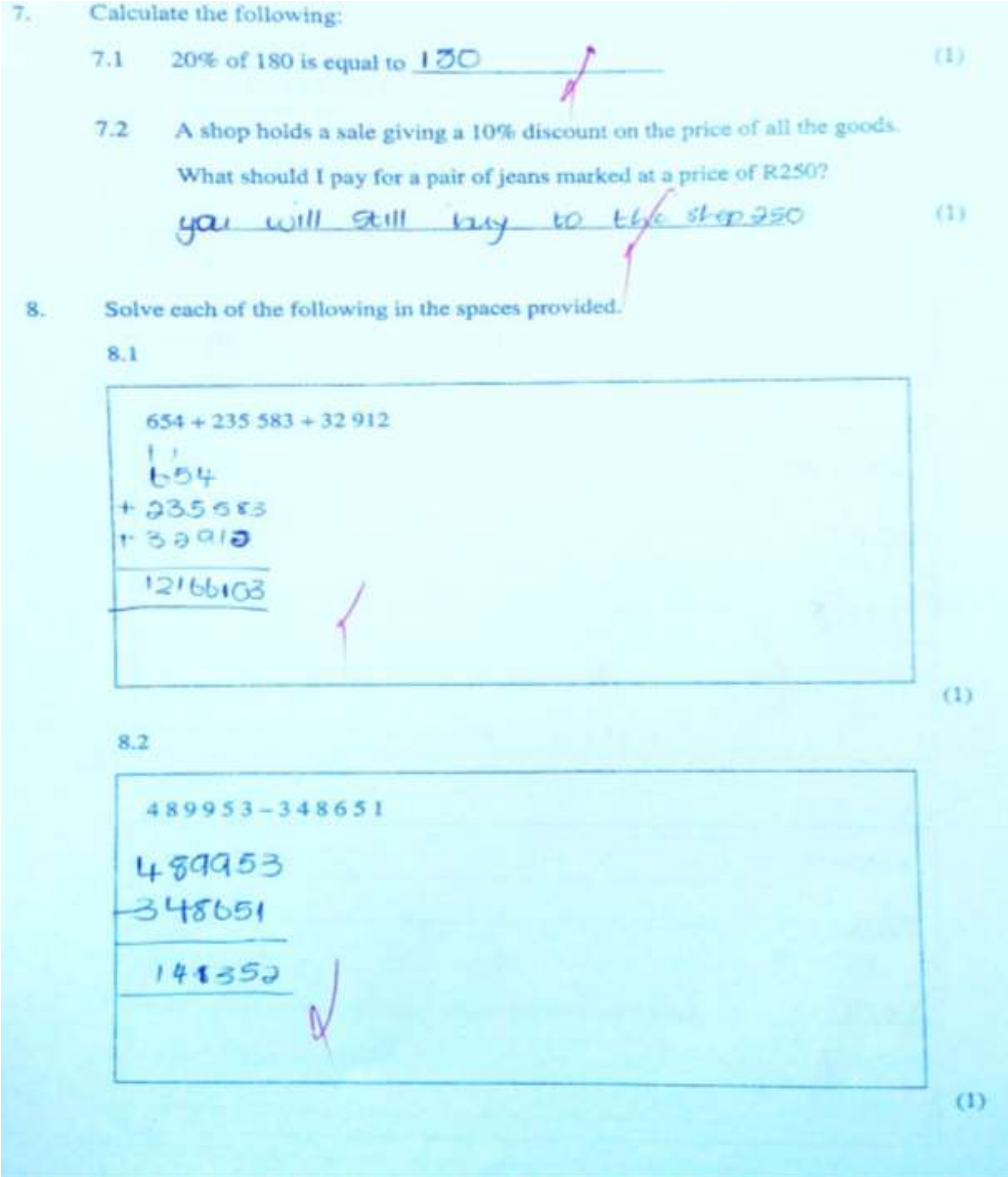
I even tried to go back to basics with them. I tried to, sort of like, explain place value but every time they end up saying that they don't know what they are doing wrong.

Here I explained it on the board:



I then tried to do word sums hoping that it will make the questions more practical and that it would help them...




7. Calculate the following:

7.1 20% of 180 is equal to 130 (1)

7.2 A shop holds a sale giving a 10% discount on the price of all the goods.

What should I pay for a pair of jeans marked at a price of R250?

you will still buy to the shop 250 (1)

8. Solve each of the following in the spaces provided.

8.1

654 + 235 583 + 32 912

654
+ 235583
+ 32912
12166103

(1)

8.2

489953 – 348651

489953
– 348651
141352

(1)

It's not all bad. As I said, I'm very passionate. I want to do the right thing. It feels as if I am not prepared for all of this. I am not sure how to plan my lessons if the kids keep getting the wrong answers. Sometimes, when school is out and everybody went home, I just sit there, wondering what it all means.

I vaguely remember that we used to have lesson study which involved group work in our didactics class during my final year. I took out my old study guide and browsed through some of the articles. I wish I had more time to look at their ideas and to share them with some of my colleagues. Maybe this would make it better for all of us. Please help me to design my lessons to make teaching this year better than the previous one.

I attached a recent newspaper clipping regarding the crisis that teachers face in South Africa. It just puts things into perspective for me. I am sending this message form my laptop while visiting my nephew from his home during the December holidays. At school, we don't have internet access because there's no reception. I just have such high hopes for this new year, I really do hope you can help me.

All my best,
The (not so) independent teacher.

Newspaper clipping:

The Need for Pre-service and/or In-service Programs
For at least seventy-five years educators have recognized
the need to prepare teachers for rural classrooms.
In 1917, Woofter [10] wrote,
For the changed conditions of rural life a new order of school
is needed, and a new type of teacher is necessary for the new
order. Too often the teacher of the rural school is a young person
with city ideals and interests and not in touch with the rural
needs, and again too often not qualified to introduce what rural
life demands (p. 24).
More recently, Dunathan [1] stated,
Small schools cannot allow themselves to be the dumping ground
for the unfit, the inept, and outrageous. They must collectively
insist that the excess capacity of teacher education programs,
particularly in universities, be put to use in training teachers for
small schools. They must demand equity in the study of urban
and rural school problems in teacher training curricula. They
must press for funds to support the installation of small school
education courses in teacher training programs and the retraining
of present faculty to teach them (p. 206).

Here is some research notes by SACMEQ (2011):

In 2007, around one in every five Grade 6 learners did not have all the three basic learning materials needed for effective participation in classroom activities. The situation in the Eastern Cape was of particular concern, because only 67 percent of the learners in this province had all the three basic learning materials. This study also showed that only 36 percent of the learners had sole use of mathematics textbooks in 2007. This means that at least three in every five learners (64%) did not have sole use of these textbooks. This study also revealed that the mean learner teacher ratio (34) in 2007 was within South Africa's benchmark of 40 learners per teacher. In addition, the study revealed that the average number of Grade 6 learners per class (44) in 2007 exceeded the national benchmark of 40 learners per class.

I have also wondered if I should leave the school and go into urban teaching where there are more support and fewer restrictions. I don't know if that is what I really want to do. One day I came upon an advertisement that encouraged me to stay on:

Laying Foundations...Building Futures Who should apply? Anyone who loves children (age: 6 - 12 years) and wants to work in an educational environment with some of the areas such as learning basic mathematics competency, and fostering healthy relationships of these most disadvantaged children.

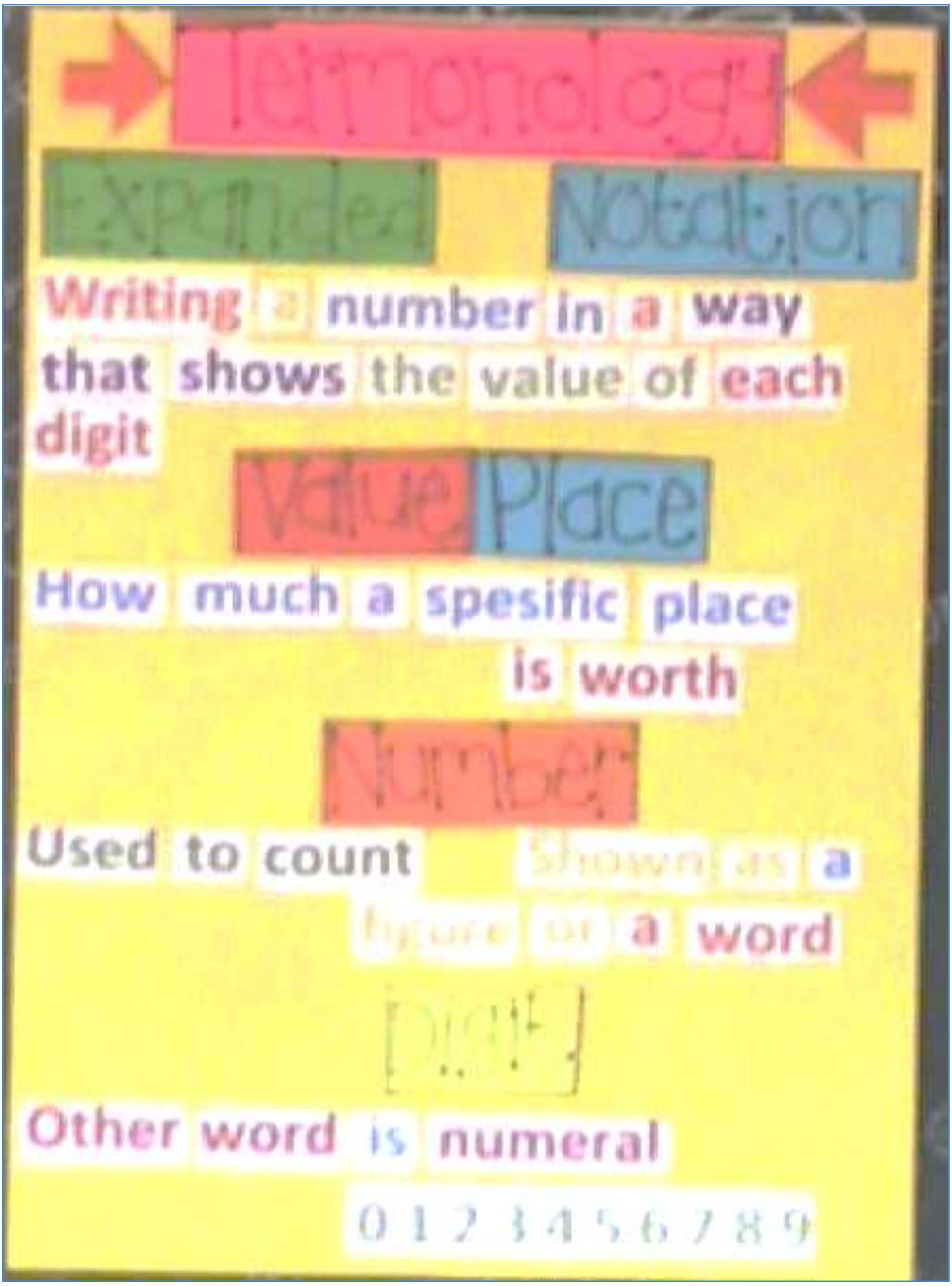
Addendum B
Design sessions' schedule

Lesson study phase		Date	Video number	Research site	Word file name
1	Planning	9 February 2014	M2U00004 M2U00005	University library	1
			M2U00246	University library	2
2	Design of lesson plan and resources	26 February 2014	20M2u00253 20M2u00254	University library	3
	Design of lesson plan and resources	06 March 2014	M2u00258 M2u00259 M2u00260	University library	4
3	Lesson presentation	25 March 2014	M2U00272 M2U00273 M2U00274 M2U00275 M2U00276 M2U00277 M2U00278 M2U00279 M2U00280 M2U00281 M2U00282	School classroom	5 6
	Lesson reflection	27 May 2014	M2U00283 M2U00284	University library	7
4	Refinement of lesson	19 August 2014	M2U00288 M2U00289	University library	8
	Representation				
1	Planning	9 February 2014	M2U00250 M2U00251 M2U00253 M2U00254	University library	1
			M2U00255 M2U00256	University library	2
2	Design of lesson plan and resources	26 February 2014	M2U00264 M2U00257	University library	3
	Design of lesson plan and resources	06 March 2014	M2U00265 M2U00286 M2U00287	University library	4
3	Lesson presentation	25 March 2014	M2U00266	School classroom	5
	Lesson reflection	27 May 2014	M2U00267	University library	6
4	Refinement of lesson	19 August 2014	M2U00268	University library	7
	Representation				8

Addendum C
Group A's posters

Group A' place value chart

Group A' terminology poster



Addendum D

100 000 000	Hundred Millions Lekgolo Mmilyone
10 000 000	Ten Millions Lesome Mmilyone
1000 000	Millions Mmilyone
100 000	Hundred Thousands Kete tse legolo
10 000	Ten Thousands Kete tse lesome
1000	Thousands Sekete
100	Hundreds Lekgolo
10	Tens Lesome
1	Ones Nngwe

Baseline assessment

Write every number in expanded notation

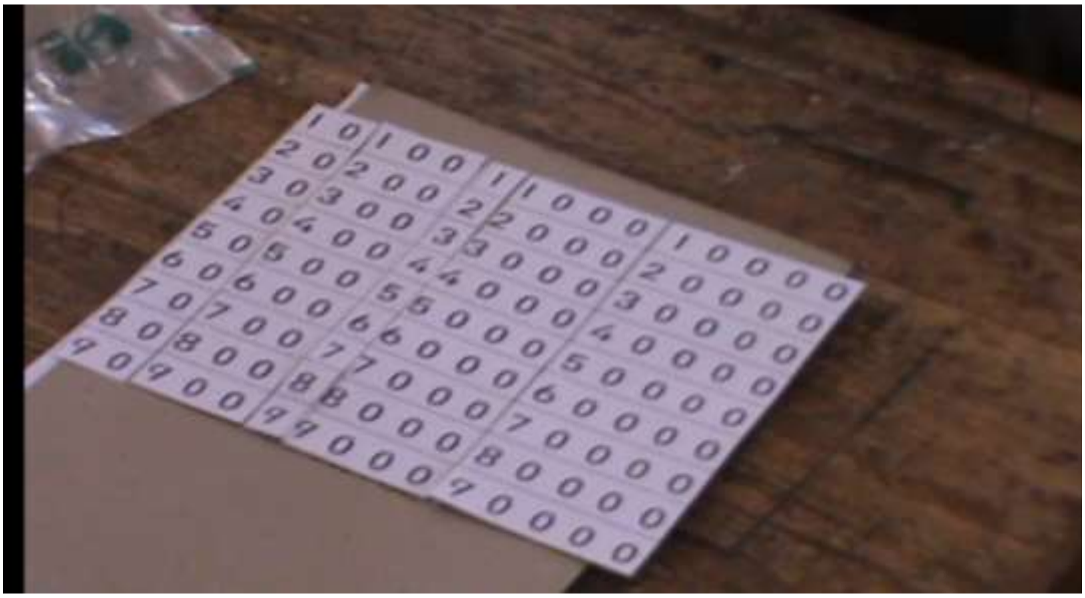
1.	48	=
2.	297	=
3.	6423	=
4.	10750	=
5.	564,663	=

Write the expanded notation as a number

6.		$= (4 \times 10) + (0 \times 1)$
7.		$= (3 \times 100) + (4 \times 10) + (0 \times 1)$
8.		$= (4 \times 1000) + (1 \times 100) + (0 \times 10) + (5 \times 1)$
9.		$= (1 \times 10000) + (6 \times 1000) + (4 \times 100) + (0 \times 10) + (4 \times 1)$
10.		$= (5 \times 100000) + (7 \times 10000) + (5 \times 1000) + (2 \times 100) + (2 \times 10) + (2 \times 1)$

Addendum E
Group B's posters

Addendum F
The table Group B used as a tool for place values



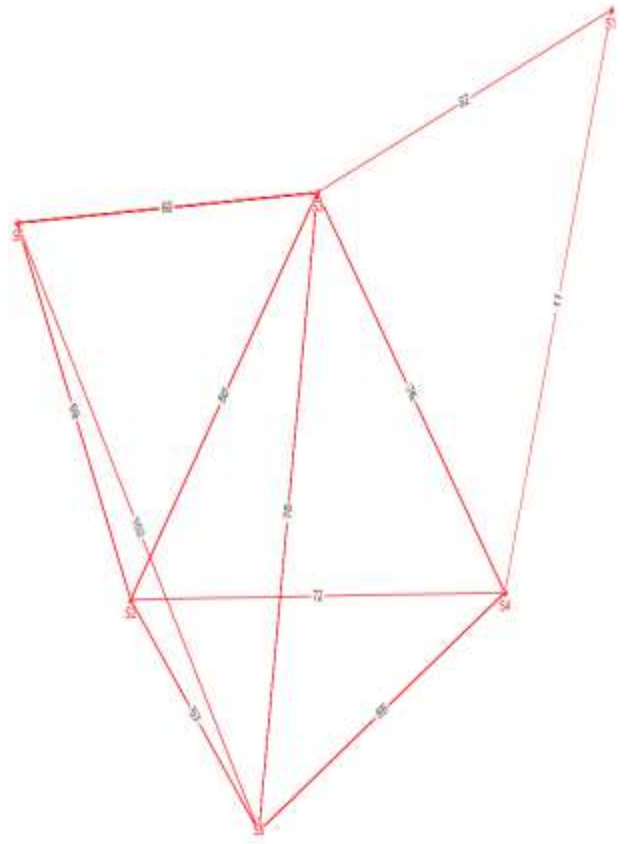
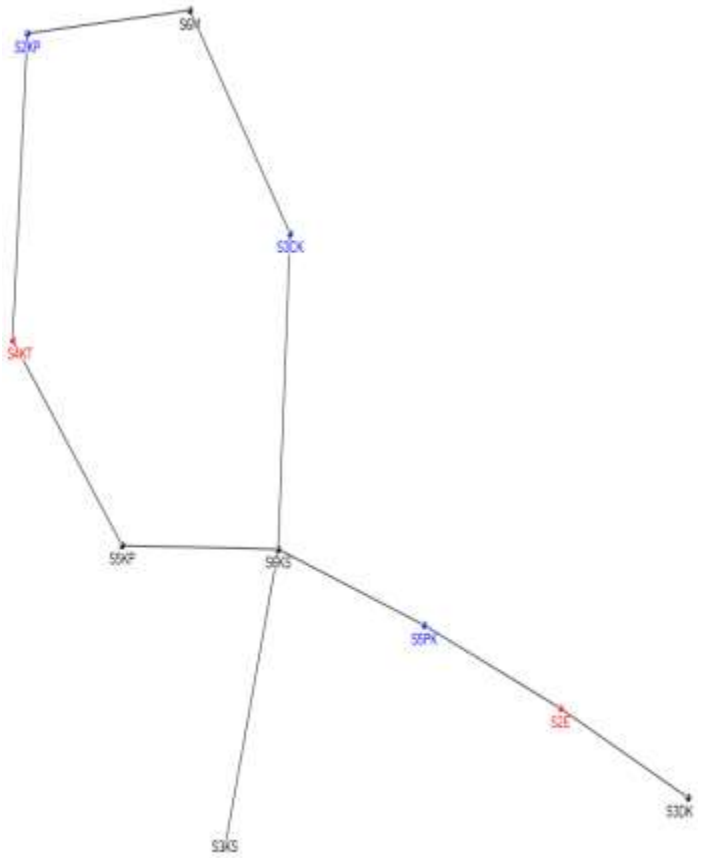
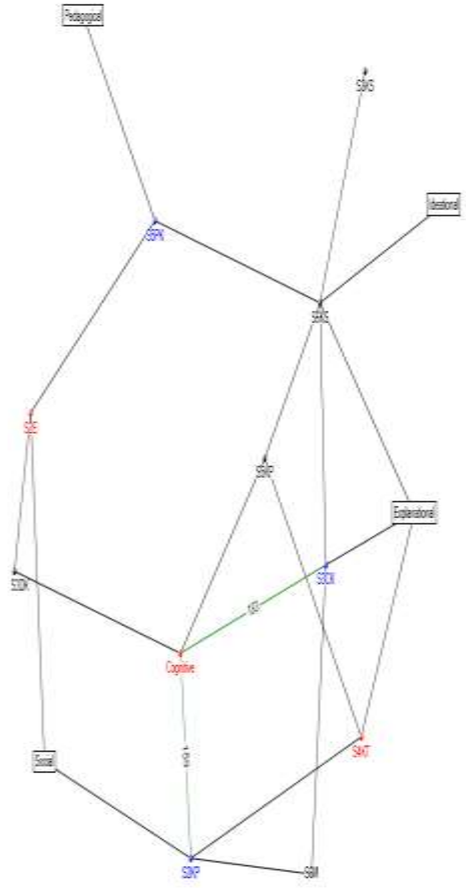
Addendum G
Journal extract of Group C: explaining why they withdrew

We met as a group and divided ourselves in two more groups...We were working together so after doing the assignments. We then schedule a meeting later on, but were too busy to attend. It felt easier to work alone than together. [Group C – S2]

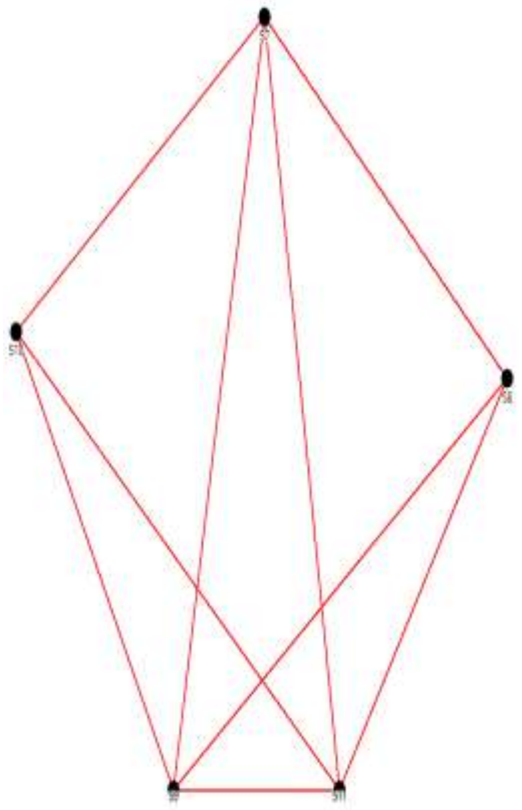
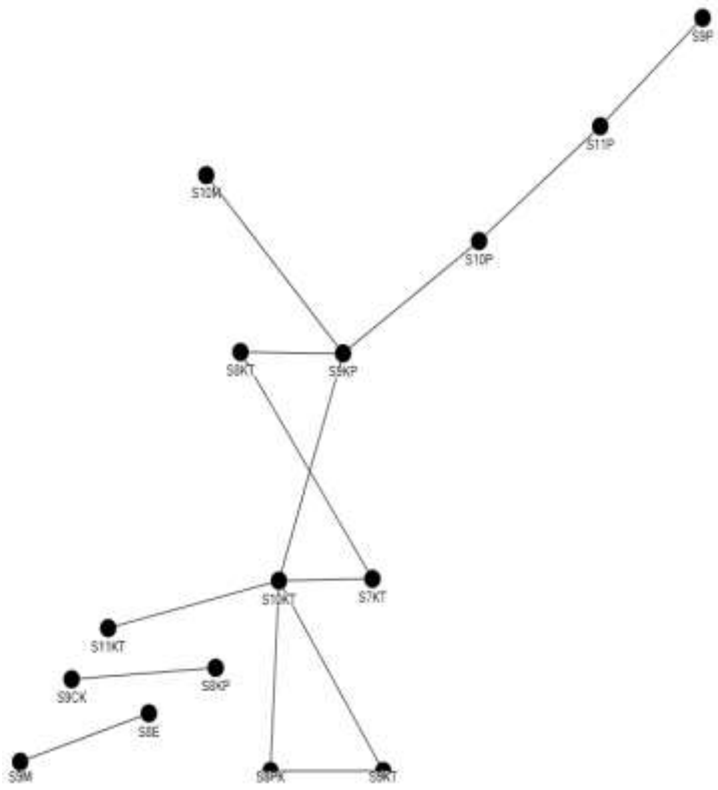
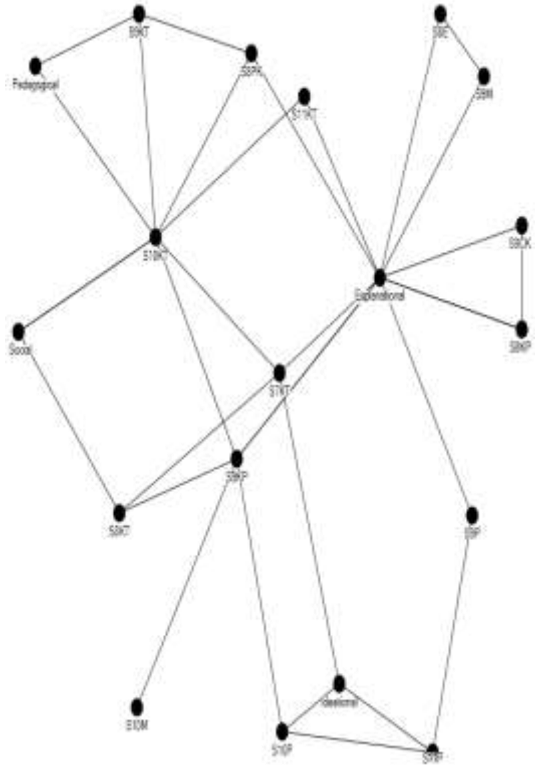
When asked: do you have any suggestions to improve the quality of the group sessions, one participants wrote: No, because we are just fine.

Addendum H
Separated networks

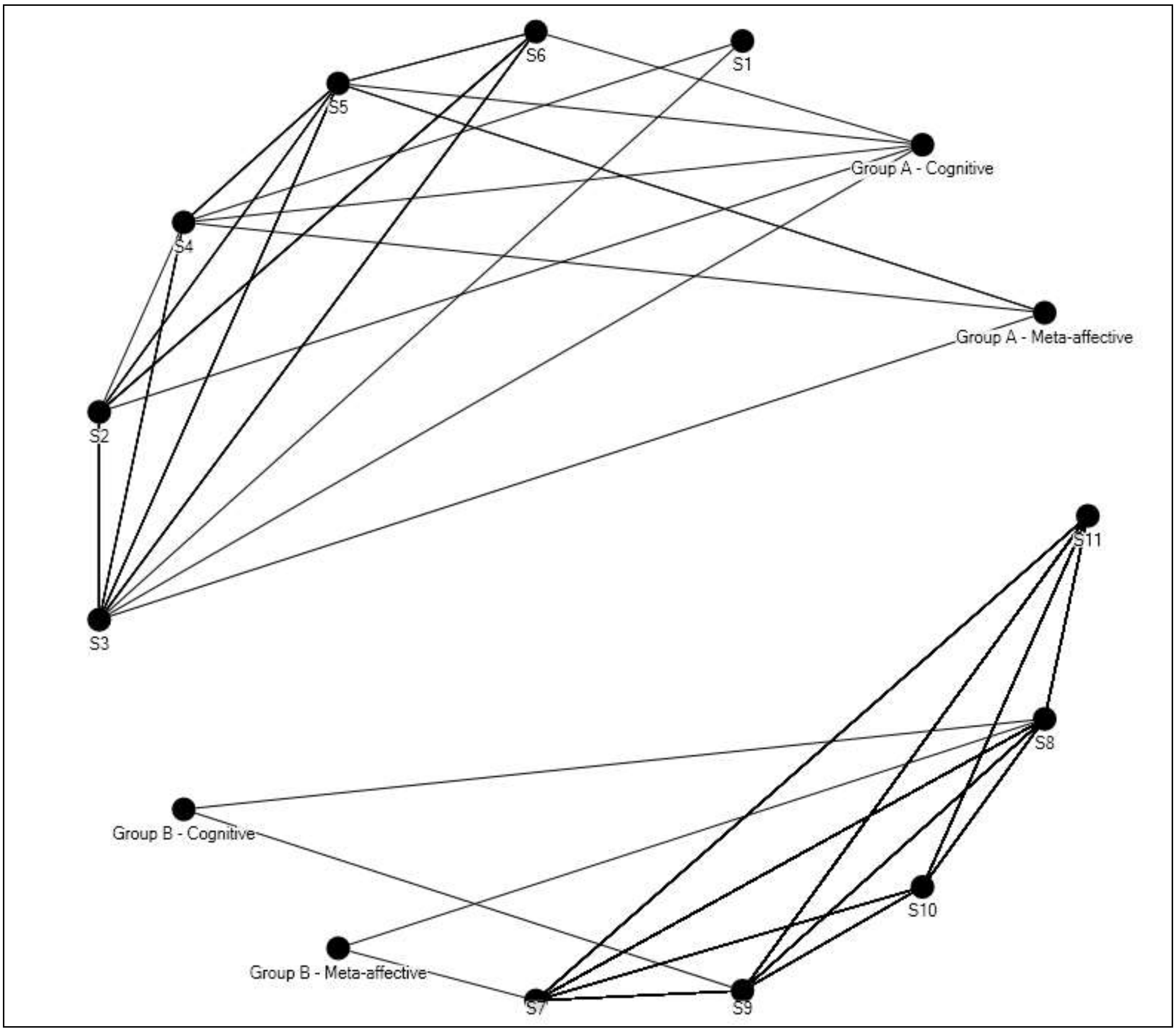
Separated networks for Group A

Social network	Interpersonal metacognitive network	Socially shared metacognitive network
		

Separated networks for Group B

Social network	Interpersonal metacognitive network	Socially shared metacognitive network
		

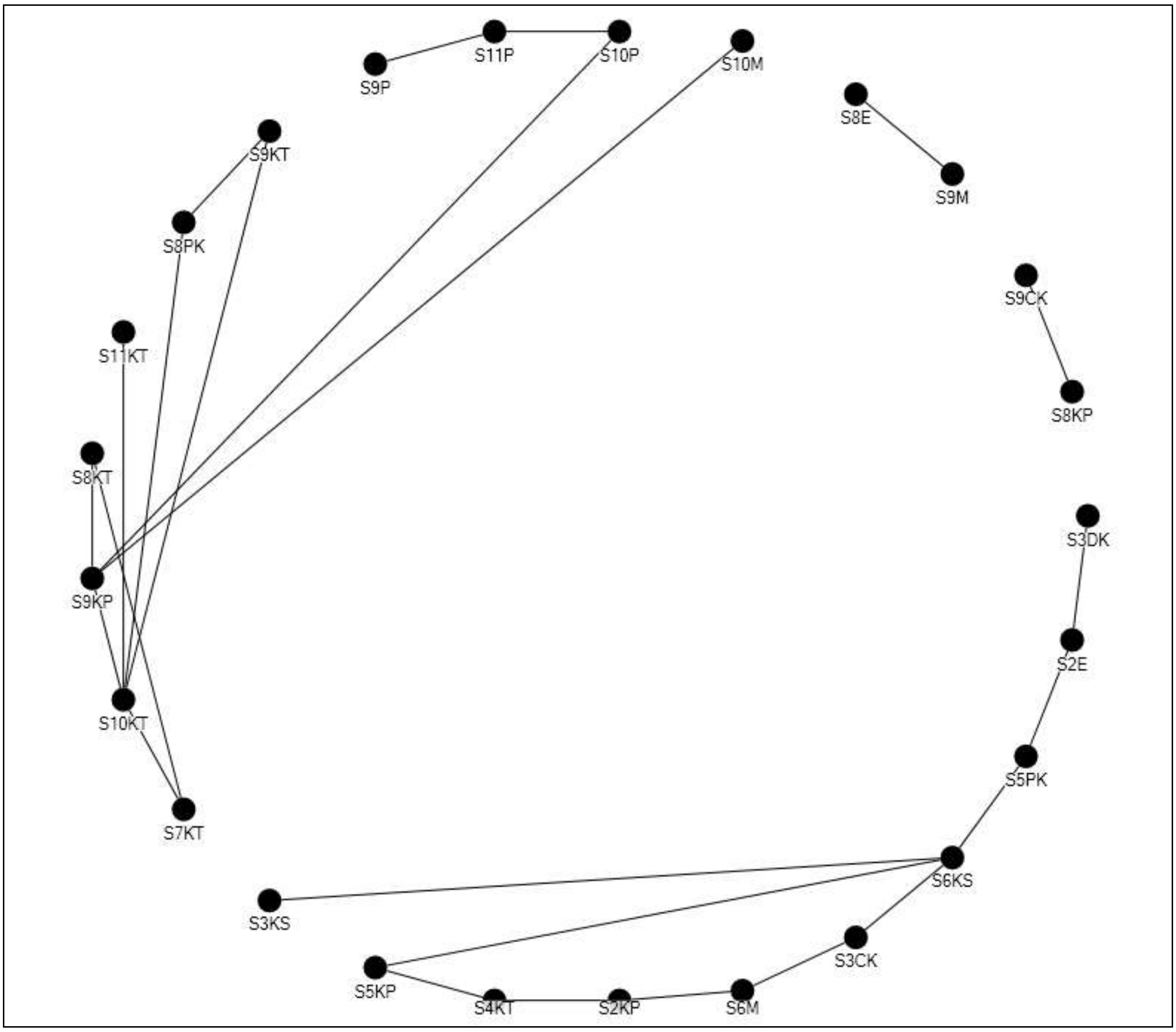
Addendum I 1
Stratum 1 - social network



Stratum 1

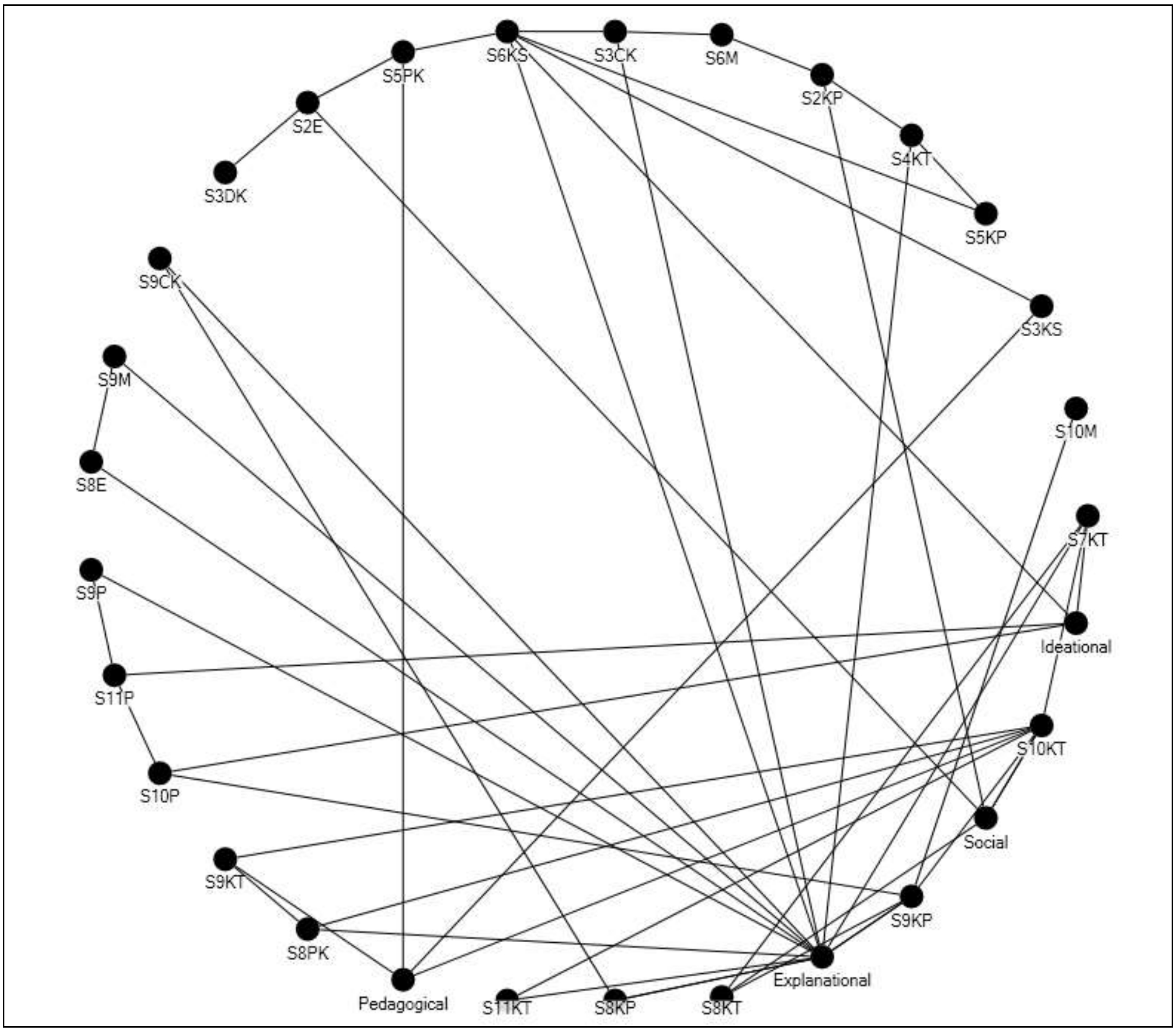
Drawn by importing the coded Atlas.ti data into NodeXL and selecting the "circle" option.

Addendum I 2
Stratum 2 - interpersonal metacognitive network



Stratum 2

Addendum I 3
Stratum 3 - socially shared metacognitive network



Stratum 3

Addendum J
Ethical clearance

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT

Private Bag X6001, Potchefstroom
South Africa 2520

Tel: (018) 299-4900
Faks: (018) 299-4910
Web: http://www.nwu.ac.za

Ethics Committee
Tel +27 18 299 4849
Email Ethics@nwu.ac.za

ETHICS APPROVAL OF PROJECT

The North-West University Research Ethics Regulatory Committee (NWU-RERC) hereby approves your project as indicated below. This implies that the NWU-RERC grants its permission that provided the special conditions specified below are met and pending any other authorisation that may be necessary, the project may be initiated, using the ethics number below.

Project title: TEACHING AND LEARNING STRATEGIES TO PROMOTE SELF-DIRECTED LEARNING

Project leader: Prof E Mentz (Main projectleader)
Prof MS van der Walt (Sub-project leader)
Student: D Jagals

Sub-project title: Metacognitive locale: a design-based theory of students' metacognitive language and networking in Mathematics

Ethics number:

N	W	U	-	0	0	0	1	0	-	1	3	-	A	2
Institution				Project Number						Year			Status	

Status: S = Submission, R = Re-Submission; P = Provisional Authorisation; A = Authorisation

Approval date: 2015/02/26 **Expiry date:** 2018/02/13

Special conditions of the approval (if any):

- Feedback for this sub-project must be submitted to Prof L Meyer (Chair: Faculty of Education Sciences Research Ethics Committee) as specified in the Minutes of the Faculty of Education Sciences Research Ethics Committee of 26/02/2015.

General conditions:

While this ethics approval is subject to all declarations, undertakings and agreements incorporated and signed in the application form, please note the following:

- *The project leader (principle investigator) must report in the prescribed format to the NWU-RERC:*
  - *annually (or as otherwise requested) on the progress of the project,*
  - *without any delay in case of any adverse event (or any matter that interrupts sound ethical principles) during the course of the project.*
- *The approval applies strictly to the protocol as stipulated in the application form. Would any changes to the protocol be deemed necessary during the course of the project, the project leader must apply for approval of these changes at the NWU-RERC. Would there be deviated from the project protocol without the necessary approval of such changes, the ethics approval is immediately and automatically forfeited.*
- *The date of approval indicates the first date that the project may be started. Would the project have to continue after the expiry date, a new application must be made to the NWU-RERC and new approval received before or on the expiry date.*
- *In the interest of ethical responsibility the NWU-RERC retains the right to:*
  - *request access to any information or data at any time during the course or after completion of the project;*
  - *withdraw or postpone approval if:*
    - *any unethical principles or practices of the project are revealed or suspected,*
    - *it becomes apparent that any relevant information was withheld from the NWU-RERC or that information has been false or misrepresented,*
    - *the required annual report and reporting of adverse events was not done timely and accurately,*
    - *new institutional rules, national legislation or international conventions deem it necessary.*

The Ethics Committee would like to remain at your service as scientist and researcher, and wishes you well with your project. Please do not hesitate to contact the Ethics Committee for any further enquiries or requests for assistance.

Yours sincerely

Linda du Plessis

Digitally signed by Linda du Plessis
DN: cn=Linda du Plessis, o=NWU, Vaal Triangle Campus, ou=Vice-Rector: Academic, email=linda.duplessis@nwu.ac.za, c=US
Date: 2015.03.10 18:04:55 +02'00'

Prof Linda du Plessis
Chair NWU Research Ethics Regulatory Committee (RERC)

Addendum J1
Letter to the students

NORTH-WEST UNIVERSITY
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NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

Private Bag X6001, Potchefstroom
South Africa 2520

Tel: 018 299-1111/2222
Web: http://www.nwu.ac.za

Dear MATD 413 - Student

INVITING YOU TO PARTICIPATE IN A RESEARCH PROJECT

I am currently busy with research focussing on metacognition, language, and networking in the teaching and learning of Mathematics education with Problem Based Learning (PBL) and Lesson Study (LS) as instructional strategies during lesson planning and problem solving.

The purpose of the study is to examine, understand and foster experiences for pre-service teachers (4th year students) in Mathematics education to conduct Lesson Study in order to teach Mathematics content to Primary school learners in township schools. The study aims to improve the teaching, learning and doing of Mathematics in township schools and to create collaborative opportunities where pre-service teachers, as students, come together and learn from each other, using a LS model. The results of this study can be used to make informed decisions regarding the use of Work Integrated Learning in teaching and learning programmes, especially in the didactics of Mathematics. The study guide for the module MATD 413 (Mathematics didactics in Foundation and Intermediate phase) will be revised according to the necessity of Lesson Study, metacognition and its use in under resourced schools as well as compilation of reading material and planned assignments that make part of this course. Moreover, this study creates awareness for pre-service teachers on the education of South Africa in diverse communities and creates equal opportunities to learn, acquire and take cognisance of the grounding challenges to develop the required knowledge, skills and competencies to facilitate teaching in Mathematics in under resourced schools.

- Through adapted lesson studies and reflection on designing and presentation of lessons, the teaching and learning experiences of the students will be improved.
- This will identify issues to address in preparing students for a diverse landscape in education to overcome the challenges that teachers and learners face within under resourced schools.
- By learning from their experiences and collaboratively, working closely with the associated schools, students will gain experience from this service opportunity to facilitate change and development within teaching and learning environments to promote lifelong learning.

By developing such skills as planning, analysing, testing, reflecting and adapting lesson studies in Mathematics, students will demonstrate an appreciation for the value of Mathematics and the nature of teaching and learning in diverse communities through action and service.

Students who participate in this study will be requested to form a Lesson Study group to communicate about their ideas in order to plan a series of Mathematics lessons and to teach these lessons in a under resourced school. From this experience, students will be invited to follow up interviews where they can reflect and refine the lesson(s). The interview will consist of four sessions held on different days. The study also consist an online component during which students are required to reflect in an online ‘chat room’ about their ideas and experiences with Lesson Study. The students are allowed to change their approach to the lesson(s) at any time even after they finished. The choices made and the reaction found during the interviews will be analysed and interpreted to better understand metacognition, language and networking through the lenses of PBL and LS.

Participation in the project is free willingly and the student is allowed to withdraw from the study at any time.

Your permission is therefore necessary to participate in the study. Information gathered will be regarded as confidential and all students will be kept anonymous.

The research will be conducted during scheduled sessions from February 2013 to June 2014. Students that are invited to the interviews will be notified and appointments will be made during this time. Sessions for the interviews will take about one hour.

Thanking you kindly,

Mr. D Jagals
Cell: 079 338 7881

Mr D. Jagals

Lecturer: Curriculum studies, Philosophy and Research Methodology

North-West University (Potchefstroom Campus)

Faculty of Education Sciences

School for Education studies

E-mail: divan.jagals@nwu.ac.za

Tel: 018 299 2154

E-mail: 12782890@nwu.ac.za

………..

Permission

I have read the above and understand the nature of the research. I understand that by giving permission the researcher will conduct the study in a professional manner and will consider my human rights. I understand that I can contact the researcher at 12782890@nwu.ac.za for any queries regarding the study. Undersigning, I hereby give my permission to participate in the above study.

Name and surname of student:_______________________________________Age:_________________

Degree enrolled for:___________Contact number:____

Student's signature:____Date:_________________

Addendum J2
Letter to the dean of the faculty

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
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POTCHEFSTROOM CAMPUS

Private Bag X6001, Potchefstroom
South Africa 2520

Tel: 018 299-1111/2222
Web: http://www.nwu.ac.za

Dean of the faculty of Education Sciences

INFORMING YOU ABOUT MY RESEARCH PROJECT

I am currently busy with a PhD-study which focuses on metacognition, language, and networking in the teaching and learning of Mathematics education. Primarily these constructs are explored through the lenses of Problem Based Learning (PBL) and Lesson Study (LS) as instructional strategies during Mathematics lesson planning and problem solving.

The purpose of the study is to examine, understand and foster experiences for pre-service teachers (4th year students) in Mathematics education to conduct Lesson Study in order to teach Mathematics content to Primary school learners. The study aims to improve the teaching, learning and doing of Mathematics in township schools and to create collaborative opportunities where pre-service teachers, acting as students and teachers, come together and learn from each other, using a LS model. The results of this study can be used to make informed decisions regarding the use of Work Integrated Learning in teaching and learning programmes, especially in the didactics of Mathematics. The study guide for the module MATD 413 (Mathematics didactics in Foundation and Intermediate phase) will be revised according to the necessity of Lesson Study, metacognition and its use in under resourced schools as well as compilation of reading material and planned assignments that make part of this course. Moreover, this study creates awareness for pre-service teachers on the education of South Africa in diverse communities and creates equal opportunities to learn, acquire and take cognisance of the grounding challenges to develop the required knowledge, skills and competencies to facilitate teaching in Mathematics in under resourced schools:

- Through adapted lesson studies and reflection on the design and presentation of lessons, the teaching and learning experiences of the students will be improved.
- This will identify issues to address in preparing students for a diverse landscape in education to overcome the challenges that teachers and learners face within under resourced schools.
- By learning from their experiences and collaboratively, working closely with teachers from associated schools, students will gain experience from this service opportunity to facilitate change and development within teaching and learning environments to promote lifelong learning.

By developing such skills as planning, analysing, testing, reflecting and adapting lesson studies in Mathematics, students will demonstrate an appreciation for the value of Mathematics and the nature of teaching and learning in diverse communities through action and service.

Students who participate in this study will be requested to form a Lesson Study group to communicate about their ideas in order to plan a series of Mathematics lessons and to teach these lessons in a under resourced school. From this experience, students will be invited to follow up interviews where they can reflect and refine the lesson(s). The interview will consist of four sessions held on different days. The study also consist an online component during which students are required to reflect in an online ‘chat room’ about their ideas and experiences with Lesson Study. The students are allowed to change their approach to the lesson(s) at any time even after they finished. The decisions made during the interviews and online conversations will be analysed and interpreted to better understand metacognition, language and networking through the lenses of PBL and LS.

Participation in the project is free willingly and the student is allowed to withdraw from the study at any time.

The research is conducted during scheduled sessions from February 2014 to October 2014. Students who are invited to the interviews will be notified and appointments will be made during this time. Sessions for the interviews will take about one hour.

Thanking you kindly,

Mr Divan Jagals
Call: 018 299 2154
E-mail: 12782890@nwu.ac.za

………......................

Addendum J 3
Letter to the school principals

Dear Principal

Permission to conduct research in your school

I hereby request permission to do research in your school. I (student number 12782890) am an enrolled PhD student at the North-West University (Potchefstroom Campus). The title of this dissertation is: *Metacognitive locale: a design-based theory of students' metacognitive language and networking in Mathematics.*

I would like to conduct research in this school environment as it fits the school profile required by the research service learning project. My research is centred on metacognition in the teaching learning and doing of Mathematics of fourth year Mathematics education students. My research will therefore require (at least) one Grade 6 Mathematics classroom for the students to present a Mathematics lesson that they have prepared collaboratively on the topic of *Place value*.

All the information that is gained from the school will revolve around the students. No information about the learners or the teachers will be required as they do not form part of the scope of this research, but serve as the context of the study. Instead, audio recordings will be made of the presented lesson whereby the students who do not teach at that moment will observe and make notes about their observations. Therefore the teachers and the learners (and students) will be handled confidentially and within the ethical rules of research determined by the North-West University. Aspects such as informed consent, voluntary participation and respect for anonymity will be adhered to.

Teachers and learners are welcome to keep, used and adapt the material that the students prepared for the class.

I sincerely hope that you will be able to accommodate this group of five students and I thank you for your assistance in this regard.

....

Mr Divan Jagals

Addendum G
Certificate of language editing

HESCOM
COMMUNICATION SERVICES

Translating.Writing.Editing

Hester van der Walt
HesCom Communication Services
Member: Professional Editors' Group

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633 Daphne Ave
Mountain View
0082

LANGUAGE EDITING STATEMENT

2015-04-30

Metacognitive locale: a design-based theory of students' metacognitive language and networking in Mathematics

by D Jagals

- Has been edited for language correctness and spelling.
- Has been edited for consistency (repetition, long sentences, logical flow)

No changes have been made to the document's substance and structure (nature of academic content and argument in the discipline, chapter and section structure and headings, order and balance of content, referencing style and quality).

Final responsibility for the thesis rests with the author of the document.

HESTER VAN DER WALT

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