

**A SUBJECT DIDACTICAL ANALYSIS OF THE DEVELOPMENT OF THE
SPATIAL KNOWLEDGE OF YOUNG CHILDREN THROUGH A
PROBLEM-CENTRED APPROACH TO MATHEMATICS TEACHING
AND LEARNING**

by

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Thesis submitted in fulfilment of the requirements for the degree Philosophia
Doctor in Subject Didactics in the Graduate School
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Potchefstroom

1997

ACKNOWLEDGEMENT

I wish to thank the following people and institutions for their support:

- ◆ Mr. H. D. Nieuwoudt and Prof D.C.J. Wessels, my supervisors, for their support and guidance.
- ◆ Mrs. I. Booysen for the recording of the video material of all the classes.
- ◆ The teachers of Fauna Park Primary school as well as the parents of the children, for their trust and permission to work with their children and for the children who taught me so much about their spatial skills.
- ◆ Mr. P. Human, Mr. C. Basson and Mrs J. A. Brönn, for the language corrections.
- ◆ The FRD (Foundation for Research and Development) for financial support in the form of a bursary.
- ◆ My parents Lena and Carel Basson, and my husband, Gerhard and two young sons, Charl-Werner and Riebeeck for their constant support.

Hebrew 13:15 Through Jesus, then let us continually offer up to God the sacrifice of praise, that is, the tribute of the lips which acknowledge His name.

I dedicate this work to Gerhard, Charl-Werner and Riebeeck.

SUMMARY

A subject didactic analysis of the development of the spatial knowledge of young children through a problem-centred approach to mathematics teaching and learning

Researchers and educators are in agreement that it is very important that the spatial knowledge of the young child should be developed from the first years of school. In order to develop these skills the appropriate materials and activities need to be designed. This can only be realised through proper research methods that not only acknowledge the cognitive abilities of the young child, but also the social and cultural backgrounds of the children.

This implies that due attention should be given to language, beliefs, cognitive skills, socio-economic background, schooling and teaching systems. The immediate worlds of the children should be used in developing the spatial skills of these children.

The work that is described in this document is an effort to describe the complexity of such a research endeavour. The development of the spatial skills of young children were investigated through three different instructional/executional media namely language, drawing/writing and physical constructions. The three major variables that were described as influencing the spatial development in the different media were the task that was given to the children, the objects that the children worked with and the dimension and viewpoints of the objects and situations.

It was clear from the research that in the development of the spatial skills of children, attention should be given to: the real-world of the children, hands-on-experiences of the children, the cultural background, the language of instruction, the socio-economic

background, the classroom culture, the media of instruction and the cognitive skills of the children.

Key words: spatial development, spatial perception, spatial sense, geometry teaching, mathematics education, pre-school children, kindergarten children, young children.

OPSOMMING

‘N VAKDIDAKTIESE ANALISE VAN DIE ONTWIKKELING VAN DIE RUIMTELIKE KENNIS VAN JONG KINDERS DEUR MIDDEL VAN ‘N PROBLEEMGESENTEERDE BENADERING TOT DIE ONDERRIG EN LEER VAN WISKUNDE.

Navorsers en opvoedkundiges is in ooreenstemming dat die ontwikkeling van die ruimtelike vermoëns van jong kinders, reeds in aanvang moet neem in die beginjare van laerskool. Om die vermoëns ten volle te ontwikkel is dit noodsaaklik dat die geskikte aktiwiteite en materiale ontwikkel moet word. Dit is slegs moontlik indien daar deur middel van die korrekte navorsingsbenadering, wat nie slegs die kognitiewe vermoëns van die jong kinders nie, maar ook die sosiale en kulturele agtergrond in ag neem, te werk gegaan word.

Dit impliseer dat genoegsame aandag gegee moet word aan taal, beskouing van wiskunde, sosio-ekonomiese agtergrond, skool-en-onderrig sisteme en kognitiewe vermoëns. Die onmiddellike leefwêreld van die kinders moet geken en benut word om die nodige ruimtelike kennis te ontwikkel.

Die werk wat in die dokument beskryf word is ‘n poging om die kompleksiteit van so ‘n navorsingsondersoek te beskryf. Die ontwikkeling van die ruimtelike vermoëns van jong kinders is ondersoek deur van drie verskillende instruksie/onderrig media gebruik te maak nl. taal, teken/skryf en fisiese konstruksies. Die drie veranderlikes wat deurgaans gemonitor is om vas te stel wat die effek op die ruimtelike vermoëns is, is die taak wat aan die kinders gegee is, die tipe voorwerpe waarmee gewerk is en die dimensies en oogpunte van die voorwerpe en situasies.

Uit die navorsing het dit duidelik na vore gekom dat daar in die ontwikkeling van die ruimtelike vermoëns van jong kinders, aandag gegee moet word aan: die werklike leefwêreld van die kinders, praktiese ervaring van die kinders self, kulturele en sosio-ekonomiese agtergrond, taalmedium van onderrig, klaskamerkultuur, onderrigmedia en kognitiewe vermoëns van die kinders.

Sleutelwoorde: ruimtelike persepsie, ruimtelike ontwikkeling, meetkunde onderrig, wiskunde onderrig, voorskoolse kinders, jong kinders, laerskool kinders, kleuterskool kinders.

“Table of Contents”**CHAPTER 1****BACKGROUND AND OVERVIEW OF THE STUDY**

1.1 Introduction	1
1.2 Problem statement	5
1.3 Aims of the research	7
1.4 Research design	8
1.4.1 Literature study	8
1.4.2 Empirical study	11
1.5 Value of the research	13
1.6 Terminology	14
1.6.1 Subject didactical approach	14
1.6.2 Spatial development of the young child	14
1.6.3 Problem-centred approach	14
1.6.4 Western worldview	15
1.6.5 Indigenous worldview	15
1.7 Progress of the investigation	16

CHAPTER 2**PHILOSOPHICAL PERSPECTIVES CONTRIBUTING TO THE
UNDERSTANDING OF THE SPATIAL DEVELOPMENT OF THE YOUNG
CHILD**

2.1 Introduction	17
2.2 Indigenous perspective	18
2.2.1 Orientation	18
2.2.2 Space and time	19
2.2.3 Animate and inanimate	20
2.2.4 Individual and society	21
2.2.5 Dreams and visions	22
2.2.6 Perception and reality	23
2.2.7 Causality and synchronicity	23
2.3 Socio-cultural perspective	24
2.3.1 Orientation	24
2.3.2 Vygotsky	24
2.4 Constructivist perspective	25
2.4.1 Orientation	25
2.4.2 Piaget	29
2.5 Realistic instruction theory	33
2.5.1 Orientation	33
2.5.2 The Van Hiele's	33

2.5.3 Freudenthal (Wiskobas)	38
2.6 Cognitive science	43
2.6.1 Orientation	43
2.6.2 Anderson's model of cognition	44
2.6.3 Greeno's model of geometry problem solving	44
2.6.4 Parallel distributed processing (PDP)	45
2.7 Kosslyn	46
2.8 Olson and Bialystok	48
2.9 Problem-centred perspective	50
2.9.1 Orientation	50
2.9.2 Objectives	50
2.9.3 The role of the teacher	53
2.9.4 Problem solving as a learning type	54
2.9.5 The role of social interaction	55
2.10 Conclusion	56

CHAPTER 3

RESEARCH APPROACHES REGARDING THE INVESTIGATION OF THE SPATIAL DEVELOPMENT OF THE YOUNG CHILD

3.1 Introduction	58
3.2 Three theoretical research models from a Western perspective	59
3.2.1 The Psychometric approach	59

3.2.1.1 Introduction	59
3.2.1.2 Research findings from a Psychometric perspective	59
3.2.2 The Experimental approach	62
3.2.2.1 Introduction	62
3.2.2.2 Research findings from an Experimental perspective	63
3.2.3 The Developmental approach	66
3.2.3.1 Introduction	66
3.2.3.2 Research findings from a Developmental perspective	66
3.3 A theoretical research model from an Indigenous (non-western) perspective	69
3.3.1 Orientation and location	70
3.3.2 Maps	71
3.3.3 Shapes and architecture	71
3.4 An agenda towards an interpretative research methodology	72
3.4.1 Introduction	72
3.4.2 Contextuality of cognition	75
3.4.2.1 Introduction	75
3.4.2.2 Experiential context	76
3.4.2.3 Cognitive context	77
3.4.2.4 Anthropological context	77
3.4.3 Role of theory	78
3.4.4 Role of the teacher/researcher	80

3.4.5 Data collection techniques	81
3.4.5.1 Introduction	81
3.4.5.2 Task specific interviews	82
3.4.5.3 Questionnaires	83
3.4.5.4 Diagnostic teaching experiment	83
3.4.5.5 Triangulation	84
3.4.6 Developmental research	84
3.4.7 Evaluation	87
3.4.7.1 Introduction	87
3.4.7.2 Validity	88
3.4.7.3 Reliability	89
3.4.7.4 Reproducibility	89
3.5 Conclusion	90

CHAPTER 4

A SUBJECT-SPECIFIC THEORETICAL FRAMEWORK OF THE SPATIAL DEVELOPMENT OF THE YOUNG CHILD

4.1 Introduction	92
4.2 Context	92
4.3 The role of worldviews in defining the spatial understanding of the young child	94
4.4 Task	95

4.4.1 Type of task	95
4.4.1.1 Movement of object	96
4.4.1.2 Movement of viewer	97
4.4.2 Order of activities (Hierarchy)	101
4.5 Object	105
4.5.1 Referent or relatum	105
4.5.1.1 Ego	105
4.5.1.2 Canonical objects	105
4.5.1.3 Noncanonical objects	106
4.5.2 Object space versus environmental space	108
4.6 Dimension of stimulus and point of view	110
4.6.1 Rotational axes	111
4.6.1.1 Vertical axis	112
4.6.1.2 Frontal axis	115
4.6.1.3 Lateral axis	117
4.7 Media	119
4.7.1 Role of different media	119
4.7.1.1 Language	125
4.7.1.1.1 Role of language for people with different worldviews	125
4.7.1.1.2 Features of the development of a spatial lexicon	128
4.7.1.1.3 Predicate	129

4.7.1.2 Drawing	136
4.7.1.2.1 The role of drawing for people with different worldviews	136
4.7.1.2.2 The dimensional aspect of drawing	138
4.7.1.2.3 Conventions	140
4.7.1.2.4 Children's drawings	140
4.7.1.2.4.1 Developmental stages in children's drawings	142
4.7.1.2.4.2 Developmental stages for drawing specific solids	142
4.7.1.2.4.3 Developmental stages for foldouts (nets) of solids	143
4.7.1.2.4 Projection system classification	144
4.7.1.3 Construction/Building/Physical activities	147
4.7.1.3.1 The role of constructions, building and physical activities for people with different worldviews	147
4.7.1.3.2 Developmental stages of children's block constructions	148
4.8 Conclusion	150

CHAPTER 5

INVESTIGATION OF THE SPATIAL SENSE OF THE YOUNG CHILD THROUGH BLOCK BUILDING

5.1 Introduction	154
5.2 Experimental background	155
5.2.1 Data collection	156
5.3 Introductory activities	157
5.4 Final activities	160

5.5 Language	163
5.5.1 Results	163
5.5.2 Discussion	171
5.5.2.1 Task	171
5.5.2.2 Object	172
5.5.2.3 Dimension and viewpoint	173
5.6 Drawing	175
5.6.1 Results	175
5.6.1.1 Coding of drawings	176
5.6.2 Discussion	186
5.6.2.1 Task	186
5.6.2.2 Object	187
5.6.2.3 Dimension and viewpoint	188
5.7 Numerical representation	191
5.7.1 Results	191
5.7.2 Discussion	192
5.7.2.1 Task	192
5.7.2.2 Object	193
5.7.2.3 Dimension and viewpoint	194
5.8 Constructions	195
5.8.1 Results	195

5.8.2 Discussion	198
5.8.2.1 Task	198
5.8.2.2 Object	199
5.8.2.3 Dimension and viewpoint	199
5.9 Conclusion	205

CHAPTER 6

INVESTIGATION OF THE SPATIAL SENSE OF THE YOUNG CHILD THROUGH THE DEVELOPMENT OF SOLIDS

6.1 Introduction	210
6.2 Experimental background	211
6.3 Language	216
6.3.1 Results	216
6.3.2 Discussion	219
6.3.2.1 Task	219
6.3.2.2 Object	222
6.3.2.3 Viewpoint and dimension	224
6.4 Drawing	224
6.4.1 Results	225
6.4.1.1 Representations of the progress of individual children through different stages for drawings for all the different objects	225
6.4.1.2 Progress of all three groups with the cube from period 1 to 4	229

6.4.1.3 Progress of all three groups with the rectangular prism from period 1 to 5	231
6.4.1.4 Progress of the bottom group with all the objects	234
6.4.1.5 Progress of the middle group with all the objects	236
6.4.1.6 Progress of the top group with all the objects	236
6.4.1.7 An accumulative version of the progress of the complete group with all the objects	239
6.4.1.8 Drawings of the progress of individual children in the development stages for all the objects	240
6.4.1.8.1 Cube	240
6.4.1.8.2 Rectangular prism	241
6.4.1.8.3 Triangular prism 1	242
6.4.1.8.4 Triangular pyramid (Tetrahedron)	242
6.4.1.8.5 Square-based pyramid	244
6.4.1.8.6 Triangular prism 2	244
6.4.1.8.7 Cylinder	246
6.4.2 Discussion	247
6.4.2.1 Task	247
6.4.2.2 Object	249
6.4.2.2.1 Cube	249
6.4.2.2.2 Rectangular prism	251
6.4.2.2.3 Triangular prism	252
6.4.2.2.4 Tetrahedron	254

6.4.2.2.5 Square-based pyramid	256
6.4.2.2.6 Cylinder	258
6.4.2.3 Dimension and viewpoint	261
6.5 Constructions	264
6.5.1 Results	264
6.5.2 Discussion	264
6.5.2.1 Task	264
6.5.2.2 Object	265
6.5.2.3 Dimension and viewpoint	265
6.6 Conclusion	266

CHAPTER 7

DEDUCTIONS, RECOMMENDATIONS AND CONCLUSIONS

7.1 Introduction	270
7.1.1 Language	273
7.1.2 Drawing/writing	276
7.1.3 Construction	278
7.2 Deductions	280
7.2.1 The execution media (language, drawing, writing and construction)	280
7.2.2 The context	280
7.2.3 The order of the activities	280

7.2.4 The type of task	280
7.2.5 The type of object	281
7.2.6 The dimension of the stimulus materials	281
7.2.7 The type of mathematical knowledge (physical, social and logico-mathematical)	281
7.3 Recommendations	281
7.3.1 Aims	281
7.3.2 Contents	282
7.3.3 Methods	282
7.3.4 Teaching aids	282
7.3.5 Assessment	283
7.3.6 Teacher preparation	283
7.3.7 Evaluation of long term effects	283
7.3.8 Researchers	284
7.3.9 Implementation	284
7.4 Conclusion	284
7.4.1 The cultural reason	285
7.4.2 The epistemological reason	285
7.4.3 The evolutionary reason	285
ANNEXURES	287
REFERENCES	291

LIST OF FIGURES

xx

LIST OF TABLES

xxiv

LIST OF FIGURES

Figure 1.1: Research design	10
Figure 4.1: The interrelationship between the different variables in the teaching-learning situation	94
Figure 4.2: Two types of movement that the viewer can undergo	100
Figure 4.3: Movement around the vertical axis for a three-dimensional object	112
Figure 4.4: Movement of parts of a three-dimensional object along the vertical axis	114
Figure 4.5: Movement of a three-dimensional object along the frontal axis	115
Figure 4.6: Movement of parts of a three-dimensional object around the frontal axis	116
Figure 4.7: Movement of a three-dimensional object along the lateral axis	117
Figure 4.8 : Movement of parts of three-dimensional object along the lateral axis	118
Figure 4.9: Drawing stages for different solids	144
Figure 4.10: A projection classification system for the cube	146
Figure 4.11: A diagram to illustrate the interrelationship of the different spatial components in the teaching-learning situation	153
Figure 5.1: A set of seven soma cubes	154
Figure 5.2: Three different instructional variables	160
Figure 5.3: Different positions of the fourth block are indicated by the arrow	164
Figure 5.4: Different orientations for soma cube 1	165

Figure 5.5: Different orientations for soma cube 3	165
Figure 5.6: Verbal explanation of the position of the top block	166
Figure 5.7: Building restrictions or specifications	166
Figure 5.8: Verbal discussions about the top of a structure	167
Figure 5.9: Left and right are dependent on the direction in which one is facing	167
Figure 5.10: The use of different deictic terms for the same position	168
Figure 5.11: The use of the deictic term “in front” for different positions in space	169
Figure 5.12: Holistic and analytic descriptions for the soma cubes	170
Figure 5.13: Category A: Line drawings	176
Figure 5.14: Category B: Erroneous number	177
Figure 5.15: Category C: Frontal view	178
Figure 5.16: Category D: Defective orientation	179
Figure 5.17: Category E: Disjunct	180
Figure 5.18: Category F: Partial occlusion	181
Figure 5.19: Category G: Height-in-picture	181
Figure 5.20: Category H: Attempted dimension	182
Figure 5.21: Different drawing categories for 3-D structures	185
Figure 5.22: Drawings of different orientations for soma three	190
Figure 5.23: Worksheets illustrating the numerical representations of the 3-D structures	191
Figure 5.24: Example of evaluation worksheet at the end of the year	192
Figure 5.25: Movement from 2-D to 3-D	193
Figure 5.26: Movement from 3-D to 2-D	193

Figure 5.27: Vertical and horizontal dimension confusion when moving from 3-D to 2-D	195
Figure 5.28: Single structures constructed with loose blocks	196
Figure 5.29a: Soma cube “pictorial manual”	196
Figure 5.29b: Soma cube “pictorial manual”	197
Figure 5.29c: Soma cube “pictorial manual”	198
Figure 5.30: Different back views of a 3-D structure	201
Figure 5.31: Progress of two children in the construction of a 3-D structure	202
Figure 5.32a: Construction of soma structures which are both in the horizontal orientation	203
Figure 5.32b: Construction of soma structures which are both in the vertical orientation	204
Figure 5.32c: Construction of soma structures which are in the vertical and horizontal orientations	205
Figure 6.1: The stimulus materials or objects for the development of solids were three-dimensional as well as two-dimensional	212
Figure 6.2: A transcription to illustrate the language use of the children	217
Figure 6.3: Progress of the drawings of a child in the development of the cube	228
Figure 6.4: The different drawing stages for the cube	229
Figure 6.5: Progress of all three groups with the development of the cube from period 1 to 4	230
Figure 6.6: The different drawing stages for the rectangular prism	232
Figure 6.7: Progress of all three groups with the development of the rectangular prism from period 1 to 5	233

Figure 6.8: Progress of the bottom group through the stages of the development of the different objects	235
Figure 6.9: Progress of the middle group through the stages of the development of the different objects	237
Figure 6.10: Progress of the top group through the stages of the development of the different objects	238
Figure 6.11: Summarised comparison of progress through stages	239
Figure 6.12: Progress of the drawings of two children in the development of the cube	240
Figure 6.13: Progress of the drawings of three children in the development of the rectangular prism	241
Figure 6.14: Progress of the drawings of two children in the development of the triangular prism	242
Figure 6.15: Progress of the drawings of three children in the development of the tetrahedron	243
Figure 6.16: Progress of the drawings of two children in the development of the square-based pyramid	244
Figure 6.17: Progress of the drawings of four children in the development of the triangular prism	245
Figure 6.18: Progress of the drawings of three children in the development of the cylinder	246
Figure 6.19: The different drawing stages for the triangular prism	253
Figure 6.20: The different drawing stages for the tetrahedron	255
Figure 6.21: The different drawing stages for the square-based pyramid	257
Figure 6.22: The different drawing stages for the cylinder	259
Figure 6.23: A method to determine the dimensions of the rectangle that comprises the curved surface of the cylinder	260
Figure 6.24: Two different ways of drawing and constructing the net of the square-based pyramid	263

LIST OF TABLES

Table 4.1: The characteristics of different axes	113
Table 5.1: Introductory activities	159
Table 5.2: Final activities	161
Table 6.1: Representations of the activities, materials, and objectives with the development of solids	214

CHAPTER 1

BACKGROUND AND OVERVIEW OF THE STUDY

1.1 Introduction

In his book "Mathematics at the Cross-roads", Van Zyl (1942:97) states that during the 1930s Euclid was followed more closely in Great Britain than elsewhere. As a result of British influence, South Africa has adhered to the same type of geometry. It was also unfortunate that the improvements in the teaching of geometry being felt in England after the 1920s, had no influence in South Africa. This led to the situation that South Africa inherited its Geometry from England at a time when its teaching was more conservative than in any other country in the world.

The result of this was that any informal approach to geometry teaching in high school was looked upon as a waste of time and theorems were introduced as early as possible. Van Zyl (1942) continues to state that informal geometry is all that the average standard 7 pupil is able to absorb with profit, unless he has been taught it in standard 6 or even standard 5. He goes on to say that the earlier introduction of informal geometry is desirable but the fact that primary school teachers are not trained at universities militated against this.

In 1944 Gevers came to the conclusion that the Euclidean geometry that is taught in South African schools is not suitable for youths. He is also in favour of combining the different disciplines namely geometry, algebra and trigonometry. He thinks that it should be tackled as a whole in school curricula. Krige (1961) joins Gevers in his view by stating that the teaching of Euclidean geometry is outdated and more could be gained by turning to analytical geometry as part of the curriculum.

Dreyer (1965) mentions the fact that several other South African authors voice the same concerns, namely that South Africa should move with the times when it comes to the development of their mathematics curricula, and especially the geometry component needs attention (Senex, 1963; Maarschalk, 1963; Dekker, 1962).

The approach to the teaching of geometry in primary school up to 1994 started with the introduction of the basic two-dimensional shapes (squares, rectangles, triangles etc.) that had to be recognised, followed very early in the beginning of the senior primary phase with the introduction of formulas for the calculation of the surface areas for these figures (DET, 1991). At the beginning of 1994 a new curriculum was prescribed (TED, 1994) against the background of the problem-centred approach that had been implemented in the majority of white schools in the then province of Transvaal. Along with this came new guidelines for restructuring the geometry curriculum that adheres to many of the international guidelines (reports of the NCRMSE and the NCTM). The obvious lack of any teacher materials as well as teacher training regarding the spatial development, that accompanied the TED (1994) document, gives an indication of the "importance" of geometry in the primary schools in South Africa.

Shuster (1975:168) points to the fact that schools in the United States of America have had, for most of the twentieth century, Euclidean Geometry as the content of their geometry curriculum. He continues to say that the focusing on the formal structure of geometry has been a serious mistake that has had some unfortunate results. Firstly the programme has failed; that is, the goal of teaching formal structure via geometry has not been achieved. Secondly the overemphasis on axiomatics has resulted in an underemphasis on application. Thirdly, not enough concern was given to three-dimensional and higher concepts.

Other important topics appropriate to secondary school work are slighted. Among these are combinatorial aspects of geometry, transformations, symmetry, and topological aspects of geometry (Freudenthal, 1971:425). Freudenthal further is of

the opinion that one of the prominent features of these approaches is, that rather than giving the child the opportunity of organising spatial experiences, the subject matter is offered as a pre-organised structure, with all the concepts and definitions preconceived by the teacher. He is of the opinion that the traditional system was a fake system, but by teaching it, teachers used to indoctrinate themselves to believe in the system. Freudenthal (1971:429) goes further, predicting a solution to this problem when he says that geometry can be saved if it is presented as a field in which students can be active.

Questions surrounding the teaching of geometry were popular in the Netherlands as far back as the 1950s (Van Hiele-Geldof, 1958a, 1958b). The Van Hieles were actively involved in the basic research and P. Van Hiele's (1959) article delineating the thought levels and the phases of learning attracted the immediate attention of Soviet psychologists. Professor Isaak Wirszup, at the university of Chicago, formally introduced the Van Hiele ideas to American audiences in 1974. Some of the studies undertaken by the Americans were the Oregon Project (1979-1982), the Brooklyn Project (1979-1982), and the Chicago Project (1979 -1982). The Netherlands also embarked on a research project in the field of initial geometry (ages 4 to 14) under the inspiring leadership of Freudenthal, called the Realistic Approach.

The model of realistic thought and the phases of learning that were developed by the Van Hieles, propose a means for identifying a student's level of geometric maturity. They also suggest ways to help the students to progress through these levels. The five thought levels are: Level 0, Visualisation; Level 1, Analysis; Level 2, Informal Deduction; Level 3, Deduction; and Level 4, Rigour (Van Hiele, 1982:214).

According to this model it is argued that instruction rather than maturation is the most significant factor contributing to the student's geometric development (Van Hiele, 1982:215). The phases of instruction are: Phase 1, Inquiry/Information; Phase 2, Directed Orientation; Phase 3, Explication; Phase 4, Free Orientation; and Phase

5, Integration. Research has supported the accuracy of the model for assessing student understanding of geometry (Hoffer, 1983:205-227). It has also shown that materials and methodology can be designed in such a way that they match these levels and promote growth through these levels (Burger & Shaughnessy, 1986:31-48). It is no longer a question whether these thought levels actually exist, but how to utilise them so that insight can be gained into the development of students' spatial abilities. Once insight is gained, it is possible to design the appropriate materials and instruction for the next teaching episode (Usiskin, 1987:29).

Although mathematics is often considered to be a collection of facts and procedures, current thinking in the field supports a view of mathematics as the activity of constructing patterns and relationships (NCTM, 1989). The NCTM have, according to Wheatly (1990:10), stressed the importance of the development of spatial sense (spatial visualisation, spatial reasoning, spatial perception, visual imagery and mental rotations) as part of the school curriculum. They have also chosen to call these spatial developments by a collective name, namely spatial sense.

Different researchers and authors who have done work on spatial development are not uniform about the terminology or the classification of spatial development. What they are uniform about, though, is the importance of spatial development as part of the school curriculum. Del Grande (1990:14) expresses the collective feeling of many people, namely that geometry for young children must not be preoccupied with structure and proof as presented in secondary school. They feel that geometry should start on an intuitive basis and should be built on objects and experiences that are part of a child's environment.

1.2 Problem statement

Educators in South Africa are in the process of implementing a problem-centred approach to mathematics teaching and learning (Human, Murray, and Olivier, 1992). This approach is characterised (*inter alia*) by a 'change and development' view of teaching and learning, and of the content to be taught and learned; furthermore, teaching for understanding can only be accomplished if the socio-anthropological context of the classroom can effectively be accounted for in the teaching for the learning of specific content by specific learners in a specific classroom setting and context (Cobb, 1988:87-91).

In practice the number component of mathematical development has been the main focus of the teaching and learning of mathematics in the primary phase. This trend has been continued in the approach advocated by Human, Murray and Olivier (1989). Booysen (1994:10) mentions that in contrast to the research on the number component, very little research has been done in the South African situation in connection with the spatial knowledge of children in primary school, viewed against the background of a problem-centred approach to mathematics teaching and learning.

The term "spatial knowledge" includes all the activities that children engage in, in order to structure the space around them. According to Freudenthal (1974-75:152), this does not start with the formulation of definitions and theorems but with the ordering of the every-day spatial experiences of the young child. In other words, the ordering and structuring of these original spatial experiences can then eventually lead to formulation and structuring of theorems and definitions that are part of the more formal spatial knowledge (geometry) to which the child is exposed at school. It is important, though, to remember that the formal geometry that children are exposed to in schools is but a part of the total spectrum of the spatial knowledge that children acquire during their lifetime. It is necessary, though, to develop this

informal or intuitive spatial knowledge of the child in order for them to be able to cope with school geometry.

The National Council of Teachers of Mathematics (NCTM) in the USA recently proposed that young children experience both two and three-dimensional geometry so that they can develop a sense of space, relationships in space and awareness of geometry in their environment (NCTM, 1989). These authors use the term “spatial sense” to refer to what has been known by a variety of other labels, from spatial visualisation, spatial reasoning, spatial perception, and visual imagery to mental rotations. According to the NCTM (1989), “investigation of geometry is a natural activity that young children engage in as they seek to make sense of their world”.

The current situation in many countries including South Africa groups children of different cultural backgrounds together in one classroom. Different descriptive modes are created to describe where in space (or space-time) objects and events occur relative to each other. Depending on what is important to them, each culture establishes its own spatial/temporal conventions. For every people or culture, their own physical and conceptual structuring of space-time is such an integral part of their world and their world view that it seems both obvious and natural (Ascher, 1991).

The aims of this research will be to investigate the development of the spatial knowledge of young children as they progress from a situation of orientating themselves in space to a situation of insight into the spatial situations that they perceive.

South Africa finds itself in a situation where the total curriculum (that includes mathematics) for the formal school years (primary and secondary) is under investigation. Research of this nature should be able to give much needed direction to decisions that need to be made about key issues like:

- What is the influence of culture and language on the spatial development of the young child?
- Do people who have different worldviews necessarily need to be guided along other paths when it involves the spatial development of the mind of the child?
- What should the nature of the spatial (geometry) contents be that needs to be taught to children at different age levels?
- Which methods should be employed to ensure optimal development of spatial learning?
- What kinds of teaching aids are practical and affordable when teaching children about spatial matters?
- What types of assessment strategies need to be employed to evaluate the progress of children?
- What will the effect of this knowledge about the ways in which young children think about space have on the training or in-service training of teachers?
- What are the benefits of following the development of the spatial knowledge of young children over an extended period of time?
- What needs to be done in terms of the training and development of future researchers in this kind or research approach?
- What is the possibility of implementing such a program on a large scale?

1.3 Aims of the research.

This investigation took place in a grade one multi-cultural classroom in a rural area of the Northern Province in South Africa. The children were between the ages of 6 and 8 years and were taught through the medium of English.

The objective will be to describe the “conditions” under which this teaching and learning took place and includes investigating the following issues:

- The nature of the classroom culture.
- The influence of the different cultural backgrounds of the children.

- The teaching philosophy employed by the researcher as well as the class teacher.
- The design and development of the appropriate materials and activities for the progress from orientation in space to insight into space.
- The utilisation of the different execution media (writing, talking, drawing and constructing) in the teaching and learning of spatial concepts. This includes: communication of events and objects through writing and drawings, direct and indirect observation of three-dimensional and two-dimensional situations, the verbal description of an object or situations, the mental imaging of situations and objects, utilising the mental images to solve problems and the taking of a viewpoint (mentally and physically).
- The evaluation of the progress (process as well as product) of the children in a qualitative as well as a quantitative way.

This implies that the researcher on the one hand had the intention of investigating the spatial knowledge *per se* of the children, and on the other hand refining a specific research methodology namely “developmental research” (see section 3.4.6). Cooney (1994:613) is of the opinion that because of the current emphasis on cognition and context, there is a dramatic shift away from the use of quantitative methodologies based on a positivist framework to that of interpretative research.

1.4 Research design

1.4.1 Literature Study

A Dialog search was done with the following descriptors: spatial development, geometry, spatial perception, pre-school children, kindergarten children and young children. The aim with this literature study was to identify all the relevant literature and research projects that had been conducted in similar areas of spatial development all over the world.

The literature study is tightly interwoven with the practical component (see section 3.4.6) and can be divided into three phases:

- First phase

A pilot curriculum was drawn up using the literature acquired. The pilot instructional units (pilot materials) were then designed by adhering to specific geometric objectives as well as didactical objectives as prescribed by the literature. It is important that the mathematical hierarchy of development as well as the didactical hierarchy be taken into consideration throughout the design of the units. The major objective with this phase was to investigate how young children's minds work when they are confronted with these kinds of materials.

- Second phase

The pilot materials were researched by the researcher in the classroom. For this type of research, it is preferable that the teacher of the specific group that is working with the materials is part of the research situation. Both the independent researcher and the teacher have a very valuable role to play. In most cases the researcher will be able to supply important mathematical knowledge, whereas the teacher who knows the specific cultural and historical background of the children will have to supply that. Both these components have a very important effect on the implementation of such a new subject.

- Third phase

During this phase the original material which was strongly guided by the literature study was changed to fit the specific needs of the group that the researcher was working with. The redesigned materials were then again tested on these children either during the next period, or at a later stage with children with a background similar to that of the original group. This is an ongoing situation

and it can be implemented from period to period or from one cultural group to another.

This phase is marked by the active participation of the teacher in formulating the goals and objectives for the specific group they are working with. If this phase is done in a proper way, the general as well as the specific character of each teaching unit will be brought to the surface. This information is very important when it comes to the INSET and PRESET phases of the research development.

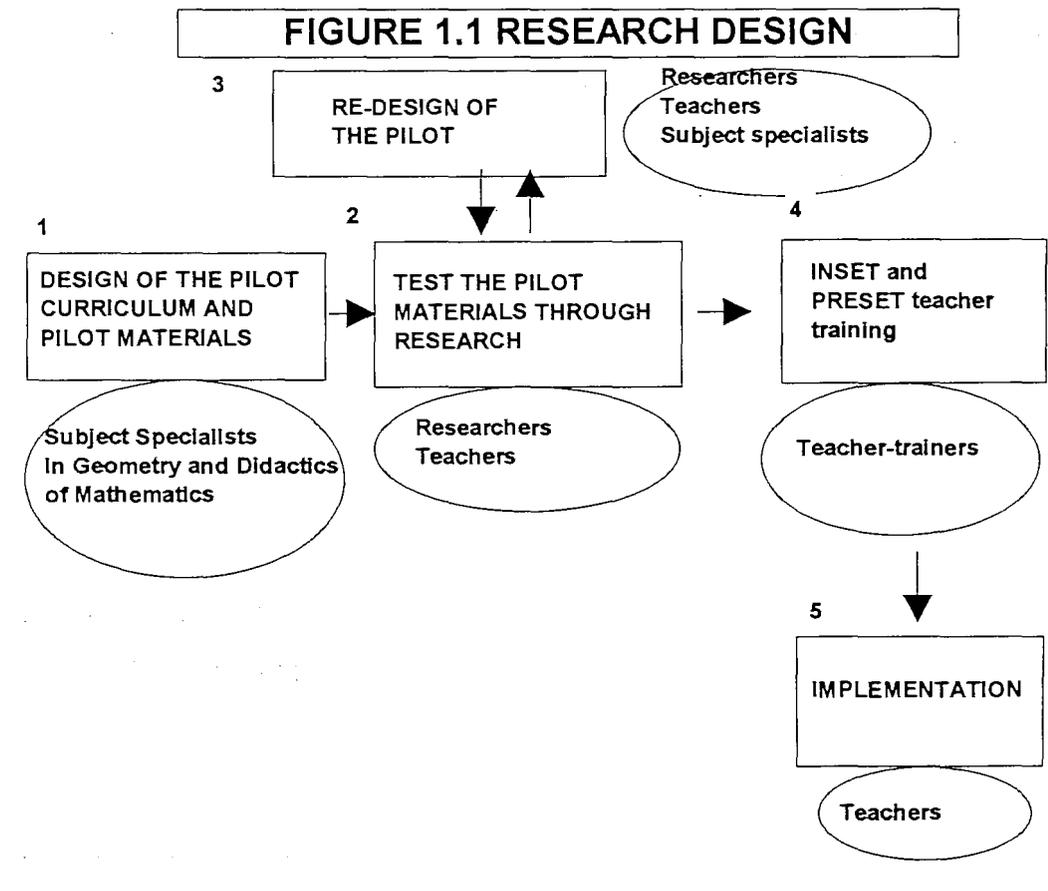


Figure 1.1 gives an outline of the cyclic nature of the different phases in the design of a developmental approach to research. The final result from such a research endeavour is the design of a dynamic curriculum in which children, teachers, subject specialists as well as theory play a crucial role.

The research was supported by two visits made to the Freudenthal Institute in The Netherlands and discussions were held amongst others with P. M. Van Hiele, K. Gravemeijer, L. Streefland and E. De Moor, working on the development of a geometry curriculum for primary schools in The Netherlands. A Colloquium at the Dortmund University in Germany was presented under the title: "Research on the spatial development of junior primary pupils". A workshop as well as an article was presented at the "11th Panama Najaarsconferentie" in 1995 with the title: "4-Kubers in Suid-Afrika".

1.4.2 Empirical study

The research was done in the form of a diagnostic teaching experiment in a multi-cultural grade one classroom in South Africa. The aim was to acquire insight into the thought processes of children while they were involved in different spatial development activities. Gravemeijer (1994:449) is of the opinion that this implies that the researcher will envision how the teaching-learning experience will proceed, and afterwards will try to find evidence in a teaching experiment that shows whether the expectations have been right or wrong. This leads to a cyclic process of development and research which is theoretically and empirically sound in the end (Gravemeijer, 1994:450). In developmental research, knowledge gain is the main concern. The focus is on building theory, explicating implicit theories.

The following procedure was decided upon:

Each teaching episode was to be video-recorded. After these sessions the video materials would be studied in order to gain insight into the child's way of thinking, so that the appropriate materials and activities could be designed for the following teaching session. At regular intervals these activities would also be supported by personal interviews with the teachers and the children.

The video-recorded classroom activities are part of the gathering of research data, apart from the field notes that the researcher makes while the class is in progress.

One such class activity takes about one hour. After video-recording the activities, the information is transcribed by the researchers and teacher that might have been involved in the classroom activities. The transcription of such data plus the time it takes for a classroom activity of one hour are subdivided into three main categories namely (i) data collection, (ii) data interpretation and (iii) materials design.

(i) Data collection

All the physical materials (drawings, written calculation, and models built) that are generated by the children are collected and categorised, together with the transcription data (written notes made by the researcher of the verbal conversations of the children). The video materials are transcribed in three different ways:

- Verbal transcription

The **actual conversations** of all the participants during the activities are written down after the researcher has listened to and looked at the video and the sound recordings of the video material.

- Construction activities transcription

The construction activities (cutting, folding, building, etc.) concerning the utilisation of the different **equipment** as well as the **physical solution strategies** that are employed by the children while solving the problems, are copied down.

- Pictorial or written material transcription

All the activities that involve the **process** of drawing and writing during the classroom activities are copied by the researcher after viewing the video recordings, in other words, not only the end product but the whole process that

the child engages in from the moment that the drawing is started until the completion of the drawing.

(ii) Data interpretation

All the data that have been gathered under point (1) are interpreted qualitatively and quantitatively, depending on the nature of the data.

(iii) Materials design

After the interpretation of the data the materials and activities for the next period are designed. This includes workcharts for the children.

1.5 Value of the research

The current trend in the rest of the world to acknowledge the importance of the spatial development of the young child emphasises the importance of such a study. South Africa has a unique multi-cultural society (with possible different worldviews) speaking different languages. Pinxten, Van Dooren and Harvey (1983:157-160) have gone as far as to identify some important differences between what they call "Western" space and the non-western Navajo (American Indian) space. Ascher (1991:133) points to the important fact that the language of a culture creates and reinforces particular shared habits of thought and shared habits of observation. Once this research has been completed in South Africa it may also serve as a "model" for other African countries which are still making use of curricula that they have inherited from their "colonial" pasts.

1.6 Terminology

1.6.1 Subject didactical approach

For the purpose of this document the definition of subject didactics of Human (1987:125) will serve to define the term. According to Human, subject didactics deals with didactical learning-and-developmental psychological-issues, philosophy of life, fundamental anthropological, historical and cultural, sociological and comparative curriculum as well as subject epistemological issues. In the subject didactics the focal point is identifying, describing and explaining the subject specific issues with the intention to effectively control the subject content, by the teachers, curriculum designers and other subject leaders in the field.

1.6.2 Spatial development of the young child

The NCTM use the term spatial sense to refer to what has been known by a variety of labels from spatial visualisation, spatial perception, and visual imagery to mental rotations. Wheatly (1990:10) suggests that it should be called spatial sense in terms of imagery. According to Kosslyn (1983:121), imagery involves the construction, representation and transformation of self-generated images. For the rest of this document the two terms namely spatial sense or spatial development will be used.

1.6.3 Problem-centred approach

The term problem-centredness, according to Human, Murray, Olivier, and Du Toit (1993: ii), could be defined as an approach which has as focus the development of problem-solving skills of both routine and non routine problems and real life situations within a mathematical framework. Problem solving should also be utilised as the basic way of learning, in other words, children should learn through solving problems.

1.6.4 Western worldview

This refers to a certain worldview that has dominated the world, both economically and through science and technology. People who subscribe to this worldview hold values that are dominated by the need for progress, development, improvement, evolution, and the linear unfolding of time (Peat, 1994: xiii). By this the author does not imply that all people who live in Western societies subscribe to this worldview.

Examples of Western scientists with views that do not subscribe to the above-mentioned Western approaches to science are (Peat, 1994:6,7):

- Quantum physicists who stress the irreducible link that exist between observer and observed.
- Physicists who speak of an order in which the whole is enfolded within each part.
- Physicists who are of the opinion that the essential matter of the universe cannot be reduced to billiard-ball atoms, but exists as relationships and fluctuations.
- Physicists who suggest that nature is not a collection of objects in interaction but is a flux and process.
- Physicians who question the current medical models and suggest that healing involves the whole body.
- Ecologists who stress that attention should be given to the interconnectedness of nature and the sensitivity and complexity of natural systems.

1.6.5 Indigenous worldview

It is a worldview in which time and space, as known from the Newtonian perspective of the universe, do not exist. Within the Indigenous world the act of coming to knowledge involves a personal transformation. The knower and the

known are linked and both are changed during the process (Peat, 1994:6). The Indigenous worldview is not something that can be reduced to a catalogue of facts. The reason for this is that it is a dynamic and living process that involves man and an ever-changing universe as a whole.

1.7 Progress of the investigation

The way in which the rest of this research will be discussed is as follows:

Chapter 2 gives an overview of the different philosophical perspectives that have influenced and are still influencing the views on the development of the spatial development of the young child.

In Chapter 3 the different research approaches that have been employed to study the spatial development of young children are discussed.

Chapter 4 gives a theoretical framework for the development of the spatial competence of the young child.

In Chapter 5 the spatial development of the young child is investigated through an empirical teaching experiment utilising wooden playblocks and soma cubes.

The work in Chapter 6 is an extension of the teaching experiment described in the previous chapter but through the use of six different solid objects (cubes, rectangular prisms, triangular prisms, square-based pyramids, tetrahedrals and cylinders). The focus is on designing the foldouts (nets) of the different objects.

In Chapter 7 the research findings are put in perspective with what is known about the spatial development of the young child through the literature and what was found during the empirical part of the research. The chapter concludes with recommendations and motivation for the extension of such a research project.

CHAPTER 2**PHILOSOPHICAL PERSPECTIVES CONTRIBUTING TO THE
UNDERSTANDING OF THE SPATIAL DEVELOPMENT OF YOUNG
CHILDREN****2.1 Introduction**

The problem of the development of spatial understanding in young children has been studied through different research approaches. The research approach that a researcher takes is always imbedded in and influenced by the beliefs of the specific researcher as well as the research community at that time. A critical component of the belief system of a research community is the theoretical framework which they consider as the guidelines for their empirical work.

The spatial development of the child is a multifaceted component of the child's total development. In the light of this it is important to take cognisance of the different views that have contributed to the vast body of theoretical information regarding the spatial development of the young child.

During the last decade of the twentieth century researchers and scholars have continued to draw heavily on the work of two authors, namely Piaget and Vygotsky (Confrey 1993:2). These authors have contributed greatly to the overall understanding of how children view the world and how humans interact within a cultural and historic setting. They have also contributed to the specific understanding of the way in which young children develop their spatial skills. The work of the different authors will be discussed within the context of the specific learning and teaching philosophies that they embrace.

In discussing the different theories of the different authors, Confrey (1993: 2) focuses on the fact that a variety of factors have created a critical need to revise such theories. Some of the factors include issues such as changing demographics, a reform climate in education, the creation of new technologies, the pressure of environment concerns, and issues of power and oppression. It is in the light of this that the theories of several other authors are also discussed in order to give a broader, more contemporary perspective about the knowledge of the spatial development the young child both in and outside of the formal school setting.

2.2 Indigenous perspective

2.2.1 Orientation

Nelson-Barber and Estrin (1995:178) state that we should consider the customary ways of knowing and acquiring knowledge of a specific group of learners. The Indigenous perspective on science differs radically from the traditional Western way of looking at knowledge in terms of factual information, information that can be structured and passed on through books, lectures or programmed courses (Peat, 1994:5). Within the Indigenous world knowledge is seen as the act of coming to know something which involves a personal transformation. One of the biggest mistakes that can be made when looking at indigenous learning is to disregard the fact that it is a dynamic and living process and an aspect of the ever-changing, ever-renewing processes of nature (Peat, 1994:6).

Space is not conceived of as separate from time and motion. Spatial boundaries are not independent of the processes of which they are part, so that segmenting space in an “arbitrary and static way” without accounting for flux over time is senseless (Ascher, 1991:129).

To enter this domain is, according to Peat (1994:4), to question what is meant by space and time, animate and inanimate, individual and society, dreams and visions, perceptions and reality, causality and synchronicity.

2.2.2 Space and time

According to Kearney (1984:99), two views of time can be distinguished amongst all cultures namely the oscillating and the linear view. Only one of these views is mostly dominant in a culture (Kearney, 1984:98).

The oscillating view of time can be compared to the swing of a pendulum between two extremes. This view, according to Kearney (1984:99), can be compared to the natural cycles of for example the seasons, which follow one another in a circular way. The linear view of time is based on the idea of non-repetitiveness and can be compared to the situation where life is viewed as the irreversible process of birth, ageing and death (Kearney, 1984:101).

This linear perspective of time, as conceived by Isaac Newton, was an ever-flowing stream that moved, without resistance or change of pace, from the past into the future (Peat, 1994:199). This is still the view of many people living in the Western world. For many Western people time is ever-flowing, linear and totally independent of man and all the workings of the cosmos (Nelson-Barber & Estrin, 1995:178). Time is seen as being independent of all physical laws and processes. For many of these people time is something that is registered by watches and clocks (Peat, 1994:203).

The opposite perspective to this view which Indigenous people have of time, is that time is alive and not independent of nature and man. Peat (1995:199) mentions that time is alive and not independent of man or nature for many of the Indigenous people in America. In the case of the American Indians they perceive time as cycles with which they seek relationships (Peat, 1995:200). The result of this approach is that in

many of these cases people do not plan for the future. The group as a whole will rather act when they together feel that the time is right to act after concensus has been reached. This implies that the very nature of time for many of these people is different from that which is experienced by people who live in a highly technological developed country. It seems that people living in closer correspondence with nature like the American Indian, have access to dimensions within the spirit of time that is not part of the way in which technologically advanced people live (Peat, 1995:203).

According to Engelbrecht (1974:32), the space of the Indigenous African people is very closely linked to the physical world in which they live. They do not only measure space, they actually live it.

2.2.3 Animate and inanimate

According to Peat (1994:231), in the world of the Indigenous people of America the important distinctions that are made within their languages are between the animate and the inanimate. In the same way that gender is assigned to objects in languages like French and Spanish, objects are seen as being animate or inanimate in the Indigenous languages of the American Indians. The problem, though, according to Peat (1994: 230), arises in discovering on what basis animate or inanimate perceptions are assigned to certain objects in these languages.

According to Peat (1994:232), in some instances objects are regarded as animate depending on the relationship of the specific object to the speaker. That same object can be regarded as inanimate when this relationship between the object and the speaker changes.

Mönnig (1967:53) states that the way in which the Pedi, which is a Indigenous group in South Africa view the sun, the moon and the stars is strongly soul-invested. They view these as having powers that are active on their own, mostly connected with the

weather and architecture. Mönnig (1967:53) elaborates on this by saying that some other objects in nature seem to have latent powers, but when they are adversely affected by man, they become active and can use their powers contrary to the desire of man.

2.2.4 The individual and society

Within many Indigenous societies there exist an insistence upon relationships rather than objects as the primary reality, and this is accompanied by the ensuing view of fluidity of boundaries and constant transformations of form (Peat, 1994:299). The indigenous person is always part of a much greater entity and each individual is an expression of the group.

Nelson-Barber and Estrin (1995:176) state that learning styles are also affected by one's environment. For example, "discovery learning" may be discouraged in an environment fraught with physical dangers as is the case for many traditionally-raised Indian children. In some cultures such as Navajo and Chipewyan, adults do not pay much attention to children to correct them, because the adult would need to continually monitor their obedience (Nelson-Barber & Estrin, 1995:177).

Kearney (1984:73) calls the relationship of the African person with his or her fellow man the "ecological" relationship. In this relationship people see themselves as very closely related to one another. Kearney (1984:74) distinguishes between two aspects of this view, namely the relationship of the individual with other people and the relationship of the individual with nature. The 'self' is seen as part of the group in such a way that this relationship forms a harmonious whole, not only within the group but with the total environment and nature. The individual is dependent on the group for his or her survival. This view is very well expressed in the phrase: "Umuntu ngumuntu ngomunye" (A person is only a person through other persons), which is very often expressed amongst African people (Muthwadini, 1990:32).

2.2.5 Dreams and visions

According to Peat (1994:272), in many Indigenous groups knowledge and initiation come through dreams. Peat states that this way of gaining insight is not restricted to Indigenous ways of thinking because it is well known that many great discoveries of Western science have come about through sudden “inspiration” or “insights”.

Harris (1991:53) states that the whole of the traditional Aboriginal life and thought is dominated by the concept of the ‘Dreamtime’ or ‘Dreaming’. This ‘dreaming’ is a view of life which explains life as a sacred, heroic time of the indefinitely remote past, which is in a sense still part of the present realities of Aborigine life (Harris, 1991:53).

Mönnig (1967:57) says that the main method of communication between the Pedi (Indigenous African people) ancestor spirits and their living descendants is through dreams. The living can communicate directly but one-sidedly with their ancestors through prayers. The ancestors, on the other hand, cannot speak directly to the living, and can only express their desires by visiting them in their dreams.

Within many of the African groups the ‘unseen’ is viewed as having both a personal and impersonal nature (Coertze, 1973:242-243). The personal view of the unseen can be linked to a belief in God as well as in some form of supernatural powers. The ‘spirits’ of the deceased are also viewed as powers that can influence people’s lives. These supernatural powers cannot be manipulated by individuals. The impersonal view of the unseen is linked to a belief in some form of ‘spirit’ that can be manipulated by individuals for the purpose of doing good or evil deeds. These different spirits are called by different names and they can communicate with the living through dreams and visions as well as through natural phenomena like changing of the seasons and thunder (Hammond-Tooke, 1974:323).

2.2.6 Perception

Within the Indigenous society the eye is not viewed as the exclusively dominant instrument of perception (Peat, 1994:276). Indigenous people rely very heavily on the ear to reveal a world of energies and vibrations. Amongst the Maori, Polynesian, and Inuit, greetings do not involve “rubbing noses” but taking in the smell of the other person. To such people smells are important dimensions through which to perceive reality.

Peat (1994:276) states that for Indigenous people the instrument of perception also involves the total being of the person, and this allows the person to move beyond what is normally called “rational thought”.

2.2.7 Causality and synchronicity

Although Indigenous people acknowledge the importance of cause, the inner nature of these causes appears to be substantially different from those in the West (Peat, 1994:259). To the Indigenous person some causes involve the action of spirits and energies (Peat, 1994:258).

For the African, not all events are explained by logical explanations in the form of formulas, numbers and concepts (Tempels, 1946:34). The role of spirits and forces as causes of events is very heavily emphasised in many African cultures (Coertze, 1973:14).

Kiernan (1981:10) warns that any worldview should be conceived, primarily, as a space-time framework for the conduct of social life. He states that it is important to remember that under modern conditions, it must be expected that a worldview will take cognisance of an expansion in the universe of social discourse. He also focuses on the fact that in many cases it would not be permissible to speak of for example, the

Zulu (Indigenous African people) worldview without qualification. There is, according to him not one single Zulu worldview, but many.

2.3 Socio-cultural perspective

2.3.1 Orientation

According to Cobb (1994a:13), two major trends can be identified in the research of mathematics education. The first one is the generally accepted view that learners are actively involved and responsible for the construction of their own mathematical knowledge. The theoretical arguments that form the basis for these arguments have an epistemological foundation which was advanced by the work of von Glasersfeld (1987). This view of constructing knowledge (constructivism) has secondly being influenced by the emphasis on the social and cultural situated nature of a mathematics activity. According to Cobb (1994a:13), the theoretical basis for this view is also inspired by the work of Vygotsky and the work of the Activity theorists like Davydof, Leont'ev and Galperin.

Theorists have focused upon different sources of influence regarding the acquisition and use of spatial knowledge. One such source of influence is that of linguistic or symbolic systems. According to Confrey (1993:11), Vygotsky's central tenet is that socio-cultural factors are essential in the development of the mind.

2.3.2 Vygotsky

For Vygotsky (1962:60) language is gradually internalised as private speech in the course of development and, as a consequence, the very structure of language becomes a vehicle of thought. He is thus of the opinion that concept formation is the result of a complex activity in which all the basic intellectual functions take place. The relevance that his theory has for application to the spatial development of the child can be seen in

the importance of the role of the visual experiences of the child which is gradually supported by an extended use of the appropriate language.

According to Taylor (1993:3), the three major themes that emerge from Vygotsky's integrated theory of mind are:

- The developmental method emphasises the origins, history and process of life-span development. This goes beyond developmental psychologists' typical focus on child development.
- Higher (uniquely human) mental functioning has social origins and a "quasi-social" nature. This is in direct contrast to Piaget's emphasis on individual rather than social functioning.
- Higher mental functioning is mediated by socio-culturally evolved tools and signs. The sign and the symbols of a culture influence individual development. This idea has been used in studies of language development; it seems equally applicable to mathematical development.

The question whether the mind is located in the head or in the individual-in-social-interaction, and whether mathematical learning is primarily a process of active cognitive reorganisation of enculturation into a community of practice, is currently in dispute (Cobb, 1994a:13).

2.4 Constructivist perspective

2.4.1. Orientation

The Constructivists tend to characterise the role of signs and symbols as a means by which students express and communicate their mathematical thinking, whereas socio-cultural theorists treat them as carriers of either established mathematical meanings or of a practice's intellectual heritage. Cobb (1994a:13) is of the opinion that

mathematical learning should be viewed as “ both a process of active individual construction and a process of acculturation into the mathematical practices of a wider society”.

According to Cobb (1994a:4) the antecedents of constructivism can be traced to Piaget’s genetic epistemology, to ethnomethodology and to symbolic interactionism. Confrey(1993:3) distinguishes between social constructivism and radical constructivism in the following ways:

“Constructivists argue for the importance of children’s active participation in the building up of concepts. They reject the view that children’s minds are blank slates, and they believe that there must be significant discussion and interaction around the variety of strategies that students propose. However for them the endpoint of instruction, the character of mathematical knowledge, is seldom questioned. Constructivists generally seek to reproduce in their students the same mathematical ideas that they themselves hold and that dominate modern mathematics. Little investigation is made of the meaning of mathematical ideas through historical, cross-cultural or cross-disciplinary methods. Generally constructivism is replicative in its goals and only modestly re-visionary. The methods of instruction are reformed, and the focus is more psychological than epistemological.

“Radical constructivism is a theory whose roots lie in a rejection of illegitimate claims for epistemological certainty. If one accepts that knowledge cannot be shown to represent reality in some iconic way, as a picture of the world, then one is left with more subjective construction of reality, subjective in the sense that one abandons the effort to factor the human subject out of the process. Although the radical constructivist is relativistic in contrast to the realist, that relativism is tempered by stability that is achieved by the individual in relation to his or her own experience. Others exert a significant influence on those experiences. The radical constructivist program assumes that the individual makes sense of experience in order to satisfy an essential need to gain predictability and control over one’s environment. Many of the efforts of researchers in this tradition have been devoted to describing how the individual builds up (rather than passively acquires) knowledge of the world. ”

Cobb (1994a:13) states that the conflict between radical constructivist and sociocultural perspectives lies in the role that is ascribed to teaching. Von Glasersfeld (1991:xvi) emphasises this view by saying that the most important features of radical constructivism are the sharp distinction that is drawn between teaching and training. He states that radical constructivists aim at generating understanding while the second emphasises competent performance.

Confrey (1993:9) suggests three limitations of the radical constructivistic programme:

- Many Constructivists assume an incremental view of knowledge construction. Most of the research in this field has focused on the primary and elementary grades. It is suggested that work need to be done in the secondary phases where a less incremental view of knowledge is needed “in which complexity can be lived in and comprehended with increasing depth”.
- Constructivist approaches can be criticised for positing a universalist or essentialist view of cognition across classification except age. It has lead to the documentation of diversity in student methods, but little or no discussion exists in the literature to explain systematic differences among classification of student participants according to culture, race or gender. One possible explanation lies in the tendency for the constructivist programme to assert heavy dependence on the autonomy of the individual.
- Constructivism may lack an adequate theory of instruction. In constructivist classrooms, the students are encouraged to articulate their views and explain how they think. In many cases though, the teachers, when required to make use of the diversity of ideas, find themselves at a loss. The teachers are also afraid of “telling” because of the belief that all constructivists commits them to refusing to inform the discussion with expert opinion or to bring the discussion to premature

closure. As a result of this, constructivist classrooms can lack direction and progress.

The above-mentioned limitations could be addressed if researchers and teachers start viewing the constructivist approach as a “guideline” for the development of personal models concerning teaching. Steffe and Kieren (1994:728) state: “Because constructivist research has sought means by which persons can construct their own mathematical knowledge structures, one of the products of such research has been the descriptions of a variety of constructs (e.g. various kinds and levels of units pertaining to natural numbers, rational numbers, and ratios) and constructive mechanisms (for example unitising operations, partitioning operations, proportionality operations, unit compositions, and decompositions).” According to Steffe and Kieren (1994:728), these descriptions can benefit the practising teacher in two ways. First, they provide a guide to teachers for listening to and observing learners and secondly, they provide potential sources for content and organisation of various mathematical curricula.

The primary contribution, according to Confrey (1993:7) that radical constructivism has made to mathematics education, “has been to challenge the stark evaluative climate of the mathematics classroom...they have legitimated diversity among individuals as a fundamental part of learning”. Confrey (1993:8) states that radical constructivism has also raised the importance of the epistemological issues and has “effectively challenged the passive mode of learning, putting into practice the use of manipulatives, contextualised problems, the use of small group work, and the co-ordination of actions, operations and representations”. Von Glasersfeld (1991:xix) adds to this view by stating that radical constructivism is a theory of active knowing rather than a traditional theory of knowledge or epistemology.

According to Confrey (1993:3), the work of Piaget is well suited to form a conceptual link between the philosophy of science and learning theories. According to her, Piaget

was also able to demonstrate the development in children of fundamental organising concepts such as space, time and number.

2.4.2 Piaget

The work of Piaget and Inhelder (1956:3-43) on children's conception of space can be divided into two major themes. Firstly, that representations of space are constructed through the progressive organisation of the child's motor and internalised actions, resulting in operational systems. Secondly, that the progressive organisation of geometric ideas follows a definite order, and this order is more logical than historical in that initially topological relations (for example, connectedness, enclosure, and continuity) are constructed, and only later, projective (rectilinearity) and Euclidean (angularity, parallelism, and distance) relations. This has been termed the topological primacy thesis.

Piaget and Inhelder's theory of children's conception of space has been widely influential and widely criticised. One criticism by Freudenthal (1983:232) has been that their use of terms such as topological, separation, proximity, and Euclidean, as well as the application of these related concepts to the development of their studies, are not mathematically correct. Freudenthal (1983:232-238) goes on to elaborate on the reasons for these in his work called "Didactical Phenomenology of Mathematical Structures". Freudenthal (1983:225) summarises the situation with these words: "If one looks only at the titles, theoretical introductions and conclusions of the chapters, and sections of Piaget-Inhelder's work, one can indeed get convinced that the authors have tried to investigate the child's conceptual approach to space or rather to find out which features of some adult conceptual approach to space can be traced in the children's mind."

According to Montangero (1976:101), Piagets' theory of intellectual development of the child went through four stages.

- 1923-1932

Piaget studied children's' language, moral reasoning, etc. and described development as a progression from a state of egocentrism to one where different viewpoints are co-ordinated and where relationships with others are based upon reciprocity.

- 1930s

Piaget conceived of development more in terms of adaptation with interaction no longer distinguished between subject and subject but rather between subject and object.

Piaget's adaptational model stresses the functional aspects of knowledge namely that to know something means that it must be incorporated into existing action systems, and to gain knowledge requires that these action systems be incorporated into more and more action schemata.

- 1940s

Piaget turned from the functional aspects of knowing to address the structural aspects of intellectual development. It was during this time that he wrote about children's conceptualisation of time, space, number, volume, etc.

- 1960s

Piaget attempted a new synthesis which encompassed some of the dichotomies in his theory between structures and functions of development, between the subject and the object of knowledge, between biology and epistemology and between action and thought. It was not until this time that his ideas were subjected to intense scrutiny and testing.

Piaget and Inhelder (1956:8) draw a distinction between operative and figurative aspects of knowledge. The function of the figurative aspect is to furnish an approximate imitation or copy of reality. Perception, imitation and mental imagery fall under this heading. The latter, which Piaget considers to be an internalised imitation, has thus an active component, but is basically suited to symbolise static aspects of reality. The function of the operative aspect of knowledge is to transform reality and this transformational character is specific of intelligence.

Two kinds of operations appear in children's thinking after the age of seven. First, the logico-mathematical operations deal with discrete elements and consist, for instance, in gathering elements into classes and including subclasses into a total class, interrelating the difference between elements, in adding or subtracting the elements. Second, spatial operations deal with continuous objects that they partition and then reconstruct. The two kinds of operations are analogous but not identical.

Another distinction that permeates Piaget's work is the distinction between the logico-mathematical and physical poles of knowledge. At the one pole the logico-mathematical structures are drawn by reflective abstraction from the subject's actions and allow for inference and ultimately deduction. They constitute structures that do not exist in the object. At the opposite pole, the causal structures or physical knowledge are in part drawn from empirical observations in the child's endeavour to explain reality. These structures give account of the structures of the outer world such as

spatial properties of objects or relations between physical variables. It must, however, be stressed that physical knowledge does not stem from mere empirical observations, but also involves reflective abstraction.

Reflective abstraction consists in abstracting something out of the organisation of the subject's own actions and in rearranging the elements or rules abstracted on a new level. The construct of reflective abstraction plays two important roles in Piaget's theory. First it implies, contrary to empiricism, that something new can be learned which is not directly abstracted from outer reality. Second, the construct of reflective abstraction contributes to explanations of the continuity of development from the biological organisation of the individual up to the higher forms of intelligence.

Piaget and Inhelder (1956:477-485) propose a model for the spatial development of the child in relation to the overall intellectual development. The four main issues that follow from this model can be summarised as follows:

- Children's representation of space arises from a gradual internalisation and co-ordination of actions.
- Knowledge of space evolves through four levels or structures of organisation namely, sensori-motor, pre-operational, concrete operational and formal operational.
- Three classes of spatial relations form the content of spatial intelligence namely topological space, projective space and Euclidean space.
- Imagery is a vehicle for representation arising from gradual internalisation of deferred imitations.

The Realistic instruction theory and the Problem-centred approach that are currently being followed in The Netherlands and South Africa respectively, are two approaches that have utilised many of the constructivist assumptions in their endeavour to implement their mathematics curricula. Gravemeijer (1991:75) points to the

relationship between the realistic approach and constructivism when he states: “But especially the idea of re-construction of knowledge relates closely to constructivism. In the realistic education theory, two sources are tapped in designing instruction courses which are meant to elicit this reconstruction process: the history of mathematics and the spontaneous, self thought up arithmetic methods of children.”

2.5 Realistic instruction theory

2.5.1 Orientation

Gravemeijer (1991:52) points to the link between constructivism and realistic mathematics when he states that constructivism is more a philosophical theory about knowing whereas the realistic theory is a self-constructed model of reality based on the experiences of the learners.

One important historical contribution to the development of the realistic approach to mathematics teaching in the Netherlands concerning the development of geometry, was the work done by the Van Hiele during the late 1950s.

2.5.2 The Van Hiele

According to the theory of the Van Hiele (1986:39), students progress through levels of thought in geometry. It starts with the most basic level where children display a Gestalt-like visual approach through increasingly sophisticated levels of description, analysis, abstraction and proof.

The theory of the Van Hiele has the following characteristics:

- Learning is a discontinuous process, which suggests that there are jumps in the learning curve. This implies that there are qualitatively different levels of thinking.
- These levels are sequential and hierarchical. In order for students to function adequately at one level, they must have mastered most of the previous level. The progress from one level to the next is more dependant upon instruction than age or biological maturation.
- Concepts that are implicitly understood at one level become implicitly understood at another level.
- Each level has its own language. This implies that two people that reason at different levels cannot understand each other. Neither can they manage to follow the thought processes of the other. Language structure is a critical factor in the movement through the levels.

The Van Hiele (1986:39-47) have distinguished between five different thought levels which can be summarised as follows:

- Level one: Visual

Initially students identify and operate on shapes and other geometric configurations according to their appearance. Children recognise figures as visual gestalts, and are thus able to represent these figures as visual images. This is an intuitive observation and at this level the objects are classes of figures, for example triangles. At this stage they do not attend to geometric properties or to characteristic traits of the class of figures represented. This means that children

identify, name, compare and operate on geometric figures (triangles, angles, intersecting or parallel lines) according to their appearance.

- Level two: Descriptive/analytical

Children at this level are now able to recognise and characterise shapes by their properties. They see figures as a whole but now also as a collection of properties rather than visual gestalts. Properties of figures are established experimentally by observing, measuring, drawing and modelling. Students at this level do not see relationships between classes of figures. The children will analyse figures in terms of their components and relationships among components and discover properties/rules of a class of shapes empirically (for example by folding, measuring, using a grid or diagram).

- Level three: Abstract/relational

Children can form abstract definitions, distinguish between necessary and sufficient sets of conditions for a concept, and understand and even sometimes provide logical arguments in the geometric domain. At this level, the objects about which students reason are properties of classes of figures. The product of this reasoning is the reorganisation of ideas achieved by interrelating properties of figures and classes of figures.

- Level four: Formal deduction

Students can establish theorems within an axiomatic system. They can reorganise the differences among undefined terms, definitions, axioms, and theorems. They are now capable of constructing original proofs. The product of their reasoning is the establishment of second-order relationships.

- Level five: Rigor/mathematical

At this level students reason formally about mathematical systems. They are now able to study geometry in the absence of reference models, and they can reason by formally manipulating geometric statements such as axioms, definitions and theorems. The product of their reasoning is the establishment, elaboration, and comparison of axiomatic systems of geometry.

According to the Van Hiele (1985a and 1985b), progress from one level to the other depends little on the biological development or maturation, but on the learning/teaching process. They distinguished five instructional phases:

- Phase one: Information

The students become acquainted with the content domain. The teacher initiates a discussion through presentation of materials. Through this discussion the teacher learns how students interpret the language and what they already know about the materials.

- Phase two: Guided orientation

Students now become acquainted with geometric ideas by handling the material. The teacher's role is to choose the materials in such a way that the targeted concepts and procedures are salient.

- Phase three: Explication

Students now become conscious of relations and begin to elaborate on their intuitive knowledge. They are now able to describe these relations in their own

language. They are also now ready to learn the traditional language for the subject matter. The teacher's role is to bring the object of study (geometric objects and ideas, relationships and patterns) to an explicit level of awareness, by leading students' discussion of them in their own language.

- Phase four: Free orientation

Students are now able to deliberately choose their activities. They learn to orient themselves within the network of relations. The teacher's role is to select appropriate materials and geometric problems, to give instructions, to permit various performances, to encourage them to reflect and elaborate on these problems and their solutions, and to introduce terms, concepts, and relevant problem-solving processes as needed.

- Phase five: Integration

Students are now able to build a summary of all the knowledge that they have learned. They integrate their knowledge into a coherent network that can easily be described and applied. They can use the appropriate language to describe all the networks. The role of the teacher is to encourage students to reflect on and consolidate their geometric knowledge.

Freudenthal, who has actually introduced the Van Hiele's to the rest of the world, together with a team of co-workers has in the meantime developed a theoretical framework to substantiate their teaching philosophy for mathematics, which is described at length by Adri Treffers' book, *Three Dimensions* (Treffers, 1978).

2.5.3 Freudenthal (Wiskobas)

Wiskobas was the project name for a group of people that was worked together under the guidance of Freudenthal, which had as its objective the investigation of the mathematics in the elementary schools in The Netherlands from 1971 (De Moor, 1997:25). The work of the Freudenthal institute is based on what Freudenthal calls his didactical phenomenology. This term is described by Freudenthal (1983:28) as follows: "Phenomenology of a mathematical concept, a mathematical structure, or a mathematical idea means, in my terminology, describing this *nooumenon* in its relation to the *phainomena* of which it is the means of organising, indicating which phenomena it is created to organise, and to which it can be extended, how it acts upon these phenomena as a means of organising, and with what power of these phenomena it endows it."

Although the researchers and developers at Wiskobas made use of the Van Hiele's levels, they also proposed some additional alterations in order to try and clarify some of the already mentioned problems that researchers all over the world were experiencing.

In the Dutch literature levels of didactical activity are sometimes described as follows (Treffers, 1978:243):

- didaxis, the level where thinking is still embedded in didactical activity.
- didactics, where acting is subjected to discussion. The reasoning is incoherent and inconsistent language is used.
- didaxology, where acting didactically is viewed in the framework of a coherent theory based on research results.

Van Hiele distinguishes similar levels in the process of learning mathematics which is analogous to the already mentioned levels of didactical activity and is called mathemataxis, mathematics and mathemaxology. The Wiskobas group at the Freudenthal Institute working under the guidance of Freudenthal agreed that the levels distinguished by Van Hiele in the macro learning process are still of great value viewed in the long term (Treffers, 1978:245).

It should, however, be pointed out that there are no strict criteria for delimiting macro-levels independent of the contents. The vulnerable spots in the materialisation of the three levels in the teaching-learning process as envisaged by van Hiele can now be more sharply identified than in the past.

Treffers (1978:245) focuses the attention on the fact that at the time when van Hiele launched his level theory, the contents of mathematics education were constantly under discussion. At that time, though, the questions like mathematics for all, mathematics in primary school, mathematics for slow secondary school students, and geometry instruction for the primary school were not the paramount issues of concern. Therefore, at the end of the sixties when some of these questions became important, this obstacle was bypassed in Dutch secondary education and elsewhere by focusing on new subject matter. In primary education, though, Wiskobas could not avoid these questions any more. It gradually became clear for the Wiskobas group that they could not find in Van Hiele's level theory and its concretisation in his textbooks a ready answer to two important questions (Treffers 1978:245):

- How to shape concretely the phenomenological exploration at the first level?
- Which didactical acts should be performed to raise the pupils as efficiently as possible from one level to the next?

In answering the question of what mathematics for all really is and how mathematics education should be structured, the Wiskobas group proceeded to formulate an answer and the results of this work can be read in Freudenthal's (1991:45-123) and Treffers' (1978:247-250) works. In these two works the emphasis is on describing and explaining the concepts of didactical phenomenology and mathematisation.

The concrete work of Wiskobas, in which Freudenthal was also strongly involved, was instrumental in guiding the formulation of the above-mentioned concepts in practice. This phenomenological orientation answers the first question posed with regard to Van Hiele's level system. Freudenthal states that the child must be put in touch with the phenomena for which the mathematical structure is the organising tool in order to let them shape these tools themselves in a process of re-invention, and to learn to handle and use these mathematical organising tools in concept formation. He also suggests that the constitution of mental objects precedes concept attainment and that the former can still be highly effective even if it is not followed by the latter. Freudenthal does not distinguish levels in the learning process in the same way as Van Hiele. In his view, there is no rigid tri-partition but rather in principle, an unrestricted progression according to micro-levels which are only relatively delimited against each other.

In answering the second question the Dutch propose a process of mathematisation (Freudenthal, 1991:41-42). They distinguish between horizontal and vertical mathematising in order to account for the difference between transforming a problem field into a mathematical problem on the one hand and processing within the mathematical system on the other.

In the horizontal component the way towards mathematising is paved via model formation, schematising and symbolising. The vertical component is concerned with mathematical processing and level raising in the structuring of the problem field under consideration. The Dutch admit, though, that the distinction between the horizontal

and vertical components is a bit artificial, given the fact that the two are strongly interrelated.

Progressive mathematisation, both vertically and horizontally, is inspired by five educational tenets (Treffers, 1978:248-250):

- Constructions stimulated by concreteness.

All kinds of applications and real situations are starting points for the children in the refining process towards the mastery of procedures. Concreteness in their sense is quite a different thing to manipulation of (Dienes) blocks by children. Reality serves both as a source for concepts, ideas, operations and structures and as a domain for application.

- Developing mathematical tools to move from concreteness to abstraction.

To learn a mathematical skill is a long-term process which passes through different levels of abstraction. There need to be a lot of learning opportunities for the learner in order to proceed along these levels of increasing abstraction. In no way are these levels of abstraction related to the tripartite sequence: concrete materials, pictures, mental actions. Instead, these levels refer to the degree of closeness to context problems, allowing for primitive informal strategies, and to moving onwards to more formalised procedures within the systematics of the subject area.

- Stimulating free productions and reflection.

Learning mathematics and especially raising thought levels is promoted by reflection. The requirements for reflection as the driving force in individual learning can be met by selecting special assignments in the course design and by promoting the pupils' own productions.

- Stimulating the social activity of learning by interaction.

In general, opportunities to compare one's own work with that of others function as opportunities to reflect on one's own method of problem solving. The requirements for interaction in order to provoke group learning can be met by respecting the learning environment at school as a social context for learning.

- Intertwining learning strands in order to get mathematical material structured.

It is the objective that from the beginning onwards, related parts of the programme are to be intertwined in order to have operations well enough understood in order to be applied successfully. If courses in different subject areas are both intertwined with each other and related to reality as much as possible, pupils will construct coherent and well-structured knowledge and skills, with a large measure of applicability.

Gravemeijer and Kraemer (1984:8) describe the approach the Dutch have developed as part of extensive research in the field of geometry for primary schools, in a book called "*Met Het Oog Op Ruimte*". The crux of their proposal is the view that geometry is built from the personal experiences that the child has had. These personal experiences are utilised to develop insight by means of a mathematical approach that will

eventually allow the child to visualise. They also make a very important distinction between visualisation as process and visualisation as product (Gravemeijer & Kraemer, 1984:102). It is important that the spatial orientation the child starts with is developed as a continuous line from the first grade to the end of their school careers. This should ideally culminate in spatial insight for the child in order to cope not only with secondary school geometry but also with real life situations.

Cognitive science is another theoretical perspective that has been applied to understanding students' learning of geometry. In this field the researchers attempt to integrate research and theoretical work from psychology, philosophy, linguistics and artificial intelligence.

2.6 Cognitive Science

2.6.1 Orientation

According to Clements and Battista (1992:434), this model brings into perspective, more precision than the theories of Piaget and Van Hiele. Clements and Battista (1992:457) also mention that perhaps the greatest strengths and weaknesses of the cognitive science approach lie in its extreme degree of specification. It provides much detailed information on cognitive processes, but in its quest for machine-codable formats, some of the theories avoid addressing such important constructs as belief systems, motivation, and the meaningful interpretation of subject matter, while de-emphasising the roles of intuition and culture.

The following three models are important to clarify the cognitive science approach (Clements and Battista 1992:434-437):

2.6.2 Anderson's model of cognition (ACT-model)

The cognitive science model of Anderson postulates two types of knowledge, namely declarative and procedural (Clements and Battista, 1992:434). Declarative knowledge is "knowing that" for example, postulates and theorems would be stored in schemas along with knowledge about their function, form and preconditions. Procedural knowledge, "knowing how", is stored in the form of production systems, or sets of condition-action pairs.

According to the ACT-model, all knowledge initially comes in declarative form and must be interpreted by general procedures. Procedural learning occurs only in executing a skill; one learns by doing. When declarative information is in the form of direct instruction, step-by-step interpretation is straight forward. Learning in this theory involves the acquisition of declarative knowledge. It also means the application of knowledge to new situations by means of search and analogy. It further means the compilation of domain-specific productions and the strengthening of declarative and procedural knowledge.

An important key to success in proof-oriented geometry problem solving is the development of data-driven rules. These rules respond to configurations of information and result in further development of the problem.

2.6.3 Greeno's model of geometry problem solving.

The model of Greeno is similar to Anderson's model of cognition (Clements and Battista, 1992:435). A computer simulation was designed, based on think-aloud protocols of ninth-grade students, solving the problems that these students could solve. The simulation is a production system within three types of productions, reflecting three domains of geometry, required for students to solve problems. First, propositions are used in making familiar statements about geometric relations, such as:

"Corresponding angles formed by parallel lines and a transversal are congruent." Second, perceptual concepts are used to recognise patterns mentioned in the antecedents of many propositions (for example corresponding angles). Third, strategic principles are used in setting goals and planning. For example, when a solution requires showing that two angles are congruent, one approach is to use relations such as corresponding angles and another is to prove that triangles containing the angle are congruent.

2.6.4 Parallel distributed processing (PDP)

Other cognitive science research suggests models with even more low-level detail. The PDP network model tries to explain the holistic template representations of the lower levels in the Van Hiele hierarchy. Such a network possesses units, representing conceptual objects, such as features, words, or concepts, and connections between these units. It is the pattern of interconnections among the units that constitute the processing system's knowledge structure in the domain. What it knows is how it responds.

The question that arises is how these networks might more precisely represent students' knowledge structures at different Van Hiele levels. It is argued that during the pre-recognition level, neural network units that recognise certain commonly-occurring visual features are formed. Shapes are "recognised" when certain patterns of links among features become established and enable the child to respond to any of a class of visual stimuli. When a sufficient number of visual features becomes recognisable and their detectors become interconnected in patterns that correspond to common shapes, the child progresses to the visual level. Figures that match visual prototypes closely enough cause certain patterns to be activated and, in turn, the figures to be recognised. The properties of figures are not recognised explicitly but the visual features that embody these properties simply activate the prototype recognises. These representations are not usually reflected upon by the child. For example, children often

encode the basic configuration of a polygon rather than the number of sides, describing a non-convex quadrilateral as a "triangle with a notch" or a "triangle with a side bent in" (Clements & Battista, 1992:435).

With appropriate instruction, property recognition units begin to form. This implies that visual features become cognisant in isolation and are linked to a verbal label. The students are now able to reflect upon the visual features and thus, recognising the shapes' properties, eventually are led to level two thought. Clements and Battista (1992:435) state that while this is all conjecture, it is meant to illustrate a possible cognitive science interpretation of the current Van Hiele theory.

2.7 Kosslyn

Although cognitive science is often regarded as a sub-field of psychology, Kosslyn (1983:205) argues that the questions it asks and the tools it uses to answer them come from the areas as diverse as linguistics, philosophy and artificial intelligence, as well as experimental psychology. A vast amount of the research Kosslyn has done focused on attempting to characterise the kinds of structures and processes that are available in the mind for using imagery to perform specific tasks.

Wheatley (1990:10) suggests that we should think of spatial sense in terms of imagery. Imagery is especially useful when one wants to reason about spatial relations because it portrays them all. Kosslyn (1983:183) argues that imagery involves three basic components namely construction, representation, and transformation of self-generated images.

The spatial properties of imaging make them a natural way of approaching and solving spatial problems (Kosslyn, 1983:10). Imaging is valuable as a way of seeing familiar things in new combinations. Apart from the fact that it is informative, using a mental

simulation of the situation can save one the effort of a real trial run. Visual thinking can also be extended to situations that would be impossible to observe.

Many mathematicians and mathematics educators have argued that spatial ability and visual imagery play vital roles in mathematical thinking (Wheatley, 1990; Del Grande, 1990). Kosslyn (1983:213) is of the opinion that elementary school children might rely even more on visual representations than adults, because they also rely more on imagery. This finding argues for presenting information graphically to young children whenever practical.

According to Kosslyn (1983:214-216), his research findings have very important implications for education. An understanding of the precise limitations of children's imagery would inform one how to use imagery in teaching. For example, if children's images are static, in that they cannot transform their images (see them melt into a new shape), they may still be able to make use of "blink" transformations (that is erasing the first image and imaging the object anew in some altered way). If so, then it would make sense to shift from trying to teach the child the rules of transformation, to teaching him/her the rules of formation. Learning how to "break up" an initial image and see it flow into a new shape is different from learning how to form a second image that is related to the first in some way. Similarly, if young children are able to form images easily, while they have difficulty in maintaining them, it would be better to try teaching by inducing a series of rather simple images. In addition, if one knew the specific limitations of children's imagery, one might be able to use the symbolic mode of imagery to teach fairly "abstract" ideas.

It is also important to remember that people have different aptitudes. An underlying idea to the already mentioned role of spatial/visual imagery is the situation mentioned by Clements and Battista (1992:443), namely that there exist two different modes of thought: verbal-logical and visual-pictorial. The theory of hemispheric specialisation of the brain corroborates the existence of two modes of thought.

Fischbein (1993:150) goes even further, saying that there exists a conflict between the two when it comes to the actual solving of some geometric problems. As a solution to this he introduces the idea of a "figural concept". He claims that one has to consider three categories of mental entities when referring to geometrical figures: the definition, the image (based on the perceptive-sensorial experience, like the image of a drawing) and the figural concept. He defines the figural concept as being a mental reality which is handled by mathematical reasoning in the domain of geometry. It is devoid of any concrete-sensorial properties (like colour, weight, etc.) but displays figural properties.

This figural construct is controlled and manipulated, in principle without residuals, by logical rules and procedures in the realm of a certain axiomatic system. Fischbein states that the difficulty with accepting this third type of mental activity is the direct awareness only of the mental representation (including the various sensorial properties like colour) and the corresponding concept. In order to grasp the figural concept, an intellectual effort is needed to understand that mathematical-logical operations manipulate only a purified version of the image, namely the spatial-figural content of the image.

The difficulty in manipulating the figural concepts, that is, the tendency to neglect the definition under the pressure of figural constraints, represents a major obstacle in geometrical reasoning.

The cognitive psychologist Olson and the linguist Bialystok have made important contributions to the understanding of the development of spatial sense in the young child.

2.8 Olson and Bialystok

The model of Olson and Bialystok (1983:234-238) is of importance to the understanding of the development of the spatial knowledge of the child in that it also

addresses two major issues influencing spatial development, namely active involvement (construction) and verbalisation (language). See section 4.3, 4.4, and 4.5 for a comprehensive discussion of the influence and role of their theory.

The model of Olson and Bialystok is of specific importance to the South African situation in the sense that it analyses the spatial language that young children use to represent their spatial experiences, up to the point of the use of specific terminology. The reason for this is that 33 different languages and 11 official languages are spoken by the children of South Africa and of the 11 official languages only one is technically on par with English.

The 9 other official languages are not technical languages and this implies that in many cases the equivalent spatial terminology in English does not exist in these languages or the same words in the indigenous languages are used for different words in English. According to Olson (1975:85), the most interesting property of the language of space is the fact that to understand the meaning of such terms you must know something about space. The question is: does the linguistic structure learned by every speaker of the language determine how he “codes”, “organises” or “perceives” space, or is the causal relation operative in the opposite direction the structure of perception of space determines the subsequent structure of the language?

The ability to solve mathematical problems develops slowly over a long period of time and it involves much more than mathematical content (spatial) knowledge (Lester, 1994:669).

2.9 The problem-centred perspective

2.9.1 Orientation

Lester (1994:669) is of the opinion that problem solving ability seems to be a function of several independent categories of factors (knowledge acquisition and utilisation, control, belief, affects, and the socio-cultural contexts). The problem-centred approach should not be equated with the constructivist approach to mathematics teaching and learning (Lester, 1994:668).

Murray *et al* (1993:193) points to the fact that the problem-centred approach currently being followed in many schools in South Africa utilises many of the constructivist views on learning when they state: “In a problem-centred learning approach compatible with a constructivist view of knowledge and learning, social interaction among students and the attempts by students to make sense of their own and each other’s constructions lead to the development of shared meanings and to individual students’ constructions of increasingly sophisticated concepts and procedures”. Human and his co-workers in South Africa have formulated this approach to learning and teaching of mathematics and are researching the development of number knowledge in the Primary school (Human *et al*, 1987, 1989, 1992, 1993).

2.9.2 Objectives

The problem-centred approach with regard to the development of the number component of mathematics, as being researched and implemented in South Africa under the guidance of Human *et al* (1993:ii), has as its main objectives (1) to develop problem-solving skills with regard to routine and non-routine real life situations within a mathematical framework; and (2) to utilise problem-solving as a learning type in order to solve problems. Apart from these they have identified several other broad objectives in this approach.

The problem-centred approach does not focus solely on the acquisition of mathematical knowledge and skills but also on:

- Understanding
- Problem-solving skills
- The ability to communicate effectively in and through mathematics
- Intellectual autonomy
- Positive self-image
- Perspective on mathematics as making sense and being of worth

The first of these objectives can be evaluated through the means of tests and examinations. The test and examinations should also contain questions evaluating the “logic” of the calculation strategies. A substantial amount of the question paper should, therefore, evaluate the systematic and notationally correct way of communicating in mathematics. In the traditional transmission approach that was followed in South Africa in the past, the focus was on the speed and mechanical (non-thinking) abilities of children to solve routine problems. In other words, a very mechanical view of calculation was expected of the children. The problem-centred approach is focused on instilling “computational know-how” in the sense that children should be able not only to calculate effectively but also to make judgements about the suitability of the different solution strategies.

In the same way that problem-solving acts as an objective as well as a learning mechanism in the problem-centred approach, calculating acts as both (Human *et al*, 1993:ix). Calculating as a mechanism is necessary for the development of:

- Understanding of numbers, computational methods, properties of computational methods and algorithms.

- The development of “algorithmic thought” includes execution, description and justification, the comparison and evaluation of the different algorithms, the design and adjustment for specific instruments (paper-and-pencil-methods, calculators and out-of-the-head-methods) and the recognition and utilisation of fluid calculation procedures (out-of-the-head and estimation).

Apart from these subject specific objectives there are general objectives focusing on the acquisition of knowledge *per se* which according to Human *et al* (1993:vi) include the following:

- Children tackle problems in an independent way, regardless of their difficulty or foreignness. They do not wait for someone to help or show them what to do.
- The children focus on the problem *per se*. They do not regard the method as the guide to solving the problem. They feel comfortable using so-called “primitive” methods like drawing.
- Children are intrinsically motivated by the solutions to the problems. They do not rely on the appreciation of the teacher for judging success. The focus is therefore on solving the problem and not on competition or reward.
- The children act as their own judges. They do not guess and then wait for judgement by the teacher.
- The children are prepared to work for long periods at a time on one specific problem.

- Children show an active and sincere interest in one another's way of solving the problems. They are also willing to share their own results with others.
- Children are not unduly disturbed about their own mistakes because they view this as a way of learning. They are also not judgmental about the mistakes that others make.
- Children work together spontaneously in small groups and they accept responsibility for one another's progress.

The role of the teacher in instilling the above-mentioned characteristics is crucial.

2.9.3 The role of the teacher

Teachers should refrain from giving hints like "Try this..." or "Remember what we have done the previous period" (Human *et al* 1993:vii). If children indicate that they have a problem to start their activities the teacher should, instead of giving hints, make sure that the children understand the task or problem that is put to them. It is of the utmost importance that children should understand the problem. If children exhibit the tendency to try and identify the method that they should use before they even try to understand the problem, the teacher could give a hint like: "Why don't you try to make a drawing?" or "Try to count". The major objective with hints of this kind is not to help the children to solve the problem but to influence their attitudes towards the problem.

At no point should suggestions be made that, in order to solve a problem, a specific computational method should be identified. Another dangerous practice that needs to be avoided is praising those children that have finished first. Any form of competition between groups or children should be avoided. One of the surest signs of children who have the wrong attitude is children who expect from the teacher to judge their work.

This facilitating role that the teacher has to play puts a tremendous pressure not only on the teacher's organisational skills but also on her specific subject didactical knowledge. One of the biggest problems that is currently hampering the proper implementation of the problem-centred approach in South Africa is the fact that all the teachers in primary schools(in service) need to be trained with regard to this specific subject didactical knowledge.

2.9.4 Problem solving as a learning type

In the problem-centred approach, problem solving plays a dual role according to Human *et al* (1993:viii). Development of problem solving skills plays a central role on the one hand and problem solving is the dominant learning type on the other hand. Children learn mathematics through solving problems. Children do not learn new mathematics when they solve problems by purely utilising existing knowledge.

To learn through the solution of problems children need to be exposed to problems which are new to them. The knowledge that develops through this process should not have been available beforehand. This, however, does not neglect the fact that there are still some mathematical knowledge that children cannot discover by themselves, namely the physical knowledge and the social knowledge. One of the biggest

differences between the way in which mathematics were taught previously in South Africa and the problem-centred method, is the emphasis that was put on the type of mathematical knowledge that children should acquire at school. Mathematical knowledge in the past was mostly viewed as social or physical knowledge, whereas advocates of the problem-centred approach in South Africa view the bulk of mathematical knowledge regarding the development of numbers, as logico-mathematical knowledge.

Another of the crucial activities that should accompany the development of logico-mathematical knowledge, is the opportunity to reflect about the process as well as the results (Human *et al*, 1993:xi). This also implies that there should be adequate opportunity for the children to interact with one another.

2.7.5 The role of social interaction.

According to Murray *et al* (1993:195-196) social interaction serves at least the following purposes in problem-centred classrooms:

- Social interaction creates the opportunity for children to talk about their thinking and encourages reflection.
- Students learn not only through their own constructions but also from one another.
- Through classroom social interaction the children also interact with the teacher.

The observations of Murray *et al* (1993:202) highlight a very specific issue concerning the nature of the social interaction among children. They have found that “observations of the nature of the collaborative interactions among students seem to contradict Vygotsky’s notion of learning through collaboration with more capable peers”. They have actually found that through their problem-centred approach to mathematics teaching, the children actually tend to prefer to interact with peers of near equal ability rather than with peers of higher ability.

One very important phenomenon that faces many of the children and teachers in the classrooms in South Africa is the multi-cultural situation. It is not uncommon to find children of four or more different cultural backgrounds grouped together in the same class. This is a situation that does not only ask a lot from the teacher but also from the designers of classroom materials and curricula.

Human *et al* (1993:xii) have also found that learning through problem solving and personal goalsetting has a radical influence on the order in which the curriculum materials are presented. The main difference is the fact that, in the case of the transmission method of teaching, the ordering of the materials is done according to a logical structuring which is guided by a thematic analysis of the content. In the case of the problem-centred approach concerning the development of the number component of mathematics, though, the order is determined by a psychological approach which is guided by the results of the previous activities. In the problem-centred approach the starting point is not the formal mathematics content of the syllabus but the informal knowledge of the children about the content.

2.10 Conclusion

There is ample evidence in literature and research that supports a constructivist/problem-centred approach on how young children learn and should be

taught spatial/geometric ideas. This evidence also points to the issue that there is progressive construction of spatial ideas from the perceptual to the conceptual plane as well as the developmental sequences in which children develop spatially.

Research that utilises the Van Hiele levels is crucial, especially in demarcating the phases of instruction concerning specific areas of geometry. Notice should also be taken of research into the three levels of thought that Van Hiele distinguishes and which Treffers denotes as intuitive phenomenological, locally descriptive, and subject-matter systematics. In this regard the Realistic approach to geometry learning has valuable contributions to make.

Research is needed to identify the specific cognitive constructions that children make at all age levels. Research from a cognitive science perspective should be integrated in this quest. In this regard the research should aim to inform how to build on the strength of children's existing ideas. Clements and Battista (1992:457) add to this when they state that the cognitive approach provides much needed details on the cognitive processes and that it may even provide details to limited aspects of students' representations at lower Van Hiele levels.

One of the major issues regarding the above mentioned approaches, is the implementation of the approach in a classroom situation. This not only involves the subject matter *per se* (mathematics) but also the beliefs and cultural "baggage" of the teachers and children in these mathematics classrooms. According to Lester (1994:671), problem solving seems to be a function of several independent categories of factors (knowledge utilisation, control, beliefs, affects and socio-cultural contexts). Previously unexplored factors such as language, schooling, and the immediate social culture are important components that need to be considered in any research program that investigates the development of mathematical (number/spatial) knowledge of young children. All these factors need to be addressed when looking at the complex issue of the teaching and learning of mathematics in a classroom situation.

CHAPTER 3

DIFFERENT RESEARCH APPROACHES REGARDING THE INVESTIGATION OF THE SPATIAL DEVELOPMENT OF YOUNG CHILDREN

3.1 Introduction

A scientist carries out his or her research and asks questions from within the limits defined by a paradigm which is "accepted" by the research community from which this specific scientist comes. Given the fragmented nature of current thinking and research about the spatial development of the child, it is important to remember that many of the research questions asked today were also asked by philosophers many years ago.

At the present time, the literature on psychological space contains a number of theoretical models and an enormous amount of empirical research. The majority of this information, though was done by researchers and research communities with bias towards a so-called "Western" perspective on the world and nature. Very little research information on the development of the spatial knowledge of the young child has come from researchers with a more non-Western approach (Briggs and Peat, 1984; Gay and Cole, 1967; Harris, 1991; Peat 1994; Pinxten, Van Dooren and Harvey, 1983).

Much of the literature forthcoming from the group with a leaning towards a Western perspective on research can be grouped and presented within either a psychometric, a developmental, or an experimental approach (Eliot, 1987:13). These three approaches differ from one another in their assumptions about the nature of psychological space, in their preferred methods of studying spatial problems, and in the kinds of measures used to assess different aspects of intelligence, spatial development, and spatial processing (Eliot 1987:13).

The research design for the investigation of the spatial development of the young child needs to utilise the appropriate components of the above-mentioned theoretical models. The result should be a cyclic research process of design, evaluation through teaching, followed by a redesign of the materials.

3.2 Three theoretical research models from a Western perspective

3.2.1 The psychometric approach

3.2.1.1 Introduction

The psychological study of human intelligence is primarily the focus of psychometricians, who assume that intelligence exists as a set of quantifiable dimensions along which people can be measured. They study individual differences by administering test batteries which demonstrate reliability to large numbers of subjects under controlled conditions. Scores from these tests are then analysed by factor analysis or multidimensional scaling to extract underlying factors or dimensions responsible for patterns of correlations. Individual differences are usually interpreted in terms of factors which differentiate between both test and subject. The models, being advanced to account for intelligence, are expressed as factor structures (Eliot, 1987:37).

3.2.1.2 Research findings from a psychometric perspective

These findings will be summarised under six general investigative issues:

(i) The emergence and decline of spatial ability

There have only been a few studies where spatial ability by itself was the focus of longitudinal comparisons (Eliot, 1987:74). The stability of factors over different ages

has also been difficult to measure because of a lack of tests that are known to require similar solution responses at different age levels. The age-differentiation hypothesis was formulated which states that abstract or symbolic intelligence changes in its organisation as age increases from a fairly unified and general ability to a loosely organised group of abilities or factors (Garrett, 1946:373).

(ii) Sex-related differences

Although the fact of sex-related differences is well documented, the extent of such differences and the explanation for these have been the focus of considerable debate. These controversies persist in large because there is no widespread agreement about the characterisation of spatial ability. Evidence for age-related and sex-related differences inevitably raise questions about the trainability of spatial intelligence (Kimbal, 1981:333; Linn & Petersen, 1985:1491).

(iii) Trainability of spatial abilities

This issue has been studied making use of three types of investigations. Early studies entailed a pre-test/post-test design, with training consisting mainly of some experience thought to require a high degree of spatial ability (Eliot, 1987:78). Later studies gradually became more controlled as researchers sought to manipulate particular variables, and recent research has looked at differences in the training of high and low ability subjects upon specific tasks.

Results from most training studies, while often encouraging, are also inconclusive (Eliot, 1987:77). When subjects are trained indirectly through courses that are thought to require high levels of spatial ability, the results have been mixed. If they are given practise on tasks that are highly similar to the tasks on spatial tests, the results can be interpreted as indicating that spatial ability, as a set of operations, can be trained. At

the present time, though, few results are available from long-term training efforts, to indicate whether training effects persist.

(iv) The role of imagery in spatial thinking

The role of imagery in spatial thinking has a long history in psychology (Galton, 1983:113). Although the nature of visual imagery was not defined by these early psychometricians, they clearly assumed that it was intimately associated with spatial ability (Poltrok & Brown, 1984:136). Despite the alleged intimacy, the relationship between imagery and spatial ability is still not well understood. Although factor analysts have repeatedly described spatial ability as entailing the use of visual imagery, there have been few efforts to develop models that have related specific spatial factors to different imagery processes or structures. A difficulty in developing such models is the fact that the terms "visual imagery" and "visualisation" are often used interchangeably. Eliot (1987:79) is of the opinion that, although these tests have been devised to measure both constructs, they are often one and the same test.

(v) The types of processing required by different spatial tasks

Researchers have distinguished between tasks that require the sensing and retention of form (perceptual processes) and those that require the manipulation of spatial relationships (conceptual processes). A somewhat different expression of this issue is the distinction between spatial tasks that appear to require analytical processing and those that require synthetical processing. Eliot (1987:80) states that analytical processing entails the successive or sequential trial-and-error checking among the various parts of a stimulus, while synthetical processing entails the simultaneous holistic response to the stimulus as an organised whole. It has been further suggested that the same task can elicit a number of processing strategies. If the process used to solve a task also determines what ability the task measures, many so-called spatial tests

may actually be measuring distinctly different abilities for different people (Lohman, 1979:191).

(vi) The issue of stimulus dimensionality

Eliot (1987:80) mentions that it is unclear whether tests of figural, object, or large-scale space are measuring the same underlying construct. If measures for different spaces do not have a common factor structure, then we will be forced to talk about a plurality of psychological spaces, and ascertain what tests will be required to assess situations where all three spaces (figural, object, and large-scale) are perceived and represented simultaneously.

3.2.2 The Experimental approach

3.2.2.1 Introduction

The experimentalists study individual differences in terms of the sensory bases of spatial perception, imagery and its relation to representation and computational models of information processing. For the most part differences are interpreted in terms of the means of response times or response accuracy on tasks requiring the recognition, discrimination and generalisation of stimuli under controlled conditions (Eliot, 1987:134).

The empiricists divide thought into three categories namely:

- Sensory perceptual space
- Imagery
- Spatial processing

3.2.2.2 Research findings from a experimental perspective

These findings will also be discussed under six general investigative issues:

(i) Age related changes in performance on spatial tasks

Age-related changes in performance on tasks have been treated simply as one of several possible independent variables (Eliot, 1987:163). As a result, while something has been learned about such changes in childhood and early adulthood, very little has been learned about changes in the latter half of life.

(ii) Sex-related differences

Eliot (1987:164) is also of the opinion that something similar can be said of sex-related differences. It was found that females generally had longer response times when making a comparison of a mental rotation task than males. It was also further found that males in elementary school grasped the principle of horizontality, but a significant number of college females could not. An important finding was that many spatial tasks were more strongly affected by the algorithm for solution embedded in task instructions than has been supposed earlier.

It is important to emphasise that most findings which provide evidence of sex-related differences on spatial tests are based upon trends rather than definite patterns of difference. Increased variance in performance is usually associated with individual differences rather than with sex-related differences.

(iii) Trainability of spatial abilities

Trainability has received less attention from experimental psychologists than one might suppose. Two solution strategies which have most often emerged during pilot studies were one of visualisation and one of analysis. Some researchers have also found that high spatial ability subjects performed best when they practised solving items and were given feedback about their performance. Subjects who were low in both spatial and verbal abilities performed best after receiving a visualisation treatment. Some researchers are of the opinion that strategy training for specific spatial tasks appears beneficial to some degree, especially when one capitalises upon the cognitive strengths of particular subjects while compensating for the weaknesses in the face of specific task difficulties (Eliot, 1987:165).

A continuing difficulty with studies that attempt to train spatial ability is the question of generalisation to other forms of perceiving or thinking about the surrounding world and the use of knowledge about the "whereness" of things.

(iv) The role of imagery in spatial thinking

A large number of questions have not been addressed. It is unclear how self-generated images are recognised as compared to the ones which have perceptual reality. While experimental research has advanced the knowledge about the functions and properties of visual imagery, very little is known about the large scale representation of those who lack visual imagery.

The controversy which continues to surround the study of imagery has not been so much with the end product but rather with the processes by which the end product is achieved.

(v) Types of processing required by spatial tasks

According to Eliot (1987:166), a large number of studies have reported that processing strategies vary. The following is a summary of the findings:

- The subject's preference for a processing mode may be related to ability level.
- There exists an interplay between stable and automatic processing routines and more labile attention-demanding processes.
- The influence of task instruction may force or suggest a particular strategy.
- The effect of task demands such as speed, complexity, and the number of response alternatives is characteristic of a particular problem.
- There exist individual differences in the knowledge base of individuals and/or previous experience with spatial problems.

It is also clear from the research that more complex tasks elicit more solution strategies than do simple ones, that providing more time to complete complex tasks appears to encourage the use of less efficient strategies, and that individual imagery differences are viewed more in terms of differences in process-based attributes than knowledge-based ones.

(vi) The effect of stimulus dimensionality on spatial performance.

It was found that adolescent boys are more successful on items requiring a combination of three-dimensional thinking and the manipulation of mental images than adolescent girls (Eliot, 1987:168). No such sex-related differences were found on items requiring two-dimensional thinking and the use of static imagery. The issue of stimulus dimensionality goes beyond observer-as-actor relationships in different environments.

3.2.3 The Developmental approach

3.2.3.1 Introduction

The developmentalists present specific tasks to individuals or to groups of subjects at different ages and then examine their responses in terms of the hypothesised stages of development. Individual differences are typically interpreted in terms of the achievement of cognitive structures which enable individuals to organise, represent, and transform relationships (Eliot, 1987:83). The findings under this heading do not include the developmental research results and approach discussed under 3.4.6.

3.2.3.2 Research findings from a developmental perspective

These findings will also be discussed under six general investigative issues:

(i) Age-related changes in spatial behaviour

Despite the emphasis in research on age-related changes in spatial behaviours, most theories have emphasised the emergence of these behaviours and ignored their persistence or decline (Kirasic, 1985:187; Mandler, 1984:452). Unfortunately, apart from a few comparisons between young and elderly subjects, very little is known about changes in spatial behaviours during middle and old age.

(ii) Sex-related differences

Some researchers are of the opinion that sex-related differences may be a function of greater hemispheric specialisation in adolescent males. An extensive review of research literature done in 1974 concluded that sex-related differences in performance on spatial tasks appear from adolescence onwards (Eliot, 1987:128).

(iii) Trainability of spatial abilities

Much of the early Piagetian research in the United States was undertaken to replicate and accelerate the achievement of developmental systems as presented in Piaget's developmental account. The results have been inconsistent and inconclusive. In 1981 an extensive list of studies was compiled which looked at the training of spatial abilities (Eliot, 1987:129). The evidence indicated that the various spatial skills are trainable, given the appropriate experience. Brief training with pictorial materials is sufficient to improve the performance of subjects on spatial test items. The teaching of various spatial conventions and exercises with diagrams helps to improve geometry performance. More successful studies were carried out with younger children whereas studies with older subjects met with less success.

Notwithstanding these findings, it remains unclear from the literature the extent to which different kinds of training are generalised to other tasks. It is also unclear to which extent the training results in permanent effects and how training can contribute to the understanding of mathematical and scientific concepts.

(iv) The role of imagery in spatial thinking

From the range of findings of different researchers it is often difficult to know whether to regard imagery as an ability, a mediating process, a mode of representation, a skill, or a stimulus attribute. An ongoing developmental controversy is whether imagery impedes or facilitates thought in childhood (Eliot, 1987:130). Researchers have noted that children appear to use imagery more than do adults, and eidetic imagery, common in childhood, appears to fade in adolescence at about the same time that spatial abilities emerge. For some researchers high imagery in children is a valuable component of thought while others regard it as a primitive way of representing information which impedes symbolic or abstract thought.

It is unfortunate that many studies of imagery have used spatial tests as measures. Not only is there a clear need for the development of new imagery measures, but there is also a need to measure different kinds of imagery across the life-span of a person.

(v) The types of processing required by spatial tasks

Early notions about processing regarded it in terms of the sequential solution response to a given problem situation. More recent approaches have incorporated information processing occurring in a programmed or given sequence of response decisions or actions to a fluid problem situation. Whereas earlier notions about processing emphasised organisational characteristics of response, later notions emphasise the task demand of different stimulus situations.

(vi) The effect of stimulus dimensionality on spatial performance

The effect of stimulus dimensionality on spatial performance, as well as the extent to which spatial representation of different surroundings, corresponds to the layout of the geographical features and locations (Eliot, 1987:132).

Researchers have found that environmental learning is not something that is taught, but is picked up by the learner. Cognitive maps of the environment are not likely to be isomorphic with physical space because it is affected by cognitive status, the nature of experiences and feelings about different places. Presumably the same reasoning may apply to representations of object arrangements and even two-dimensional configurations.

A persistent issue has been the problem of accounting for apparent stability and changes of behaviour, both within and between age groups. Although some significant advances have been made, there is still considerable debate over what constitutes a

"stage" in descriptions of spatial development and how perception, imagery and representation interact within stages to maintain or further development.

3.3 A theoretical research model from an Indigenous (Non-Western) perspective

A scientist carries out research and asks questions from within the limits defined by a paradigm. By the end of the nineteenth century the map of the Newtonian paradigm looked almost as detailed as the nineteenth century maps of the Americas (Briggs & Peat, 1984:28) because it seemed that only a few areas were left to explore. Through the continued probing of these areas the puzzle-solving map makers of normal science began to experience disturbing difficulties. The appearance of "anomalies" which could not be fitted into the classical Newtonian picture of the world was bringing on what Kuhn (1976) calls a "crisis" in the paradigm. He noticed that during times of crisis new theories arise to explain anomalies. These theories vie with one another for the honour of becoming the new paradigm.

The question that many people, according to Peat (1994:41), ask is: "Are Western scientists trapped within their paradigms and worldviews?" He states that the real problem experienced with a paradigm or worldview is when it is held by a society that wields a considerable amount of political and financial power (Peat, 1994:42). Once this is the case, one culture (the financially and economically advanced) can then move into a position of total dominance regarding the way in which other cultures should think about issues.

Indigenous science, and for that matter research, cannot be reduced to a catalogue of facts because it is a dynamic and living process which is an integral part of ever-changing ever-renewing nature (Peat 1994:6). Research results that are discussed under this indigenous perspective should be understood against the above-mentioned background.

Findings from Indigenous groups from three different continents are discussed under the headings orientation and location, maps, and shapes and architecture.

3.3.1 Orientation and location

The Kpelle of Liberia have a complex set of terms which gives an indication of their relations to location. These terms belong to special classes of dependent nouns (Gay & Cole, 1967:59). These terms have specialised meanings in particular contexts and are all more or less related to the root meaning of the term.

The Warlpiri culture in Australia places great emphasis on the knowledge and skills that are associated with children's ability to handle directional and spatial terminology (Harris, 1991:22). It appears that not only do Aborigines know North, South, East and West, but they also employ them in the place of left and right. Harris (1991:23) states that Aborigines using their own language use these compass directions in a great variety of situations and journeys in both familiar and unfamiliar surroundings.

Harris (1991:25) states that it is not only the Australian Aborigines that place a high emphasis on cardinal directions but also the ancient cultures of Central America, the American Indians and the great civilisations of China.

In the environment of the people of the Central Caroline Islands in Micronesia, water is the predominant feature (Ascher, 1991:140). These Caroline navigators do not use any navigation equipment such as rulers, compasses and charts to navigate them between the islands. They travel only with what they carry in their minds (Ascher, 1991:142). These people have two spatial models; the one is an approximation of reality and the other they do not believe to be a statement of reality at all. According to Ascher (1991:142) these navigators know more than these models, but the models are crucial. For many Westerners it is still not exactly clear how these mental spatial models enable these navigators to get exact results (Ascher, 1991:142).

3.3.2 Maps

Aboriginal people have difficulty in interpreting Western style maps printed on paper (Harris, 1991:27). The “realism” of maps are interpreted in a different way by Aborigines. The realism of the map depends on the purpose of the map, in particular on whether it is ‘only a map’ for the purpose of showing a journey or whether it has mythical or spiritual significance (Harris, 1991:28). For Aborigines the context of the map and the relationships that are involved determine how the map should be interpreted.

The Inuit do not share any of the conventions of Western artists when it comes to drawing and making maps (Ascher, 1991:136). For them time and space remain unified, and the content of the picture is not confined to what can be seen from a fixed position in space-time. This, combined with the lack of background, means that there are no changes in size due to differing distances; everything in the picture or map is relatively at the same distance (Ascher, 1991:137).

3.3.3 Shapes and architecture

Many of the African art forms and, for that matter expression and utilisation of shapes, are closely related to religious, social and domestic customs (Zaslavsky, 1973:154). A good example of this is the way in which the African adapts his home to the availability of materials, and to the requirements of the climate. The circular home in its many versions is found throughout the African continent (Zaslavsky, 1973:155). The circle is also the geometric shape in the plane which encompasses the greatest area within a given perimeter. In many cases the changed use of familiar shapes of housing has a profound influence on the African society. Zaslavsky (1973:163) states that for the older people of a society, the transition from round to square or oblong houses is more than just an innovation in geometry. This represents a passing of the old order as well as the loss of security in many cases.

Zaslavsky (1973:174) also states that there exists a tendency to distort natural forms for the purpose of stressing certain characteristics. The wide application of symbolic motifs in decoration leads to emphasis on the geometric aspect in African art.

Gay and Cole (1967:53) found that the language used by the Kpelle of Liberia does not indicate the preciseness of the figure or shape that is named, but more the way in which space is divided. Thus the term for 'path' (*pere*) can refer to a straight line but it can also apply to a curved or jagged line. In other words, the distinctions that are required in English when using words like "straight line" are unimportant to the Kpelle. The important thing about that which they term "*pere*" is that it extends from one place to another without crossing itself.

According to Harris (1991:41), no abstract terms for 'shape' can be found in any of the Aborigine languages. The Aborigine languages also show differences between Western and Aboriginal categories. There seem to be far more descriptions for round or circular shapes than of shapes with straight lines, such as squares and cubes (Harris, 1991:42). When some Aboriginal women were asked to give names for four basic shapes, a square was described as: "flat, smooth", a circle was described as "round", a rectangle was described as "long" and a triangle was described as "having sides" (Harris, 1991:42).

3.4 An agenda towards an interpretative research methodology

3.4.1 Introduction

It is the view of Schoenfeld (1994:697) that the development of our notions of mathematical understanding plays a major part in directing the type of research that is needed for the future. Schoenfeld continues by stating that Richard Skemp's (1976) much quoted article on instrumental versus relational thinking triggered the interest of

many researchers to take on the arduous task of building theoretical models of mathematical thinking.

Understanding began to be thought of in terms of a spectrum rather than a single dimension focused on rightness or wrongness. This is one of the most important contributions of Indigenous science, namely that there should be a greater need for balance, and this can only be achieved by looking at phenomena as a whole (Peat 1994:8). This evolution, according to Steffe and Kieren (1994:721), in concert with the constructivist theories of Piaget, resulted eventually in the notion of learners' conceptual frameworks.

Research on mathematics education is subject to many changes, because views on mathematics education are changing and at the same time research paradigms are changing.

In the current community of mathematics researchers and educators, the view of mathematics as a system of definitions, rules, principles, and procedures that must be taught is changing (Gravemeijer, 1994; Cobb & Steffe, 1983; Human *et al*, 1993). The general view is that mathematics is a process in which children must engage. This view is very strongly expressed in the United States by the National Council of Teachers of Mathematics Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989).

Much of the research that took place in the classroom was to measure the effects of teaching on a class of individual students, rather than to study the role of the social processes in the construction of mathematical knowledge (Steffe & Kieren, 1994:728). One of the reasons for the development of the new classroom orientations of the research trend is related to a limitation of past constructivist, cognitively oriented work. Researchers were hard-pressed to reconcile the theory that all learning is individually constructed with the evidence of commonalities found across individuals.

Constructivists had to admit to the social dimensions of learning which led to what has recently been called social constructivism. The impact on research of this theoretical response was the beginning of the attempt to account for learning as it occurs in classroom environments.

Some of the methodological implications of this view of mathematics learning are still in the process of being developed, as the field attempts to craft a research approach that combines some of the desired features of laboratory studies of individual cognition with the ethnographic-sociological methods to which the classroom lends itself.

The complexity of such research is due in part to the inherent difficulty of capturing not only the interactions between teacher and learner but also between one learner and another. The specific nature of individual learning that occurs in the social environment of the classroom also complicates the issue further. Bishop (1994:17) elaborates on this when he states that care should be taken in order to structure the curriculum in such a way that it relates to the local culture. By this statement he does not suggest that a uniform multi-cultural curriculum should be developed, but that a start should be made to determine the criteria for evaluating curricula. Ascher and D'Ambrosio (1994:40) raise two related matters concerning the research and development of an appropriate mathematics curriculum for different cultural groups. They state that the one aspect seeks understanding of the relationship between mathematical ideas and culture and the other aspect is the question of implementation of this new knowledge.

Cooney (1994:625) states that the emergence of constructivism as an epistemological framework for mathematics education has significantly influenced the research in mathematics education. Constructivists are of the opinion that the activity of exploring children's constructions of mathematical knowledge must involve teaching. Theoretical analysis by the researcher does play an important part in understanding the significance of children's mathematical behaviour but knowledge gained through

theoretical analysis can at best intersect only part of the knowledge gained through experiencing the dynamics of a child doing mathematics (Cobb & Steffe, 1983:84). Another reason why the researcher must act as teacher is because the experience children gain through interactions with adults greatly influences their construction of mathematical knowledge.

Some researchers have developed instructional programmes based on constructivist principles that explicitly focus on the culture of the classroom. For these researchers, teacher development consists of teachers developing a deeper knowledge of children's mathematical thinking, but in the context of the "community" of the mathematics classroom (Cobb, Yackel & Wood, 1991; Cobb, Wood & Yackel, 1992; Murray *et al*, 1993).

3.4.2 Contextuality of cognition

3.4.2.1 Introduction

Constructivists attribute great importance to the context within which a child constructs mathematical knowledge (Cobb, 1990:200). With context is meant the cognising subject's own constructions. This is distinguished from the situation or setting. For example, the setting might be mathematics instruction in which the children are asked to complete a set of tasks. However, the context within which students attempt to complete the task might differ radically. For some it might be better to try recalling what the teacher has told them what they are supposed to do, whereas for others the focus could be on mathematical sense making. This implies that *purposes* and *intentions* are crucial aspects of context (Cobb, 1990:200).

Cobb (1990:200) is of the opinion that the contextuality of cognitions are as applicable to researchers as to mathematics students. The researcher continually switches from one context to another, from one way of making sense to another. The three primary

contexts that Cobb (1990:200-215) wants to focus on are the experiential, the cognitive and the anthropological. It is argued that these contexts of sense making are non-intersecting domains of interpretation. Cobb(1990:201) argues that it should actually be the goal of the researcher to find ways of *co-ordinating* analyses developed in the different contexts.

Analyses that focus solely on individual children's construction of mathematical knowledge tell only half the story. The issue that needs to be addressed is the form that the process of mathematical acculturation should take and how it can be co-ordinated with what is known about mathematics (and in this case the development of spatial knowledge in children).

3.4.2.2 Experiential context

The purpose of the experiential context is that of attempting to infer what another's experience might be like. In order for the researcher to do this, the researcher must elaborate on his or her own understanding of the specific task. Through reflecting on their interactions with students, teachers formulate models of their students' mathematical knowledge.

Constructivists and non-Constructivists' views of teaching differ in the emphasis they place on the activity of modelling children's realities. In the course of interaction, both the teacher and the children attempt to make sense of each others verbal and non-verbal activities. The researcher's purpose is to call into question his own taken-for-granted ways of knowing mathematics and penetrate beyond his own symbolisations and objectifications.

Within the experiential context there is a distinction to be made between actualities and potentialities. In this case consideration must be given to potentialities as well as to actualities, to knowledge-in-action as well as to objects of knowledge.

3.4.2.3 Cognitive context

If we look at mathematical cognition, the purpose that structures the cognitive context is to explain how it is that students have the mathematical experience that they are inferred to have. In other words, children's inferred mathematical worlds are the data of cognitive explanations (Cobb, 1990:213).

3.4.2.4 Anthropological context

Cobb (1990:213) has suggested that mathematics learning and teaching can be analysed from six different vantage points of contexts. He subdivides each of the three previously mentioned contexts under a mathematical and a social component. These are the mathematico-experiential, mathematico-cognitive, and the mathematico-anthropological and the companion, socio-experiential, socio-cognitive, and socio-anthropological contexts. This framework of complementary though irreducible contexts are applied to the problem of truth and certainty in mathematics.

From the anthropological perspective, mathematics theorems can be seen as emergent truths that are institutionalised by the co-ordinated activity of members of mathematical communities. Cobb (1990:201) feels that severe difficulties are faced if restricted to regarding the cognitive and experiential contexts only, even if the primary focus is on mathematics learning. Cobb continues by proposing that the best way out is to complement cognitive constructivism with an anthropological perspective that considers that cultural knowledge (including language and mathematics) is continually regenerated and modified by the co-ordinated actions of members of a community. This characterisation of mathematical knowledge is compatible with findings that indicate that self-evident mathematical practices differ from one community to the other (Ascher, 1991:195-196). It furthermore captures the evolving nature of mathematical knowledge revealed by historical analysis.

This situation might at first seem paradoxical; mathematical meaning can be in the world (mathematico-experiential), in the individual's head (mathematico-cognitive), and in social interaction (mathematico-anthropological). Learning appears to be a paradox, as progress is made and solutions to problems are worked out. According to Cobb (1990:203), teaching also appears to involve a paradox. From the perspective of the Constructivists, their objective would be trying to maintain the tension between pushing students to achieve and providing a comfortable learning environment, between covering the curriculum and attending to individual understanding.

This complementary that seems endemic to mathematics education theorising, expresses the apparent paradox between mathematics as a personal, subjective construction and a mind independent, objective truth. Accounts of students' mathematical learning typically emphasise one extreme or the other. Cobb (1990:215) is of the opinion that this problem can only be solved through learning to cope with it in local situations by reflecting on the underlying antagonistic relationships and mutual interactions of the two positions.

It is against the background of this paradoxical situation that exist in the teaching and learning of mathematics, that a choice has to be made as to the most suitable research method.

3.4.3 Role of theory

Lesh (1976:188) feels that it is perhaps unrealistic for mathematics educators to continue to search for "outside" theories that can be used without modification. He feels that perhaps the emphasis should shift from theory "borrowing" to theory building. One of the main benefits to be derived from theory building is that the theory seldom has to be completely rejected when conflicts are detected or when difficulties occur. Theory building is not necessarily a quick fix to a difficult problem. It can for starters simply involve organising a point of view that can form the basis for

communication among educators. In this way, individuals can profit by and extend the work of others. However, to avoid errors and inconsistencies, theory building inevitably attempts to describe the range of applicability of a view. Consequently, when difficulties arise, a theory should be more than a point of view that is simply accepted or rejected. It should be an explanatory and predictive "model" that can (and must) be gradually modified and reorganised to deal with progressively more complex situations.

While the history of science is filled with examples to illustrate the power of theory building, many mathematics educators would point out that mathematics is more a profession than a scientific discipline and that "the best practice of the best practitioners is still better than the best theories of the best theorists" (Lesh, 1976:189).

Theoretical analysis by the researcher plays an important role in understanding the significance of children's mathematical behaviour, according to Cobb and Steffe (1983:88). It should, however, be kept in mind that the knowledge gained through experiencing how children construct knowledge while actively doing mathematics, gives a more complete picture of how the child thinks. It is these experiences that allow the researcher to test and revise his understanding. Cobb and Steffe (1983:92) state that the continual tension created by observations that might seem contradictory, leads ultimately to a knowledge of the child that supersedes the initial theoretical analysis.

The idea of adapting, improving and adjusting continuously is characteristic of educational development, where the development never stops. In constructing a set of educational activities that makes sense, the developer is guided by beliefs about what mathematics is, how it is learned, and how it should be taught. This belief system of the developer then functions as a background theory against which all instructional activities are evaluated. Gravemeijer (1994:448) elaborates on this when he states that the theory is at first like a "philosophy" which has a theoretical core. This core theory

is embedded in a framework of theories or theoretical notions on learning, instruction, and instructional design. He argues that it is this set of theoretical notions that guides the developmental work from the outset, not just in thinking out the appropriate instructional activities but also in relation to classroom trials.

3.4.4 Role of the teacher/researcher

The role of the teacher/researcher in a mathematics classroom can be compared to a pendulum swinging between two poles, namely that of interaction and non-interaction with the children. It is precisely this movement between these poles and the factors that influence them that are so difficult to identify and manage in a mathematics classroom. One of the most important aspects that influences this “swing” of the pendulum is the views of the teacher about what mathematics learning is and what mathematics teaching should result in.

An important theoretical view that will swing the pendulum to one of the two sides is the type of knowledge that comprises mathematics. Piaget and Inhelder (1965) identified three types of mathematics knowledge namely: (1) Physical knowledge, which is the knowledge of objects in their external reality. An example of this is that if you put a ball on a slope it will roll. (2) Social knowledge, which is conventional knowledge, taught by social transmission. An example of this is the names that are given to the different geometric figures namely rectangle, square, circle, etc. (3) Logico-mathematical knowledge, which is the relationships constructed by each individual mentally. An example of this is the realisation that a square is also a special parallelogram. This logico-mathematical knowledge cannot be transmitted from one person to another, but must be constructed by each individual. It is the task of the teacher to distinguish between social knowledge (which can be transmitted to children) and logico-mathematical knowledge (which can only be constructed by the children).

There exists a participation-dissociation balance in role taking. The participation-dissociation balance indicates that the researcher must find the middle course between too much dissociation and too much involvement. Role taking implies that the researcher or teacher has to take the “actor’s” point of view, in the words of Cobb (1987:5-7). Adequate role taking will prevent a researcher or teacher from being too detached, and the realisation that the “actor” can be aware of his or her position can prevent too much involvement.

3.4.5 Data collection techniques

3.4.5.1 Introduction

The methods that are used for collecting data distinguish between two categories of research methods, namely interactive and non-interactive methods (Le Compte & Preissle, 1993:159). In the interactive method the researcher and the participants interact and that might influence the results. In the non-interactive methods there is very little interaction between the researcher and the participants.

According to Human (1987:128,) research about the subject worlds of children imply that information is also gathered about the subject-related thoughts of the learners, as well as their affective experiences with the subject. Three techniques that are utilised in gathering information of this type are: (1) task specific interviews (2) questionnaires and (3) diagnostic teaching experiments (Human , 1987:128).

3.4.5.2 Task specific interviews

Task specific interviews, also called clinical interviews, had their birth in the work of Piaget (Human, 1987:129). In such an interview the child is given a specific task by the interviewer or researcher and the child is supposed to complete the task in the presence of the interviewer.

According to Wiersma (1986:246), interviewing can be done from two distinct viewpoints, namely quite casual and informal or very structured. In the former instance, questions of those being observed might be asked in an attempt to capture the feelings of those observed or to get clarity on what is happening.

In the case of the formally structured questions a phrase was coined by Piaget, namely clinical interviews. The objective with this method is that the learner is expected to perform certain tasks on request from the interviewer. The learner is then given time to solve the problem without being disturbed by the interviewer. The role of the interviewer is to observe the actions of the learner and try to deduce as much information as possible. When the learner has finished, the interviewer then questions the learner about the observed actions that were made.

One problem with this way of collecting information is the nature of the didactical contract that exists between the interviewer and the learner. Elbers (1991:36) states that in many of these interviews there exists a discrepancy between the expectations of the person to be interviewed (the child) and the interviewer. In many cases children count on the fact that they will be assisted in some way while the interviewer has as his or her major objective that the child should solve the problem on his or her own.

Another problem with clinical interviews of this kind is that, in the case of young children, there exists a verbal incompetence to articulate their thoughts properly. This leads to a distorted evaluation and analysis by the interviewer (Elbers, 1991:38).

3.4.5.3 Questionnaires

Written questionnaires or tests are another form of gathering information about the subject worlds of learners. According to Human (1987:130), this method is used in cases where sufficient information is available about the total background of the learners. This enables the researcher to formulate very pertinent questions. He warns though, that in the case of written tests or questionnaires the levels of validity and reliability are more difficult to attain in comparison with personal interviews. The advantages, though, of these questionnaires or tests are that they can be administered to larger numbers of participants at a low cost.

3.4.5.4 Diagnostic teaching experiment

According to Von Glasersfeld (1987:12), the term “teaching experiment” could easily be misunderstood. He states that: “It is not intended to indicate an experiment in teaching an accepted way of operating, as for instance, the adult’s way of adding and subtracting. Instead, it is primarily an exploratory tool, derived from Piaget’s clinical interviews and aimed at discovering what goes on in the student’s head.”

In a diagnostic teaching experiment participants are taught with the aim of eliciting a response which will shed light on the hypothesis that has been formulated about the subject worlds of the learners. It can even lead to the discovery of new information about the subject worlds of the learners. Human (1987:129) emphasises the fact that the objective is not to evaluate the teaching modalities but purely to gain more information about the subject-worlds of the children.

Human (1987:130) comes to the conclusion that the quality of research will be improved if a combination of the three above-mentioned approaches are used.

3.4.5.5 Triangulation

The term triangulation comes from the word trigonometry. Just as in trigonometry, where two angles are needed to define the third angular point, two sources will tell more about a certain phenomenon. Wiersma (1986:246) states that triangulation is qualitative cross-validation. It assesses the sufficiency of the data according to the convergence of multiple data sources or multiple data collections.

In other words, triangulation can take many forms, but its basic feature will be the combination of two or more different research strategies in the study of the same phenomena. Basically, it is a comparison of information to determine whether or not there is corroboration. It is to search for convergence of the information on a common finding or concept. Wiersma (1986:246) also points to the fact that the triangulation process assesses the sufficiency of the data. If the data are inconsistent or do not converge, it is insufficient. The researcher is then faced with a dilemma regarding what to believe.

3.4.6 Developmental research

Gravemeijer (1994:444) distinguishes between developmental work and developmental research. According to him, developmental research makes use of a large range of methodologies. Lijnse (1995:196) draws attention to the fact that there are different stages in the development of the research that lead to different methods being employed. The first stage emphasises the interpretative qualitative methods like introspection, interviews, classroom observation, protocol analysis of learning processes, historical analysis of concept development, content analysis, etc. In the later stages more quantitative methods are used as well. Lijnse (1995:196) emphasises the fact that the intent with this type of research approaches are “ not to prove anything, but to make it possible for others to judge what has been done and to enable them to reconstruct for themselves the processes described ”.

According to Gravemeijer (1994:445), in the conventional research model, development and implementation are separated. He continues to say that this is in stark contrast to the current views that educational development is more than just curriculum development in the sense that it also contains the end goal of changing educational practice. This implies that pre-service and in-service teacher training, counseling, test development, and opinion shaping is built into this approach.

This highly exploratory nature of educational development which is supported by a definite philosophy of mathematics education, structures the developmental work, the implementation, and the research.

Because a new curriculum cannot be developed from scratch, the developer looks for examples of instructional activities that can be adapted to his overall concept of mathematics education and can be fitted into the total structure. Curriculum development, however, is primarily product-orientated, whereas developmental research is theory-oriented. In addition, curriculum development is self-contained and time-restricted, whereas developmental research accumulates knowledge over a long-term research process (Gravemeijer, 1994:446).

In constructing a set of educational activities that makes sense, the developer is guided by beliefs about what mathematics is, how it is learned, and how it should be taught. This belief system of the developer then functions as a background theory against which all instructional activities are evaluated. Gravemeijer (1994:450-451) elaborates on this when he states that this theory is at first more like a "philosophy" which has a theoretical core. This core theory is embedded in a framework of theories or theoretical notions on learning, instruction, and instructional design. He argues that it is this set of theoretical notions that guides the developmental work from the outset, not just in thinking out the appropriate instructional activities but also in relation to classroom trials.

In developmental research the evolutionary aspect is much more important. Gravemeijer (1994:451) elaborates on this by saying that it is not in the sense of a random process channelled by natural selection, but as a goal-oriented process of improvement and adjustment. He feels that this process can only mean something if it is guided by a theory that grows during the process.

At the start this theory consists of a global framework, with key concepts such as mathematics as a human activity, mathematising, and reinvention. This global theory is then elaborated in prototypes that represent local theories. Gravemeijer (1994:449) is of the opinion that this implies that the developer will envision how the teaching-learning experience will proceed, and afterwards he will try to find evidence in a teaching experiment that shows whether the expectations have been right or wrong. This leads to a cyclic process of development and research which is theoretically and empirically sound in the end (Gravemeijer, 1994:450). In developmental research, knowledge gain is the main concern. The focus is on building theory, explicating implicit theories.

This acceptance of the close interaction between the theory and the practice is accompanied by a specific legitimisation of the research approach (Gravemeijer, 1993:26). According to Gravemeijer, the proof of the research can be ascribed to the extent to which the argument can be negotiated in an open discussion. The reason for this paradigm shift is that in the rational empirical approach the objective is to evaluate the success of the approach after implementation. This is very different in comparison with a developmental approach where the objective is to find out under which conditions a specific approach can work.

3.4.7. Evaluation

3.4.7.1 Introduction

To the forum of researchers the justification of the new theory will be of major interest. In the case of developmental research, theory is not put to the test after the development has been concluded. Instead, it is the developmental process itself that has to underpin the theory. In the cyclic process of development and research, discovery and justification are closely interwoven (Gravemeijer, 1993:21). Discovery is not restricted to the thought experiment, and justification is not merely found in the results of the trials. Some of the discoveries are made in the trials phase and part of the justification is not empirical.

Justification is also found in the thought experiment. However, it is then justification based on arguments. In the positivist interpretation, justification is confined to empirical teaching. In the case of developmental research, however, the rationale for one's choices and the interpretation of the empirical data are part of the justification as well. The positivist rationality, which only takes into account means-end relations, is exchanged for a broader kind of rationality based on argumentation and comprehension (Gravemeijer, 1993:22). As the deliberation contributes to the justification, so the trial phase can provoke discoveries. In the trial phase the focus is not only on justification, but one also looks for discoveries that will enable theoretical progress. One of the cornerstones of the positivist research paradigm is prediction. This is true for fundamental research, where experimental experiences are subjects of an interpretative process. In other words, the researcher tries to make sense of what is going on in the classroom against the background of the thought experiment that preceded the instructional activities.

The empirical evidence in developmental research is more often than not qualitative in nature, in the sense that the analysis of the data stays close to the original meaning of

the data. The data are not projected onto a mathematical-numerical system with the objective of doing analytical reasoning within that system. Smaling (1990:6) states that analysing is often a process of interpreting, in which data which are gathered in research are compared with other data, in which each item is interpreted in the light of the data as such. An important step in the transition from the data as such to the interpretation is the construction of categories of data and the construction of concepts.

3.4.7.2 Validity

Internal validity concerns the correctness of the findings within the actual research situation (Smaling, 1990:7). Gravemeijer (1993:24) is of the opinion that researchers can improve the quality of their judgements and the interpretations by seriously searching for counterexamples or by searching for alternative explanations. They could also ask fellow researchers to play the devil's advocate.

External validity concerns the bearing of the results on other situations. In the case of qualitative research this external validity does concern generalisability as such. Here a rather differentiated generalisability is more important. The question that needs to be addressed is how certain elements of the results will apply to other situations. This is exactly what developmental research strives for.

3.4.7.3 Reliability

Reliability refers to the absence of accidental errors and is often defined as reproducibility. For qualitative research this means virtual replicability. Here the emphasis is on virtual, because it is important that the research is reported in such a manner that it can be reconstructed by other researchers. In other words, there should be reports on failures and successes, on the procedures followed, on the conceptual framework and on the reasons for the choices made. Furthermore, internal reliability can be interpreted as intersubjective agreement among the researchers on the project.

Reliability and validity are indicators of objectivity and therefore the researcher must strive to do justice to the object under study. Gravemeijer (1993:25) feels that this should be done in reference to a certain question, problem, or goal, within a certain framework (like culture or an underlying philosophy of mathematics education). Doing justice in this instance has two aspects: the positive aspect, which concerns the object to reveal itself, and the negative aspect, concerning the avoidance of a distortion of the image of the studied object.

3.4.7.4 Reproducibility

According to Freudenthal (1991:161), one of the most important differences between physics and social sciences is the possibility or impossibility, respectively, of replication. In the natural sciences, it is easy to present new knowledge as the result of an experiment, because such an experiment is easily repeated. In educational development, replication in a strict sense is impossible. An educational experiment cannot be repeated in the same manner and under the same conditions. Therefore new knowledge will have to be legitimised by the process by which this new knowledge was gained.

Van Oers (1994:7) questions whether the developmental research method should be seen as a method that wants to side-step the issues of objectivity, validity and reliability. This is confirmed by Streefland (1995:21) when he states that the standard methodological views of validity, reproducibility and reliability would only put the system in a "straight jacket". The objective with the developmental research is to define as clearly as possible the new situation that develops in practice. Streefland hastens to add that this implies that the research should be repeated and that the professional quality of the approach should be reported objectively, without the intention of any hidden agendas.

3.5 Conclusion

Cooney (1994:613) is of the opinion that, because of the current emphasis on cognition and context, there is a dramatic shift away from the use of quantitative methodologies based on a positivist framework to that of interpretative research methodologies.

One of the main problems in developmental research is found in reporting on this complex issue. If the whole learning process is explicated in detail, the report will be a thick, indigestible book. The question that remains is which experiences, conditions, and deliberations should be reported and which omitted. Gravemeijer (1994:456) suggests that it would probably be best to be guided by the intended audience for the report. Publications that focus on teachers should be different from those that are intended for researchers. If results are presented to researchers, next to accessibility, trackability and the possibility for verification are of utmost importance. Fellow researchers must be able to retrace the learning process of the developmental researcher in order to enter into a discussion. The justification will include the relations with the theory, notions, crucial moments in the developmental process, and so on. What makes matters complicated is that the theoretical arguments by which empirical data are valued evolve in the developmental process itself.

Van den Heuvel-Panhuizen (1993:108) emphasises the situation that in developmental research there is not a dependency on methodology *per se*, but that the methods that are employed serve to be proof enough. She is further of the opinion that the basis for this assumption can be linked to three pointers, namely the inherent power of persuasion, internal and external consistency, and finally the ability to correct itself.

Lijnse (1995:197) summarises the aim of developmental research as being “not aimed at building grand theories, such as, for example, understanding the human mind, but at understanding and developing good teaching practice”. The actual classroom scenarios serve as a description and justification of the methods and results, describing and

analysing them in such depth that it is convincing in itself (Lijnse, 1995:196). The objective with the approach is not to prove anything but to make it possible for other researchers to judge the activities against the background of the total experiment.

Teaching/learning research is needed that will lead students to concept formation through a meaningful synthesis of diagrams and visual images on the one hand, and through verbal definitions and analyses on the other.

CHAPTER 4

A SUBJECT-SPECIFIC THEORETICAL FRAMEWORK OF THE SPATIAL DEVELOPMENT OF THE YOUNG CHILD.

4.1 Introduction

During a teaching session there is interaction between the teacher and the children where a multitude of variables are present. The influence of these variables and their interrelationship with each other has a profound effect on the outcome of the teaching-learning situation in general, but also on the specific subject matter that is taught. The three major instructional variables that will form the focus of this study are the task that is given to the children, the objects that are used, and the dimension of the objects that are used. The execution of these activities can take place in different media where the different modes of expression modes are talking, drawing, writing and building.

The manipulation of these instructional variables, coupled with the specific features of the different execution media, leads to an intricate and complicated network of interrelationships which can, coupled with the different worldviews and expectations of the children and the teacher, have a profound outcome on the development of the spatial competence of young children.

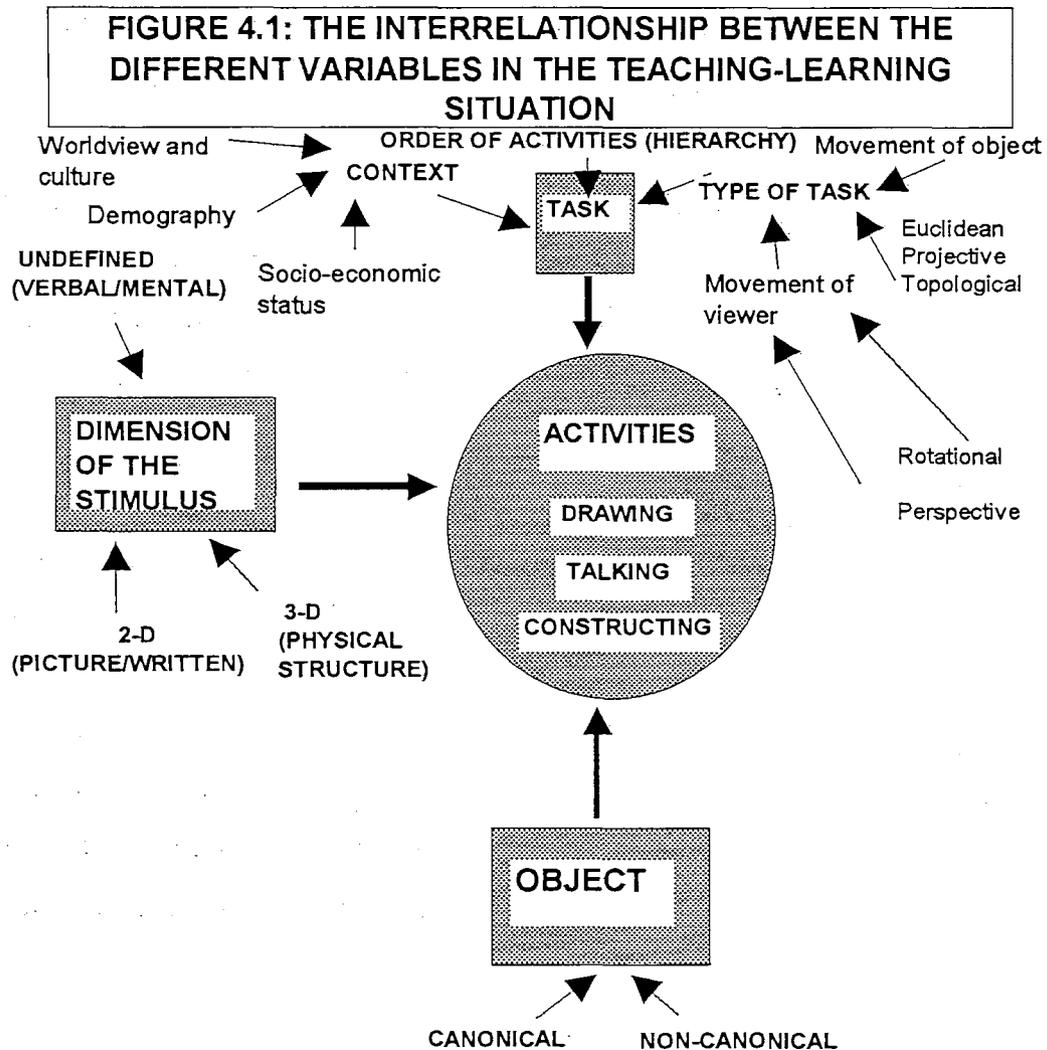
4.2 Context.

The way in which a child interprets a specific task is influenced by the contexts he brings from home. Cobb (1990:201) elaborates on this by stating that it should be remembered that purposes and intentions are crucial components of these contexts. Apart from the socio-economic and demographic background which can be more readily determined, another very important influence is that of the worldview of the particular cultural group to which the child belongs.

According to Peat (1994:234), it should be remembered that for Indigenous people the total context in which things happen is always important. The importance of this context is not only a crucial factor for Indigenous people but also for Western scientists who work with concepts concerning research in quantum theory. According to Peat (1994:233), in quantum theory the context is constantly emphasised. These researchers have found that if an experiment is set up in one way the result is waves, and if it is set up in another way the result is particles.

Peat (1994:234) makes a very important observation, namely that it is impossible to separate the phenomenon from the context and that categories no longer exist in the absence of contexts. Freudenthal (1983:245) agrees with Peat when he states that contexts should never be taken for granted. He illustrates it by stating that a child understands early what objects are to be classified as for instance chairs, but in a certain context these chairs can be appointed as either a locomotive or a ship. Freudenthal (1983:245) warns against coming to conclusions about what is understood by children without taking into consideration the context in which activities or observations take place. It is therefore of the utmost importance to take cognisance of the different variables which constitute a particular context in which children are taught.

Figure 4.1 gives an outline of the interrelationship between the different variables described further in this chapter. The first variable that will be discussed is the effect of the nature of the task on the outcome of the spatial activities. This variable will be theoretically investigated through discussion of the effects of the context, the order of the activities, and the type of task on the outcome of a spatial investigation.



4.3 The role of worldviews in defining the spatial understanding of the young child

Nelson-Barber and Estrin (1995:176) state that all cultures generate mathematical and scientific knowledge, but that knowledge needs not look the same from one group to the other. The specific way in which mathematics is viewed within a community determines the manner in which it is taught and understood. Peat (1994:189) is of the opinion that for many philosophers of mathematics the subject should be culture free because it is the study of the basic logical relationships of the world. For people holding this view the highest levels of mathematics is identical for Europeans, Mayans, Arabs, Chinese, and Indians. There is another view, though, that mathematics is

inseparable from language and culture (Peat, 1994:190). People who subscribe to this view also believe that mathematics is a particular formal expression and extension of the different relationships, transformations, and interconnections that exist within language. These same people are of the opinion that language is to some extent linked to culture and the particular way people live (Peat, 1994:190). Moore (1994:13) makes a strong case by stating that the differences in culture are so vital that it is very important that every teacher of mathematics makes the necessary effort to understand any cultural diversity that may exist between himself and the students.

For the purpose of the rest of this discussion the above-mentioned two views will both be presented during the discussion of the different variables which could have an influence on what goes on in the classroom. These two views could be seen as two different positions in a continuum of views which exist about the purpose and role of mathematics within different societies. For the purpose of the discussion which will follow, the former approach will be termed the “Western” approach and the latter the “Indigenous” approach (See chapter 1 for a clarification of the context in which these terms are used).

4.4 Task

4.4.1 Type of task

The type of task that is presented to children has specific features which may contribute to the degree of difficulty of the task. The task types can be divided into two categories, namely tasks which lead to specific movements of the objects and tasks that lead to movements of the viewer (Olson & Bialystok 1983:150).

4.4.1.1 Movement of object

For Indigenous people the basic element of their worldview resides in the balance that exists between flux, transformation, and change (Peat, 1994:174). This view is accentuated by the way in which Indigenous people do not seek to control or hold on to stability. The reality which Indigenous people experience goes beyond surface forms and it involves a much deeper level of process and transformations, according to Peat (1994:283). This view of transformation is also shared by some Western scientists who are involved with quantum physics research (Peat, 1994:283). Indigenous people from the Ojibawa society take this transformation idea into a physical level when they state that they believe certain people have the power to change themselves into animals (Peat, 1994:279).

When objects are changed or altered in some way, mathematically it is said that the object is transformed. In the Western approach to mathematics several types of transformations exist, but for the purpose of this work only three will be considered namely topological, projective and Euclidean.

Properties are the characteristics of a figure that is conserved (not changed) under transformation. Important aspects that need to be explained in order to understand transformations are the geometric properties enclosure, separation, order and proximity. Enclosure describes the position of an item between two others in a line, within a region, or within a figure or space. Separation describes the place where one object breaks from another. Order describes an arrangement of objects by pattern or classification. Proximity describes the nearness of one object to another. Under topological transformations (stretches and shrinks), shape or size of the objects can change, but proximity, order, enclosure, and separation cannot change. Projective transformations, which occur from changes in visual perspective, can change the shape and/or size of a figure but not its straightness. Euclidean transformations of slides,

flips, or turns may change location and/or orientation of figures but not their shape or size.

4.4.1.2 Movement of viewer

From an Indigenous perspective no clear separation exists between the observer and the observed. Briggs and Peat (1984:33) state that from the quantum physical work performed during the past few years the idea has arisen that observer and observed appear to have an influence on one another. This implies that movement of the viewer is influenced by what is viewed and vice-versa. Briggs and Peat (1994:33) come to the conclusion that this implies that the universe is a whole. There is also no separation between what is observed and by whom, as will be seen from the rest of the discussion as viewed from a Western perspective.

Abels, Bloem, Van den Brink, Goddijn, Gravemeijer, Van den Heuvel, Ten Hove, Kalmijn, Kok, Krabbendam, Meeder, Obdeijn, Querelle, Riemersma, Schoemaker, Staal, Verhage, Van der Werf, Wijers and Van der Zwaard (1992:74), on the other hand, have divided the movement of the viewer into two categories, namely the role of the participant and the role of the observer. They clarify this further by stating that the participant is the one who sees and the observer is the one who thinks about what is seen. In most cases, children understand situations better if they take the role of the participant rather than the role of the observer. The reason for this can be attributed to the fact that in the role of the participant the child is more exposed to the context of the problem as opposed to the situation where the child can only imagine the context when in the role of the observer.

Olson and Bialystok (1983:151) complicate the issue even further by distinguishing between two different types of display namely arrays, which involve unrelated constituents, and objects which involve related constituents. They note that if only two

features are considered, namely the type of display and the type of transformation, then already four types of problems of spatial transformations can occur namely:

- Viewer transformation relative to an object display

This requires viewers to judge what a single object would look like from various perspectives. The difficulty of the task could be explained on the basis of the assignment of the distinguishing constituents of the objects and positioning them relative to ego space - front, back, left and right (see figure 4.2).

Furthermore, the question “What would it look like for the observer?” requires determining only what would be in front of the observer.

- Viewer transformation relative to a fixed array (Perspective problems)

This approach is described in the work of Piaget and Inhelder’s (1956) classic three-mountain problem. It involves a fixed array of constituents having no obvious internal spatial structures, such as fronts and backs. The task is to select a picture correctly representing the view from the various positions around the display.

- Array transformation relative to a fixed viewer

Huttenlocher and Presson (1973, 1979) were responsible for this research. An array of objects is rotated through various distances relative to a viewer who is then required to select a picture showing what the transformed array would look like.

- Object rotation relative to a fixed viewer (rotation problems)

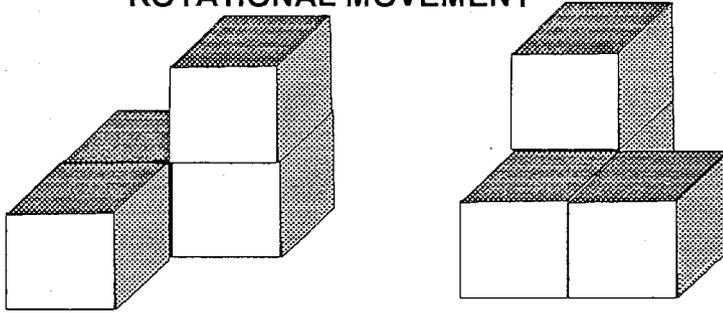
In the experiments of Metzler and Shepard (1974) they presented subjects with schematic displays of a complex object and asked the subjects to judge whether the two displays represented the same object in different orientations, or whether the displays represented two different objects.

The type of transformations that results from any activities based on rotational axes involves two different types of movements (figure 4.2). Either the viewer moves (or pretends to move) relative to the display, or else the display moves (or is taken to move) relative to the viewer. The first is typified by Piaget and Inhelder's (1965) perspective problems and the second by Metzlers and Shepard's (1974) rotation problems. According to Olson and Bialystok (1983:151), the Piagetian perspective problems are the most difficult for young children.

Cox (1986:107) mentions another factor that might influence the task, namely the inherent biases that are present in the different media which are utilised by the teacher when it comes to expression. She has found that the pictorial mode of representation may lead to a drastic underestimation of young children's knowledge about observers and what they can see. She has shown that there is a tendency towards production bias which is influenced by the children's knowledge of what they are drawing. If they, for instance, know that the surface of a table and the surface of a cube are rectangular, then these features may be preserved in their drawings of a cube and rectangle.

FIGURE 4.2: TWO TYPES OF MOVEMENT THAT THE VIEWER CAN UNDERGO

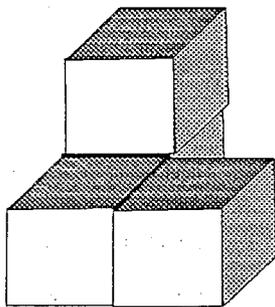
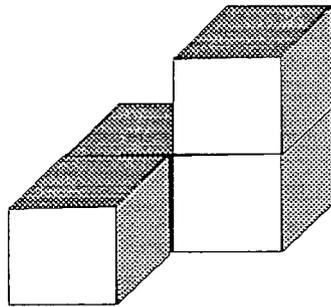
ROTATIONAL MOVEMENT



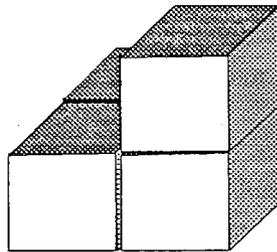
ARE THE TWO PICTURES ABOVE OF THE SAME OBJECT?

REPRESENTATIONAL MOVEMENT

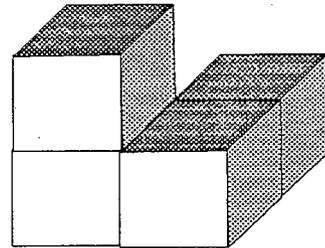
A



B



C



D

WHICH ONE OF B, C, OR D GIVES THE CORRECT BACK VIEW OF A?

4.4.2 Order of activities (Hierarchy)

One of the first attempts at classifying or structuring geometry was done more than a century ago by Felix Klein with his so-called "Erlanger Program". In this approach Euclidean space is viewed as dealing with distances, angles, straight lines, circles, planes and spheres. A first step towards impoverishing this structure is to neglect the general comparability of distance and angles, while preserving rectilinearity and parallelism. The result of this is called affine geometry. In affine geometry all parallelograms are the same: rectangles and squares cannot be distinguished from other parallelograms and circles cannot be distinguished from ellipses. The next phase is to neglect parallelism, while preserving rectilinearity, which results in projective geometry, where all quadrilaterals are the same and all conicals are the same. If rectilinearity is neglected as a structuring property the topological structure emerges.

According to Piaget and Inhelder (1956:9), development progresses from the poorer to the richer structures, as can be found in Felix Klein's hierarchy. This progresses from Topological via Projective and Affine to Euclidean geometry, and according to Piaget and Inhelder, this is true as well as for the perspective, the representative, and the cognitive aspects of development.

In their explanation of the construction of space, Piaget and Inhelder (1956:6-8) give a detailed explanation of the way in which more sophisticated notions are linearly deduced or construed in a systematic one-to-one progression from notions acquired earlier. According to them the notion of 'distinctness' is built upon that of 'neighbourhood'. The notion of 'order' builds on a combination of 'distinctness' and 'neighbourhood'. 'Border' builds on 'order' and so on. Higher order concepts eventually consist of a number of linear and systematically progressing procedures. The Piagetian approach has formed the basis for the design and instruction of geometric material and curricula for most children in South Africa.

In contrast to the neatly regular hierarchical structure and development present in Western conceptualisation of space, Navajo conceptualisations are quite different. Three equally important basic notions dominate: movement, volumeness/planeness, and dimensions (Pinxten, Van Dooren & Harvey, 1984:160). All three are topological in character and none are primary in the Piagetian and Western sense.

Freudenthal (1991:64) is of the opinion that, viewed developmentally, geometry is the direct opposite of arithmetic. Space and the objects around people are early mental objects, the result of structuring and being structured. According to Freudenthal (1991:65), choices have to be made, not as in arithmetic of sources and approaches, but of subject matter, when it comes to teaching. He cautions by stating that there is so much material available that the danger looms of choosing too much. The obvious danger that emerges from this situation is the problem of structuring the materials and activities in such a way that it results in a long term learning process. This learning process should have the quality of lifting the children to higher levels of deduction as time passes. Freudenthal (1991:66) sounds a definite word of warning when he states that this way of teaching geometry should be introduced when children are quite young because it can come too late for older children.

Gravemeijer and Kraemer (1984:8) suggest that the development of children's spatial sense should start with the real world of the child. Own experiences of children should be utilised to build insight. There should be a movement from spatial orientation where the starting point is the physical world of the child, to insight which depends on the abstract perception of the world. Gravemeijer and Kraemer (1984:114) further suggest that in the design of the spatial activities for children, care should be taken to have a progression line run through the activities starting with orientation, and finally culminating in insight. They go further by suggesting that the following phases should be present in order to achieve this hierarchical build-up namely:

- The direct and indirect viewing of objects and photos of objects
- The taking of a viewpoint (mentally)
- The verbal and written description of an object
- The formation of a mental image of the object
- Acting on and manipulation of the mental image

The types of objects that are utilised when developing the spatial knowledge of young children through the above-mentioned hierarchy are crucial. Freudenthal (1983:227) suggests that the objects that are utilised when teaching children about geometric concepts must be very carefully chosen.

According to Pinxten, Van Dooren and Harvey (1984:161), “objects” cannot be defined in a similar way and “form” cannot be understood in quite the same way as in the Western outlook since all aspects of reality in Navajo knowledge are “process-like” and not “thing-like”. In the first place fluxes should not be “cut up” and considered as statically defined combinations of statically defined chunks of processes or fluxes.

Harris (1991:37) states that the Indigenous people of Australia are not traditionally concerned with abstracting, classifying and naming 2-D and 3-D objects. Harris (1991:41) also found, when analysing the languages of the Indigenous people of Australia, that there were major differences in the way in which objects were perceived and described in comparison with the more Western approach. Another example of this can be found amongst the Kpelle of Liberia who name only those geometric shapes that are in common use in their culture (Gay & Cole, 1967:61).

Another very important aspect that is different between the way in which many Indigenous and Western people view objects, lies in the classification of objects. According to Peat (1994:230), these important distinctions are made within their language and it concerns their views of the animate and inanimate.

According to Peat (1994:231), the Western person is used to living in a world of objects and most of the objects that are encountered are put into Aristotelian categories. From this perspective, if something is not animate then it must be inanimate. For the Indigenous person, there is no such thing as an object that must at all times be classified as being either animate or inanimate. This means that a particular object which at one stage is classified as animate can at another time be classified as inanimate (Peat, 1994:231).

According to Peat (1994:231), the Mic Mac language of the American Indian is not really designed to talk about objects but rather enters more deeply into the realities of the world. Aboriginal languages have a very rich vocabulary for expressing ideas about spatial relations. The important differences, though, between their language and English is that their's does not have words that can be called prepositions to indicate position (Harris, 1991:45). They make use of locative suffixes which they add to the basic word stem. The result of this is that the Aboriginal children have considerable difficulties in learning the position of things in space through the medium of English (Harris, 1991:46). Peat (1994:234) says that one of the biggest differences between the languages of the Indigenous people of America and English, is that English is a noun orientated language whereas the Indigenous language is verb orientated.

The rest of the discussion that follows takes a closer look at the way in which "object" is viewed from a Western perspective.

4.5 Object

4.5.1 Referent or relatum

Although it has been noted that a spatial relation necessarily implies the existence of a relatum or a frame of reference, the consequence of the nature of the relatum for the representation of spatial relations has been less well documented (Olson & Bialystok, 1983:72). Nevertheless, there are three categories of events that can serve as the relatum in a spatial proposition, each of which has different properties reflected in the resulting spatial proposition (Olson & Bialystok, 1983:72-73). The three categories are egos, canonical objects including observers, and non-canonical objects including frameworks.

4.5.1.1 Ego

The ego of a person is used as the reference point for assigning predicates like top, bottom and so forth. A distinctive quality of ego space is that it subsequently provides a means for assigning spatial relations to other objects in the environment, such as trees, cars, and so on, but the assignment of these descriptions nonetheless honours the spatial structures established for ego space.

4.5.1.2 Canonical objects

These are objects such as desks, cars and houses, which by virtue of their usual orientation in space have parts that are intrinsically tops, fronts, and backs. The origins of these parts are the same as the origins of the parts of ego space. This implies that they were assigned spatial predicates from the same set and in the same manner as was used to determine ego space. Once these have been assigned plus the fact that these objects have particular spatial orientations with distinguishing features which correspond to those orientations, the spatial descriptions such as top, back, etc. become

intrinsic parts just as motor and roof are intrinsic parts of the car. Because these intrinsic spatial descriptions have become part of the object, their specification no longer depends on the position of the ego. This will also apply even though the correspondence between the ego top and the car top may have been responsible for the initial assignment of that description. This has as a result that the spatial system behaves independently and as a consequence could come into conflict with the spatial system established by the ego.

In many ways observers, that is other people, are treated just as canonical objects (Olson & Bialystok, 1983:73). The use of the observer as the relatum, though, is more complex than is generally the case for canonical objects. Observers are complex because they can be used either as canonical objects, or they can be used as frameworks to assign representation to noncanonical displays. The difference between the observer as a canonical object and the observer as a framework is in the degree of elaboration required to specify a spatial relation. For the former the spatial relations may take the form of "The apple is in front of the observer". For the latter role of the observer a spatial relation between two or more entities is interpreted in terms of the space imposed on the display by the observer for instance "The apple is in front of the tree that is in front of the observer".

Observers are assigned spatial features which become intrinsic properties of those people, and they provide the basis for constructing the spatial relations. The use of the observer as the relatum, however, is more complex than is generally the case for canonical objects.

4.5.1.3 Noncanonical objects

Noncanonical objects are objects in the environment, which have no particular spatial orientations and are also likely to have no distinguishing features marking the spatial position (Olson & Bialystok, 1983:73). Noncanonical objects are also called arrays

when they cannot be assigned any obvious internal spatial structure such as fronts or backs. These objects such as boxes, balls, balloons and rocks can be assigned spatial predicates, again from the same set, and again on the same basis to correspond to ego space, but quickly lose those descriptions when the situation changes. This means that the spatial predicates are assigned temporarily, and during that time the object may be treated as if it were canonical. An important difference between a canonical and a noncanonical object is that the canonical object can assign space independently of the ego, but noncanonical objects must take account of the position of the ego. While noncanonical objects may thus appear more complex since more features of the environment need to be attended to, namely the position of the ego, they are also simpler since there is no possibility of conflicting descriptions as was the case for the canonical objects.

A special case for noncanonical objects is provided by frameworks or displays (Olson & Bialystok, 1983:73). These are confined spaces to which the spatial predicates top, bottom and the like may be assigned in much the same way as they are assigned to the relatum. Similar to noncanonical objects once assigned, these displays are treated as canonical for the purpose of solving the particular problem, thus encoding the particular event. They frequently become relevant in situations where it is necessary to keep track of changes in spatial relations relating a stimulus object to some fixed reference system.

In contrast to the use of gravity for the assignment of an invariant canonicity, spatial predicates may also be temporarily assigned to noncanonical objects, but here the assignment may proceed from a different basis of ego, or other canonical objects, including the world (Olson & Bialystok, 1983:75). The one chosen for this assignment may depend on such factors as the relative salience of this relatum in the situation.

There are objects which share some of the properties of each of the canonical and noncanonical categories. These are objects such as bottles, cups, trees, tables, etc.

They all have intrinsically specified tops and bottoms but, are noncanonical with respect to fronts and backs and lefts and rights. It is interesting to note that there are few objects which have canonical fronts and backs but no tops or bottoms. Some examples however are, mirrors, arrows, carbon paper, etc.

Olson and Bialystok (1983:234) have argued that the complexity of a spatial relation arises from an interaction between the properties of the predicate and the properties of the argument of the spatial proposition, thus, between the dimension presupposed by the predicate and the properties of the argument of the spatial proposition.

4.5.2 Object space versus environmental space

The spatial terms used to describe the structural properties of objects can be expressed proportionally as one-place predicates, and linguistically they are called nouns, for example: "The top/bottom/front/back of the car". In these descriptions the "top" is assumed to be the proper part over the horizontal midline. The "bottom" is the proper part under the horizontal midline, and so on. Since a particular horizontal and vertical midline are not defined, the meanings of "top" and "bottom" may be ambiguous.

Changing the criteria to a different set of horizontal and vertical axes will change the meaning of the spatial proposition. This conflict is particularly apparent for canonical objects. The usual interpretation of these axes is to use the environmental space which through gravity indicates an asymmetrical vertical axis. For canonical objects in their usual orientations and for all noncanonical objects, this strategy provides a means for interpreting the appropriate vertical axis and hence assigns meaning to the structural vertical relationship. Top, in these cases, are both the uppermost surface and the intrinsic top of the object. When canonical objects are disorientated, the objects' space, although originally given by their usual orientation, is preserved as an intrinsic structure of the object and this object space may now provide an alternative means of specifying a vertical axis.

A conflict can develop between using environmental space and using object space to establish the vertical and the horizontal axes. While the environment will indicate the top to be the uppermost surface, the object will indicate the top to be a particular part of the object which is environmentally uppermost only when the object is in its canonical orientation. In other words, the structural, canonical top may conflict with the more general environmental top when the object is in a novel orientation.

Ordinary language makes some differentiation between interpretations in that the structural part, that is, those intrinsic to the object tend to be marked by the definite article. The article in "the top" tends to mark the intrinsic or structural top not the current environmental top. However, the ambiguity arises more strikingly when this noun phrase is embedded in a prepositional phrase, yielding "on top" and "on the top". Olson and Bialystok (1983:82) assume that there is a tendency to interpret "on top" on the basis of environmental space and "on the top" on the basis of object space, partly because of the presence of the definite article in the latter expression.

A similar conflict exists in determining the horizontal structural relations for canonical objects. In these cases, however, the ambiguity involves the use of object space and the use of ego space, since the environment does not clearly signal a front/back or a left/right horizontal axis. Using the object space interpretation, the front of the object is the part so designated in the object's usual orientation and use, for instance the headlights of the car, the main entrance to the house, etc., and is described as "the front". Using the ego space interpretation, however, the front of the object is the side closest to the speaker.

Olson and Bialystok (1983:84) conclude by adding that other factors might also contribute to the interpretation of the propositions which are potentially ambiguous. Some of these factors are the likelihood of encountering the object out of its usual orientation, the use of the spatial term as the name for an intrinsic part of the object (compare the bottle top with the top of the chair) and the degree of difference between

two frames of reference. Small discrepancies may be ignored and transformations sharing some aspects of the predicates, such as those of 90°, 180° and 270°, may produce more conflict than others.

Terms that indicate the relational propositions like the two-place predicates “over, under, in front, and behind” are prepositions. These terms must also be interpreted in terms of specified sets of vertical and horizontal axes and may be ambiguous where more than one set of axes can satisfy the proposition. As in the case of the structural propositions, the problem is particularly apparent when these propositions are applied to canonical objects. For the verticality predicates “over” and “under”, the conflict is between using object space and environmental space, especially when the relatum is disorientated. For the horizontality predicates “in front” and “behind”, the conflict is between object space and ego space.

4.6 Dimension of stimulus and point of view

Cox (1986: xvii) emphasises the importance of the development of point of view when she says that it is important if we are to operate effectively in our social and natural environment. Our perception of the world is organised in part in terms of a system of three co-ordinates, each axis perpendicular to the other. Olson and Bialystok (1983:50) have described the representation of these dimensions in the use of language. Cox and Isard (1990:481) divide the use of the different spatial terms, which link relationships among objects and observers' point of view, between the deictic or the non-deictic ways. According to Cox and Isard (1990:487), if children place an object according to the observer's point of view, rather than to the back or front of the features of the object, the response is called deictic. For example, if children have to place a play-doll “in front” of a play-car and, they place it in front of themselves and not to the front of the play-car, it is called deictic. If they place the play-doll near the car's front, they act in a non-deictic way.

Just as gravity is the primary cue to the assignment of verticality, so direction of motion and positioning of perceptual apparatus are the cues to the assignment of front and back (Olson & Bialystok, 1983:50). The third dimension is logically determined once the first two have been specified since no degrees of freedom remain in three-dimensional space. These may be further simplified into two dimensions namely a vertical and a horizontal one. Whereas gravity specifies a vertical orientation common across activities, no counterpart reliably describes the horizontal.

Several researchers have found that spatial tasks which require children to make judgements in these two dimensions show differences attributable to the vertical/horizontal dichotomy. A number of factors serves to assist in the explication of the vertical features from the unanalysed structural description prior to that for horizontal ones (Olson & Bialystok, 1983:70). The bilateral symmetry of the body is important in this regard. Whereas the left-right symmetry of our bodies makes horizontal discrimination difficult, the top-bottom asymmetry of our bodies facilitates vertical discrimination. This means that vertical information may be more simply represented because it corresponds to our biological and perceptual biases. Another critical difference between vertical and horizontal relationships is that vertical relationships are invariant to the perceiver while horizontal ones are not.

Olson and Bialystok (1983:51) conclude by stating that in constructing structural descriptions for objects and events and in explicating these structural descriptions in the form of explicit spatial propositions, we may expect to find that vertical relations are more basic than horizontal ones.

4.6.1 Rotational axes

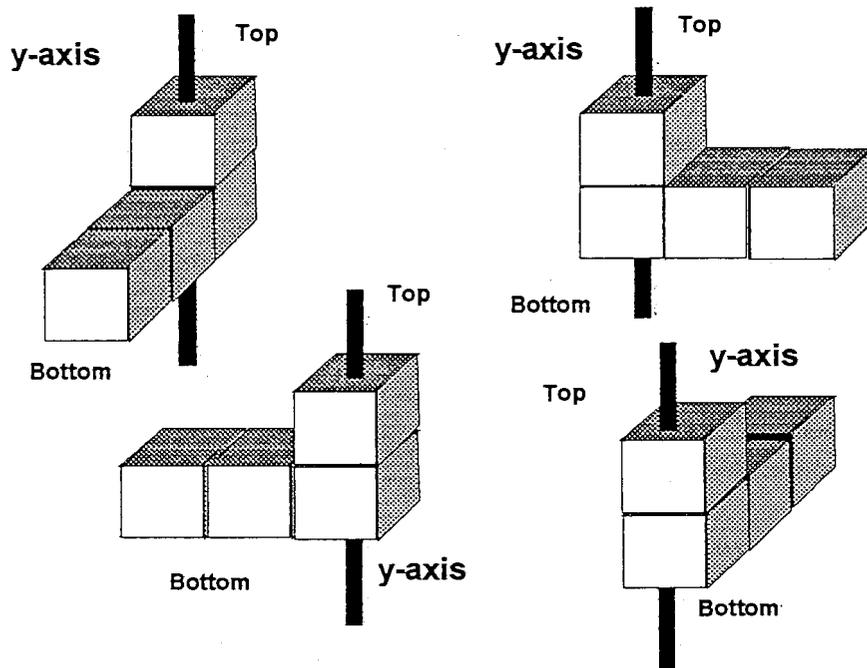
Just as there are three dimensions in terms of which features of objects can be specified, so too are there three axes around which rotation can occur. Furthermore, the rotation can occur in one of the two directions for a certain distance. The axes are

the Cartesian x-axis, y-axis, and z-axis, but the axes are assigned, as Olson and Bialystok (1983:150) suggest, on the basis of ego space with top, front and sides.

4.6.1.1 Vertical axis

The first dimension is the vertical axis defined by gravity (vertical) and represented in language by the terms such as “up/down”, “top/bottom”, “over/under”, “above/below”, “high/low”.

FIGURE 4.3: MOVEMENT AROUND THE VERTICAL AXIS FOR A THREE-DIMENSIONAL OBJECT

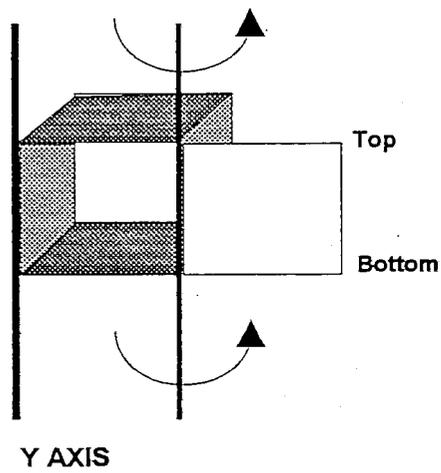
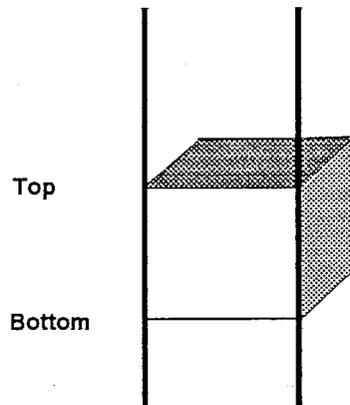
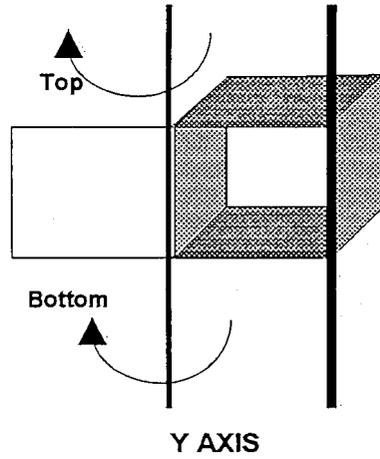


The *y-axis (vertical axis) rotation* involves depth rotation around a vertical axis in one of two directions, either right towards (anti-clockwise), or right away (clockwise) from the viewer, for any distance.

TABLE 4.1: THE CHARACTERISTICS OF THE DIFFERENT AXES (Olson and Bialystok, 1983:150)

TYPE OF AXIS	CHARACTERISTICS	AXIS	INVARIANT DIMENSION	VARIABLE DIMENSION
LATERAL	DEPTH AROUND HORIZONTAL AXIS	X	LEFT/RIGHT	TOP/BOTTOM
VERTICAL	DEPTH AROUND VERTICAL AXIS	Y	TOP/BOTTOM	FRONT/BACK LEFT/RIGHT
FRONTAL	PLANE ROTATION	Z	FRONT/BACK	TOP BOTTOM LEFT/RIGHT

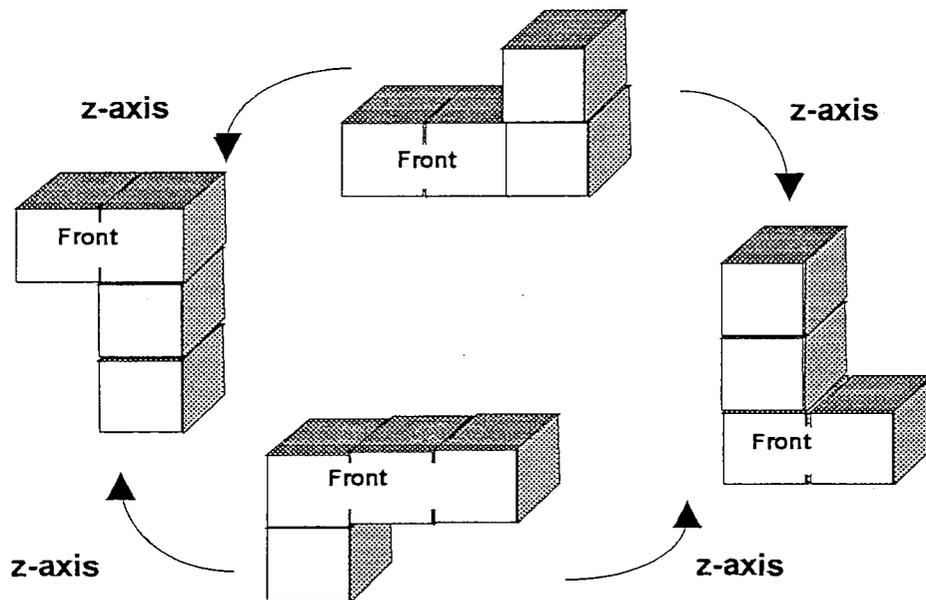
FIGURE 4.4: MOVEMENT OF PARTS OF A THREE DIMENSIONAL OBJECT ALONG THE VERTICAL AXIS



4.6.1.2 Frontal axis

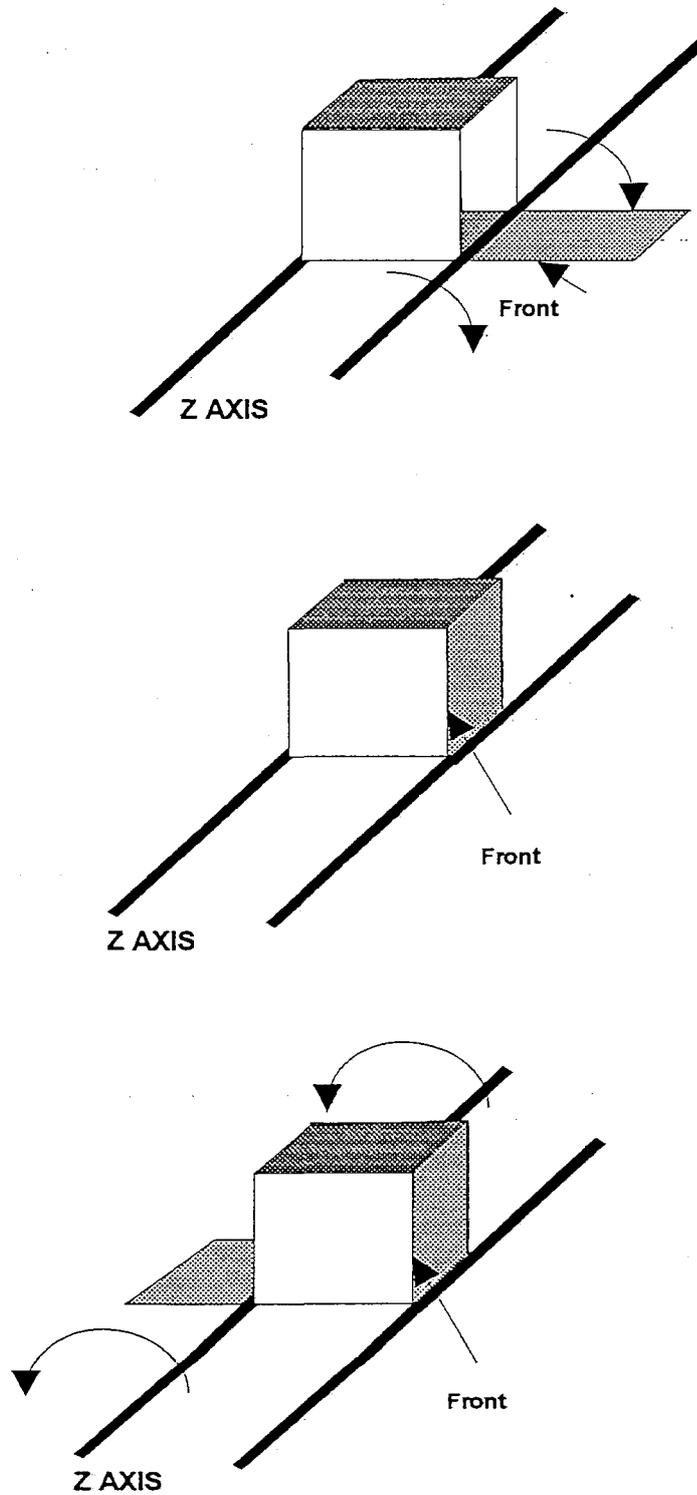
The second dimension can be described as the plane extending through the centre (frontal) of our bodies and corresponds to the descriptions “front/back”, “ahead/behind”.

FIGURE 4.5: MOVEMENT OF A THREE-DIMENSIONAL OBJECT ALONG THE FRONTAL AXIS



The *z-axis (frontal axis) rotation*, involves rotation in a plane around a frontal axis, either top to the right (clockwise), or top to the left (anti-clockwise) for a particular distance. Any rotation keeps the values in one dimension invariant while allowing the other two to vary (see table 4.1).

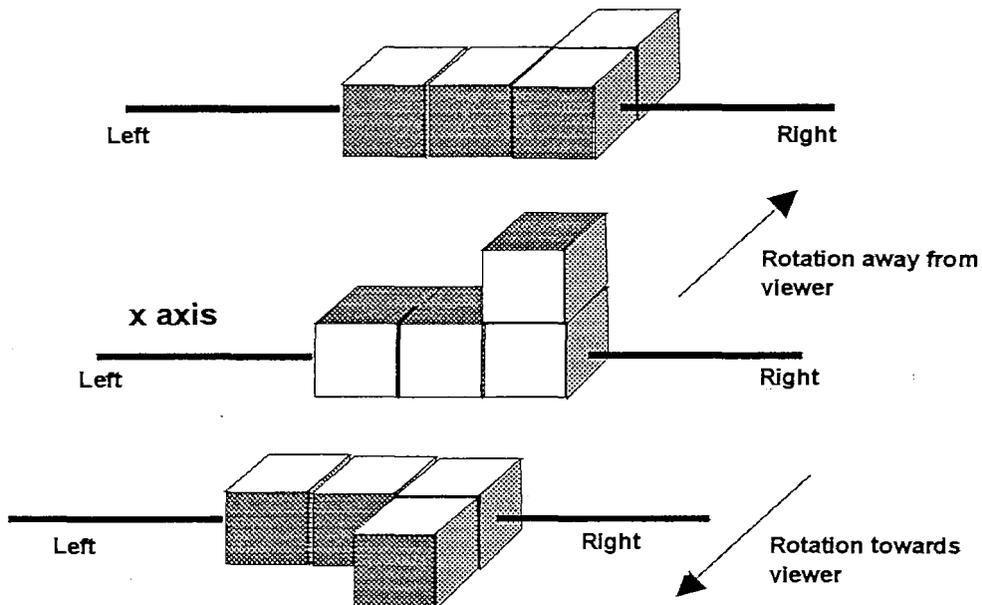
FIGURE 4.6: MOVEMENT OF PARTS OF A THREE-DIMENSIONAL OBJECT AROUND THE FRONTAL AXIS



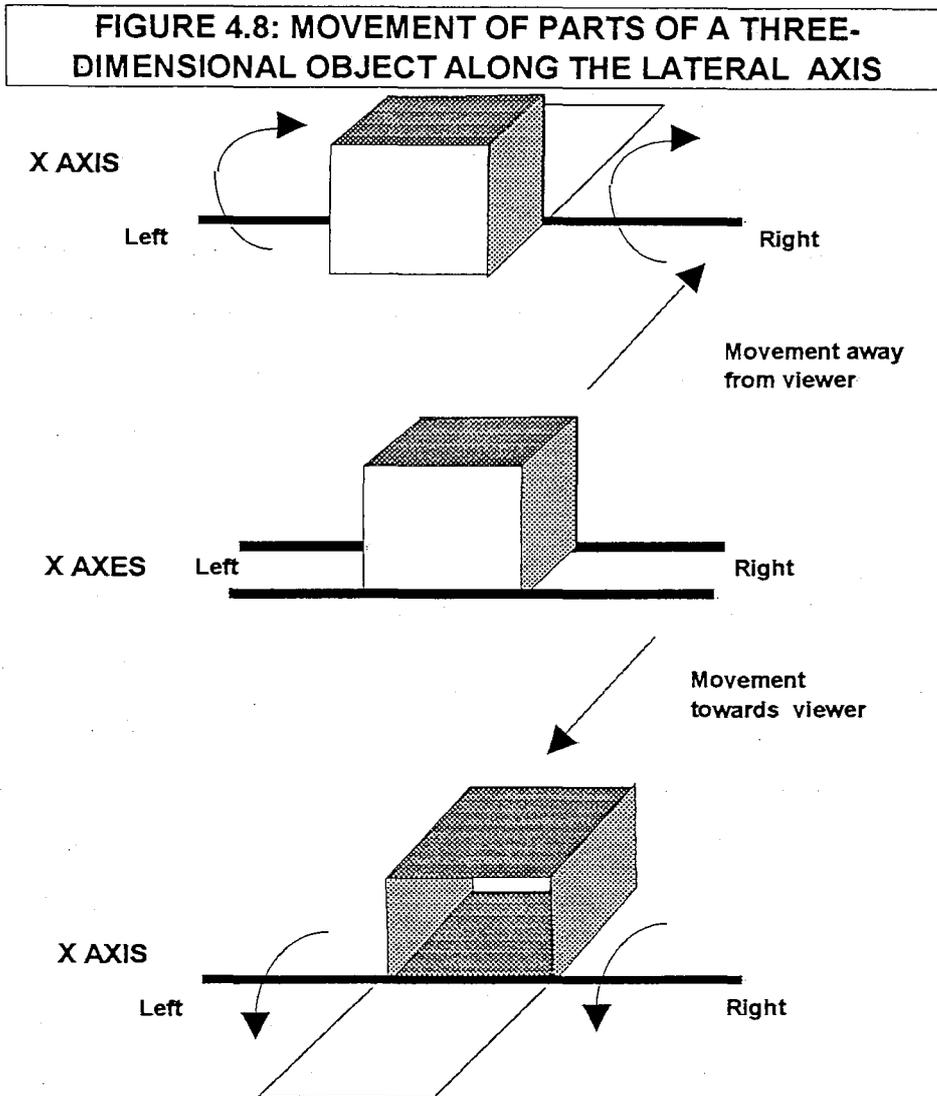
4.6.1.3 Lateral axis

Finally, the plane which is perpendicular (lateral) to and symmetrically bisects our bodies, is called “left/right”. Research has shown that children first master the lateral axis followed by the frontal and lastly by the vertical.

FIGURE 4.7: MOVEMENT OF A THREE-DIMENSIONAL OBJECT ALONG THE LATERAL AXIS



The *x-axis (lateral axis) rotation* involves rotation in depth around a horizontal axis. Rotation can be either top towards, or top away from the viewer and it can go any distance.



Any rotation keeps the values in one dimension invariant while allowing the other two to vary. From the development of spatial terminology regarding dimensions it follows that "up/down" is the easiest followed by "front/back" followed by "left/right". This implies that the vertical axis rotations are the most difficult followed by the frontal axis rotation, and the lateral is the easiest for children to handle. In the case of the vertical axis rotation, front, back and left and right are the invariants and are acquired last by children.

4.7 Media

4.7.1 The role of different media

Media cannot be chosen simply in terms of their ability to convey certain content, but must also be chosen in terms of their ability to develop the processing skills that make up such an important part of human intelligence (Olson, 1974:13). Olson proposes that each medium, if its bias is properly exploited, reveals and communicates a unique aspect of reality.

According to Olson (1970:184), spatial perception takes place in what he calls two performatory domains. By media he means a range or a domain of performatory activities (Olson & Bialystok, 1983:18). The first domain is more general and exists as a result of being human and being kept upright by earth's gravitational forces. This domain is called locomotion and grasping or prehension. The second domain is culture specific and could be subdivided into language and art (drawing and constructing).

If one looks at the performatory act of locomotion and prehension it stands to reason that the human being will select features from the visual world which are primarily topological because those are the features permitting the guidance of those movements. Cues that are not invariant such as left/right orientation, provide little information for those performatory acts and hence they will not be selected. Moreover, "animals" that specialise in such performatory acts should be biased towards cues that are invariant for the guidance of such activities. Olson and Bialystok (1983:59) therefore argue that there are not two independent systems for perception and representation, but rather one perceptual one which is altered substantially by performatory acts in different media. This is somewhat different to Piaget's view of two systems (Piaget & Inhelder, 1956:30). According to Olson, Piaget is right about the role of activity in the child's development but wrong about the nature of the effect of this activity.

Olson (1970:187) is adamant about the fact that the media in which children work determine and require different perceptual information. In the representational drawing some of the portrayals true to natural life depend on information that was not given in original perception. It is in the context of the performatory attempts in the medium of drawing and painting that the alternatives arise for which additional information is then sought. For example, in the drawing of a house or square, information is required to guide each component of the performatory attempt in the medium to which it is unique. This means that children have to pick up new cues from the stimulus event.

In other words, the difference in "perceiving" or "recognising" a figure and performing or copying it, is that different perceptual information is involved in both cases. The alternatives, being confronted at each point in time are to some extent unique to the performatory medium. For instance, to draw a diagonal does not require the same information as to define in words the concept of the diagonal (Olson, 1970:184). In both cases, however, the information is perceptual, and in both cases the information must be appropriate to the alternatives with which the children are faced if they are to succeed. They differ primarily in the fact that each activity, namely discriminating, drawing, constructing on the checker board, and defining, involves somewhat different sets of alternatives for which different cues, features, or information must be selected from the model.

It is important to note that it is the performatory which enables children to pick up certain other invariants from the perceptual world. Similarly linguistic decisions require information. The decision whether to use the past or the present tense of a verb, or to understand the differential usage of other speakers, requires information of a kind which has not previously been selected, or attended, or perceived, because there has been no occasion for it. In this way language as a medium provides the occasion for obtaining not only information, but different types of information from one's perceptual world than would have been the case if the medium did not exist.

In order to understand the effect of mastering and utilising specific media in the process of solving a spatial problem, it is necessary to have a closer look at the features of the different media along with the different variables which contribute to the complexity of acting in that media.

The structural descriptions (Olson & Bialystok, 1983:18) which a human being assigns to objects, events and layouts will reflect the activities in which he is urgently engaged, planning to engage, and prepared by nature to engage in. Predominant in these activities are such things as locomotion, balance, prehension, recognition of recurrent events, and the like. To perform these tasks successfully would require some sensitivity to gravitational vertical, to invariants of shape across changes in position, to the spatial ordering and spatial relations, as well as to sizes, distances and so on.

Such predispositions for representing events in a particular way are the biases which children assign to experience and yet these biases anticipate and reflect the functions that these representations are serving. As a result the representations are restricted by the predispositions of the observer rather than being neutral and objective copies of reality (Olson & Bialystok, 1983:7).

Olson and Bialystok (1983:24) caution by saying that we cannot claim that the spatial structures which are turned up in the examination of the language of space, roughly spatial terminology, or those which are turned up in representational art, will in fact precisely reflect the child's structural descriptions for the recognition of the objects or events. Children have good reasons for using spatial language and drawings as models for structural descriptions implicit in ordinary perception.

Olson and Bialystok (1983:25) argue that the information in the structural description by which we recognise objects, is largely implicit. The colours, shapes, sizes, locations and so on, which provide the basis for object identification and discrimination are usually neither known nor attended to as such. Yet it is these perceptual features which

are responsible for identification of objects. The question that now comes to mind is, what determines the selection of the features which will assign explicit form or linguistic representations?

According to Olson and Bialystok (1983:48-49), three factors that may contribute to this selection are:

- The perceptual and functional biases of the child

Humans are differentially biased to perceive the distinctions necessary for their usual daily activities (e.g. vertical position to stay upright). Human perceptual biases determine which features of the environment are possible to encode in the structural descriptions. This importance may then be reflected in the speech habit of the person.

- The restriction of the linguistic system *per se*

The bias of the linguistic system is to represent information in a contrastive, binary fashion. Perceptual information not suitable to that format cannot be assigned a simple lexical description (Messick, 1988:14-15). The same argument can be used for the representations in drawing or painting or construction. They all allow for different aspects of the structural descriptions to be made explicit.

- The particular process recruited by the task

Just as alternate structural descriptions could be assigned to a display as a function of the perceptual requirements, so alternate linguistic representations can be extracted from structural descriptions as a function of communication requirements. More generally it is reasonable to expect that the spatial lexicon will be more elaborate in some cultures, such as those involving carpentry, than in others such as those not so involved. These are indications that the features which are made explicit by language are determined in part by situational and in part by communicational demands.

The limitation of the medium is shown in children's attempts to give complete verbal descriptions for guiding instructing performance in a medium such as drawing or reconstructing. In these cases the use of language fails to differentiate all the alternatives found by children in their attempted reconstruction and hence language is not a completely successful means of instruction. This point simply reflects the fact that different media involve different sets of alternatives, and therefore require different information.

A very important issue in the development of spatial representations according to Olson and Bialystok (1983:64), concerns the order in which the implicit features of the structural descriptions can be made explicit. They postulate two principles describing the restrictions on the child's early spatial representations, namely the Principle of Invariance and the Principle of Information.

- **Principle of Invariance**

Spatial propositions differ in their extent of generalisability. The proposition, which reflects the position of a cup of coffee with respect to the desk, for example "The cup is on the desk", will serve as an accurate description from any position around the desk. It would stay appropriate even if the speaker has moved around the desk. A similar situation, though, relating the chair to the desk, "The chair is in front of the desk", depends critically on the position of the speaker. This situation depends on where the speaker is standing and where we assume the "front" to be. The first example illustrates a situation that is more generalisable than the last, since a single proposition preserves the relation across a variety of spatial positions. This generalisability of a proposition is called the invariance of the relationship.

Invariant predicates remain appropriate when assigned to different aspects of the environment in spite of the changes in the relationship between the speaker and

the environment. In other words, predicates based on vertical dimensions such as “up”, “over”, “top” are relatively impervious to changes in the position of the speaker. The predicates such as, “in front”, “behind”, “to the right” are however, dependent on the position of the speaker. The invariance of the spatial proposition is considered to be an important aspect of complexity in that the spatial relationships, which remain constant across activities, are by hypothesis simpler to represent and operate upon, than those which alter as a function of activity or change in spatial location. This implies that the detection of invariant properties should then be simpler than the detection of variable relationships. As a consequence, spatial predicates presenting these invariant relations should then also be less difficult than predicates presenting variable relations.

- **Principle of Information**

A second aspect of the complexity of a spatial relation is the amount of information required to represent an object or event. The spatial proposition reflects this quantitative difference in two ways:

Firstly, the proposition may require more than one predicate-argument construction to represent a particular spatial relation, thus increasing the number of predicates in a given proposition. For example, horizontal and vertical line segments are each based on a referential system requiring only one dimensional axis, and therefore only one predicate-argument structure in the proposition: “The line is *horizontal*”. Research has shown that these lines are easily constructed, represented and described by children. Diagonal lines, however, must be related to at least two axes and thus require two predicates in the proposition: “The line is *up to the right*”. Research has shown that these segments are problematic for children to recall, discriminate and reconstruct (Olson, 1970:179).

Secondly, predicates themselves vary in the number of arguments which they require in a given proposition. Predicates such as *between*, which require two

arguments to complete a proposition, should be more complex than similar predicates, such as *beside*, which requires only one. Examples of two-place predicates are *above*, *beside* and *behind*. Example of a three-place predicate is *between*.

4.7.1.1 Language

4.7.1.1.1 Role of language for people with different worldviews

Different cultures, as reflected in their languages, draw conceptual and linguistic distinctions in radically different ways. Language evolves to serve the distinctions which the culture takes as critical and evolves to correspond to those distinctions and groupings which are critical to the culture. This means that language develops to the extent that it is useful to differentiate events from those alternative events with which the culture does not want it to be confused (Ascher, 1991:130). Language is a formative factor in intelligence to the extent that it is used for the purpose of instruction, and in directing the child's attention to those ecologically valid cues the culture has selected for attention.

Peat (1994:220) states that when people move between different languages when they translate ideas, it could mean that it is not only a question of switching from one language to the other but actually switching between different worldviews. Nelson-Barber and Estrin (1995:174) emphasise this with the finding that many American Indian students have extensive knowledge of mathematics, but the knowledge is deeply rooted in naturalist traditions common to the Indigenous communities.

The European languages (Germanic and Italic) are closely related to an even larger family of languages called Indo-European which includes Celtic, Slavic, Albanian, Greek and Iranian languages as well as the vast family of Indo-Aryan languages such as Hindi, Bengali, Sinhalese, Panjabi and the sacred Sankrit language. The people who

speak these languages not only speak languages that are closely related but they also share a worldview that is common (Peat, 1994:221). According to Peat (1994:221), all these languages have their origin in a single proto-European language that was spoken before 3000 BC. Many languages that are spoken by Indigenous people, for example the Native American languages, are very different from the Indo-European languages and even within the North American continent there are several language families as different from each other as English is from Chinese (Peat, 1994:221).

Peat (1994:222) states that when the world of these Indigenous American people is investigated, it is not only a different language that is encountered but also profoundly different concepts and worldview. Moore (1994:11) also stresses this by stating that the language of the American Indigenous people represents an importantly different mode of thinking. Peat (1994:223) has found that, not only are the concepts enfolded within the language radically different, but even the function of language itself is different. Peat (1994:223) states that one of the reasons that non-Indigenous people find it so hard to grasp the significance that Indigenous people give to their language, is that the view from the western perspective takes language to be simply a medium in which to express thoughts and feelings. Indigenous people do not regard language in the same way. Instead, for them language has a power of its own and to speak it is to enter into alliance with the total world in which they live.

According to Peat (1994:227), another significant difference between Indigenous languages and European languages lies in what is revealed about the nature of thought. Moore (1994:11) emphasises this by stating that it was found that students whose thought processes are rooted in a language which stresses the dynamic rather than the static aspects of reality, find it difficult to apply the logic of conservation when faced with an apparently contradictory situation.

Within the European languages and thinking there exists a tendency to group things together in categories. Words are grouped together generically for example birds, fish,

trees, rocks, etc. Yet this form of categorising is not an inevitable feature of the human mind, for Indigenous languages support totally different forms of logic and reasoning (Peat, 1994:228). This major difference between the functions of language is explained by Peat (1994: 231) when he quotes the words of an Indigenous American named Sa'ke'j which says : “ English is a language for the eye, while Algonquin language is a language for the ear.” Peat (1994:231) says that the Algonquin (American Indian) languages deal in vibrations in which each word is related directly, not only to a process of thought, but also to the animating energies of the world.

Ascher (1991:133) points to the important fact that the language of a culture creates and reinforces particular shared habits of thought and shared habits of observation. The above is very well illustrated in one of the aspects of the Inuit language that particularly deals with space, namely their system of using “localizers”. The Inuit “localizers” are used when pointing out an object, place, or event. When describing the place of events, the “localizers” are adverbs comparable to our words, “here” and “there” in the sentences “He is sitting there” or “He is sitting here”. When we use “here” and “there” it is to distinguish between locations near and far from a certain point. The Inuit have very little interest in time as an entity but there is an emphasis on being as specific as possible about spatial location because of the world in which they live. They cannot afford to make mistakes when it comes to describing a location in their world which is covered with snow most of the year and a visibility close to zero.

Several of these localizers exist and in sum the considerations involved in picking the appropriate form of a “localizer”, designating the place of an event are the following: (1) whether the event begins, ends, remains, or traverses the place; (2) whether the spatial point of reference is to be the speaker or some other person; (3) whether the place is near or further away and, if away, whether it is up, down, inside a boundary, outside a boundary, or horizontally the same with no boundary involved and for all of these possibilities with the exception of a place inside a boundary; and (4) whether the place is extended or restricted. Habitual use of this system of localizers clearly

reinforces precision in observing and transmitting information about location. However, from this it can clearly be seen how time and motion, as well as configuration, become part of the clear specification of space for this specific group of people (Ascher, 1991:134).

Cox (1986:152) focuses the attention on findings that in the Western culture, most people who use the same terms are indicating the same situations. For example, for the majority of people in the Western cultures the terms “in front” and “behind” are mutually exclusive. A child should learn from these terms that they refer to the opposite sides of a referent object, and that they lie along the horizontal-frontal axis emanating from the observer and that “in front” refers to the observer’s side whereas “behind” refers to the far side of the object. Cox (1986:152) states that research has found that the Hausa-speakers of Nigeria operate rather differently. “In front of” refers to the far side of the object and “behind” refers to the near side from the observer’s viewpoint.

The research for this work was done with children coming from a multi-lingual environment, speaking first languages that belong to Indigenous as well as Indo-European families. The actual teaching and investigation of the spatial knowledge of these children were done through the medium of English. It is against this background that the role and function of English is the focus of further elaboration.

4.7.1.1.2 Features of the development of a spatial lexicon

According to Olson and Bialystok (1983:47), the language of space has been examined by several authors, and one of the most interesting properties of the language of space is the fact that to understand the meaning of such terms one must know something about space. It is therefore important to look at the different spatial terms that are used in the different language groups. English has a fairly large number of spatial terms which fall into the form classes of adverbials, for example here/there,

prepositions: up/down, on the left/on the right, at the front of/at the back of, in/on, by, above/below; adjectives: big/small; long/short; wide/narrow; thick/thin; deep/shallow; and nouns: top/bottom; left/right; front/back; side/end.

These spatial terms have several common properties (Olson, 1975:85):

- Firstly, they are organised around particular *dimensions* such as verticality or length.
- Secondly, they all assume a *point of reference*, this being, the speaker or some other object.
- Thirdly, these terms usually come in *contrastive pairs* specifying the dimension as in the following:

<u>Dimension</u>	<u>Contrastive pair</u>
where	here/there
vertical	up/down; high/low; above below
horizontal	right/left
frontal	front/back
length	long/short
distance	far/near
height	tall/short
width	wide/narrow
depth	deep/shallow
thickness	thick/thin

4.7.1.1.3 Predicate

The spatial predicates mentioned above are critical to the proposition. Most important of these predicates are those which are based on three-dimensional Euclidean space defined by the three axes, producing such predicates as front, back, top, bottom, left, right, etc. Others specify relations on one of those dimensions such as over, beside. Still other predicates fall within and are derived from these basic predicates without

altering their basic dimensional properties, such as open, closed, in, out, near, and far, reflecting topological relationships. These are spatial relations which are somewhat independent of particular dimensions and directions

Spatial propositions are applied to objects for two different purposes, namely structural and relational. In their use as structural propositions, the spatial predicates are applied to the internal structure of egos, objects, rooms and so on, in order to relate the constituent spatial properties of these relatum by means of such predicates as top, bottom, front, right, etc. (Olson & Bialystok, 1983:80). These are usually in the form of nouns. For canonical objects these descriptions are initially based on the object's usual orientation in space. The top of the car is its uppermost part in the environmental space. These descriptions continue to apply when these objects are disorientated. For canonical objects these intrinsic spatial components are critical to their recognition.

Spatial propositions may also be used to describe the relationship between ego and object, relationally. These propositions define the location of one object with respect to another object or relatum, for example, "The person is in front of the tree". Such relationships are expressed in ordinary language by means of prepositions.

These spatial predicates may be assigned to three major classes of relatum: ego, canonical, and non-canonical. The interpretation of the relation in each case depends on assigning meaning to the proposition in terms of some axes, or set of axes since the specification of an axis is fundamental to each spatial proposition. The axis is based on either the environment, the ego, or the canonical object. When more than one of these bases may generate the axes, the meaning of the proposition is ambiguous. Resolving the ambiguity requires selection from the possible axes to arrive at a consistent interpretation for a particular spatial relation (see table 4.1).

The particular interpretation chosen in each case is expected to be constrained by the extent of invariance offered by each type of axis.

One set of spatial terms which pertains to directions of movement of ego without specifying a goal, is the adverbials:

1. up / down
2. forward / backward
3. left / right

According to Olson (1975:90), “up/down” should be first acquired given that it is asymmetric. Gravitational cues utilised in maintaining vertical orientation are asymmetrical. “Front/back” should follow “up/down” because they are also asymmetrical given the position of our perceptual apparatus. “Left/right”, on the other hand, because of its bilateral symmetry would be acquired last.

A second set of spatial terms pertains to positions of objects relative to some specified referent object:

4. here / there (where)
5. by, next to, beside
6. near / far
7. above / below, over / under
8. in front of / at the back of, ahead / behind
9. on the left / on the right

Most of these terms can take ego as one of the two objects being related. “Here/there” is an adverbial which is exclusively related to ego. “There” has existential use: “There are seven days in the week”, as well as a locational one. While “here” is the more egocentric term and hence may be perceptually salient, its usage is not congruent across speakers. “Here” for the ego as speaker is not the same as “here” for the ego as listener. The terms from 5 to 9 are prepositions specifying position relative to the object of the preposition. Proximity relations are marked by “by”, “next to”, “beside”, “near” and “far”. These are the relations that Piaget and Inhelder (1956) and Olson (1970) found to be critical in a child’s attempts to copy a pattern. Important to note is

the fact that these proximity terms are undifferentiated as to the particular direction in space which the relation holds. This implies that “next to” could be applied horizontally or vertically. “By” appears to be restricted to horizontal proximity but is undifferentiated as to “in front of” and “beside”. The terms mentioned under 7, 8 and 9 preserve their proximity relations but are more complex in that they specify, in addition, a direction of application. “Above/below” and “over/under” apply vertically.

A third set of spatial terms are prepositions that are specified not by their direction of application, but rather by the spatial properties of the objects to which they relate:

10. at, on, in
11. to / from
12. onto / of
13. into / out of
14. via, across, through

According to Olson (1975:92), while “at”, “on”, and “in” specify locations, in the same way they presuppose dimensions in one, two and three-dimensional space respectively.

An important set of spatial terms are adjectives which refer explicitly to the spatial properties of the objects that they modify:

15. big / small
16. tall / short
17. long / short
18. wide / narrow
19. thick / thin
20. deep /shallow

In the above-mentioned list 15 assumes non-dimensionalised space while 16 to 20 assume a dimensionalised space of at least one dimension for 16 and 17, at least two

dimensions for 18, and three dimensions for 19 and 20. Another important fact about the predicates 16 and 20, is that they could all reflect measurement expressions.

There are some spatial terms which occur as nouns:

21. top / bottom
22. front / back
23. left / right
24. side, end, etc.

Since the predicates assigned to ego and canonical object relatum are invariant across spatial transformations, these should be simpler than noncanonical relatum according to the principle of invariance. It has been shown, though, that this invariance is not necessarily an advantage in that the inherentness of the descriptions may produce conflict when there is competition between relatum for expressing a particular relationship. Thus the assessment of the complexity of a spatial proposition must include, as well as other features, the spatial relationship, in particular those which describe the predicate. Olson and Bialystok (1983:234) propose that complexity of a spatial proposition is determined by the interaction between the relatum of the argument and the spatial dimension presupposed by the predicate for a particular proposition. Cox (1986:137) emphasises this view when she states that the referents (person, place, time) of these deictic terms (from the Greek word for pointing or indicating) will change as the speaker changes. Deictic terms, then, shift referents depending on the identity of the speaker. Thus the point of view shifts from speaker to speaker.

Cox (1986:71-87) elaborates on this use of the deictic terms by subdividing them into two categories namely: (i) person deixis and (ii) place deixis.

(i) Person deixis

English speakers usually refer to themselves as “I” or “me” and to things belonging to themselves as “my” or “mine”. Similarly, “you”, and “yours” refer to the addressee(s). An important question is, how do young children manage to grasp the notion that the referents change when another speaker is involved? Although most normal children have learned the adult use by age 2 to two- and-a-half years there is a number of cases in the literature of children with otherwise normal linguistic development who do make errors. What seems to be happening, is that a child first understands the pronouns used by others which refer to themselves (You, yourself, etc): later, they understand the pronouns used by others to refer to themselves (I, me, etc.). When the child comes to use the pronouns “themselves” it is those which refer to “themselves” (I, me, my) which they use first, and then later those which refer to others (you, your, etc.). In the meantime, they use the proper names that refer to other people. The earlier acquisition of the I/me form is clearly not directly related to adult frequency since adults use far more “you” forms to children. The reason would seem to be that when the child uses “I”, they are the only referents. Many people can be addressed by the child as “you”, so this second-person pronoun has multiple referents. The explanation, though, for the child’s understanding of “you” before “I/me”, but the use of “I/me” before “you” is in terms of the restricted set of possible referents. In speech directed at them, the children themselves are the only referents for you, but many speakers can themselves be “I/me”. When the child speaks, they themselves are the single referents for “I/me”, but many people could be called “you”. The task in which the set of referents is limited should be easier to master.

(ii) Place deixis

Deictic terms such as “here” and “there” are just two of a number of linguistic terms used to specify location. Whereas “I” and “you” refer to the speaker and the addressee respectively, “here” and “there” are not necessarily related to a place near the speaker

and a place near the addressee; rather, they refer to a place near and a place farther from the speaker. "I" and "you", then, are mutually exclusive (except in cases where you is used in the sense of "one").

In contrast, although "here" refers to a place near the speaker it may also include the addressee's position; similarly there may be a place far from both of them. Were two participants at opposite ends of a particular distance, then the positioning will be distinguished as "here" and "there", whereas those in relative proximity will be described as "here" and the place farther away from both of them will be described as "there". Another problem with "here" and "there" is that, unlike "I" and "you", there is no clear boundary between them. It is difficult to say when "here" becomes "there". Furthermore, whereas "I" and "you" are primarily deictic terms and have no other uses, "here" and "there" may be used in utterances like "There are three books on the table". "Here" and "there" are used with reference to the speaker's position, but the boundary between "here" and "there" is not as definite as that between "I" and "you". Furthermore, both terms may be non-deictical as well as deictical. It is not surprising to find that the correct use of "here" and "there" is achieved later than the correct use of "I" and "you". It should be remembered, however, that when "I" and "you" are normally used, they are not backed up to any extent by gesture. The role of gesture is believed to be crucial in the use of "here" and "there". A classic situation often found amongst young children is the discrepancies they experience concerning left and right (Figure 5.9).

Cox (1986:114) stresses the importance of studying the child's acquisition of deictic terms in that these provide an opportunity to tap the child's understanding of the changing roles of conversational participants and the shifting referents to which the terms relate.

4.7.1.2 Drawing

4.7.1.2.1 The role of drawing for people with different worldviews

Many of our pictorial conventions began in Europe in the fifteenth century (Ascher, 1991:136). They were influenced by and reinforced with particular mathematical ideas. The study of optics, going back to Euclid in 300 B.C, depended on rays travelling in straight lines between the eye and objects being viewed.

The fifteenth century concern was the actual point where these lines would intersect a perpendicular plane placed between what was being viewed and the viewer. Time became frozen, and hence the issues of both time and motion were eliminated (Ascher, 1991:136). Not only was time frozen on what was being viewed, but the position of the viewer was also frozen. The entire picture could only show what a viewer could see from a single fixed place in a single fixed instant. It could not be shown what was seen simultaneously from above, below, behind and inside as well as outside (Ascher, 1991:136). Related to the fixed vantage point in front of the picture plane are the horizon line and the “vanishing point” of a picture. These are sets of parallel lines which would appear to the viewer to meet again. Based on these are the sizes of objects which are to be seen as being at different distances from the viewer. To show recession, the objects diminish proportionally. Westerners, whether they be creators or viewers of a picture, are taught these conventions (Cox, 1986:51).

There is considerable evidence to indicate that the comprehensibility of shapes and colours varies depending on the people and the type of cultural background these people have (Arnheim, 1970:31). In other words, what is rational for the one group could be irrational for another group.

Different conventions are used by different people. The Inuit are one example of such a people for whom time and space remain unified and the contents of pictures are not

confined to what can be seen from a single fixed position in space-time (Ascher, 1991:136).

For the Inuit, an event through time can be depicted by showing its interrelated parts, each in its most significant aspect, despite the fact that they could not necessarily be seen at the same time nor from a single place. In general a picture has no background and no borders. Whatever is shown stands out in a featureless space which extends indefinitely in all directions. What is most distinctive is that different parts of a picture can have different ground lines and, for any part, there can be multiple vantage points. As a result of these, a picture need not be orientated in any particular way for the Inuit. The vantage points, however, are sufficiently far away so that horizon lines and vanishing points, even if there were any, would be at infinity. This, together with the lack of background, means that there are no changes in size due to differing distances and everything in the picture is relatively at the same distance (Ascher, 1991:137).

According to Changuion (1989:35-36), in the African context symmetry is one of the most basic features of African mural art (drawing and painting on the walls of their living structures). Compositional schemes are often bilaterally symmetrical on either side of a central axis, as are the individual forms within the composition. One of the reasons for this is that the painting functions as an extension of human action and echoes the structure of the body. When painters prepare for their artwork on a wall, they use the adjacent ground level as the medium for their blueprints. They often draw with both hands, beginning at the top of an imaginary vertical, and the resultant forms on either side of this are simultaneously realised and are mirror images of each other.

The movement of the hands is physiologically determined. The right and the left are apt to move symmetrically and the motions of the arms are often performed rhythmically. In this way, gesture, dance and language pass in a fleeting moment, but in painting they are given greater permanence.

These artists of Africa use the bilateral symmetry of their bodies as the vertical and as their central element of organisation. African mural painting is related to this principle of bilateral symmetry and this occurs even when the lines vary from the vertical and horizontal (Changuion, 1989:35). Angles of 45° are prevalent and every straight line that varies from the two main directions is coupled with either a vertical or a horizontal line.

In addition to bilateral symmetry, two other types of symmetry can be identified, namely shifting and rotational. Like so much of the art of the black people in South Africa, wall painting is submitted to rhythmic formalisation (Changuion, 1989:45). Forms are abstracted from the visual world in a way which has much of its inspiration originating from music and dance. The rough-hewn rectangles which are frequently found in wall paintings can be compared to the rhythms of a drum beat. Its forms and colours are limited to those which make for harmony rather than contrast.

4.7.1.2.2 The dimensional aspect of drawing

If children want to represent objects on a flat surface, they have a problem. In the real world they deal with three dimensions of space, but in a picture only two are available.

Bremnen and Batten (1991:375) point to the difficulty which is part of the drawing process, which requires spatial transformations of three-dimensional reality onto a two-dimensional surface. While horizontal and vertical dimensions can be translated directly onto the page, the child must develop particular means of portraying the third dimension. Bremnen and Batten (1991:375) mention the fact that several other authors have found that young children tend to omit this third dimension (depth) from their drawings, but with increasing age they use progressively more sophisticated depth cues, beginning with height in picture and then incorporating partial inclusions.

According to Cox (1991:376), the issue of viewpoint portrayal is intimately connected with that of depth portrayal, since in order to indicate viewpoint it is necessary to show the depth relations that exist in the scene relative to the artist's position.

While earlier accounts suggested a fairly rigid sequence of development, it is now clear that young children can be provoked into showing their view, for instance, by stressing depth relations between objects before asking children to draw (Cox, 1991:375), by making an explicit request for the child's view of an object, or by making it important that the children communicate their viewpoint information. Cox (1991:84) concludes that it also seems likely that young children do not identify portrayal of viewpoint as a primary requirement in their drawing.

Cox (1986:57) mentions the fact that the top of a page held vertically is generally regarded as "up" and the bottom as "down". If the page is flat on the table, the part furthest away is "up" and the part near to us is "down". The left-right dimension of the page corresponds to the horizontal-lateral dimension of the observer's space and because of this, the problem arises with the near-far or depth dimension of the real world. There is also no equivalent on a two-dimensional flat surface. The solution that was proposed by artists as far back as 1377 by Brunelleschi and 1404 by Alberti, was the system commonly known as perspective. This implies that the artist draws only what can be seen from one viewpoint and everything else is omitted. The system of linear perspective is generally accepted as the normal method of projecting aspects of the three-dimensional scene on to the two-dimensional surface of the picture. Children only gradually develop the ability to draw in perspective.

4.7.1.2.3 Conventions

Parzysz (1988:79) is adamant about the fact that the rules for drawing spatial figures should be made explicit to children. He is of the opinion that this type of representation is not the concern of more or less hazy conventions, but of definite projective geometrical properties.

The problem of coding (producing) a 3-D geometrical figure into a single drawing has its origin in the impossibility of giving close representations and in the subsequent obligation of falling back on a distant representation, in which there is an additional loss of information. The artist is actually confronted by an absolute dilemma due to the fact that what one knows of a 3-D object comes into conflict with what one sees of it. This unavoidable dilemma has been constant in the history of art.

In order to interpret correctly what another person has drawn, or to draw a picture from others, a set of conventions or rules is needed to avoid ambiguity and misunderstandings (Cox, 1986:51). As with language, if communication is intended through pictures, there must be predetermined conventions about how they are constructed and what they mean, otherwise their communicative usefulness breaks down. It should also be kept in mind that these conventions are but conventions and will not necessarily be understood immediately by people from different cultures, or by young children who have not yet had time to assimilate them.

4.7.1.2.4 Children's drawings

According to Arnheim (1970:260), one must bear in mind what the child is likely to have seen while observing a specific scene. Once this has been established one can appreciate the freedom with which the data of experience are transformed into an independent visual interpretation, executed with the resources of the two-dimensional medium. In the universe of the flat picture space, its visual logic is immediately

convincing and appropriate. The situations elucidated by visual thinking never concern the outer world alone. As the child grasps the characteristics of the situation, he also finds and clarifies in them elements of his own being.

The type of processing required by spatial tasks has been the central concern of several theorists. Early notions about processing regarded it in terms of the sequential solution response to a given problem situation, as in Olson's pattern of decisions affecting children's drawings (Olson & Bialystok, 1983:204).

Children's copying of visual forms involves more than what is immediately perceived - it reflects their private experiences and their cultural history as well (Olson & Bialystok, 1983:7). Children's drawings are simpler than those of adults because children respond visually rather than conceptually to the surroundings. Their drawings lack content, not because they fail to see content, but because they lack the ability to draw what they see.

Freudenthal (1983:239) points to the fact that young children go through a development of moving from the iconic to the symbolic when it comes to understanding and representing objects and situations. He is of the opinion that care should be taken when interpreting the drawings of children because of this lack of knowledge regarding the stage of representation that the child is experiencing, namely iconic or symbolic. He states that there exists a big gap between recognising two figures as being congruent or similar and being able to copy them as such (Freudenthal, 1983:238). Before something can be copied it is important to know what matters, and the majority of people who investigate children through their drawings are not in the habit of telling their subjects (Freudenthal, 1983:241). Freudenthal (1983:242) concludes by stating that seeing, interpreting and producing perspective drawings is something that children should be taught and not something that they just acquire by themselves.

4.7.1.2.4.1 Developmental stages in children's drawings

One of the most influential thinkers about children's drawings, according to Cox (1986:104), was Luquet, and his ideas were later taken up by Piaget and Inhelder. Essentially Piaget and Inhelder adopted Luquet's ideas without revision. They simply incorporated these into their own theory of a child's developing concept of space.

Luquet postulated a series of three developmental stages of drawing (Cox, 1986:103-104). In the first scribbling stage from approximately 2 to 4 years of age the child experiments with marks on a piece of paper. At first this is fortuitous realism, since children interpret the lines that have already been made. They do not intend to draw a particular object in advance. Later in the second pre-schematic stage, from approximately 4 to 7 years, children do have prior intentions about which objects they want to draw. They nevertheless encounter many difficulties and this is characterised by failed realism. The latter part of the pre-schematic stage is characterised by intellectual realism. This corresponds with Piaget and Inhelder's (1956) early projective spatial relationships where children cannot successfully adopt a particular viewpoint. After a transition period, children move on to the third schematic stage, at approximately 8 to 9 years, in which they attempt to draw a scene from a particular viewpoint and try to show the depth of individual objects and the depth relationship between these objects. This stage is the endpoint of the development and is characterised by visual realism. According to Piaget and Inhelder, children have now developed an understanding of both projective and Euclidean relationships. A scene is drawn from a particular viewpoint, and distances, proportions and relationships between objects are worked out correctly in relation to the viewpoint.

4.7.1.2.4.2 Developmental stages for drawing specific solids (compare figure 4.9)

Mitchelmore (1978:233) has shown that children's drawings of solids can be divided into four stages (figure 4.9):

- Stage 1 (Plane schematic). The figure is represented by a single face drawn orthoscopically, i.e. as if viewed orthogonally or by a general outline.
- Stage 2 (Space schematic). Several faces are shown, but the faces are either drawn orthoscopically or hidden faces are included.
- Stage 3 (Pre-realistic). The drawings attempt to represent the view from a single viewpoint and to depict depth.
- Stage 4 (Realistic). Parallel edges in space are represented to be near-parallel lines on paper.

4.7.1.2.4.3 Developmental stages for foldouts (nets) of solids

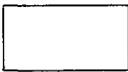
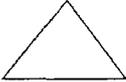
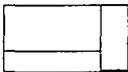
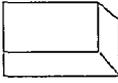
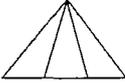
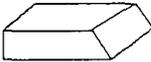
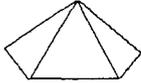
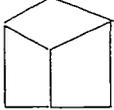
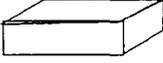
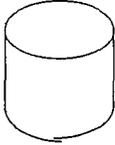
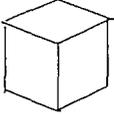
Research has shown that young children's drawings of the nets (foldouts) of solids can be divided into five categories, namely holistic models, models with elements of projection, incomplete geometrical models, complete geometrical models and physical models (Potari & Spiliotopoulous, 1992:39).

- Holistic models: In the drawings of the nets the different faces of the solids are not distinguished and the dimensions are not taken into consideration. This is an indication that children have difficulty representing the image of the solid in a conventional way.
- Models with elements of projection: The child tries to draw the solids as they would be when they are opened up. They draw only parts of the solids.
- Incomplete geometric models: The children are now starting to see the nets of the solids, but they are only considering some of the elements.
- Complete geometrical: The children are now able to draw the correct nets for the different solids.

- Physical models: Children are not only able to draw complete nets but they can also consider their physical construction.

FIGURE 4.9: DRAWING STAGES FOR DIFFERENT SOLIDS

FIGURE

STAGE	Cuboid	Cylinder	Pyramid	Cube
1				
2				
3A				
3B				
4				

(A typical drawing at each stage of representational development, from Mitchelmore, 1978:235)

4.7.1.2.4 Projection system classification

According to Bremnen and Batten (1991:391), the way in which children portray a three-dimensional object or scene from a particular viewpoint, is an important indicator of the development of the projection systems that children employ while drawing. Bremnen and Batten (1991:377) described a developmental progression in the use of projection systems, beginning with the absence of any clear system around

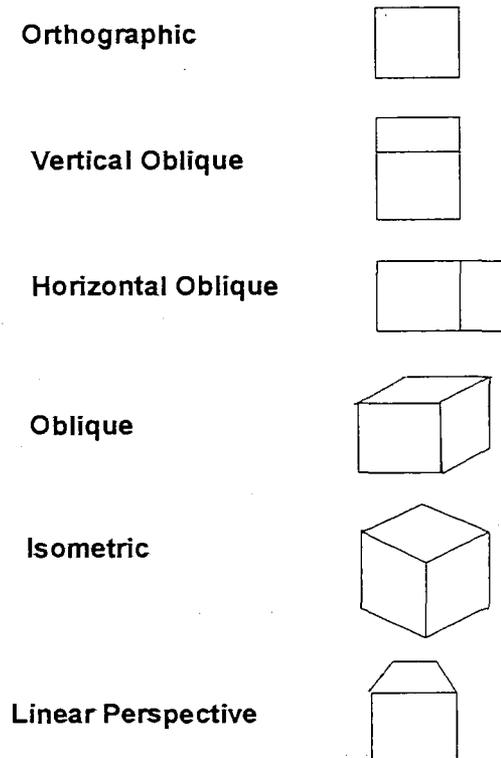
the age of 6 to 7 years and proceeding through orthogonal and oblique projection to forms of linear perspective around the age of 14.

Figure 4.10 illustrates a representation of the classification of a projection system where children start with the most primitive orthographic representations to the more advanced oblique isometric and linear perspective drawings.

Bremnen and Batten (1991:390) found in their analysis of the projection systems that children older than six used that there was a clear sensitivity to viewpoint. The viewing conditions had an effect on the choice of the projection system used for single cubes from 8 years onwards. For instance, isometric projection only appeared in the corner view condition, and horizontal oblique only appeared in the eye-level condition. In addition, there were clear differences in frequency of use of other projection systems between conditions. Vertical oblique and oblique appeared most in the frontal condition, whereas orthographic and linear perspective appeared most in the eye-level condition. According to Bremnen and Batten (1991:391), the result for orthographic is to be expected since that projection system assumes a horizontal view.

Cox (1991:84) concludes by mentioning that children seem to draw stereotyped pictures that may include more of the object than can be seen if a model was actually present. She also found that presumably the way that the scene is described will cue the child as how it should be drawn. She also found that features of the object in the array itself may influence the children's choices of their views. Further more, children are more likely to draw what they see if there is a definite contrast between the two objects in the scene or conversely, the more similar the two objects, the less likely children are to draw a visually realistic picture. She cautions by saying that the idea that children draw what they know rather than what they see cannot be accepted unconditionally.

FIGURE 4.10: A PROJECTION SYSTEM CLASSIFICATION FOR THE CUBE



(Projection system classification taken from Bremnen and Batten, 1991 :381)

What seems to be difficult for children, is to draw the object exactly how it appears. There may be strong biases towards producing rectilinear forms, and there is also the knowledge that the object being drawn is in fact made up of rectilinear surfaces. Bremnen and Batten (1991:392) sound a word of caution, though, when it comes to the evaluation of these drawings when they note that, despite the clear developmental progressions and differences between conditions, children are far from consistent in their application of a particular projection system.

4.7.1.3 Construction/Building/Physical Activity

4.7.1.3.1 The role of construction building and physical activities for people with different worldviews

Rudofsky (1977:358) notes that it is a well-known fact that many African children construct models as part of their early normal lives. "Because the early life of these children is a series of discoveries above and beyond play and work, their image of the world is compounded of their own observations and experiments. With nature as their inexhaustible toyshop children in primitive societies get far better ideas of how to have a good time than the most zealous kindergarten teacher."

Children in Uganda build miniature villages that are almost exact copies of their own villages in which they live. Among the Kpelle (a Liberian tribe) the children build houses which they then furnish with household objects made by themselves. In Borneo and the Gilbert islands children build their own houses and canoes. Ethiopian children playfully put up small scale huts. The children of the Warega, a Congolese tribe, do not only build entire model villages but act out an accelerated version of a full day's community life. Rudofsky (1977:361) notes that even in a cold climate children build their own snowhouses. Constructions include all activities involving building, cutting, pasting, and folding of different materials.

Gay and Cole (1967:61) mention that the Kpelle of Liberia know constructions which in our society would be dignified by the name theorem. They are capable of constructing circles, right angles, rectangles and cones. Gay and Cole (1967:61) mention the important fact that their technology is not severely limited by the absence of abstract geometrical terminology and knowledge.

Arnheim (1970:38) voices the opinion of many educators when he states that a great deal of active exploring, involving more than the sense of sight, is necessary before the

principle of the matter is understood. It has been implied that it is through children's performatory acts themselves, that they encounter the choice points which lead them to look for or select further information.

4.7.1.3.2 Developmental stages of children's block constructions

According to Guanella (1934:16), building and constructing differ from drawing and painting in that they are tri-dimensional. The child is therefore not expected to solve a problem in perspective, but it should be remembered that he still encounters problems, in this case with the mechanisms of construction.

Guanella (1934:17) divides the phases of development through which the child goes through in his construction activities with building blocks, and categorises these under two headings namely (i) space expression and (ii) symbolism.

(i) Space expression

According to Guanella (1934:17), space expression is the ability of the young child to use the materials as a means of self-expression. The young child develops a number of capacities when he uses the materials as a medium of expression. The capacities are: muscular strength, kinaesthetic impression, hand-eye co-ordination, space perception, memory for space impression, space imagination, space feeling or the appreciation of beauty in space relations, knowledge of the building materials (an appreciation of the size and shapes of the individual blocks and the relationship between the individual blocks), knowledge of the adaptability of the blocks to various uses, familiarity with the mechanics of construction, and the ability to construct remembered or imagined space relations (Guanella 1934:17; Reifel 1984:61).

(ii) Symbolism

According to Guannela (1934:18), symbolic representation can be divided into two categories, namely that of form and that of function. Reifel (1984:62) mentions that by the age of approximately three children start to represent blocks symbolically. He also points to the important issue that, when the child symbolically represents things such as houses and cars, the child is also using and representing spatial relationships. Del Grande (1990:14) defines this perception of spatial relationships as the ability to see two or more objects in relation to oneself or in relation to each other. In building with figures or with blocks, children must perceive the position of the cubes or figures in relation to themselves as well as the position of the cubes or figures in relation to each other. The actual construction of figures or buildings enables the child to anticipate the subsequent operations. Pallascio, Allaire, and Mongeau (1993:11) argue that the construction and handling of objects by children are essential if the child is to internalise reproductions in order to comprehend completely .

Reifel (1984:63-65) has found developmental differences in the block construction activities of young children. He divides them into two main stages, namely the early developmental progression in spatial representation with blocks and the later spatial developmental progression in spatial representation with blocks. The early stages are subdivided into: (a) stack, for on (vertical), (b) row, for by (horizontal), (c) stack and row combination, (d) pile, three dimensions with no interior space, (e) enclosure (flat), (f) enclosure (arches), (g) enclosure (combination), and (h) combination of many forms. The later spatial representations are divided into: (a) pile, comprised of objects, (b) pile, one object, (c) enclosure, each form is one object, (d) enclosure, form is more than one object, (e) enclosure (arch), (f) represents interior space not totally formed, (g) enclosure (combination), interior objects placed outside, (h) enclosure, with inside and outside objects separated, (i) objects separated, some sense of scale, and (j) complex, configuration-interior space, landmarks, routes, a sense of scale.

Freudenthal (1991:76) emphasises the importance of tangible materials in the teaching of geometry. He sounds a word of warning, though, namely that children must be allowed to structure the concrete materials for themselves: “ If we condemn the learner to the prison of pre-structured blocks... whether counting blocks or logical we should not wonder why they are not able to put the mathematics learned to good use. The best palpable material that you can give the child is his own body”.

Abels *et al* (1992:98) echo this viewpoint when they state that construction is not an objective in itself but an essential “building block” in allowing the children to progress towards more advanced cognitive activities like calculating and reasoning. They argue that if the construction activity does not expect the children to think while they are busy, it is useless. They are further of the opinion that the children should be free to decide for themselves whether they want to make use of construction activities in solving a specific problem.

4.8 Conclusion

As a whole, mathematical ideas are rich and multifaceted. There is no single path along which a particular culture must develop its specific spatial knowledge. Ascher (1991:186) states that a straight line is far too simple to serve as an image of how these mathematical ideas of the different cultures are related to one another. Ascher (1991:186) suggests that a possible visualisation of the totality of these facets can be compared with “the revolving, lighted, mirror-faceted globe that is suspended from the ceiling of some ballrooms. Each of the thousand or so small mirrored facets is contiguous to some, but widely separated from others. Which facet catches and reflects the light at any given moment depends on where you are in the room”.

In this chapter mainly three instructional facets were highlighted, namely the task that is given to the child, the objects that are used and the dimensions of the objects that are used. The teacher has control over these facets to the extent of manipulating some of

the variables. In choosing the context of the problem, the teacher can make sure that it fits in with the socio-cultural background and worldviews of the specific children. It is, however, sometimes difficult to define precisely what the worldview of a specific group of children is. In setting the task the teacher can differentiate by choosing different combinations concerning the movement of the viewer and the transformation (movement) of the objects. The order in which the different activities are presented to the children is partly guided by the inherent genetic structure of geometry, as well as the individual progress of the different children. The identification of the latter issues depends on subject knowledge of the teacher.

The teacher can also differentiate by the choice of objects and situations. Research projects have shown that young children find the use of noncanonical objects more problematic than canonical objects. This, coupled with the different types of transformations that can be expected when working with the objects (topological, projective and Euclidean), are factors that can influence the degree of difficulty of working with objects.

Another instructional variable that can be manipulated by the teacher is the dimension of the materials that are presented to the children with the ensuing point of view implied. Studies have shown that, in order for children to be able to handle the abstract geometrical concepts, they need to start with the concrete three-dimensional materials that are part of their everyday lives. It has also been shown that very young children find the different axes along which materials are manipulated, varying in the degree of difficulty.

The manipulation of the above-mentioned instructional variables can be induced by the specific use of executional media. The three different media discussed in this chapter are language use, drawing and construction. Each of these media has its own inherent features that allow for the development of different skills. Each of these three media

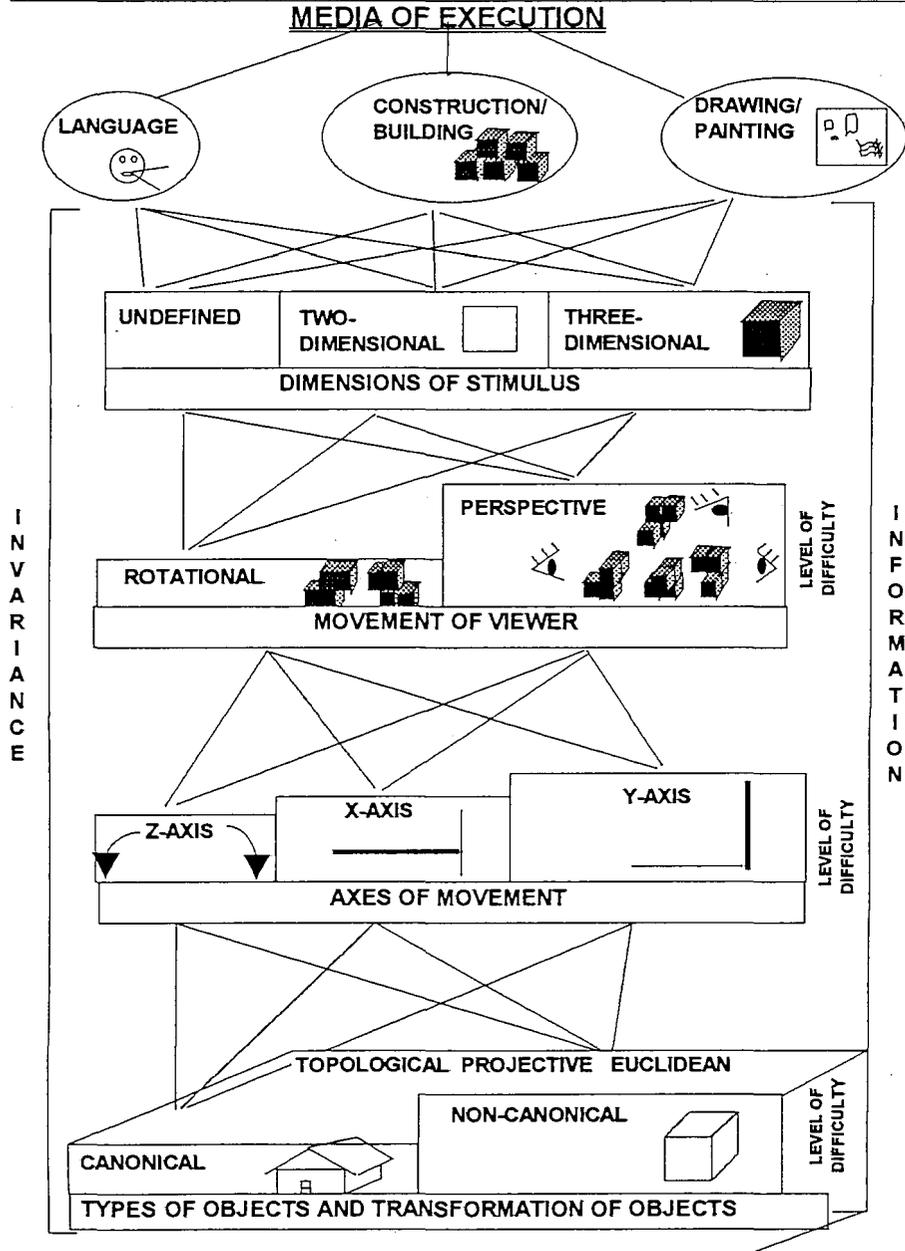
also has a specific dimensional component which is a decisive factor in the development of specific spatial skills of the young child.

The correct use of the different spatial terminologies, as well as the ability to develop the understanding of the different rotational axes, is the outstanding feature of the development of an appropriate spatial terminology. Not only does the use of correct spatial terminology allow the development of more abstract reasoning skills of the child, but it is also an indication to the teacher of the level of development at which young children are finding themselves.

The outstanding feature of communication through the medium of drawing is to overcome the problem of depth-representation. Literature has shown that this is not only a problem that manifests itself among children but it is also common among adults. It has been shown that children go through different developmental stages when it comes to drawing different objects or parts of objects. The use and introduction of conventions in the media of drawing is one of the possibilities of overcoming the depth representation problem. The actual question that needs to be answered is not whether conventions should be introduced but when and how they should be introduced.

Another medium of execution is through the physical building and handling of the materials. It was shown that the skills that are needed and developed through this activity differ quite drastically among the different cultural groups in the world and even between children from the same cultural group. Even though many researchers agree that the media of construction should be the starting point of the activities that are given to the children, caution should be taken when decisions are made about the time of introduction and time of termination of physical materials as part of the development of the spatial sense of the child.

FIGURE 4.11: A DIAGRAM TO ILLUSTRATE THE INTERRELATIONSHIP OF THE DIFFERENT SPATIAL COMPONENTS IN THE TEACHING-LEARNING SITUATION



The interrelationship of a number of the different variables that were described in this chapter are represented in figure 4.11. In the empirical part of the research that will be described in the next two chapters, the aim will be to describe the conditions under which these variables interact, for the different activities that the children engage in.

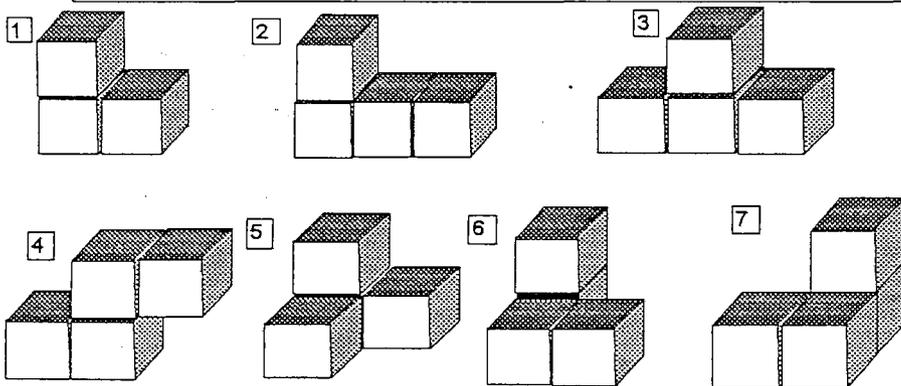
CHAPTER 5

**INVESTIGATION OF THE SPATIAL SENSE OF THE YOUNG CHILD
THROUGH BLOCK BUILDING**

5.1 Introduction

The use of simple wooden cubes to develop the spatial insight of young children has been described by several researchers. According to Browning and Channell (1992:447), the NTCM (1989:112) has stressed the importance of exposing children to experiences which are both at the perceptual and the representational levels. Gaulin (1985:54) has reported on the importance of the development and the need for emphasising various graphical representations of three-dimensional figures (in his work soma cubes were used) when teaching children. Cooper and Sweller (1989:202) have also conducted an investigation into the representation skills of children by making use of soma cubes as three-dimensional solids.

FIGURE 5.1: A SET OF SEVEN SOMA CUBES



The theoretical design of the experiment has been inspired by the work done in The Netherlands and described in the article of Van den Brink (1974:296-301) called "Bouwen met Blokken". This work gives a theoretical guideline to a

hierarchy that can be followed in the development from spatial orientation to spatial insight of young children when working with simple objects like building blocks. The different objectives that are realised through activities with block building are: working systematically, endeavouring to find a system to describe data graphically or numerically; investigating differences and similarities; reasoning spatially from different viewpoints; movement between the different dimensions; and representation of objects in different media.

5.2 Experimental background

The experiment was carried out in a state Primary school in the countryside of South Africa with the permission of the Transvaal Education Department and the parents of the individual children (see Annexures A & B). Twenty-one grade one children (between the ages of six and seven) took part in the teaching experiment. Of the two grade one classes in this school the class that took part in the experiment was considered by the school to be the slower class (see Annexure C, for end of the year report of the progress of the children in mathematics and English). They were also grouped according to their age-group, because all these children only turned seven in the second half of the year. For eleven of the twenty one children English was their second language. This means that more than half the children were not taught in their first language.

These children were divided into three ability groups according to their competence in numbers. The reason for this was that, according to the problem-centred approach that had been implemented in this school since 1992, these children were required to work in ability groups. The groups were formed after the teacher had done a diagnostic evaluation of all the children's number competence and problem solving skills. (The top group had the best number concept, the middle group had average number concept and the bottom group had poor number concept.) It is important to mention that all but one of the children in the bottom

group received instruction in their second language. The groups remained like this for the duration of the experiment with the exception of only two children who were moved from the middle to the top group and one from the top to the middle group. This happened at the beginning of the experiment and was done by the researcher after consultation with the teacher. All the children received mathematics instruction through the problem-centred approach (compare 2.9.4) concerning the development of numbers during their normal classroom mathematics periods. The groups were not changed for both the normal classroom mathematics periods as well as for the experiment for the rest of the year. The teaching experiment only started at the beginning of the second term after these children had completed a compulsory six weeks introductory course as part of the formal schooling curriculum for school beginners. This course focuses on the perceptual abilities of the children and it includes activities for the development of fine motor skills, grossmotor skills, visual skills, auditory skills, spatial skills and form perception.

Each group received instruction for one hour per week. The researcher planned and presented the classes and all the instruction periods were video-taped for analysis and transcribed afterwards. The transcription of the data can be subdivided into three main categories namely (1) data collection, (2) data interpretation and (3) materials design.

5.2.1 Data collection and interpretation

The following method was adhered to:

All the physical materials (drawings, written calculation, and models that were built) generated by the children were collected and categorised together with the transcription data. The video materials were transcribed in three different ways:

(i) Verbal transcription

The **actual conversations** of the participants during the activities were written down or field notes were made of the content of the discussions (compare figure 6.2).

(ii) Construction activities transcription

The construction activities concerning the utilisation of the different **equipment**, as well as the **physical solution strategies** that were employed by the children while solving the problem were written down.

(iii) Pictorial or written material transcription

All the activities that involve the **process** of drawing and writing during the classroom activities are written down (compare figure 6.3), in other words, not only the end product but the whole process that the child engaged in, from the moment that the drawing was started until the completion of the drawing.

All the data that had been gathered were interpreted qualitatively (e.g. figure 5.20) and quantitatively (e.g. figures 6.8, 6.9, and 6.10) depending on the nature of the data. After the interpretation of the data the materials and activities for the next period were designed.

5.3 Introductory activities

The children were given a variety of activities (see table 5.1) at the start of the experiment in order to get an indication of their capabilities. After analysing these activities it was possible to start with the actual experiment. This part of the experiment was very important because there were no guidelines as to what

children in South Africa are capable of when it comes to activities of this kind. It should be stressed that this was a very essential part of the experiment and it took up a substantial quantity of research time. Throughout the experiment the activities are indicated by means of a numerical number which only reflected the order in which the different activities were performed. In some cases more than one activity was done in one period (every period lasted about one hour). The different groups determined their own pace as will be discussed later on.

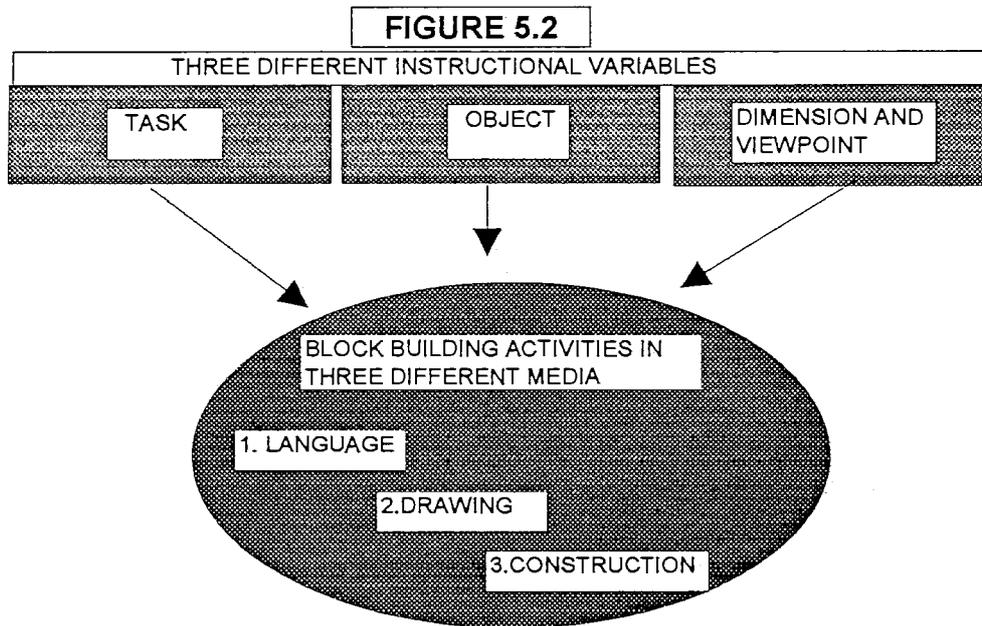
After the analysis of the results of the introductory activities, the final activities were planned (table 5.2). It was not possible to plan more than one period ahead since the results from each period determined the activities for the next period.

INTRODUCTORY ACTIVITIES

ACTIVITY	OBJECTIVES
<p><u>Activity one:</u> All the children were given soma-cubes. Each child's construction was inspected by the other children in the group and they discussed what were represented.</p>	<p>To give the children time to familiarise themselves with the cubes but also to find out to what extent they were able to verbalise what they thought.</p>
<p><u>Activity two:</u> The children had to look at physical structures that were presented to them and then rebuild them with loose blocks.</p>	<p>Their construction skills were investigated as well as their ability to copy a three-dimensional structure.</p>
<p><u>Activity three:</u> After the children had built their own structures with four loose blocks, they had to draw them on paper.</p>	<p>To investigate the children's familiarity with representation of a three-dimensional object in two dimensions.</p>
<p><u>Activity four:</u> The children had to select one soma cube from their set of seven. They had to hide it from the other children in the group and then describe verbally what the hidden soma cube looked like.</p>	<p>To investigate the children's knowledge concerning their position in space, orientation in space, as well as the use of a specific number of blocks in other words their linguistic skills as well as their skills to handle more than one variable.</p>
<p><u>Activity five:</u> The children had to look at pictures of the soma cubes and select the corresponding cubes from their own sets of seven individual pieces.</p>	<p>To investigate the children's abilities to interpret a two-dimensional representation and to recognize its equivalent in three-dimensions.</p>

5.4 Final activities

The final activities were designed and executed while keeping the cyclic nature of the developmental approach in mind (compare 3.2.3). The results and discussions for the block building experiment are discussed under the three different representational media (compare 5.5-5.8). Although table 5.2 gives the actual order in which the activities were performed, the discussions will not be presented in that order. Language as the media of instruction will form the first main heading. The influences of the three instructional variables namely task, object and viewpoint and dimension will be discussed as they manifests themselves in this media. This will be followed by the discussion of the results in the drawing media followed by the construction media.



ACTIVITIES	MATERIALS
1.1 Movement of children into different positions on instruction from the teacher and one another. The children had to move around different objects like tables and chairs.	Chairs and tables.
1.2 Look at the position of the block indicated by the arrow in figure 5.3 and describe where it was situated.	Figure 5.3.
1.3 Construct a building of the previous structure and point to the block as indicated with the arrow in the figure.	Figure 5.3.
1.4 Draw the structures of the pictures and buildings from the previous activity.	
2.1 Use three loose blocks and by fitting them side by side with no gaps or holes in between, build a structure and describe it verbally to the group. Figure 5.2 was used to illustrate what was meant by "holes" and side-to-side".	Figure 5.7.
2.2 Look at the structure that has just been built from the verbal description and rebuild it by copying the existing one.	

2.3 Draw a picture of the physical structures in the previous activity.	
3.1 By making use of figure 5.4, rebuild the structures as indicated in the figure using three loose blocks.	Figure 5.4.
3.2 Draw every completed structure.	
4.1 Use four loose blocks and construct a structure and describe it to the group.	
4.2 Look at the previously described structure and rebuild it.	
4.3 Draw the buildings by using the physical structures as models.	
5.1 Use figure 5.5 and build the structures with blocks.	Figure 5.5
5.2 Describe the structures in figure 5.5 verbally to the group.	
5.3 Draw the structures built in the previous activities.	
6.1 The physical pieces of a set of seven soma cubes are shown to the children and they are requested to select the corresponding pieces.	Soma cubes
6.2 Describe the different pieces verbally to the rest of the group.	

6.3 Rebuild every soma piece by making use of loose blocks.	Loose wooden blocks.
6.4 Use figures 5.29a, 5.29b and 5.29c to construct the "soma-cube-puzzle".	Figures 5.29a, 5.29b and 5.29c
7.1 Rebuild the structure in figure 5.30 that is given at the top of the page	Figure 5.30
7.2 Discuss the different outcomes.	Figure 5.30 (the rest of the pictures on the page).
7.3 Construct the physical structures that are represented in figure 5.28.	Figure 5.28
8.1 The children are given grid-paper and soma cubes and asked to communicate the pictures by making use of numbers or drawings to describe the physical structures.	Figure 5.23 Figure 5.26

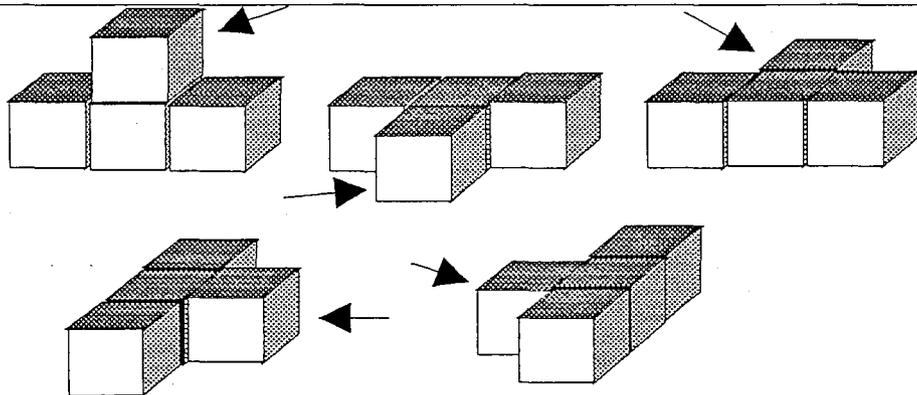
5.5 Language

5.5.1 Results

The language activities in which the children were engaged can be divided into two areas, namely the language used for pure communication purposes and the subject specific language. The discussion which follows will focus mainly on the language utilised in connection with spatial matters *per se* even though the two are interrelated in practice. The former which constituted, together with all the classroom activities, the classroom culture (compare 2.6.4), was constantly taken into consideration in the analysis of all the events. The results were gathered in the form of written transcriptions of the discussions the children engaged in on the video-recordings (compare figures 5.6, 5.8, and 5.12).

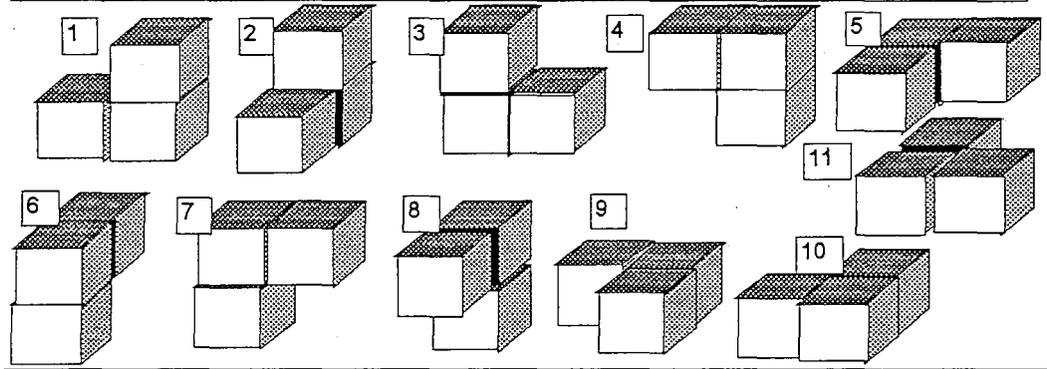
The activities that dealt specifically with language as expression medium were activities 1.1, 1.2, 2.1, 4.1, 5.2, 6.2, and 7.2, (table 5.2). During activity 1.1 it was required of the children to link the different spatial terms with different positions in three-dimensional space using their chairs (canonical) as objects to work with. During this activity a range of terminology was used namely, “on top” “behind”, “underneath”, “to the left”, “to the right”, “in front”, and “at the back”. It already became clear that most of the children in the bottom group were having difficulty with terms like “to the left”, and “to the right”.

FIGURE 5.3: DIFFERENT POSITIONS OF THE FOURTH BLOCK ARE INDICATED BY THE ARROW



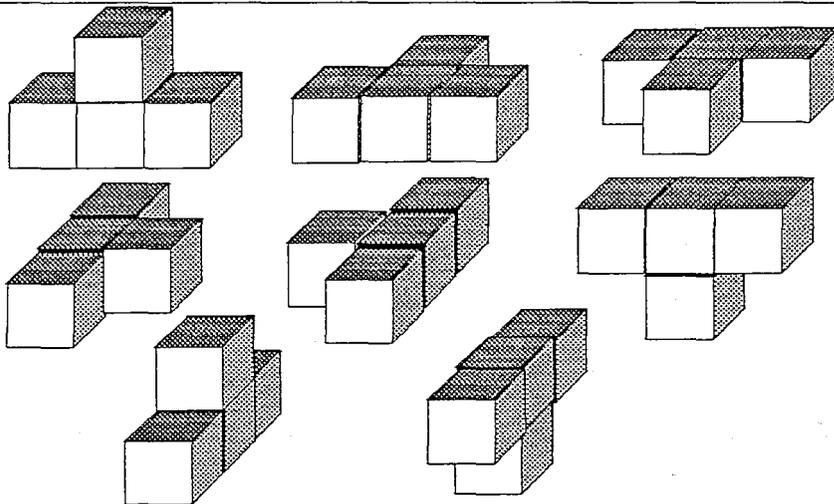
Some of the children in the middle and even in the top group (the second language children) were able to physically do the correct activity, but they could, however, not give (verbalise) the correct instructions using the correct spatial terminology. Some of the children in the bottom group could not do the correct activities when instructed in English. It was explained to the rest of the group in their own language and only two children still had difficulty with the terms “left” and “right”. The other terms did not seem to pose a problem.

FIGURE 5.4: DIFFERENT ORIENTATIONS OF SOMA CUBE 1



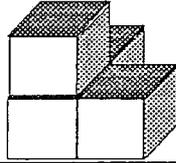
During their descriptions of the different positions of the blocks in the structures as presented in figures 5.3 and 5.4 the children had to make use of the words “on top”, “at the back” “at the bottom” and “in front” (see figure 5.5 for the actual range of positions that were presented to the children).

FIGURE 5.5: DIFFERENT ORIENTATIONS FOR SOMA CUBE 3



Most of the second language children left out the “on”, “at”, and “in” respectively. They also substituted “on top” with “up”, and “at the bottom” and “behind” with “down”. They reasoned that down indicated that the block had fallen down.

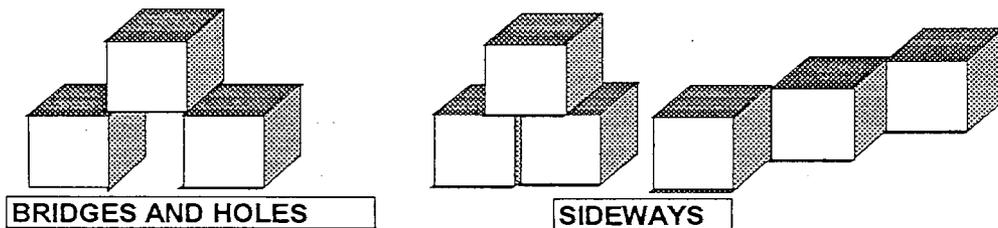
FIGURE 5.6: VERBAL EXPLANATION OF THE POSITION OF THE TOP BLOCK



Child one: "The one block is **down** and another is **up** and another is **down** and another is also **down**".
 Child two: "How many are at the bottom and how many are at the top".
 Child one : " One is at **top** and three are **down**".

From the beginning of the experiment it was agreed between the researcher and the children that the structures to be built should not have "holes", or "bridges" and the blocks should fit side by side with no overlapping of the sides (compare figure 5.7).

FIGURE 5.7: BUILDING RESTRICTIONS OR SPECIFICATIONS



There was no confusion, though, for any of the children, as to the physical position of the blocks which were indicated by the words "top" and "bottom", regardless of the dimension of the stimulus. In other words, they indicated the words "on top" correctly in both the physical building (3-D) and the pictures (2-D) of the structures (compare figure 5.8).

FIGURE 5.8: VERBAL DISCUSSIONS ABOUT THE TOP OF A STRUCTURE

The teacher, pointing towards the top block in a physical structure that she has built, asks: "Where is that block?"

Child one: "Up top."

Child two: "Top."

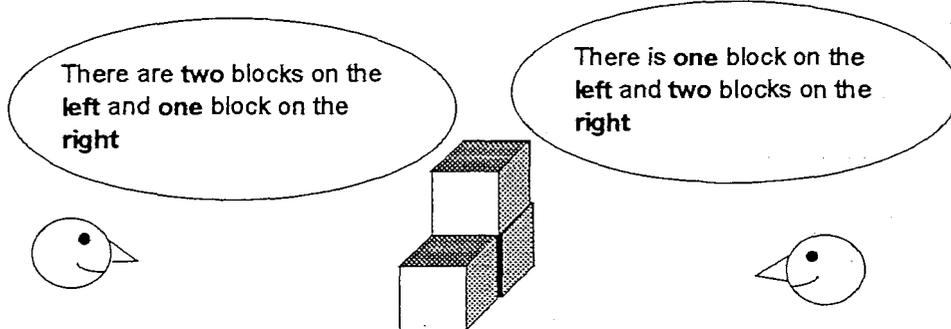
Child three: "At the top."

Child four: "It could be above."

Child five: "On top."

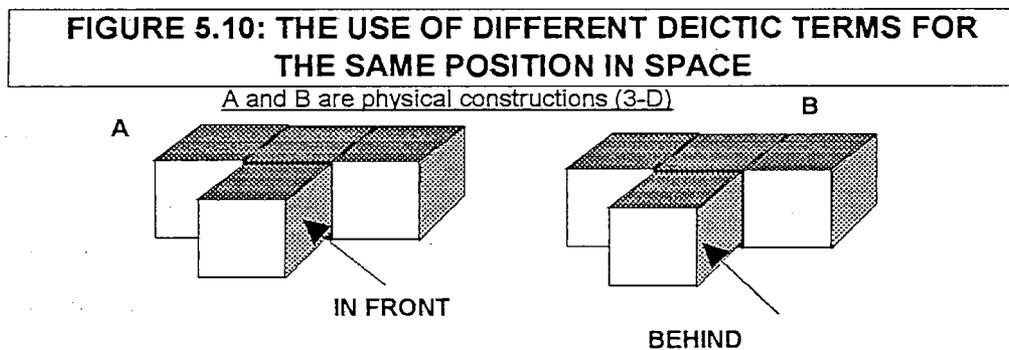
The situation with "left" and "right" was also a problem, especially when the children who did the description were facing one another. This situation was not only limited to second language speakers. The children were faced with the situation that "left" and "right" are reversed if children sit opposite one another. This situation was thoroughly worked through in the classroom situation in the sense that the children eventually came to the conclusion that it was not one's position in space which determined "left" and "right", but the actual direction in which one was facing (figure 5.9). Many of the children also used their body parts (right hands) to guide them through this experience.

FIGURE 5.9: LEFT AND RIGHT ARE DEPENDENT ON THE DIRECTION IN WHICH ONE IS FACING.



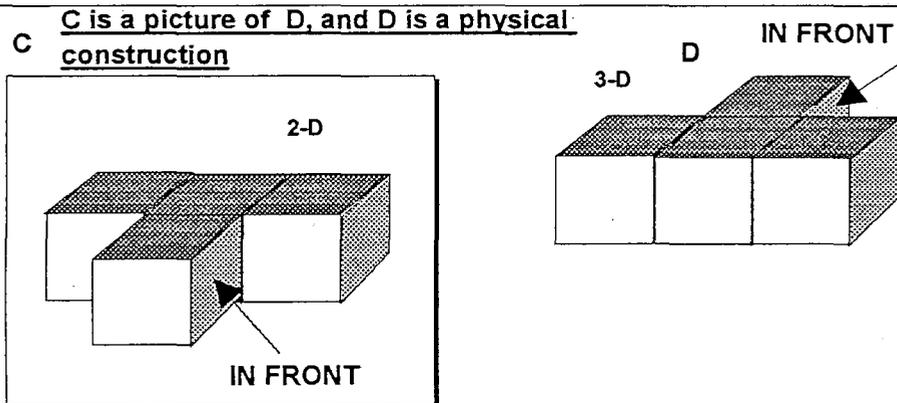
It was a different situation, though, for the use of "in front" and "behind". In figure 5.10A and B all three groups of children gave the blocks which were indicated by arrows different linguistic positions. Both first and second language speakers were switching the terms around. During the discussions the children who called the block in figure 5.10A "in front", explained that they did it on the

grounds that that block was in front of them (ego space, compare 4.5.2). Some of the children who called the block in picture 5.10B “in front”, indicated on inquiry from the researcher that they used the three horizontal blocks as reference points and stated that the marked block was in front of the horizontal three blocks (compare 4.7.1.1.3). Some of the children also said that the block in figure 5.10B was in front because the children who were looking at it (the children were all sitting in a circle) from their positions in the circle, were seeing it as being “in front” (compare 4.7.1.1.3).



After a lengthy discussion and debating session about the actual meaning of the terms, it was agreed among the children that something which is partly obscured could be called “behind”. It was also discussed why it was important that everybody should actually agree on using the same terminology. It was agreed among the children that, in order to communicate effectively about certain aspects of a structure, it is necessary that everybody uses and understands the same terminology (see section 4.7.1.2.3 on the role of conventions in mathematics).

FIGURE 5.11: THE USE OF THE DEICTIC TERM "IN FRONT" FOR DIFFERENT POSITIONS IN SPACE

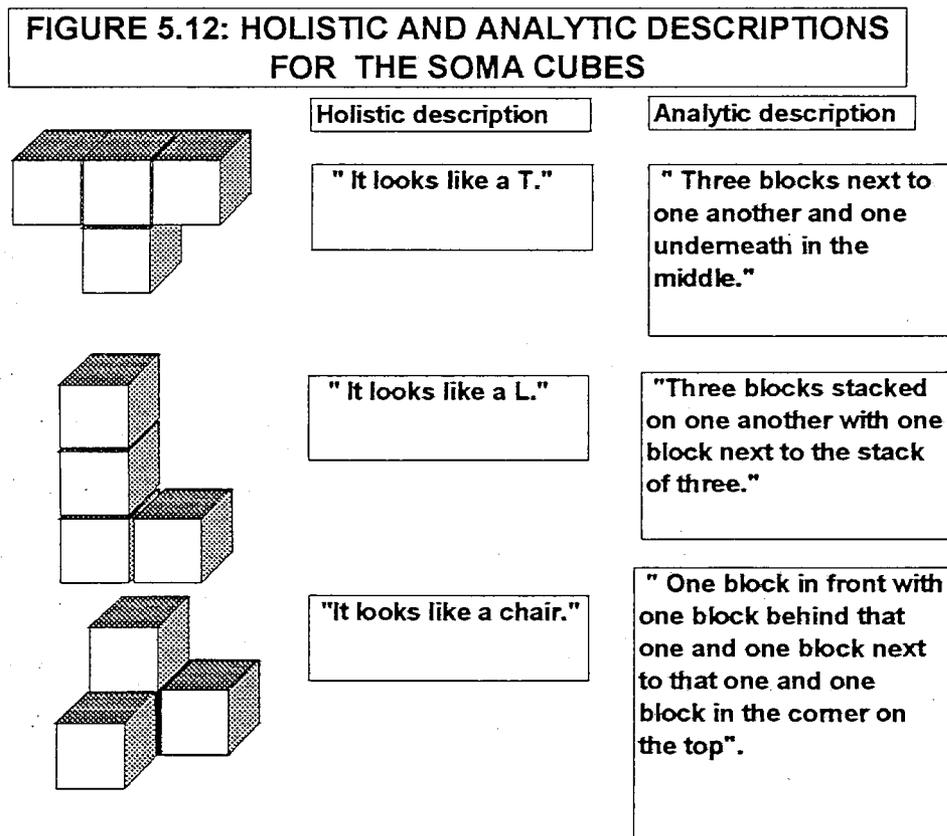


The children were exposed to three-dimensional representations (figure 5.11C) as well as two-dimensional representations (figure 5.11D) of the same objects throughout the experiment. In activities 1.1, 2.1, 4.1, 6.2, and 7.2 the dimension of the stimulus was always three-dimensional, whereas the dimension of the stimulus in activities 1.2 and 5.2 were two-dimensional.

Two different situations presented themselves during these activities. Firstly the children differed when it came to ascribing the deictic terms “in front” and “behind” to the same physical structures. Figure 5.10A and B show how children nominated the blocks which have the same positions as indicated by the arrows, “in front” and “behind” respectively. Secondly, when the same structures were presented in two dimensions (Figure 5.11C and D), the same child (children) who identified the block indicated by an arrow in figure 5.11D to be “in front”, nominated that the block in figure 5.11C indicated by an arrow, to be “in front” as well!

During activity 6.2 the children had to verbally describe to the other children in the group, what the different soma cubes (figure 5.1) looked like and the friends then had to select the corresponding piece from their own sets.

At this stage the majority of the children were using the different terms in such a way that all the children could understand what they were saying. The children used two types of descriptions. The first one was a holistic description where the children equated the overall shape of the soma with a known shape. For example soma number 3 was called the "T", soma number 2 was called the "L" and number 5 was called "a chair" (see figure 5.12). The children ran into difficulty, though, with the other soma cubes which did not have distinctive forms to which they could relate. Not all the children reverted to descriptions of this kind but they resorted to describing it in a more analytical way.



It could be called a type of analytic description in the sense that the children actually described how such a shape should be put together. During this exercise it was still clear that some children in the bottom group were not sure of "left" and "right". From this discussion it also became clear that most (all three groups) of the children had difficulty distinguishing between somas numbers 6 and 7 (figure

5.1). Many children could describe the soma cubes verbally and then indicated that they were the same.

Activity 7.2 involved discussions about the possibility of having hidden structures behind the structure which was shown in figure 5.28. The discussion continued as to how many different structure would be able to fit behind the given structures before it could be seen by the observer.

5.5.2 Discussion

5.5.2.1 Task

One of the first problems which surfaced during the initial work with these children was trying to distinguish whether they merely had a language-translation problem or a "concept" problem. Messick (1988:14) describes this problem by stating that children's comprehension of spatial terms is influenced by their conceptual understanding of the terms. Initially their juvenile concept may include variables (i.e., size of the stimuli) which are not relevant in the adult concept of the term. Throughout the design of the activities it was the objective to clarify what those variables were which influenced these spatial concept formations. If the children had only indulged in activities where the medium of response was linguistic, it would have complicated the matter, but during this entire experiment the different media of expression were utilised in order to clarify the above-mentioned situation. It is believed that the different types of media utilise and develop different skills (Olson, 1974:16).

At the beginning of the activity 1.1 it was unclear whether the children who had difficulties with the terms "on top", "at the bottom", "in front" "behind", "left" and "right", were only having a language-translation problem, or whether they also had a conceptual problem. As the activities progressed it became clear from their

discussions that the majority of those children merely had a language problem concerning the designation of “on top” and “at the bottom”. This was substantiated by the fact that the children could identify “top” and “bottom” when they were given the equivalent terms in their own languages.

For all these activities where the children had to look at pictures or three-dimensional structures and then describe them, the type of task was merely a rotational one (see section 4.4.1.2). In most of these cases the children had little difficulty. In the cases where the children had to move around their objects in their “minds eye”, or put themselves in another child’s position (a perspective problem), the children in the bottom and middle groups indicated that they had difficulties with the spatial terminology, especially “left”, “right” “in front” and “at the back”.

5.5.2.2 Object

Messick (1988:15) stresses the influence of the object on the acquisition and use of spatial terms by stating that children use different kinds of contextual information to differentiate among spatial terms, including the physical characteristics of an object, and the typical use or function of the object. Cox (1986:168) is of the opinion that “behind” is used and understood earlier than “in front of”. She thinks that the reason for this is because “behind”, rather than “in front of” conveys more information about the location of hidden objects.

Some children saw the objects as being canonical with respect to “top” and “bottom” because they argued that if the object can stand on the floor, then the part touching the floor was the bottom. In some cases they also gave the objects canonical status if the structures were asymmetrical. The parts to the left of their own bodies were then seen as the left and the parts on the right of their bodies were then seen as their right. In other words, they used their egos to assign these

specific properties. This would mean that if they changed the direction in which they were facing, these attributes would also change.

The complexity of the objects also influenced their problem-solving abilities. In the case of the number 6 and 7 soma cubes, most of the children had difficulty in either recognising that the structures were different, and those who had recognised them, had difficulty explaining why they thought them to be different.

5.5.2.3 Dimension and viewpoint

The different points of view of speaker and listener are marked in all languages by deictic terms, "I" and "you", "here" and "there" and so on. The referents of these terms change as different people take on the roles of speaker and listener (Cox, 1986:137). This was clearly a major problem for many of the children. Many of the children, regardless of language status, had a problem with the concepts of "left" and "right". Olson (1975:91) mentions that the concept of laterality is achieved only after verticality and frontality.

The fact is that in some cases the type of movement (compare 4.4.1.2) which was involved in solving the problem also seemed to influence the outcome. The children had difficulty imagining how they would move around to their friend's positions and see from there. Some of them could only solve the problem if they physically moved over and took in the position of their friends. This corresponds with the findings of Olson and Bialystok (1983:151) that, if the type of movement which is required to solve a problem, is more a perspective one than a rotational one, then children find it more difficult to solve.

The results with the designation of "in front" and "behind" were mixed for the different groups. It became clear from the discussions which the children had amongst themselves, that there were a number of variables which influenced their

choices and the contributions of each variable varied from child to child. These findings are echoed by several researchers, namely that the appropriate term is influenced by contextual characteristics (e.g. Bernstein, 1984; Harris & Kolch, 1985; Clark, 1973, in Messick, 1988).

From the explanations of the children it followed that they used different viewpoints as their reference points. It followed that ego, environment, object type as well as the dimension of the stimulus, had an influence on this assignment and the subsequent use of the different terminologies. In some cases when the stimulus was on paper (2-D), the block facing the child was called "in front" because of the influence of object space, while in the three-dimensional equivalent the environment or the object dictated the determination of the specific space (figures 5.10 and 5.11).

Cox (1990:481) has found that children from 4 to 9 years, generally treat these spatial locatives "in front" and "behind" in a non-deictic way when a referent object with an obvious front is involved. In other words they use the word "front" if the object has a definite front regardless of the position of the observer. Cox has further found that if the viewpoint of the observer is emphasised, the children will use these terms in a deictic manner. This implies that the front of the object is determined by the observer and not by the intrinsic features of the object. Cox (1990:488) concludes that the deictic and non-deictic interpretations (see section 4.7.1.1.3) are actually in competition and that the children are split between the two possible responses. She states that an important aspect that might influence this uncertainty can be ascribed to the actual use of the different locatives during the instruction of the task.

Once the groups had reached consensus on the use of the different spatial terminologies, the biggest difference between the groups was that the top group was able to complete the same exercises far quicker than the middle and bottom

groups. As the experiment progressed the ease with which the second language children expressed themselves, increased noticeably.

The appropriate use of spatial terms involves the gradual acquisition of abstract relational concepts as well as knowledge of the influence of contextual factors. Another medium in which these contextual features can be expressed is the medium of drawing.

5.6 Drawing

Throughout the experiment the children were asked to draw the structures which they had built. There was no specific mention as to the viewpoint that should be taken or any cues as to how the cubes should be drawn. At the end of every period all the drawings were gathered and analysed. In some cases the researcher discussed the drawings with the children in order to find out what they meant with some of the components.

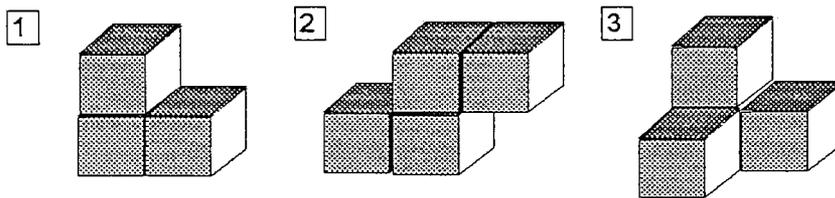
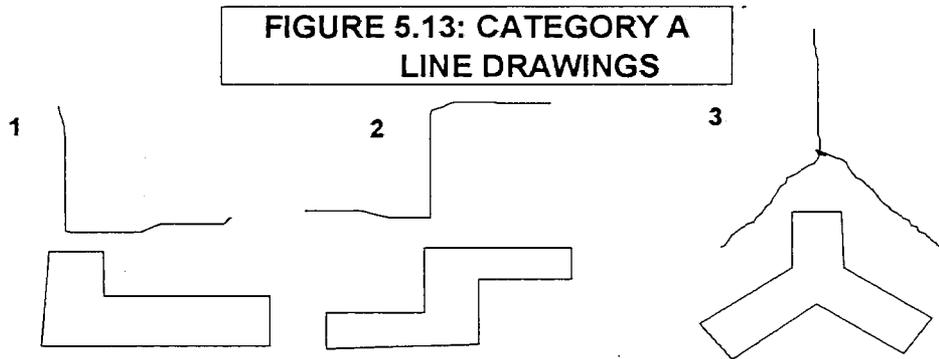
5.6.1 Results

The following activities involved drawing: 1.4, 2.3, 3.2, 4.3, 5.3, and 8.1, as described in table 5.2 The drawings which the children made are represented in table 5.20 and are divided into seven categories. The results were gathered by analysing the actual drawings which the children made. In some cases the children were asked to explain or clarify some of the drawings that they had made.

5.6.1.1 Coding of drawings

(i) Category A (line drawing)

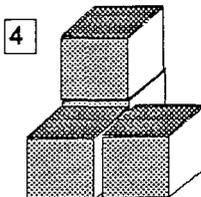
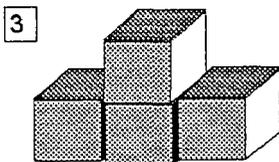
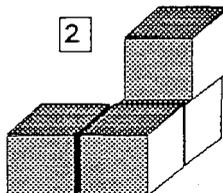
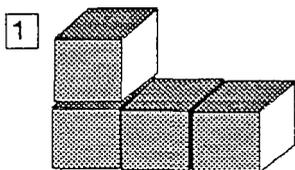
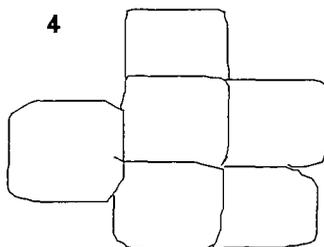
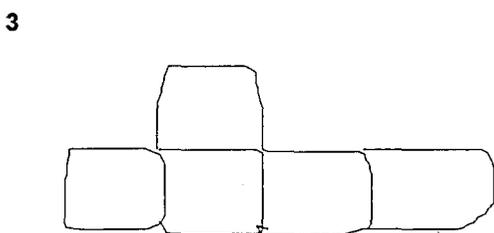
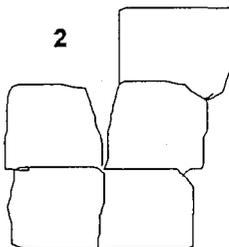
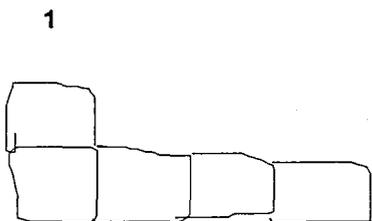
The child has drawn the basic overall shape of the structure without any detail as to the specific individual components of its construction.



(ii) Category B (erroneous number)

This is a front view depiction. The shapes of the individual components are correct but the *number* of components as well as the orientation of some of the components are *incorrect*.

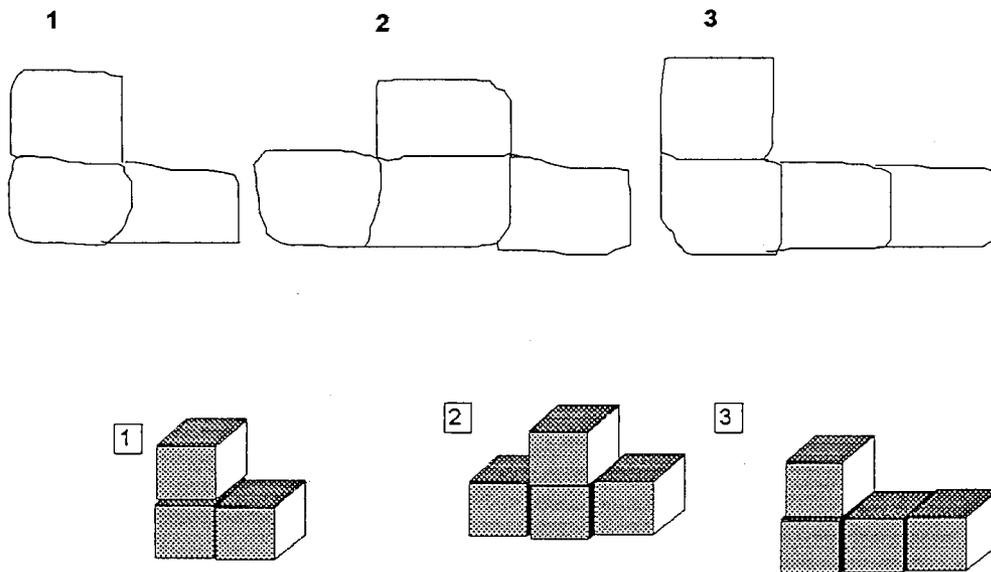
**FIGURE 5.14: CATEGORY B
ERRONEOUS NUMBER**



(iii) Category C (frontal-view)

This is a front view depiction. The shapes of the individual components as well as the correct number of individual components are drawn. The result is mostly a front view of the structure.

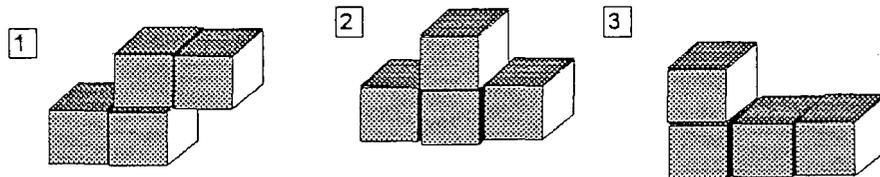
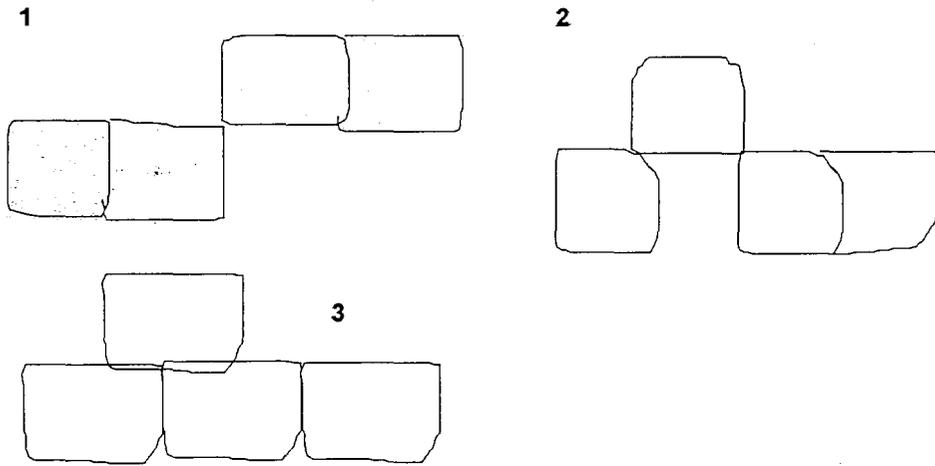
**FIGURE 5.15: CATEGORY C
FRONTAL VIEW**



(iv) Category D (defective orientation)

It is mainly a front view depiction. The shapes of the individual components as well as the correct number of components are given, but the components are in the wrong orientation.

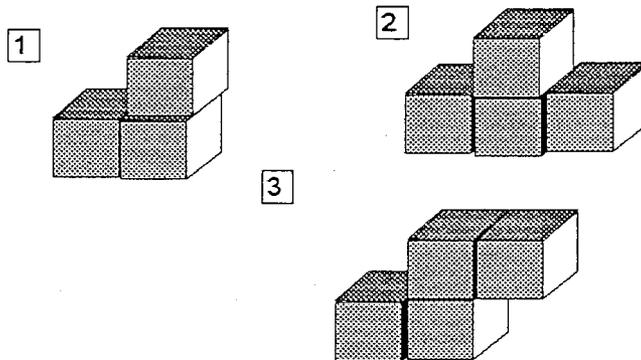
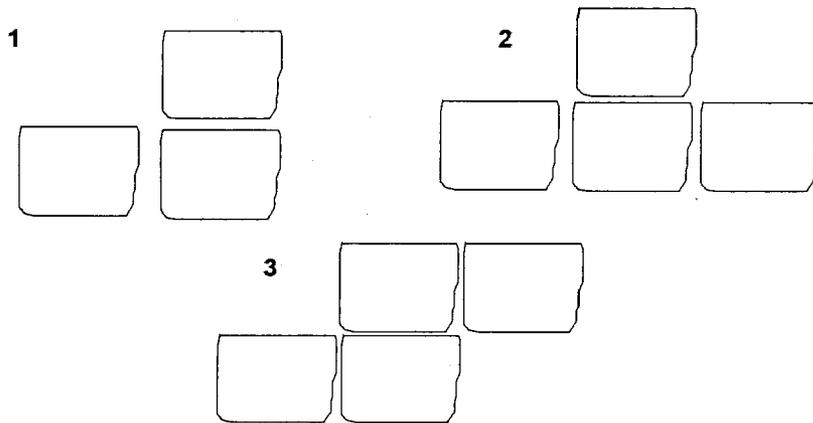
FIGURE 5.16: CATEGORY D
DEFECTIVE ORIENTATION



(v) Category E (disjunction)

It is mainly a front view depiction. The individual components are presented in a “disconnected” way. In other words, the individual components are presented as if they are not connected to one another and there are gaps between the individual components.

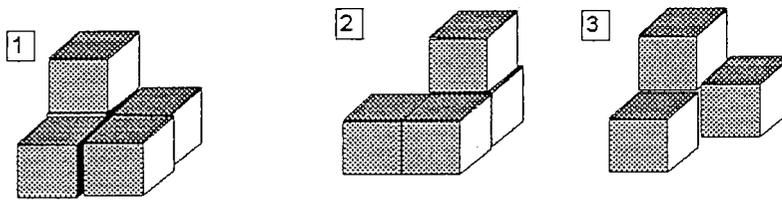
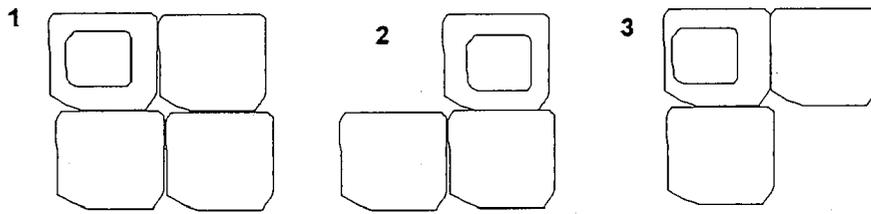
**FIGURE 5.17: CATEGORY E
DISJUNCT**



(vi) Category F (Partial occlusion)

It is a top view depiction. It is an attempt to illustrate the position of the hidden block underneath the top block, which is out of view.

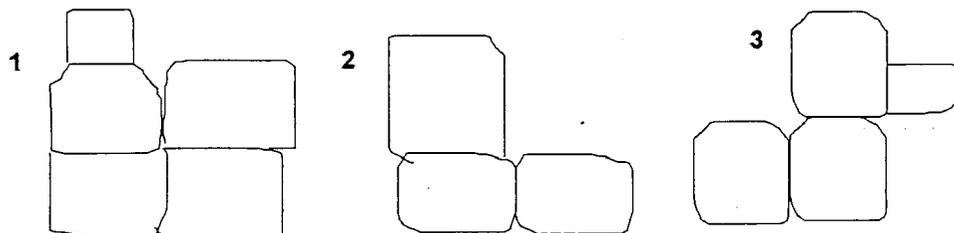
**FIGURE 5.18: CATEGORY F
PARTIAL OCCLUSION**

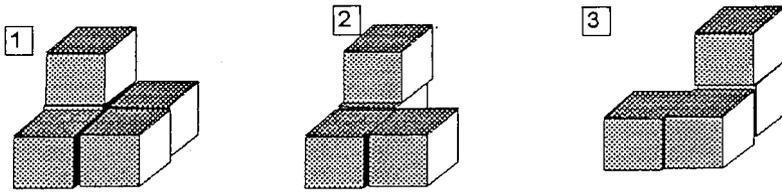


(vii) Category G (height-in-picture)

It is an attempt to illustrate depth by changing the height of the blocks.

**FIGURE 5.19: CATEGORY G
HEIGHT-IN-PICTURE**





(viii) Category H (Attempted dimension)

The first attempts are made to show some kind of dimension in the drawing of the different objects. In the majority of the cases only parts of the objects are shown in dimension.

**FIGURE 5.20: CATEGORY H
ATTEMPTED DIMENSION**

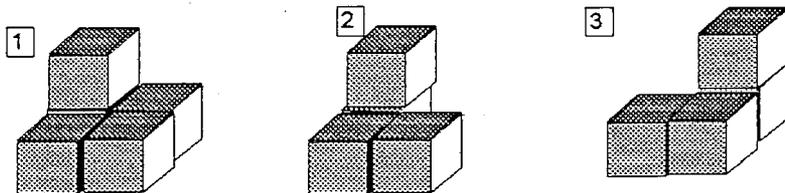
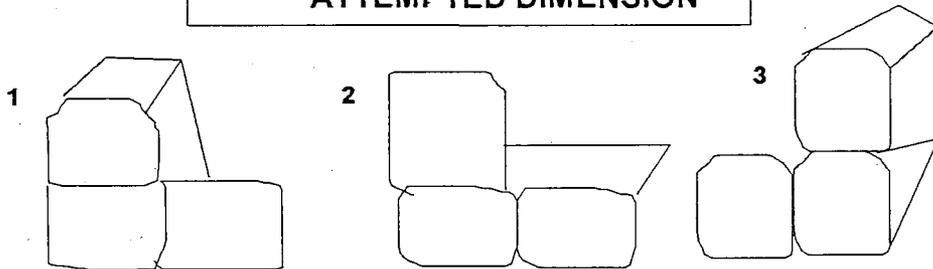


Figure 5.21. is a schematic representation of the types of drawings in each category for the different three-dimensional block constructions. The categories should not be regarded as an order of hierarchical development. What is interesting

though is the fact that the child who attempted to draw the dimension into the pictures was also the top achiever at the end of the year and the children that drew the disjunct figures falling into category E were children that were recommended for extra help.

The drawings which are categorised as line drawings (category A) were only made by three children right at the beginning of the entire research project (one child was in the top group and the other two in the bottom group). It was also noted that the same children made them for the different figures up to number 5 only. These same children though did not make any line drawings for figures 6 to 8, but reverted to other categories of representation.

The majority of the pictures drawn by the children can be placed in categories B and C (figures 5.21). The children who made category B drawings made mistakes with the number of objects which were represented in their drawings. These children were mostly in the bottom group and they were also having problems with their number work during the number development periods. The children attempted to make front view drawings of figures 5 to 8 in this way but they all reverted to other forms of representation for figures 5 to 8.

Drawings which can be classified under category D were mostly made by children in the bottom group as well as by the weaker children in the middle group.

The disjunct representations (category E) of the figures were made by 3 children in the bottom group and 4 children in the middle group. At the end of the year, 5 of these 7 children were allocated to the aid class on the grounds of their overall poor academic performances.

Drawings that fit into category F were only made for objects 5 to 8, shown in figure 5.21. The children who resorted to this way of representing hidden objects,

were all children who were being taught in their second language. The children were from all three ability groups. These children also used this way of representing the hidden objects for objects 5 to 8.

The drawings which are represented in category G are representations of the figures where depth is illustrated by means of the difference in the heights of the individual components of the figure. Representations of this type were made by the majority of the children in the top group. Only one child had attempted to make drawings that fall into category H.

FIGURE 5.21: DIFFERENT DRAWING CATEGORIES FOR 3-D STRUCTURES

FIGURE	A	B	C	D	E	F	G	H
1								
2								
3								
4								
5								
7								
8								

5.6.2 Discussion

5.6.2.1 Task

During the different activities the children were asked to “make drawings of the block buildings you see”. The children did not at any stage indicate that they were unsure about the task that was set before them. All their drawings were made by looking at three-dimensional models and by then drawing them. In all the cases children were required to make drawings while utilising the three-dimensional objects which were in front of them. They never had to imagine what the complete structures looked like and they were allowed to rotate the objects any way they wanted to.

Cox (1986:65) found that young children interpret the request to “draw all that is seen”, as to draw all visible objects. Bremnen & Batten (1991:389) have indicated that if the children were asked to do their best drawings, they included the hidden objects. According to these authors it is quite possible that instructions which ask for the child’s view, lead to omission of entirely hidden objects. Bremnen & Batten (1991:376) report that the degree to which viewpoint is explicitly requested has been shown to effect the number of view-specific drawings produced by young children. Bremnen and Batten (1991:376) interpret these findings as being indicative of the fact that the way in which the task is put, could influence the outcome of what children would draw.

The children who took part in this experiment drew pictures which fell into different categories and from their discussions it followed that the task *per se* was not guiding them into making only one category of drawing.

5.6.2.2 Object

The type of objects that the children had as models were combinations of cubes fitted together. Although the different objects consisted of noncanonical cubes, the children handled them as if they were canonical objects. They all allocated definite “tops” and “bottoms” to the objects according to their ability to “stand” on the floor. The fact that the objects 1 to 7 (figure 5.21) were introduced as having a specific name, namely “soma cubes”, might also have led to them being handled as canonical objects when it came to drawing them.

During their discussions the children made definite comments about the “difficulty” of some of the objects. For the first 4 objects the children indicated that they only needed to deal with two dimensions namely the horizontal and the vertical ones in their representations. Soma cubes number 5 to 8 had an added depth dimension and they had lots of discussions amongst themselves as to how to solve this problem when it came to representing them on paper. For the first four figures there were no contributions to categories F and G. The majority of the contributions for these figures fell in categories B and C. Cox (1986:69) has found that when an object like a cube is hidden by another object placed in front of it, then most children assume that the cube is not meant to be drawn. She also found that, in cases where one object is occluded by another, children are more likely to draw what they see if there is a definite contrast between the two objects in the scene. Conversely, the more similar the two objects are the less likely the children are to draw a visually realistic picture.

During the discussions all the children were happy with their use of a square to represent a cube. During the entire experiment not one of the children indicated dissatisfaction with this representation and not a single child tried to represent the individual cubes further than stage one of Mitchelmore’s (1978:233) classification for the drawing of a single cube.

The children in the bottom group were not at first disturbed by the fact that their representations did not show the correct number of individual components. Cox (1986:70) found that when there is no necessary structural link between the objects, most children, like adults, will omit the object they cannot see and only draw the one they can see. The children in the bottom group seemed to be more concerned about the overall shape. As the experiment progressed, though, they became more concerned about the number of individual components. According to Bremner & Batten (1991:389), the number of cubes that children draw is a measure which effectively indicates the degree to which children use total occlusion. All the children in this experiment, though, had difficulty drawing the correct number of individual components of objects 5 to 8, even though they knew what the correct number of components was. This stresses the fact that children might sometimes represent objects with the wrong number of sub-components, not because they are not aware of them, but because they do not have the capability of representing it in two dimensions or alternatively that they want to emphasise another attribute, namely the overall shape of the objects.

Arnheim (1970:211) stresses that numbers are a relative late acquisition of the mind and they are not necessarily the best instruments for describing, understanding or dealing with objects or situations that involve quantity. A shape of a geometric figure or pattern may be known, recognised, and reproduced without any awareness of the number of elements it contains.

5.6.2.3 Dimension and viewpoint

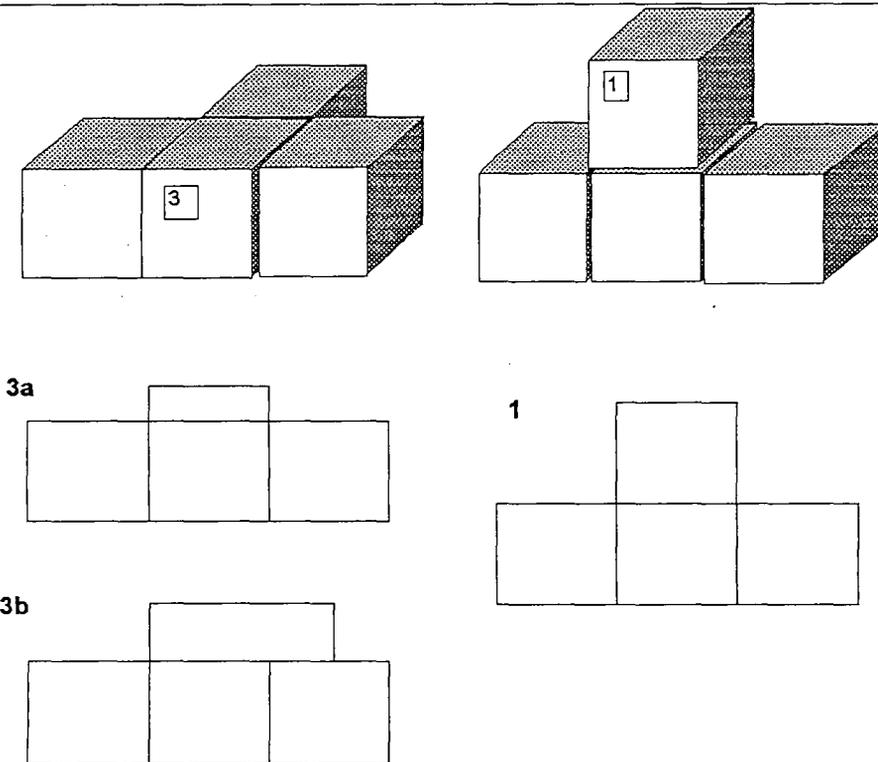
The issue of viewpoint portrayal is closely connected with that of depth portrayal, since in order to indicate viewpoint it is necessary to show the depth relations that exist in the scene relative to the child's position (Bremner & Batten, 1991:376). During their discussions and activities all the children indicated, that they found objects 5 to 8 more difficult to draw. They all indicated that the blocks that were

“deeper into the objects” (the third dimension of these objects), had to be portrayed in another way. Most of the children attempted to draw representations that fell either within category F or G. This was never an issue with the first four more “simple” objects. Arnheim (1970:54) points to the fact that to see an object in space is to see it in context. More generally, it means to see in relation, and the relations encountered in percepts are not simple. The appearance of anything in the visual field was shown to depend on its place and function in the total structure and to be modified fundamentally by that influence.

During the children’s discussions they also started to mention the fact that looking from different viewpoints could lead to different representations of the same object.

During activity 5.3, where some number 3 was presented in different orientations in two dimensions, not one of the children indicated through their drawings that there was an actual difference between the way in which figure 5.22(1) and figure 5.22(3) should be represented in drawing, even though they had indicated through their physical constructions and their discussions that there was a difference in the actual way in which these objects were orientated. They expressed the difference verbally, namely that object 5.22(1) was “standing up” and that object 5.22(3) was “lying down”.

FIGURE 5.22: DRAWINGS OF DIFFERENT ORIENTATIONS FOR SOMA THREE



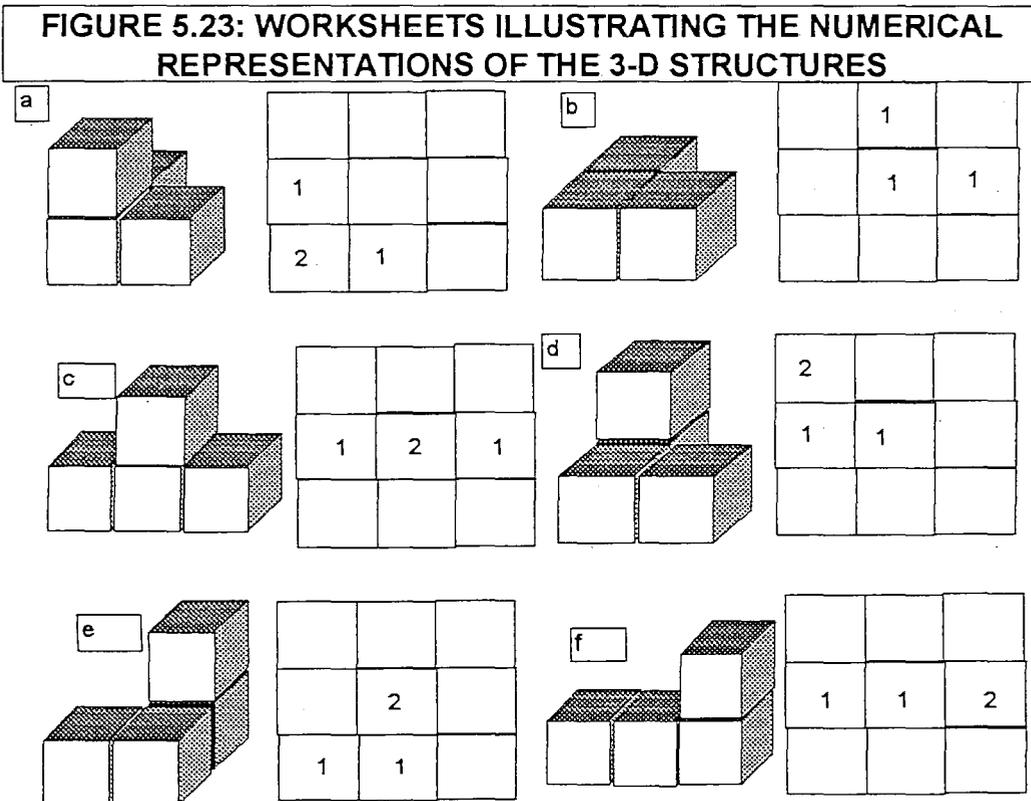
During the discussion of the results where the drawings of the two figures were compared (figure 5.22, 3a, 3b and 1), only two children (one in the middle group and one in the top group and both second language) attempted to show that there was a difference. In figure 5.22 (3a) it was attempted by changing the height of the back block. In figure 5.22 (3b) the height was changed as well as the length of the back block. The rest of the group, though, were happy with their representations (figure 5.22 (1)) of both figures (1) and (3).

This apparent incompetence of the majority of the children to represent or even attempt to represent the other dimension, was reversed when it came to representing figures 5, 6, 7, and 8. The children all attempted to show in some way that there is a definite depth dimension. Then the majority of the children's drawings fell into categories F and G.

5.7 Numerical representation

5.7.1 Results

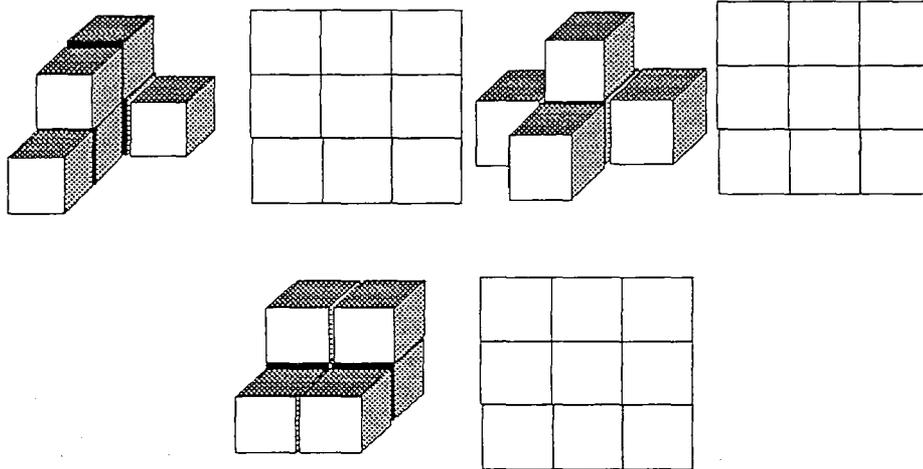
The children were given their soma cubes (figure 5.1) as well as loose blocks and papers with gridlines. The results were gathered by looking at the video recordings of the actual activities as well as by analysing the worksheets which the children handed in. Figure 5.23 is an example of the completed worksheet that the children handed in. They had to look at the pictures of the soma cubes and could then either write down the result directly or first construct the picture with loose blocks and then work out the numbers in the blocks.



After they had built their soma cubes on the grids they calculated the number of blocks that occupied the squares in the grids. It was discussed that this was another way of communicating to someone else how to build a similar structure without

giving them any pictures or actually showing them the physical construction. All the children in the different groups were able to communicate the different objects in figure 5.23. They were also given the grids with only numbers and then they had to make the physical structure with loose cubes. At the end of the year the children were presented with workchart 5.24 and asked to communicate the objects on the left by means of numbers. They were not given any concrete materials to work with. Only one child (middle group) was able to do it correctly.

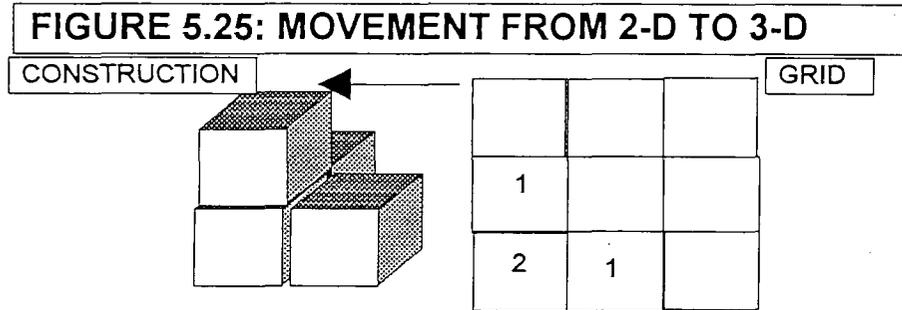
FIGURE 5.24: EXAMPLE OF EVALUATION WORKSHEET AT THE END OF THE YEAR



5.7.2 Discussion

5.7.2.1 Task

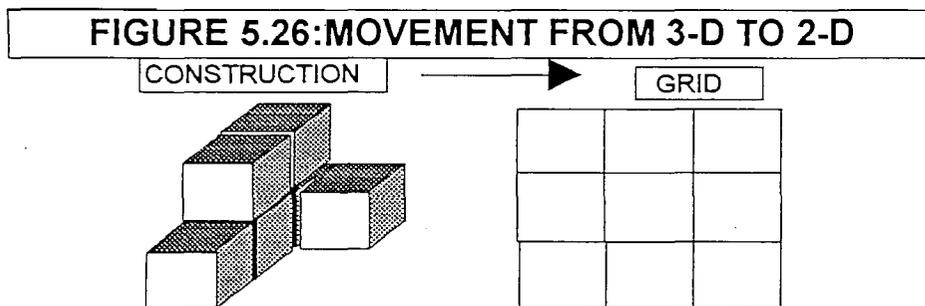
The children in the bottom group were having considerable difficulty with the new terminology on the one hand but also with the idea that the numbers in the grid were a way of communicating the information. They had less difficulty when they were given only the numbers on the grid and they had to reconstruct an object (Figure 5.25), in other words, moving from the two-dimensional grid back to the three-dimensional construction.



The fact that the researcher originally showed them this method of communicating with others, without themselves designing it, might have complicated their understanding of the purpose of filling in the grids. At the end of the year when the children were asked to complete figure 5.24, most of the children indicated that they did not understand what was expected of them.

5.7.2.2 Object

The children did the actual work with the soma figures and after two periods they could solve the numeration system if they were allowed to rebuild the structures on the grid. They had difficulty, though, getting the correct numbers when they were only given the picture of the object and were required to fill in the grid (figure 5.26). During their activities with the soma cubes they had more problems with soma cubes 6 and 7.



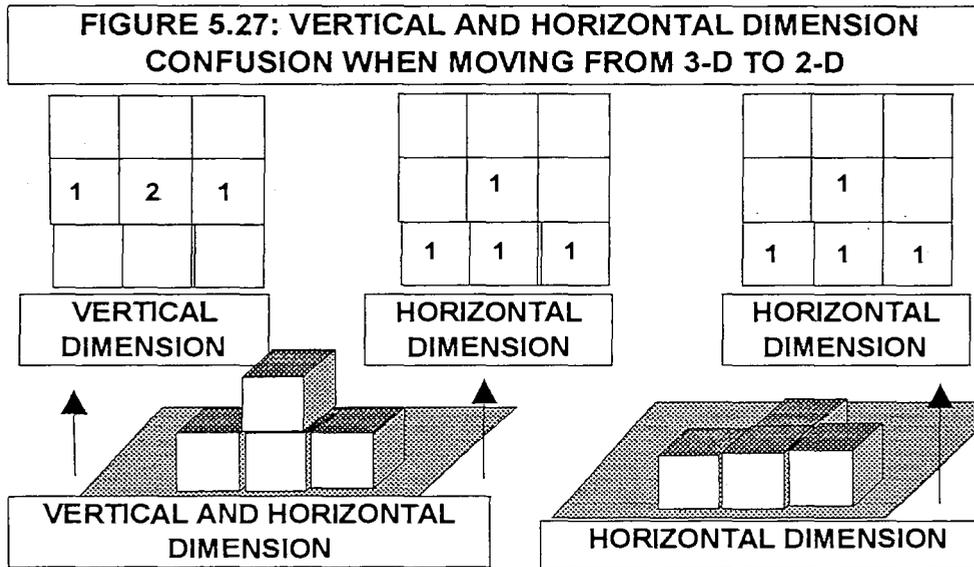
At the end of the year the researcher presented the children with more difficult structures (in picture form) than the simple pictures of the soma cubes that the

children had worked with during the beginning of the year. They all indicated that the fact that they were not allowed to rebuild the structures handicapped them in solving the problems. The fact that the structures on this worksheet (figure 5.24) were also more complicated than the soma cubes, might also have contributed to their problems, even though no one mentioned it.

5.7.2.3 Dimension and viewpoint

During the activities in the class the children constantly switched from a two-dimensional stimulus (pictures of the structures) via a three-dimensional stimulus (physical building blocks) onto a two-dimensional grid and vice versa. During the activities at the end of the year it was expected of the children to work from a two-dimensional stimulus (pictures of the structures) onto a two-dimensional grid, not via a three-dimensional stimulus. This in itself could have complicated the situation.

Another interesting comment that was made by some of the children concerned the actual dimension of the grids. It seemed that many children found that the horizontal placement of the grids on their desks confused them with the actual vertical dimension of the blocks on their desks. In figure 5.27 many children represented the upright structure of soma cube number three in different ways when they had to represent it on their grids. Many of the children "saw" the dimension of the grid as being vertical in the same way as the physical structures on their desks. This was even more the case when they only had pictures of the structures to work from as the stimulus materials. The interesting feature about this finding was the fact that the few children that could cope with the change of dimension from the vertical 3-D structure to the horizontal 2-D grid could not convince the other children through negotiation of this fact. It seemed that the visual stimulus was just too strong for these children.



All but one of the children found it difficult to represent the structures in a numerical way if they had no access to the concrete materials.

5.8 Constructions

Perception of spatial relationships is the ability to see two or more objects in relation to oneself or in relation to one another. In building with blocks, the children must perceive the position of the cubes in relation to themselves as well as the position of the cubes in relation to one another (Del Grande, 1990).

5.8.1 Results

The construction component was part of every activity that the children went through during this experiment (Activities 1.1; 1.3; 2.1; 2.2; 3.1; 4.1; 4.2; 5.1; 6.1; 6.3; 6.4; 7.1; 7.3; 8.1). At first the children only constructed single objects (figure 5.28) by utilising loose cubes. At the end of the experiment the children had to utilise soma cubes and, by “reading a pictorial manual”, combine them into final orientations (figures 5.29a, b and c).

FIGURE 5.28: SINGLE STRUCTURES CONSTRUCTED WITH LOOSE BLOCKS

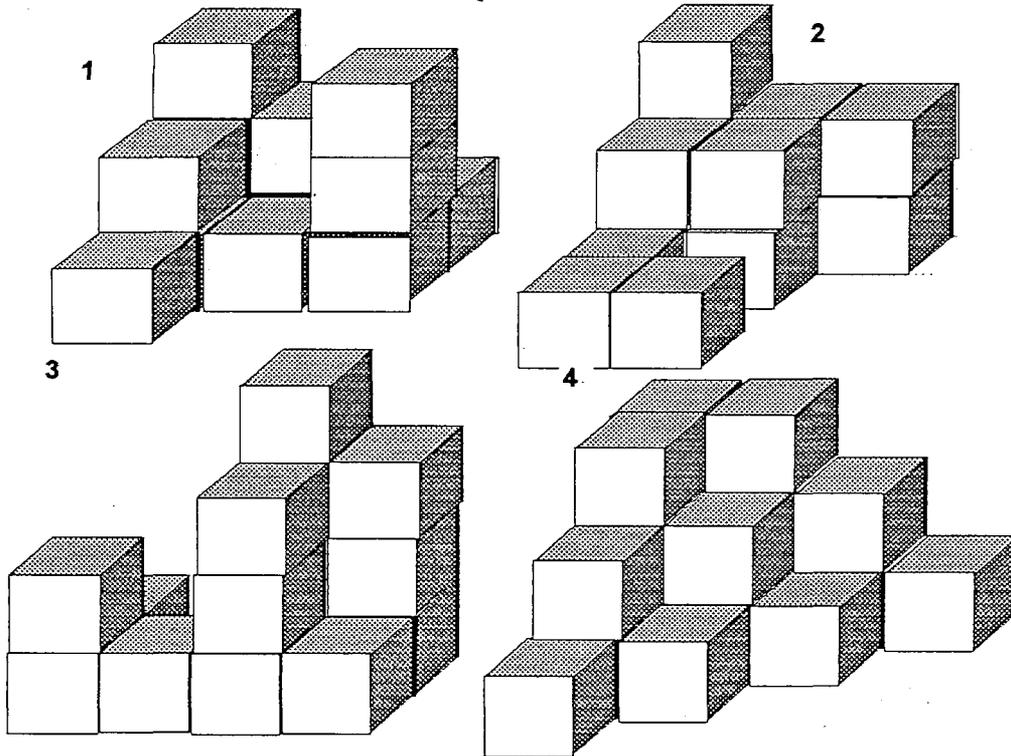
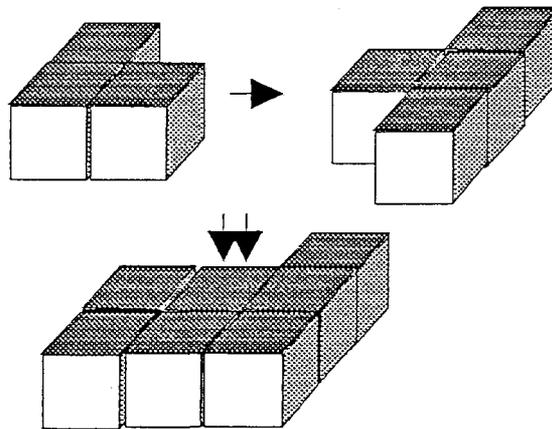


FIGURE 5.29a: SOMA CUBE "PICTORIAL MANUAL"



The instructions were given in different media (different dimensions as well) namely verbally, physically, pictorially and numerically. The constructions were made, utilising different stimuli dimensions. Children had to make constructions

while listening to verbal descriptions (undefined dimension). They had to make constructions while looking at the physical constructions in front of them (three-dimensional). They had to make constructions from looking at pictures (two-dimensional). Finally they also had to make constructions utilising a written code.

FIGURE 5.29b: SOMA CUBE "PICTORIAL MANUAL"

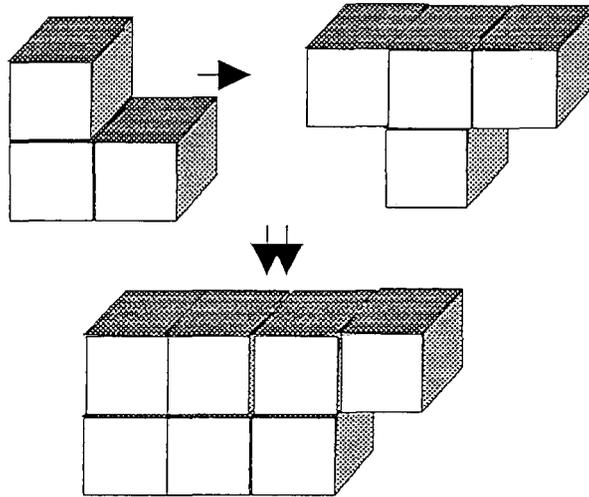
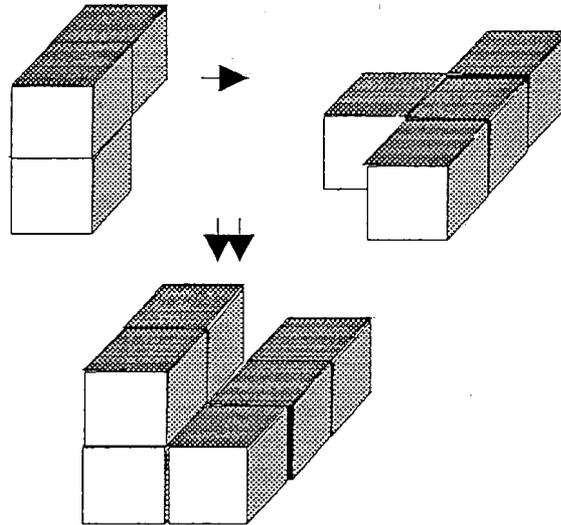


FIGURE 5.29c: SOMA CUBE "PICTORIAL MANUAL"



The constructions were initially made with only three loose cubes (figure 5.4) and eventually ended with the construction of complicated figures with more than ten loose blocks (figure 5.28).

5.8.2 Discussion

Pallascio *et al* (1993:11) emphasise the important role of construction when they say that spatial competencies cannot be developed adequately by simply analysing and drawing or writing descriptions of three-dimensional objects. The construction and handling of objects are of paramount importance.

5.8.2.1 Task

The construction medium was the one medium in which it was possible to see if the children really understood the task that was given to them. Another important feature of this medium was that children spontaneously reverted to the construction activity if they were unsure of the outcome in another medium like

drawing or verbal description. This was also the one medium in which the children were exposed to both a rotational type of problem and a perspective type of problem. The nature of the medium allowed the children to move freely between these types of problems.

The actual construction of models enables the child to anticipate the subsequent operations. Pallascio *et al* (1993:11) argue that the construction and handling of objects by the children are essential if an internalised reproduction of perceptive-exploration acts is to be accomplished.

5.8.2.2. Object

During the initial activities the children had to construct simple objects like soma cube structures with a maximum of four blocks. Once they were able to work with these blocks, regardless of the dimension of the stimuli, they progressed towards complicated objects ending with combinations of soma cubes in different orientations. From their discussions and handling of the soma cubes it became clear that they were seeing these cubes as canonical objects with specific tops and bottoms. If the objects were presented in different orientations by varying the axes along which they rotated, the children identified the objects in the vertical position as “standing up” and the objects in the horizontal position as “lying down”.

5.8.2.3 Dimension and viewpoint

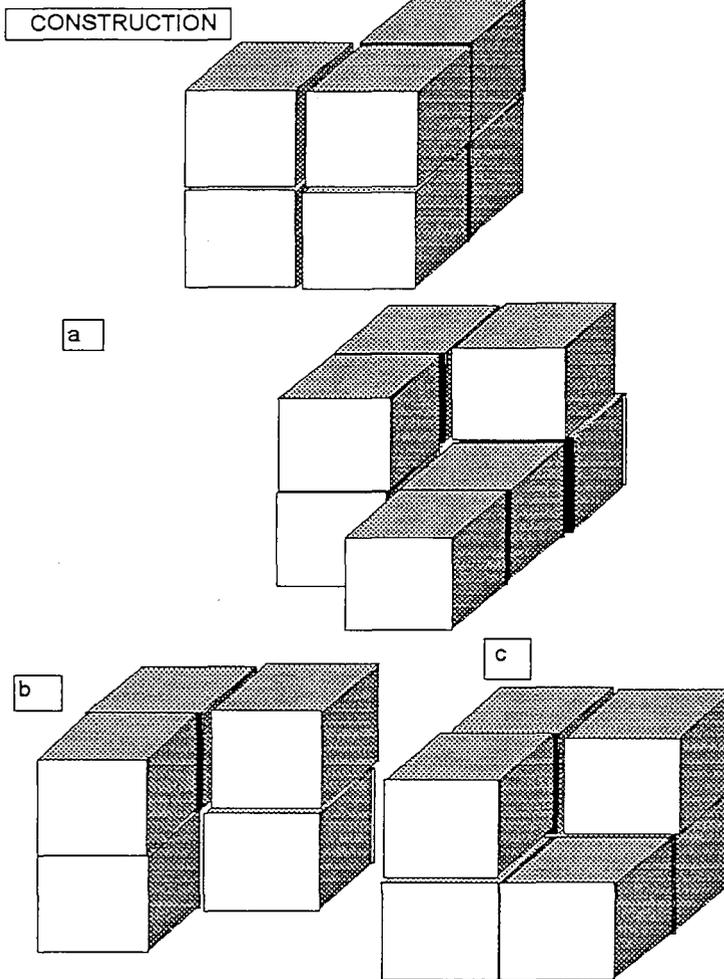
Activities 2.2, 4.2, 6.1 and 6.3 were activities where all the constructions were made while using a three-dimensional stimulus. Activities 1.3, 3.1, 5.1, 6.4, 7.1, 7.3, and 8.1 were activities where the dimension of the stimulus was two-dimensional. Activities 1.1, 2.1, 4.1 and 8.1 were activities where the dimension of the stimulus was undefined (either verbal descriptions or mental “images”).

The children had little difficulty working with the activities where the dimension of the stimulus was 3-D. At first, though, the children did not pay much attention to the actual orientation of the final constructions in comparison to the original ones. As long as they designed the overall structure they were quite happy. This implies that they built mirror images in the case of some cubes number 6 and 7. As the activities progressed, and the discussions amongst the children got under way, the children noticed that the mirror images did not have the same orientations, with the result that in the end they were very particular about the fact that buildings should not only have the correct structure but also the correct orientation in space.

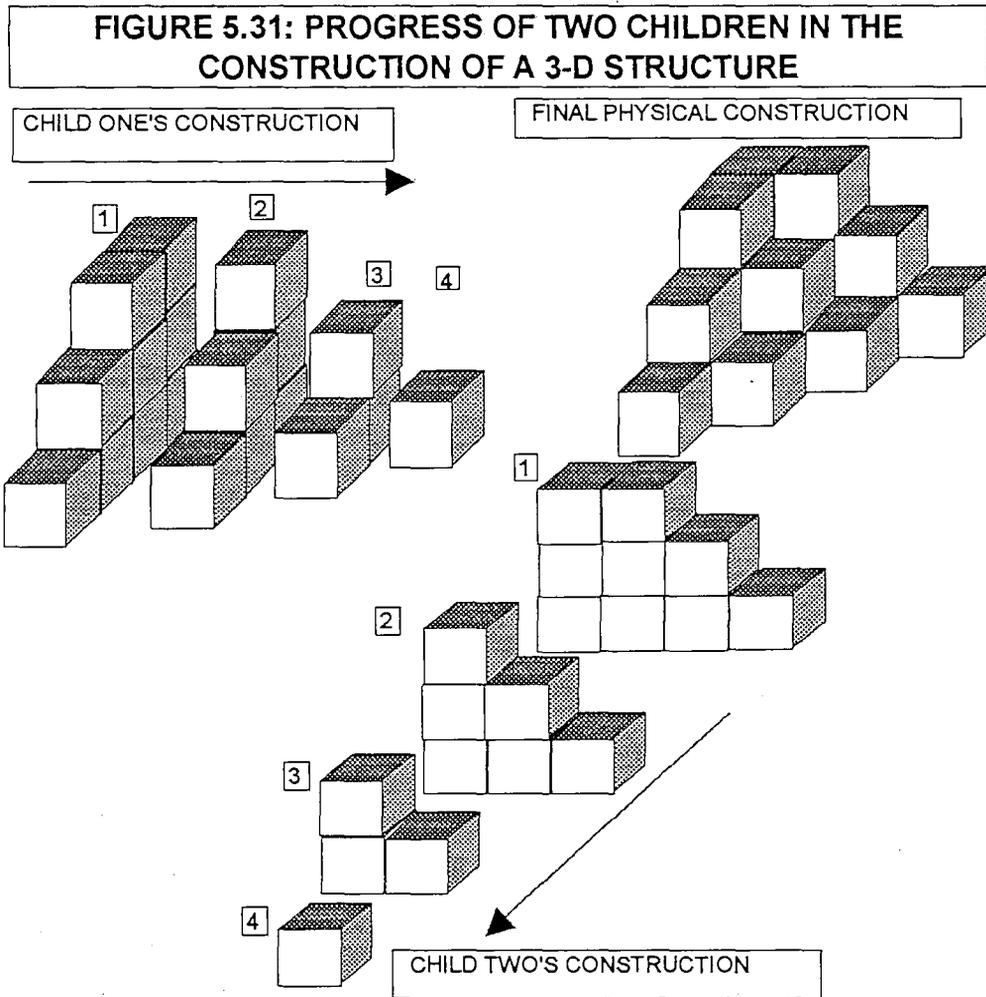
Most of the children in the middle and bottom groups at first had difficulty with the activities where the dimension of the stimulus was 2-D. It took them a while to realise that in a picture some of the blocks might not be visible. As an introductory activity to illuminate this concept they were given a picture of a structure (figure 5.30) and asked to rebuild it.

The outcomes are represented in figure 5.30 (a) (b) and (c). There were several different outcomes as to what the view from the back actually looked like. This led to the very important finding that in a picture some of the information might be obscured or even left out. The children also found that the information which is presented in a picture can sometimes not be physically reproducible. For example, in the case of figure 5.4, pictures number four, , six, seven, and eight cannot be reconstructed by using loose blocks unless there are some means of supporting the blocks that are suspended in the air (by sticking them together with some kind of glue).

FIGURE 5.30: DIFFERENT BACK VIEWS OF A 3-D STRUCTURE



The activities that were executed with the loose blocks did not seem to be difficult for children once they had sorted out the conventions that applied to two-dimensional representations. What was interesting, though, was that when it came to the more complicated 2-D representations (figure 5.31 child one and child two), the children broke up the pictures into smaller more manageable parts which they could handle. They then rebuilt the pictures according to a sort of mental blueprint which consisted of parts which were put together, even though they had the physical picture as a guide to copy. In other words, they looked at the picture and then selected a structure that they could handle.



The children then constructed the physical building without referring to the picture until it was finished. Once it was finished they made sure it was correct and then they went on to select the next structure and repeated the motions. This ties in with Guanella's (1934:27) and Reifel's (1984:63) findings that young children in their extension of space first fill in the vertical area and then move on to the horizontal area.

The activities with the soma cubes were different in the sense that it can be seen as three-dimensional puzzles since the primary aspect of the activity was discovery through trial and error. This activity does not allow for free creation as the objects are rigid. The activity is therefore analytic in nature (Pallascio *et al*, 1993:8)

FIGURE 5.32a: CONSTRUCTION OF SOMA STRUCTURES WHICH ARE BOTH IN THE HORIZONTAL ORIENTATION

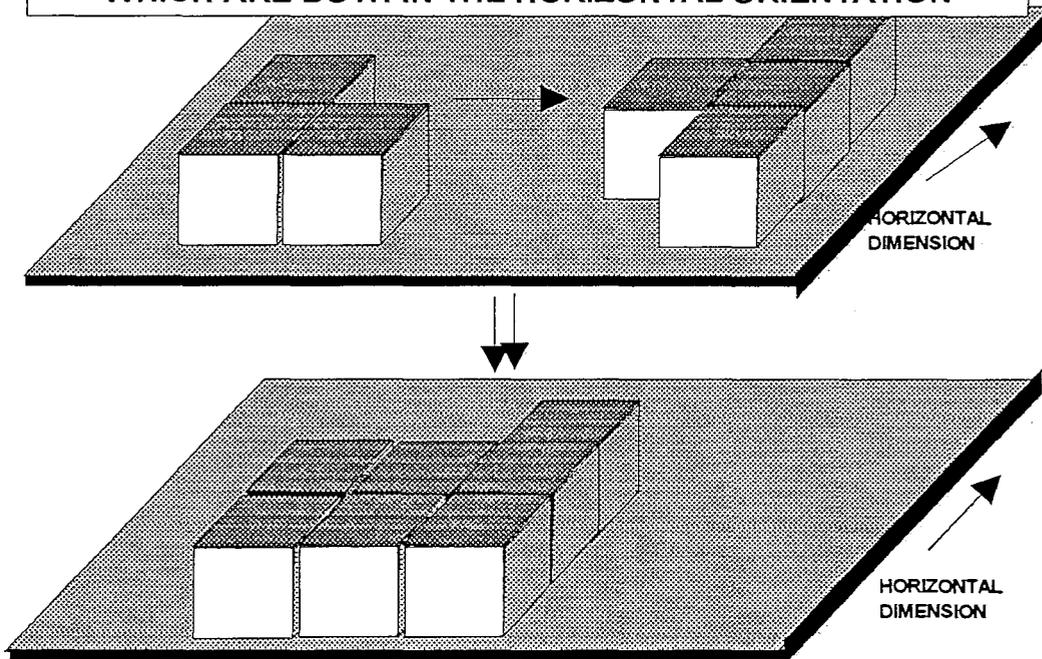
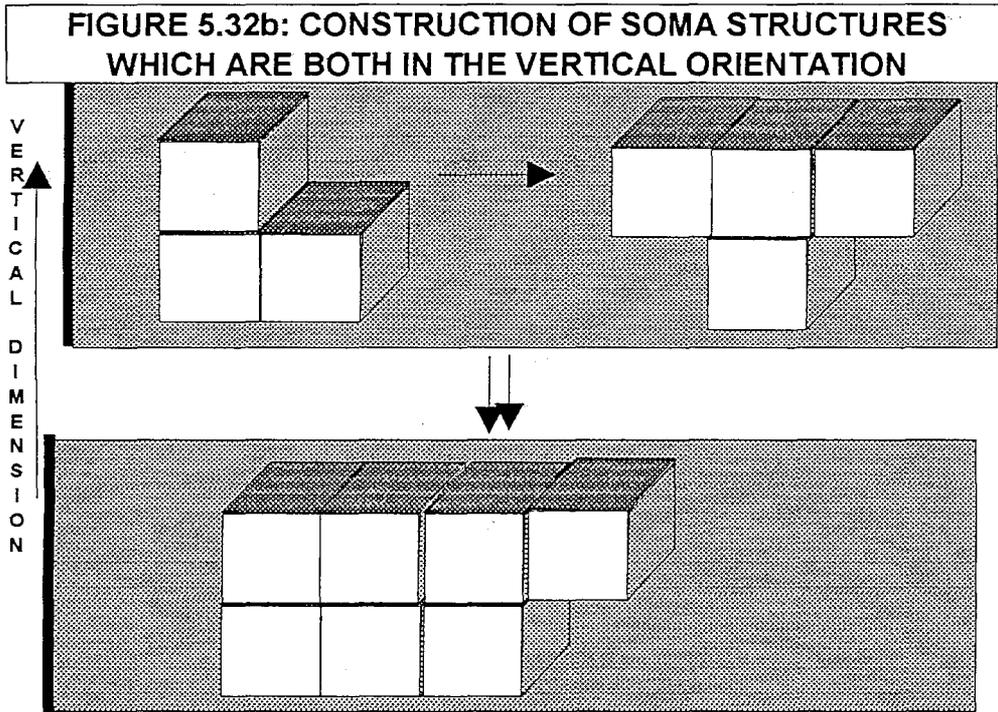
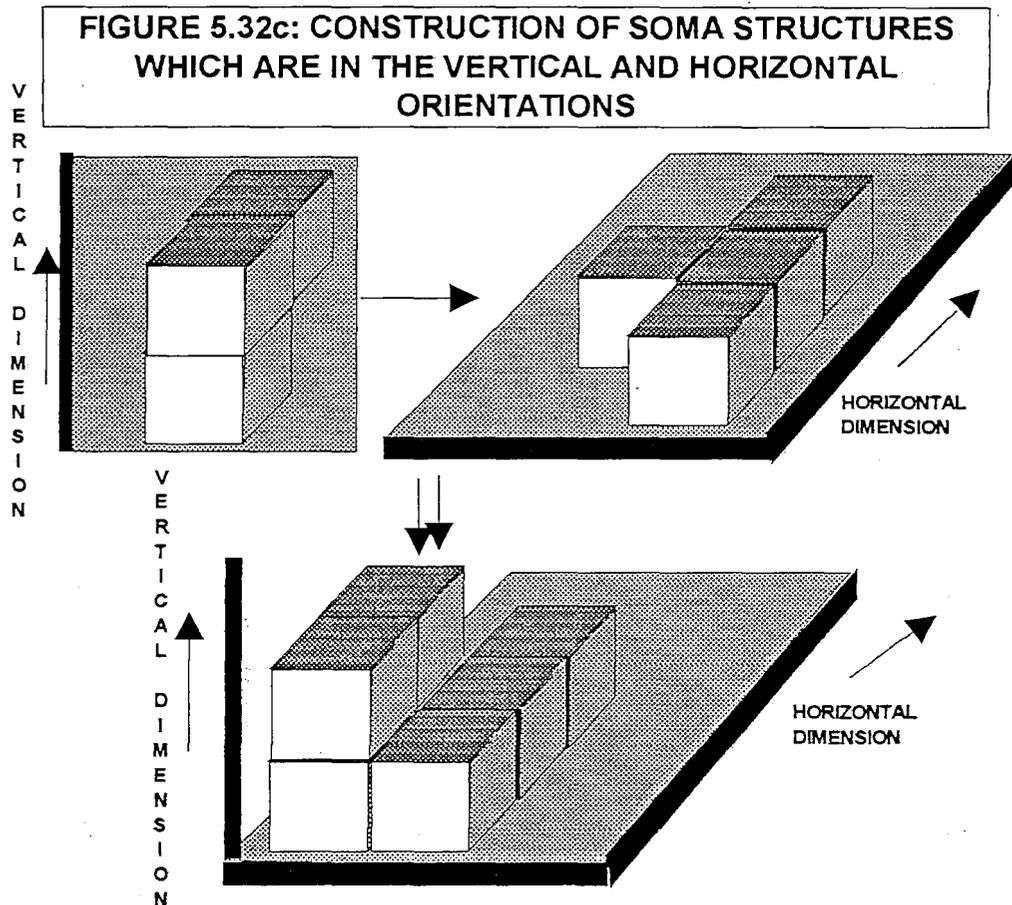


Figure 5.32a was handed out to the children first and quite a few children in the middle and the bottom groups did not realise that the blocks were actually "lying" down. When figure 5.32a was given to them most of the children in the bottom group and a few of the middle group children indicated that it was the same as figure 5.32b. It was only when they held the two figures next to each other that they realised that there was a difference in the orientation. Some of the children still insisted that it was the same and it was only after a lengthy discussion that the children in the bottom group agreed that there was a difference in the orientation of the blocks.

The same children who had problems with the previous two charts also indicated that they had difficulty with chart 5.32c. The first problem was the orientation of soma cube number one and the second problem was getting the result of moving the two blocks into each other.



In figure 5.32a and 5.32b the two soma puzzles that need to be put together have all their parts represented in the same dimensional axis. In the case of figure 5.32a both the parts are “lying down” or in the horizontal orientation. In the case of figure 5.32b all the parts are “standing up” or in the vertical dimension. This is in contrast to figure 5.32c where some one is “standing up” vertically and some two is lying “down” horizontally.



This was again an indication that it was not only the medium *per se* (two-dimensional for pictures) that was problematic for the children, but also the axes along which the rotation took place within that specific media.

5.9 Conclusion

In the above-mentioned chapter the spatial development of the children was investigated through the use of building blocks. The development of the different skills was looked at through the utilisation of three media namely language, drawings and constructions. It is important to take cognisance of the influence of the three instructional variables namely task, object, viewpoint and dimension on the development of skills in these media.

Different tasks were put to the children. The biggest stumbling block at the beginning was for the teacher to determine if the children understood the language in which the tasks were presented to them. Through the utilisation of language the teacher also increased the complexity of the task moving from a rotational problem situation to a perspective problem situation. The increase in the fluency of the use of the different spatial terms (in front, behind, left, right) that in some cases had to be negotiated was an indication of the progress that the children were making in deciphering the task, not only linguistically but also conceptually.

In putting the task to the children when it came to drawings, the teacher tried not to impose a preconceived idea as to the view from which the children should draw the three-dimensional objects they were looking at. None of the children questioned these tasks and in their discussions they were quite confident that the rest of the group would understand which structures were communicated by the drawings. The different interpretations given of the same objects nevertheless reflect the fact that the drawings were made from different viewpoints. In the task that led to the numerical representation of the blocks, some of the children indicated that they were confused by the use of this different representation. One of the reasons might have been the fact that the teacher through the task “forced” the solution onto them and did not allow for a development phase.

Throughout all the tasks, the construction activity was the activity which gave the clearest indication if children understood what was expected from them. It was also the one activity that acted as an “intrinsic” judge of the task. In other words, it was not necessary for any other person to judge the success of a construction because the solution was obvious in the final construction.

The children allocated specific canonical features to the objects through the specific use of words like “top” and “bottom”. These qualifications were made on the basis that certain structures could “stand up” and therefore had specific parts.

They had difficulty, though, with describing the differences between some cubes 6 and 7 even though they knew that they were not the same.

The type of objects influenced the way in which children chose to represent the objects in drawings. From all the results it was clear that children realised that there was a dimensional problem in representing some objects and they then chose to draw the objects from a specific view which they felt would illustrate the dimensional problem in the best way. There was a definite change in the view from which the children chose to represent objects five to eight in comparison with objects one to four. The same problem manifested itself in the numerical representation with objects five, six, seven and eight.

The construction activity was once again a measure for evaluating the difficulty the children experienced with the objects. It was clear from these activities that it was more complicated to distinguish between mirror type objects like number 6 and 7 than objects between 1 to 5.

The dimension of the objects and the subsequent viewpoints that the children were exposed to in all these activities were very clearly articulated by their use of the different deictic terms. There was a mixed response with regard to the continued use of "in front" and "behind". The different groups justified their use of these terms in the light of their decisions about the point of references that they were working from. The use of "left" and "right", though, was something that the group in total agreed upon as being a constant using the direction that one faces, as the reference point. In other words, left and right correspond with the left and right body parts of the person that is doing the viewing.

The problem that has to be overcome when drawing is the one of representing depth in a two-dimensional medium. In their drawings children made use of different techniques to try and overcome this problem. In one case they made use

of a change in the height of the objects by means of elongation (category G) of the hidden objects in order to show them. In another case they changed the view and drew the object smaller (category F). In many cases, though, they simply chose to draw the object from another viewpoint which allowed them to ignore the problem of representing parts of the object in depth. The children also indicated that they experienced problems with the representation of the objects on a two-dimensional grid if they could not physically construct it in a three-dimensional way on the grid.

The construction activity was complicated when the dimension in which the object was represented as part of the task was complex (in other word the stimulus dimension). The children did not have difficulty when the stimulus dimension was three-dimensional. If the stimulus dimension was given in the form of a two-dimensional representation (picture), increasing difficulty was also experienced regarding the different axes along which the objects in the representation were rotated.

This part of the experiment assisted in highlighting the effects of the different instructional variables on the skills that are needed, as well as developed in the three media. An outstanding difference between the activities in this part of the experiment and the following part is the fact that the stimulus materials were always three-dimensional and there occurred no dimensional changes of the materials *per se*. The children started out with cubes and they ended with cubes. The dimensional changes that took place were restricted to the different media which were utilised in executing the different tasks.

In the next part of the experiment the dimension of the stimulus materials *per se* is also changed. The objects that are started with are three-dimensional (cubes, rectangular prisms, tetrahedrons, square-based pyramids and cylinders) and they

are transformed into two-dimensional objects (squares, rectangles, triangles and circles) and back to three-dimensional objects.

CHAPTER 6

INVESTIGATION OF THE SPATIAL SENSE OF THE YOUNG CHILD THROUGH THE DEVELOPMENT OF SOLIDS

6.1 Introduction

According to Bourgeois (1986:222), topics like the foldout shapes (nets) of solids that are included in textbooks for young children appear to have a weak research base. The few studies that have been done on solids and their foldout shapes also seem to indicate that the children's responses are affected by the complexity of the geometric figures. Although some solids appear to be more difficult than others, factors capable of influencing children's recognition remain unidentified.

In the development of solids (designing the foldouts) it is required that, in order for children to be able to draw a "net" of a solid, the child must be able to visualise the three-dimensional configurations and also comprehend the geometrical relations among the various parts (Mitchelmore, 1980:83). Potari and Spiliotopoulous (1992:38) also find it a stimulating problem for children because it not only provides opportunity for visualisation, but it also has application to many aspects of everyday life, like the design and construction of various industrial products (shoes, boxes, hats, etc.). In other words, constructing the correct net of the solid implies that children have to co-ordinate the mental representations that they have in their minds with the analysis of the separate components of the solid.

The aims with the design of the teaching materials for this experiment were to alternate between analytic and synthetic intellectual operations, whilst fostering dynamic exploration in order to arrive at deductive reasoning and effective problem solving. Activities, which are at the analytic level, involve the mental activity of the recognition of forms and their properties, whereas the activities at the synthetic level

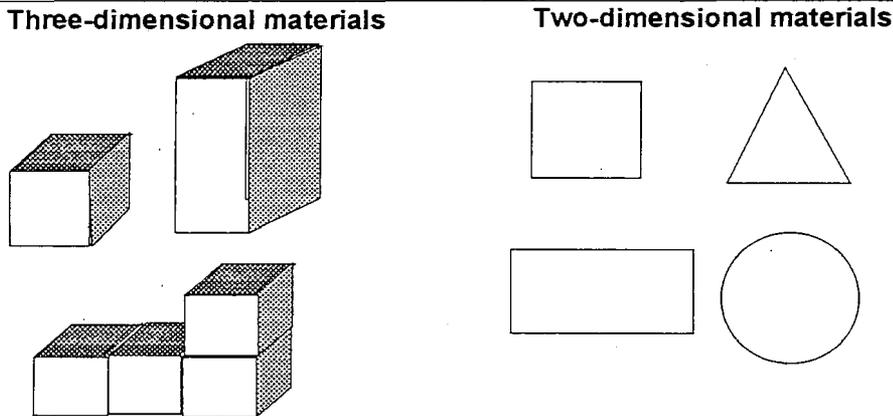
involve the transformation of shapes (Pallascio *et al* 1993:8). The base premise was that children would gain understanding by creating mental representations (via synthetic activity) and using these for the purpose of analysis and, conversely, by using increased competencies in analysing geometric objects to construct figures and forms (Pallascio *et al*, 1993:10).

6.2 Experimental background

During these studies the drawings which the children made were the major medium that was analysed in an attempt to gain insight into their spatial development. It should be stressed that these drawings were widely discussed amongst themselves and the drawings were always cut out and reassembled at the end of each session to see if the results were satisfactory. Although the majority of the results are of a pictorial nature, it should be evaluated against the total background of the classroom activities (Table 6.1).

The second part of the experiment differs in an important way from the previous part (activities described in chapter 5) in that the stimulus material that the children used could be manipulated or transformed. The stimulus materials for the development of solids were not only three-dimensional but also two-dimensional (compare figure 6.1). In a strict sense the squares, triangles, rectangles and circles that the children physically manipulated were three-dimensional but the materials also acted as a kind of metaphor for the two-dimensional equivalent.

FIGURE 6.1: THE STIMULUS MATERIALS OR OBJECTS FOR THE DEVELOPMENT OF SOLIDS WERE THREE-DIMENSIONAL AS WELL AS TWO-DIMENSIONAL



At the start of the activities there was little indication of what the children were capable of. The activities of each period were designed after the previous one had been analysed. In other words, the outcome of every period was a pointer and guide for the design of the activities for the next period. All the activities up to activity 5.2 were worked through with the cube as object and involved all three groups. Only the top group did activity six because they were starting to move ahead. In contrast the activities up to activity 5.2 were worked through with the object of the rectangular prism, but only with the bottom and middle groups. The top group spontaneously solved the problem of designing a net for their rectangular prisms after activity 2.1. For the rest of the objects (triangular prism, tetrahedron (triangular pyramid), square based pyramid and cylinder), none of the groups needed structured activities as described in Table 6.1. They all immediately proceeded to construct their nets after the problem had been put to them. The classroom activities for the first nine periods are presented in Table 6.1 and can be summarised as follows:

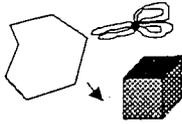
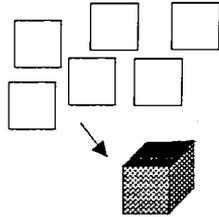
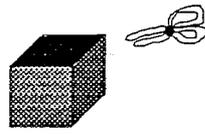
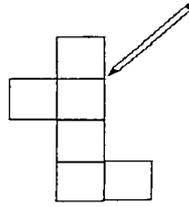
Activity 1.1 was given at the start of the first period. After one period all the children were basically at stage (1) and stage (2a) (figure 6.4). A few of the children in the top group started to copy the squares on the paper and cut them out, even though it was stressed that they should have only one piece of paper to cover their cubes. Activities

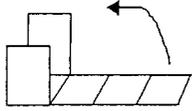
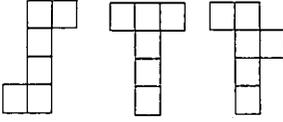
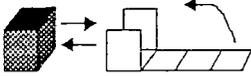
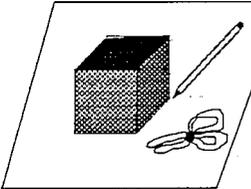
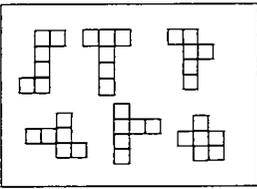
2.1, 2.2, 2.3 and 2.4 were introduced by the researcher in an attempt to break the “deadlock” that had occurred during the first period. Activity 2.1 was given at the start of the next period. Once the children had put their squares together to construct their cubes, they proceeded to activity 2.2 where they carefully cut open their cubes so that it resulted in only one piece of paper. They proceeded to trace around the edges of the net (which showed the separate pieces of the six squares clearly) to create a new net which consisted of one piece of paper of which the sub-parts were not visible. They folded this up to find that it formed a cube (activity 2.4). After the children had made different nets for their cubes they had to compare and discuss the nets with their neighbours (activity 3.1). The teacher can use this opportunity to slip in some nets that have not been made by a specific group. The reason for this is to make sure that in the case of the bottom group where the variety was not more than two, the children are exposed to more than the two types. It is also important that in the case of the top group the teacher should slip in some nets that are imperfect to elicit discussions.

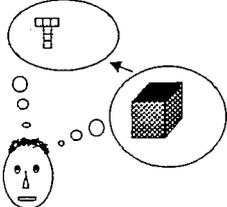
During the next period the original task that was given in activity 1.1 was again given as stated in activity 4.1. The children were all capable of designing at least one net for their cubes. In the majority of the cases the children came up with at least two different nets for their cubes. After a lengthy discussion once again the children were presented with worksheet 5.1 with the drawings of the different nets which they had been constructing during the past few periods. They were encouraged to decide which ones were workable and which ones were not without actually cutting them out. Once they had made a decision they were allowed to cut them out and test them by folding them up into cubes. The last activity 6.1 with the cube was done with no physical material but only with pencils and rulers and paper and only by the top group.

The activities are characterised by movement between two and three dimensions as well as the demand of the usage of their different skills namely verbal, visual, tactile and imagery.

TABLE 6.1: A REPRESENTATION OF THE ACTIVITIES, MATERIALS, AND OBJECTIVES WITH THE DEVELOPMENT OF SOLIDS.

ACTIVITIES	MATERIALS	OBJECTIVE	GRAPHICAL REPRESENTATION
<p>1.1 The children are instructed to cut out one piece of paper, that will cover the cube completely. There should be no overlapping.</p>	<p>1.1 Packaging materials that are cubes. Loose sheets of paper that will cover the cubes.</p>	<p>1.1 The child must intuitively discover the dimensions of the cube.</p>	
<p>2.1. The children are asked to build a cube similar to the object that they have by making use of the loose cardboard pieces.</p>	<p>2.1 Pre-cut cardboard pieces that are the same size as the different sides of the cube objects.</p>	<p>2.1 To discover the shape and the number of pieces needed for the construction of a cube.</p>	
<p>2.2 After the children have built their objects they are required to cut them open in such a way as to produce one piece of paper.</p>			
<p>2.3 They should trace around the edges of the space that they have just cut open and cut them out.</p>			

<p>2.4 They should fold it up to see if it makes the object of the cube.</p>			
<p>3.1 The children should compare the nets that they have made during the previous period, by grouping the similar ones together.</p>	<p>3.1 The cut-out objects of the previous period.</p>	<p>3.1 The discovery of the dimensions of a cube as well as the appropriate language development.</p>	
<p>3.2 All the nets must be inspected by the children through discussion to find out if they form a cube after being folded up.</p>		<p>3.2 Moving from 2-D to 3-D and vice-versa.</p>	
<p>4.1 The children are requested to make a net for their cubes without taking the cube apart. They are allowed to use the objects of the cubes to measure the dimensions.</p>	<p>4.1 Paper large enough to cut out nets for the cube objects.</p>	<p>4.1 To test the ability to construct a 3-D cube from a 2-D net of the cube.</p>	
<p>4.2 The children are asked to make as many different nets for their cubes as possible.</p>			
<p>5.1 The children are given a workchart with the 2-D drawings of different nets for cubes. They are asked to identify the correct nets for the cubes.</p>	<p>5.1 Workchart 5.1</p>	<p>5.1 To determine the ability of the children to recognise the workable and unworkable nets if only represented in 2-D</p>	

<p>5.2 Once the children have identified the nets they are requested to cut the nets out and fold them to see if they work.</p>			
<p>6.1 The children are asked to think of any cube-shaped box and design a net for it, without referring to any physical object.</p>	<p>6.1 Paper, pencil and scissors.</p>	<p>6.1 To test if children are able to design the net of a cube without having any visual cues.</p>	

Even though the bulk of the information was done in the media of drawing and construction, the importance of language as a representation media was still of the utmost importance.

6.3 Language

6.3.1 Results

During these activities with the solids the language results consisted mainly of analysis of the transcriptions of the video recordings of the conversations which the children had during their activities. Figure 6.2 gives an idea of the kind of transcriptions that were made. This piece of verbal transcription also illustrates why it was impossible to include all the transcriptions. The total number of video hours that was made were approximately 30 hours and the transcription illustrated in figure 6.2 was of 3 minutes of conversation. All the video recordings are available for analysis from the researcher.

FIGURE 6.2: A TRANSCRIPTION TO ILLUSTRATE THE LANGUAGE USE OF THE CHILDREN

TIME:12:21

RESEARCHER: "What is this shape? Is it a square?" (The teacher is pointing to the top of the rectangular prism in her hand.)



CHILD 1: "No it is a rectangle."

CHILD 2: "Yes it is a square."

RESEARCHER: "Is it a square?"

CHILD 1: "No,."

RESEACHER: "What is this shape?" (She is pointing to the top of the cube.)



CHILD 2: " A square."

RESEARCHER: " And this is it a square?" (She is pointing to the top of the rectangular prism again.)



CHILD 1: " No."

CHILD 2: "Yes."

RESEARCHER: "Why Child 2?"

CHILD 2: "Look these two sides are the same." (He indicates that the length and breadth are the same length.)

(The researcher holds the rectangular prism so that all the children can have a look at the lengths of the sides.)

CHILD 1: " They are not!" (Some of the other children join in a chorus.)

TIME 12:22

CHILD 1:" These two are longer than these two" (He motions that the two lengths are longer than the breadths. He also takes the cube and fits the square face on the rectangular face of the rectangular prism to prove his statement.)

CHILD 2: (This child indicates that the two shapes cannot be compared because the model of the cube is smaller than the model of the rectangular prism.)

CHILD 1 (He agrees that it is actually a bad comparison and he reverts back to the rectangular prism.)

CHILD 2 (He indicates that the starting point from which child 1 is taking his measurement is incorrect.)

RESEARCHER: " How are we going to find out if the sides are the same length?"

CHILD 1:(Takes the rectangular prism and shows through the use of his fingers that the lengths are not the same. In other words, he uses a natural measurement by using a part of his body.)

CHILD 3:" This nearly overlaps so actually it is a square" (This child is also commenting about the position from which the measurements are done and he joins the verdict of child 2. This is the first time that anyone in the group is joining the argument of child 2.)

TIME 12:23

RESEARCHER: "Is it a square?" (The researcher now takes the rectangular prism and cuts the rectangular part loose from the rest of the model.)

RESEARCHER: "What is a square? Can you tell one another anything about a square?"

CHILD 1: "Its....mmmm."

RESEARCHER: "What about the sides of a square?"

CHILD 1: "They have to have six sides ." (He takes the cube and starts counting the sides.)

RESEARCHER: "No that is a cube."

CHILD 4: "It has to have four sides."

RESEARCHER: "Four sides? Are they the same or not?"

CHILD 2: "They are the same."

(The children are again taking both models and comparing them and measuring them with their fingers.)

RESEARCHER: "So what will you call this shape?" (She points to the top of the rectangular prism again.)

GROUP: "A rectangle."

(The group have now reached agreement through negotiations about the fact that the top of the rectangular prism is actually a rectangle, but they feel that it is a bit of a "sloppy" model.)

6.3.2 Discussion

The discussions of the activities for the work with solids will be done in the same way as for the work with blocks (compare figure 5.2).

6.3.2.1 Task

The verbal reaction of the children during every period, as well as the results of their activities, guided the author in designing the activities for the next period. The reason

why there was a range of activities was that the children did not progress after the initial instruction was given to them, namely to design a net. At first the author did not use the word “net”. The problem was stated in the following way: “ Try and cut out one piece of paper that is large enough to cover the complete cube without having any overlapping.” The first problem that occurred was defining the word “overlapping”. The children went into a lengthy discussion about the actual meaning of the word “overlapping” in this context. They also proceeded to illustrate to one another what “overlapping” should look like. Pinxten *et al* (1983:195) points to the fact that different cultural systems will be specific on differential treatments of the notion “overlapping”. He also mentions that the Dogon have a word for shadow that conveys the meaning rather physically “blocking something from sight by casting darkness over it.” The common English meaning of “overlapping” would rather point to “sharing identical elements” in two or more objects or sets of elements. In the classroom situation this meaning was negotiated successfully to the extent that later in the experiment, it was very effectively used by many of the children to indicate that a net would be classed as being faulty because of the overlapping situation or the area of contact was too small (see figure 6.21 2bii). In order to clarify the word “net” in this context the children were asked to give their definitions of the nature and the functions of a net. Here are some examples:

- * A net is used to catch fish with.
- * A net can be used to cover a tea table with cakes on it.
- * A net is used to catch butterflies with.
- * A net is used for laying eggs in!

The last comment was made by a second language child. After discussions where the children had also used the plural form of the word, namely “nets”, this child had somehow heard the word “nests”. Nevertheless, the children all agreed that the major feature of a net is that it is one piece of material and that it should cover something. The researcher later on used these descriptions to draw the parallel between a “net” in mathematical terms and one in general terms.

At one stage the researcher even went as far as to illustrate what was expected while also taking care that the “how to” solution was not revealed. This was done by showing the children how an actual net was reassembled into a cube. Care was taken though that the children had no way of copying the net from what the researcher showed. The objective with this was to make sure that the children understood what was meant by “one piece” of paper as well as “overlapping”. The children still did not show any progress. During the first few periods many of the children, who could cope well with the second language, translated the problem into the vernacular for their peers who had difficulty understanding what the problem was. Although it was difficult to make sure that the “translators” were not actually telling the others how to solve the problem, it became clear that this was not much of a problem during the first few periods, because no one really knew how to solve the problem.

An important question that is always asked in a situation like this, where there are learners who are not taught in their first language is: “How far should the teacher go in order to clarify the task *per se*, without giving cues as to how to solve the problem!” The fact that even the first language children, even in the top group, could not progress with the first activity, might be an indication that this was more of a concept problem than a language problem.

Very little progress was made with the cube after activity 1.1 was put to them. The series of activities from activity 2.1 to 5.2 that are described in table 6.1 were designed and worked through. When the cube was substituted with the object of the rectangular prism, the children in the top group immediately started to solve the problem after activity 1.1 was put to them. They needed no further structured activities. The other two groups (bottom and middle) reacted in the same way as with the cube, indicating that they did not know what was expected of them. The activities from 2.1 to 5.2 were then worked through for the rectangular prism. When the next object (triangular prism 1) was presented, though, it was not necessary to introduce any structured activities for any of the groups. The children from the bottom and

middle groups was now actively engaged in solving the problem without any structured activities. The only difference between the groups was that the children in the top group took less time to reach the final solutions (stage 4) and they also produced fewer solutions which fell into the lower stages, namely (1), (2a), (2b), and (3) (compare figures 6.8, 6.9, and 6.10).

From this reaction it seemed that at first it might have been a combination of not understanding the task, as well as not being able to solve the task. As the activities progressed the clarity of the task was not a problem any longer but rather the way to solve the task.

The activities that are described in Table 6.1 were a result of what Freudenthal (1991:158) calls a “thought experiment”. This implies that the researcher envisions how the teaching-learning experience will proceed, and designs activities accordingly, after analysis of every teaching episode. The designer will try and find evidence in the teaching experiment that shows whether the expectations were right or wrong. This is what, according to Gravemeijer (1994:449), leads to a cyclic process of development and research that is theoretically and empirically sound.

An important consideration throughout this approach was that the children should be able to have “constructed” the mathematical knowledge that they possess in the end (Murray *et al*, 1993:194). Mathematics only makes sense when the learner can see the purpose of the activities and the activities and materials should be designed keeping this in mind.

6.3.2.2 Object

From the beginning the children called objects by certain names. Some children kept on calling a “cube” a “square” even though they were capable to select the squares when they were asked, and they did not confuse them with the cubes.

Two aspects needed to be considered and addressed, while working with these young multilingual children. Firstly, they might have had the correct object in their minds but they called it by the incorrect names because of their linguistic shortcomings. Secondly, even though they called objects by the correct name they might only be on level one (visual) understanding, with no concept of the features and attributes of the objects (Van Hiele, 1986:41-42). It was important for the researcher to clarify which of the two situations was the prevailing one for every activity, because they had to be addressed in different ways.

A possible solution to this problem was to employ another medium such as construction or drawing to clarify the unsolved situations which arose during the verbal discussions. In most cases the medium of construction proved to be very effective in clarifying many of these unsolved verbal issues.

Apart from calling the objects by the correct names the children were also very capable of “remembering” certain pieces of information about the objects. On closer observation, though, it became clear that many of the children were only “parroting” the information without any deep understanding of the qualities or features of the objects they were discussing. This is why Van Hiele (1982:207) warns against testing young children in a formal way through standard pencil and paper procedures. For example, at the end of the second period in working with the cube, all the children could *tell* (or write it down on an examination paper) that six squares are needed to construct a cube. When this was expected from them during a later stage in order to solve a problem, they still did not collect six squares for their *construction* of the cube. Most of the children in the bottom group still collected four squares.

6.3.2.3 Viewpoint and dimension

By the time the children had started with the development of solids, they had established a classroom culture where they had reached consensus on the use of the different deictic terms (compare 4.7.1.1.3). The nature of the activities for the development of solids was more focused on the mediums of construction and drawings. The children still used the different spatial expressions as previously described, with the only difference now being that they were all agreeing on the role the expressions had in describing an event.

6.4. Drawing

From analysis of the video recordings, two distinct problem-solving strategies were evident. The one that was used by most of the children, especially at the beginning of the experiment, will be termed “mechanical”. The second one was employed later in the experiment. Not all the children used it and this will be called “mental-visual”.

Characteristic of the mechanical procedures was that the children utilised the physical objects and manipulated them while they traced the different parts. They went through a number of trial and error stages where they reassembled incorrect nets. They did not indicate any pre-planned structuring of the objects and they were also not sure if the solutions in two dimensions were correct until they had cut them out and reassembled them to see if they fitted the object.

In the case of the “mental-visual” strategy children indicated that they had a “plan”. Some children even went as far as to say: “... if you see it in your head...”. They also indicated the solution by using their hands and showing in the air or on the paper, along which lines the object need to be “cut” to result in the correct solution. They also discussed the solution with their partners before any drawings of the shapes were started. In almost all the “mental-visual-strategy” cases, the children indicated that they

were quite sure that a net would be correct or incorrect before they had to “test” it, through cutting out and reassembly.

6.4.1 Results

The results of the drawings were interpreted not only by analysing the finished products (drawings) of the individual children, but also by analysing the process of drawings that the children went through while they were busy with the activities. This was made possible by replaying the video recordings and reconstructing the actual design phases which the different children employed. It implies that the drawing results were reconstructed with a graphics software on the computer. Figures 6.4, 6.6, 6.19, 6.20, 6.21 and 6.22 are the representations of the different stages of drawings that the children worked through in the process of reaching the final solutions for the nets of the objects. Figures 6.5 and 6.7 are graphs which represent the different stages that the children worked through in their design of the cube (figure 6.5) and rectangular prism (figure 6.7). The information of the progress of the different groups through all the objects and stages is given on separate graphs. Figures 6.8, 6.9 and 6.10 are representations of these final solutions (stages of drawings) for the nets of the different solids after one period.

6.4.1.1 Figures 6.3, 6.12, 6.13, 6.14, 6.15, 6.16, 6.17 and 6.18 are the representations of the progress of individual children through the different stages for the different objects

The drawings of the different objects were compiled after the video recordings of the different activities had been viewed and transcribed. The stages (categories) that are described are a combination of the stages prescribed by Mitchelmore (1978:235), Potari and Spiliotopoulous (1992:39) and this researcher’s findings. Five stages can be recognised for the different objects, namely:

Stage 1. Holistic

Drawings representing the basic overall structure. Once the children understood the problem not a single child reverted to representations of this type. Holistic representations were mainly found for the cube at the beginning of the experiment.

Stage 2. Elements of projection

Stage 2a. Wrong number, shapes and orientation

Only parts of the nets are represented, mostly showing the wrong number of individual parts, shapes or orientation.

Stage 2b. Wrong orientation

Only parts of the nets are represented. The correct shapes and numbers are now represented but the relationship or orientation of the parts are wrong.

Stage 3. Incomplete geometrical

The nets are correctly represented by showing the correct parts and numbers, as well as the orientations, but some of the parts are freehand drawings and do not correspond to the exact geometrical shape.

Stage 4. Complete geometrical

The nets of the objects are represented in the correct way and fold up to form the exact geometrical objects.

All the different stages for all the different objects were represented except in the case of the cube, rectangular prism and the triangular pyramid where no examples of incomplete geometrical object-representations of stage (3) were drawn by any of the children (compare figure 6.11).

There was a definite hierarchy in the stages which the children represented. Stage (1) drawings were only made at the beginning of the experiment and only by the children

in the bottom group. The stage (1) drawings were still found among some of the drawings later on in the experiment, but they were never presented as the final solution to the problem.

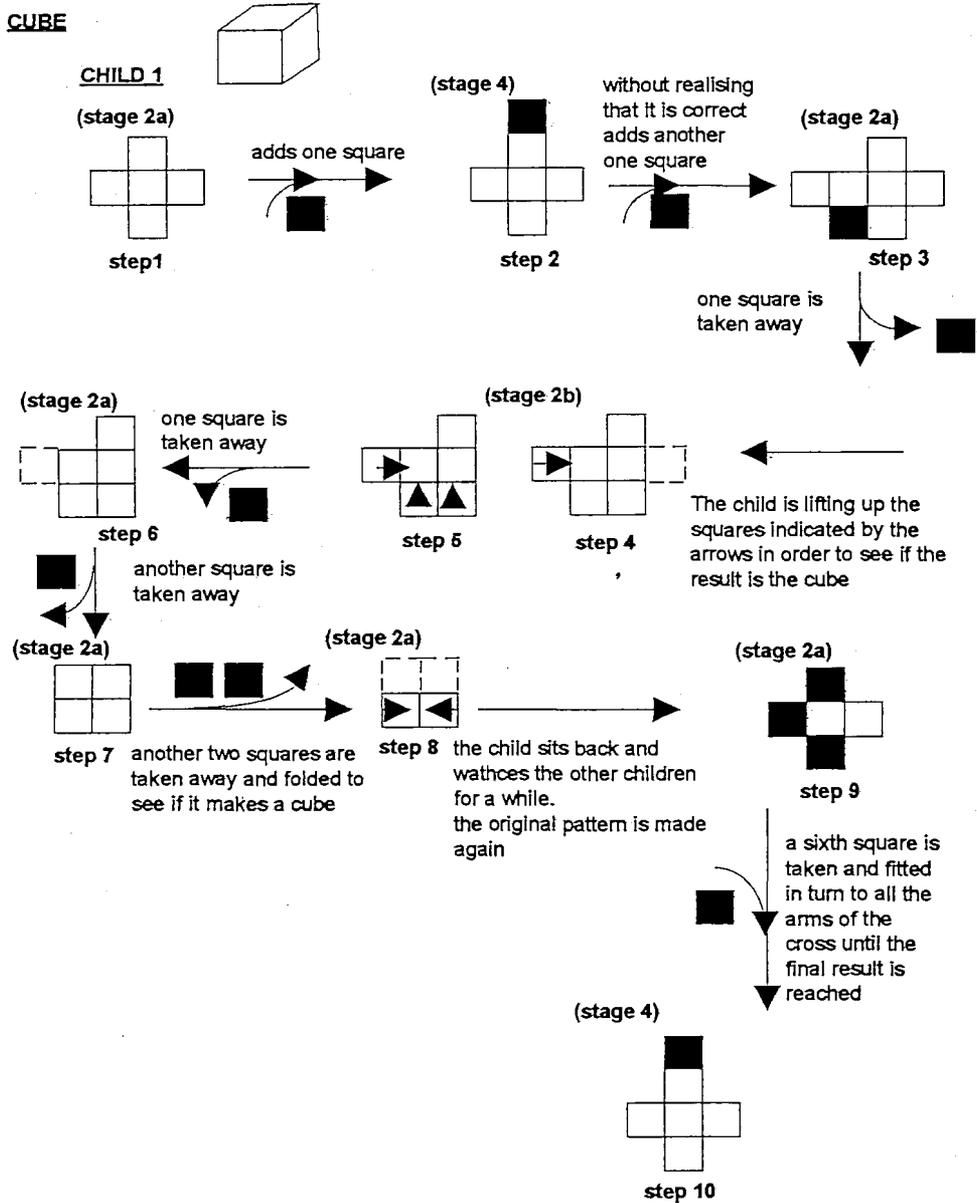
Stage (2a) drawings were mastered before stage (2b) drawings in most of the cases because the children could physically count the number of individual parts. Stage (2b) was more complicated because this involved the issue of orientation representation from a three-dimensional situation to a two-dimensional situation. This stage could not be mastered through mere physical manipulation because the children were not allowed to take the objects apart or unfold them in order to gain information about their nets.

In the stage (3) drawings the children had managed to overcome part of the 3-D-2-D representation problem, in that the orientation of the individual pieces was now correctly represented but some of the parts were not drawn to scale.

Stage (4) drawings were a correct version of the nets in all aspects.

There was not always a step by step progression from stage (1) to (2a) to (2b) to (3) to (4) for all the children. In some cases intermediate categories were either skipped or children sometimes moved back to a previous category before reaching the final category, as illustrated in figures 6.3, step 1, 2 and 3. It was also evident that some children drew variations of certain stages (compare step 1, 3, 6, 7, 8 and 9 of figure 6.3), which resulted in many different representations which can be categorised under the same stage.

FIGURE 6.3: PROGRESS OF THE DRAWINGS OF A CHILD IN THE DEVELOPMENT OF A CUBE



The cube (figure 6.4) had the most variations in the final outcome (foldouts) namely five, whereas the object of the square-based pyramid (figure 6.21) and the cylinder (figure 6.22) had only one final outcome each. The figures 6.21 (4ii) to (4vi) (the shaded area) that are shown as final outcomes for the square-based pyramid, were not achieved by any of the children that took part in this particular experiment.

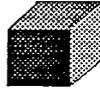
6.4.1.2 Progress of all three groups with the cube from period 1 to 4

Figure 6.5 shows the stages the children in the different groups reached after the first period of instruction using the cube as object and then again at the end of the fourth and third periods. The stages are classified according to figure 6.4

FIGURE 6.4: THE DIFFERENT DRAWING STAGES FOR THE CUBE

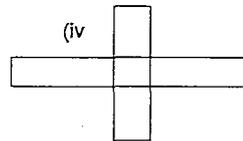
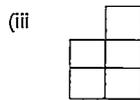
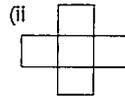
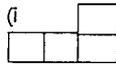
OBJECT: CUBE

(1) HOLISTIC

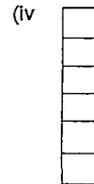
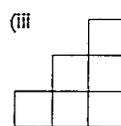
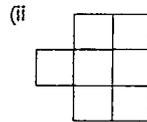
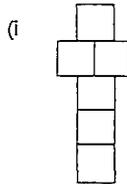


(2) ELEMENTS OF PROJECTION

(a) WRONG NUMBER/SHAPE



(b) WRONG ORIENTATION



(4) COMPLETE GEOMETRICAL

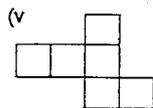
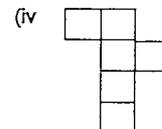
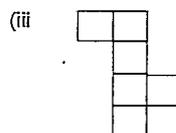
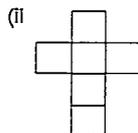
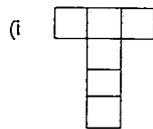
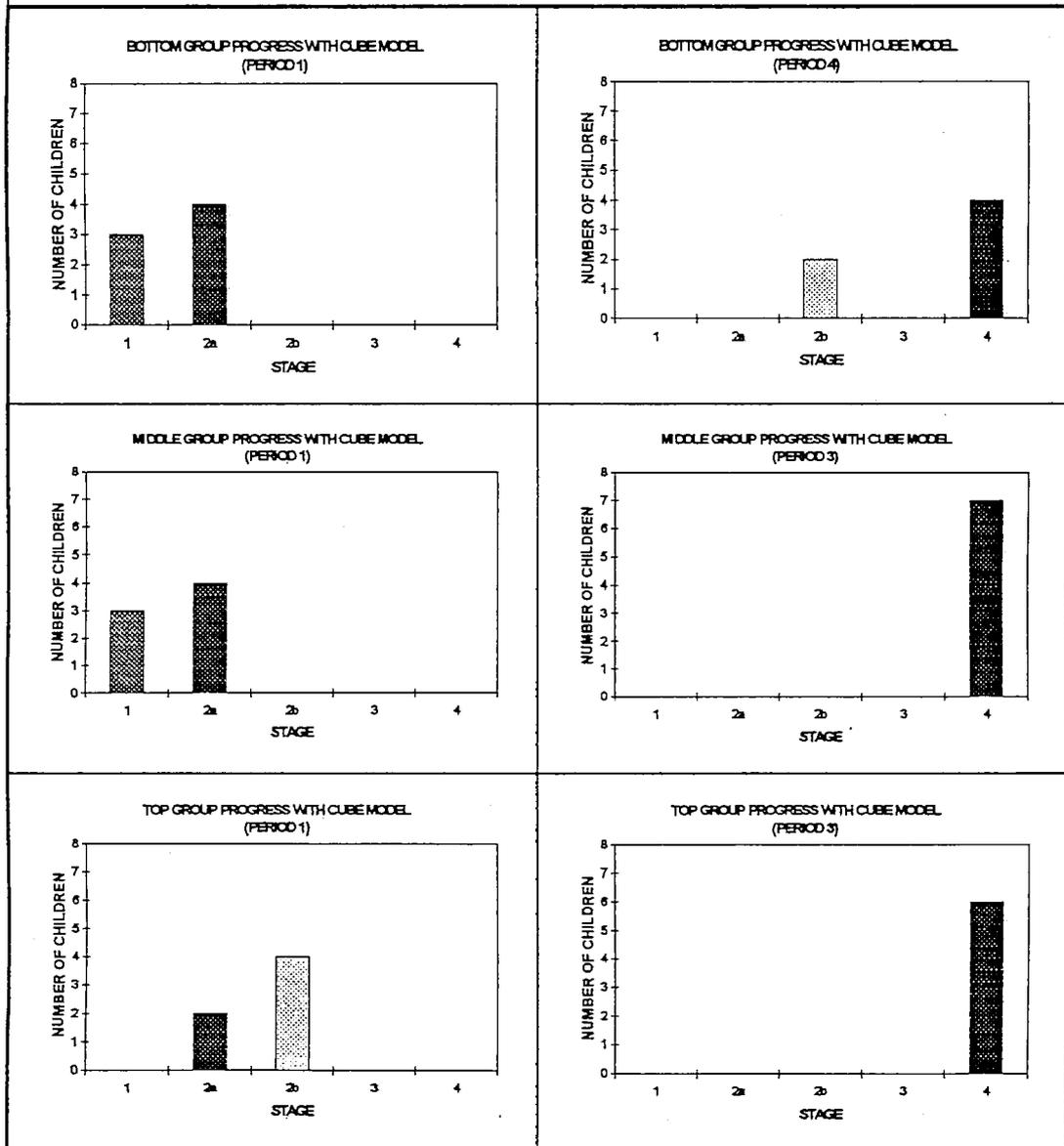


FIGURE 6.5: PROGRESS OF ALL THREE GROUPS WITH THE DEVELOPMENT OF THE CUBE FROM PERIOD 1 TO 4.



At the end of the first period very little progress had been made by all the children from the bottom to the top group (figure 6.5). At the start of the second period the activities as described in Table 6.1 were introduced. The majority of the children in the bottom group were able to produce complete geometrical representations at the end of the fourth period. All the children in the middle and top groups were able to produce complete geometrical representations at the end of the third period.

6.4.1.3 Progress of all three groups with the rectangular prism from period 1 to 5

Figure 6.7 shows the stages the children in the different groups reached after the first period of instruction using the rectangular prism as object and then after the second period (for the top group) and the fifth periods of instruction for the bottom and the middle groups. At the end of the second period the activities according to Table 6.1 were introduced for the middle and the bottom groups. The stages are classified according to figure 6.6.

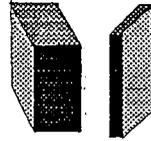
Not one of the children in, the bottom or middle group could produce complete geometrical representations of the rectangular prism (figure 6.6) at the end of the first period, whereas four of the children in the top group could already produce complete geometrical drawings at the end of the first period with the rectangular prisms.

It took the children in the middle and the bottom group children another four periods before all of them were able to make complete geometrical drawings of the prisms. The top group needed only one extra period.

FIGURE 6.6: THE DIFFERENT DRAWING STAGES FOR THE RECTANGULAR PRISM

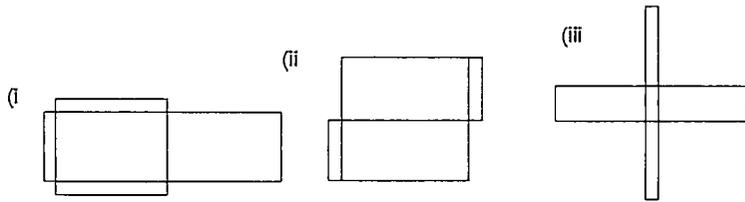
OBJECT: RECTANGULAR PRISM

(1) HOLISTIC

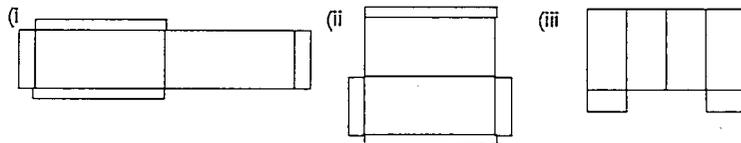


(2) ELEMENTS OF PROJECTION

(a) WRONG NUMBER/SHAPE



(b) WRONG ORIENTATION



(4) COMPLETE GEOMETRICAL

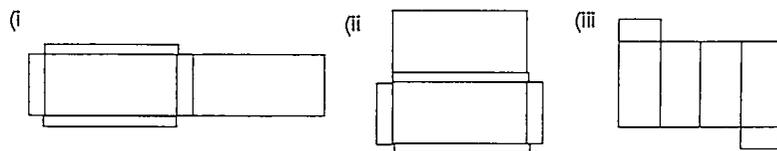
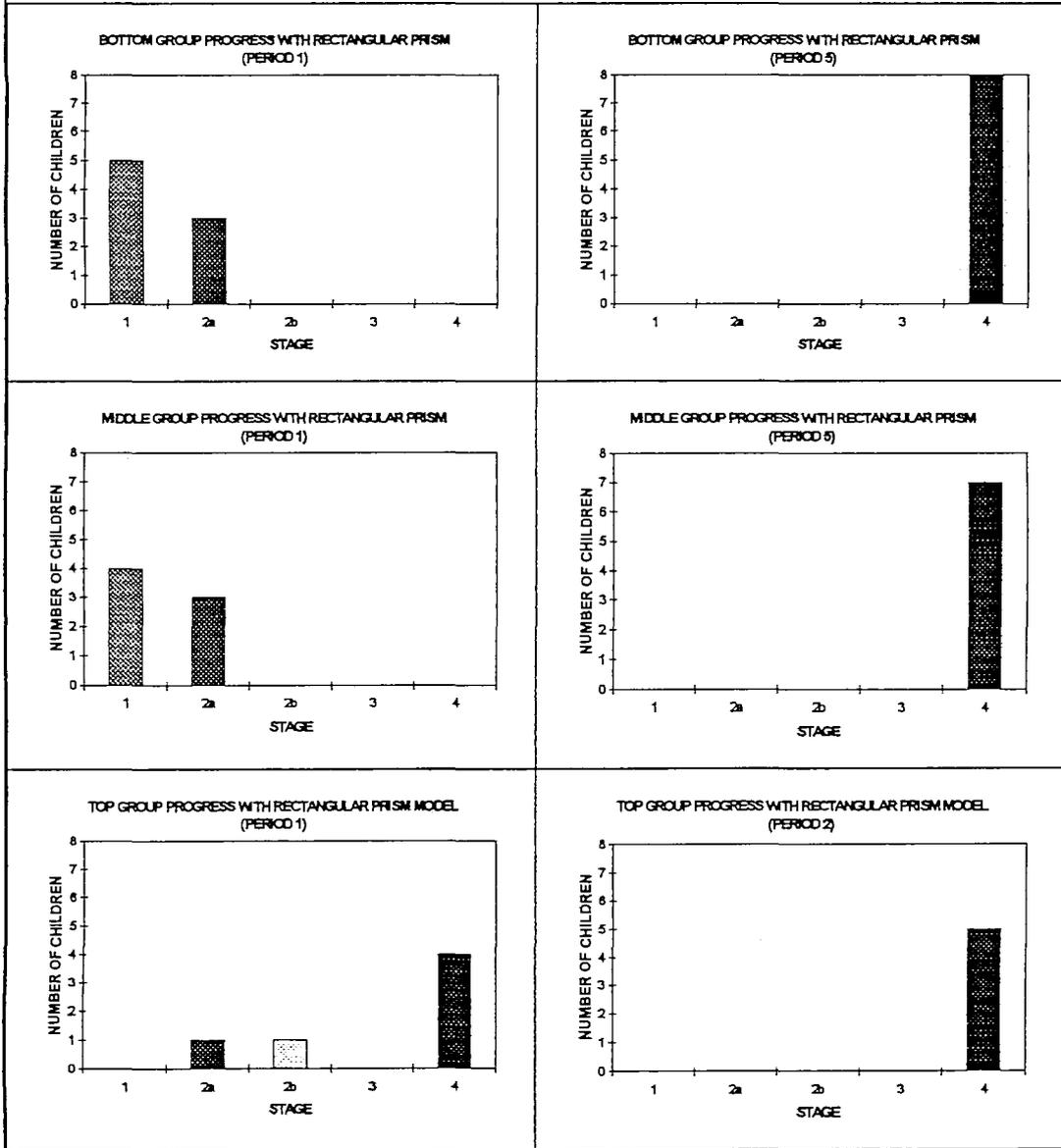


FIGURE 6.7: PROGRESS OF ALL THREE GROUPS WITH THE DEVELOPMENT OF THE RECTANGULAR PRISM FROM PERIOD 1 TO 5

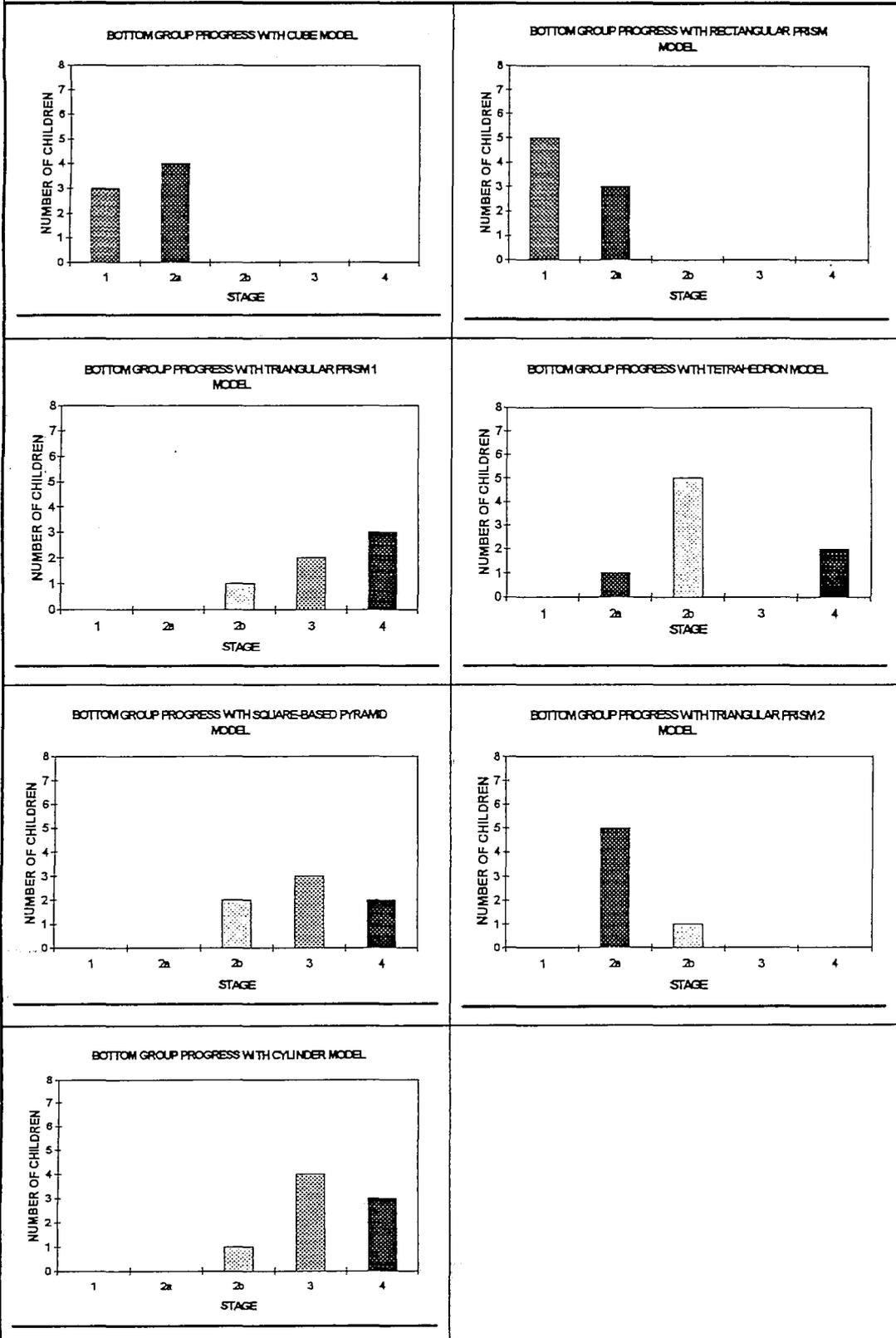


6.4.1.4 Progress of the bottom group with all the objects

Figure 6.8 is an accumulative representation of the progress of the bottom group through all the different stages for all the different objects. The order in which the objects were represented to the children was: cube, rectangular prism, triangular prism 1, tetrahedron, square based pyramid, triangular prism 2, cylinder.

From figure 6.8 it can be deduced that the order of increasing difficulty for the bottom group is: cylinder and triangular prism 1 (approximately the same), square based pyramid, tetrahedron (triangular pyramid), triangular prism 2, cube and rectangular prism.

FIGURE 6.8: PROGRESS OF THE BOTTOM GROUP THROUGH THE STAGES OF THE DEVELOPMENT OF THE DIFFERENT OBJECTS



6.4.1.5 Progress of the middle group with all the objects

Figure 6.9 is an accumulative representation of the progress of the middle group through all the different stages for all the different objects. The order in which the objects were done was: cube, rectangular prism, triangular prism 1, tetrahedron, square based pyramid, triangular prism 2, cylinder.

From figure 6.9 it can be deduced that the order of increasing difficulty for the various objects for the middle group was: triangular prism 2 and triangular prism 1 (approximately the same), then the cylinder, square-based pyramid, tetrahedron, cube and rectangular prism.

6.4.1.6 Progress of the top group with all the objects

Figure 6.10 is an accumulative representation of the progress of the top group through all the different stages for all the different objects. The order in which the objects were done was: cube, rectangular prism, triangular prism 1, tetrahedron, square based pyramid, cylinder and triangular prism 2.

From figure 6.10 it can be deduced that the objects in increasing difficulty for the top group started with the triangular prism 1 followed by the square-based pyramid, cylinder, tetrahedron, triangular prism 2, rectangular prism and cube.

FIGURE 6.9: PROGRESS OF THE MIDDLE GROUP THROUGH THE STAGES OF DEVELOPMENT OF THE DIFFERENT OBJECTS

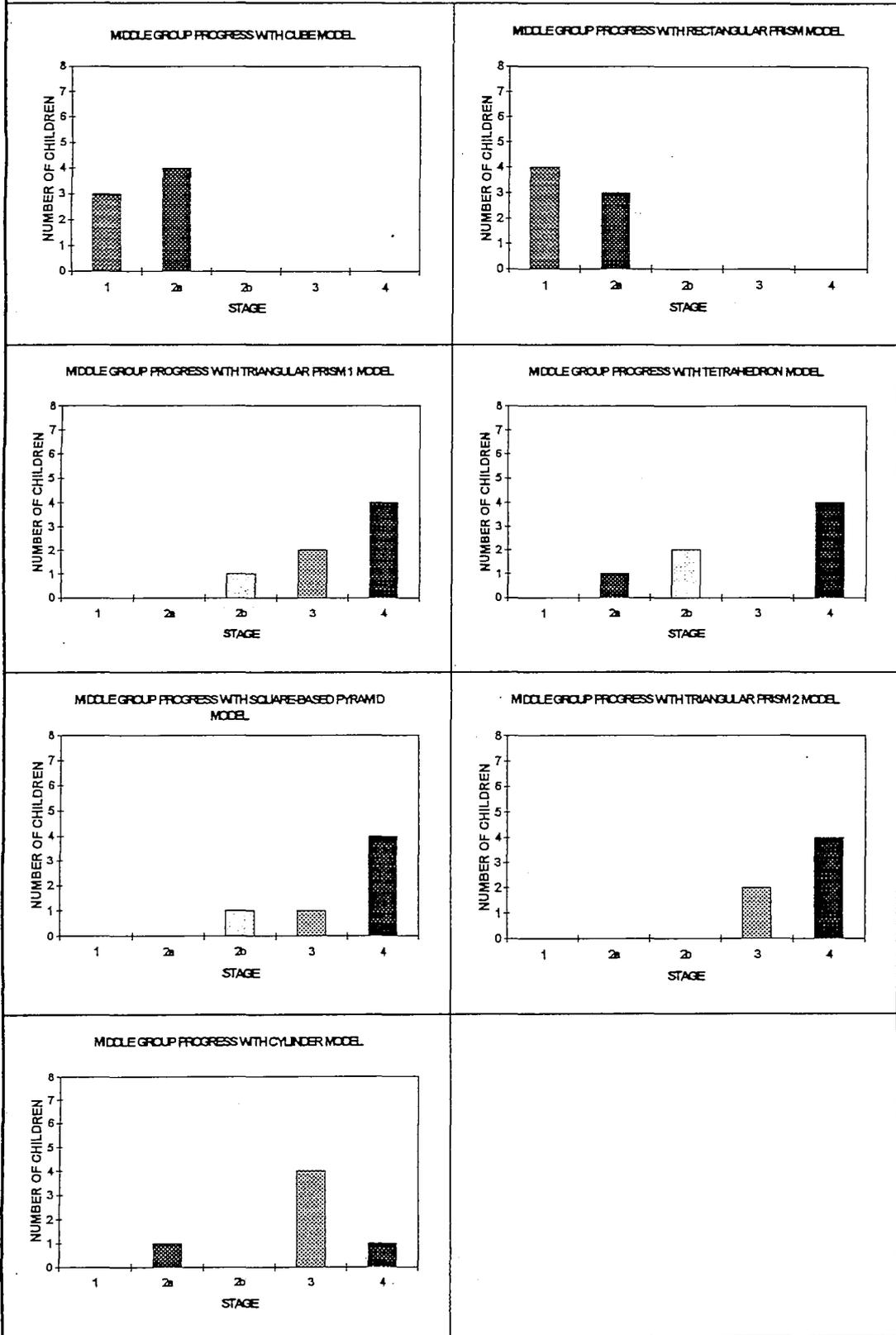
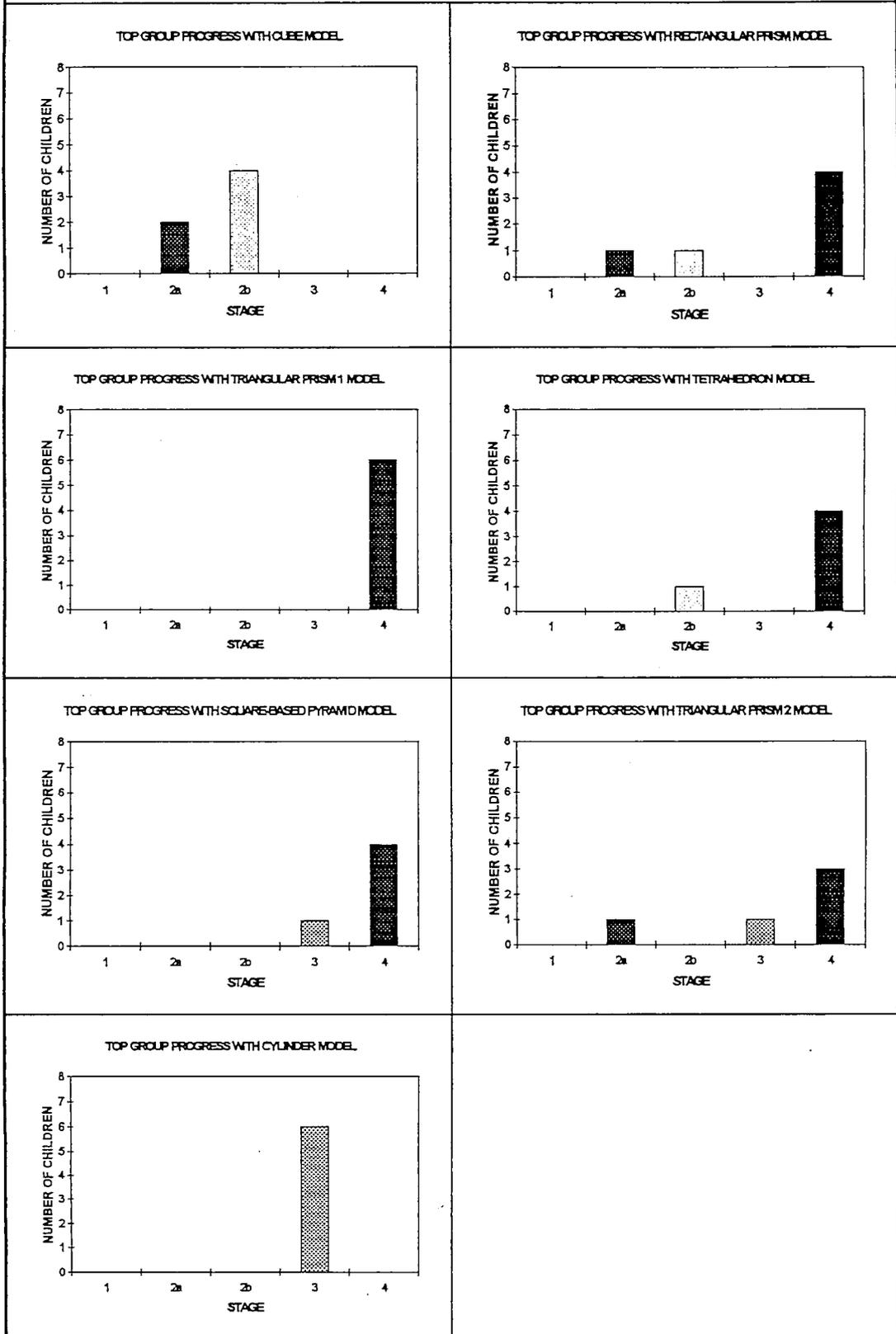
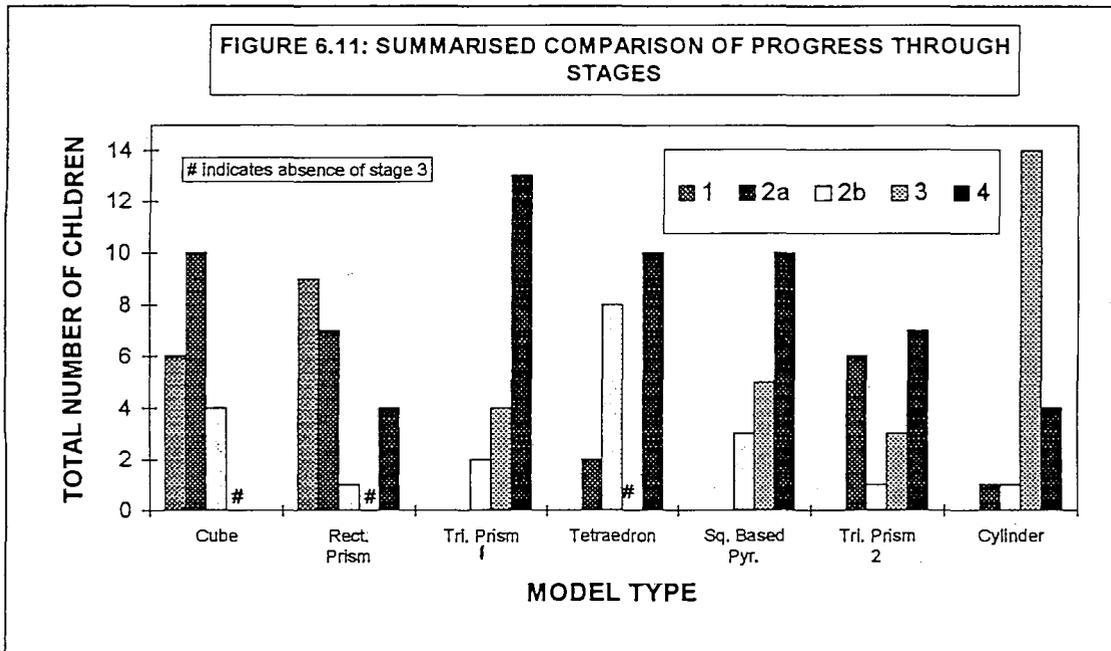


FIGURE 6.10: PROGRESS OF THE TOP GROUP THROUGH THE STAGES FOR THE DEVELOPMENT OF THE DIFFERENT OBJECTS



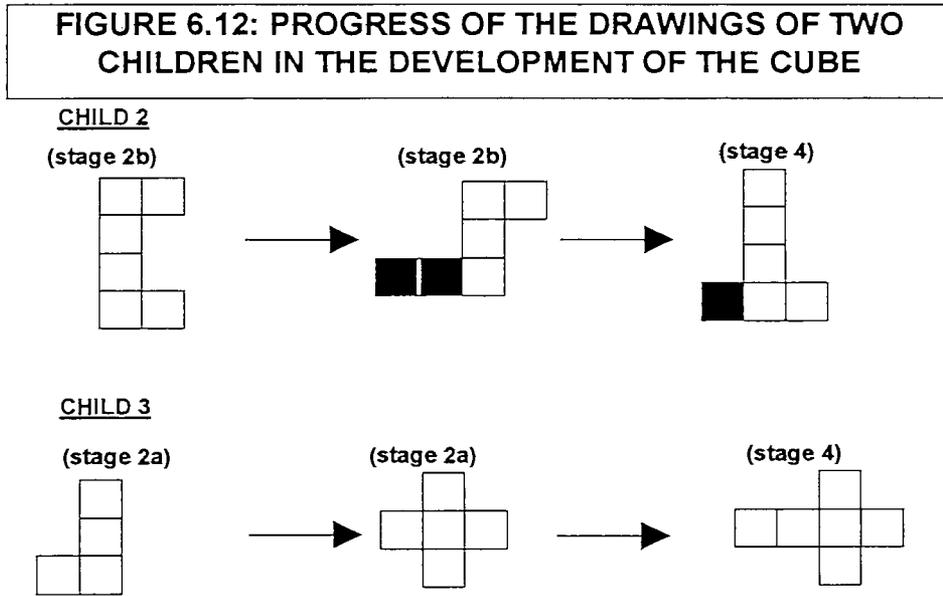
6.4.1.7 An accumulative version of the progress of the complete group with all the objects.

Figure 6.11 represents the progress of the entire group (bottom, middle and top) through the different objects, arranged with increasing difficulty. It shows that the easiest objects were the cylinder and the triangular prism 1, square-based pyramid, triangular prism 2, tetrahedron, rectangular prism and cube.



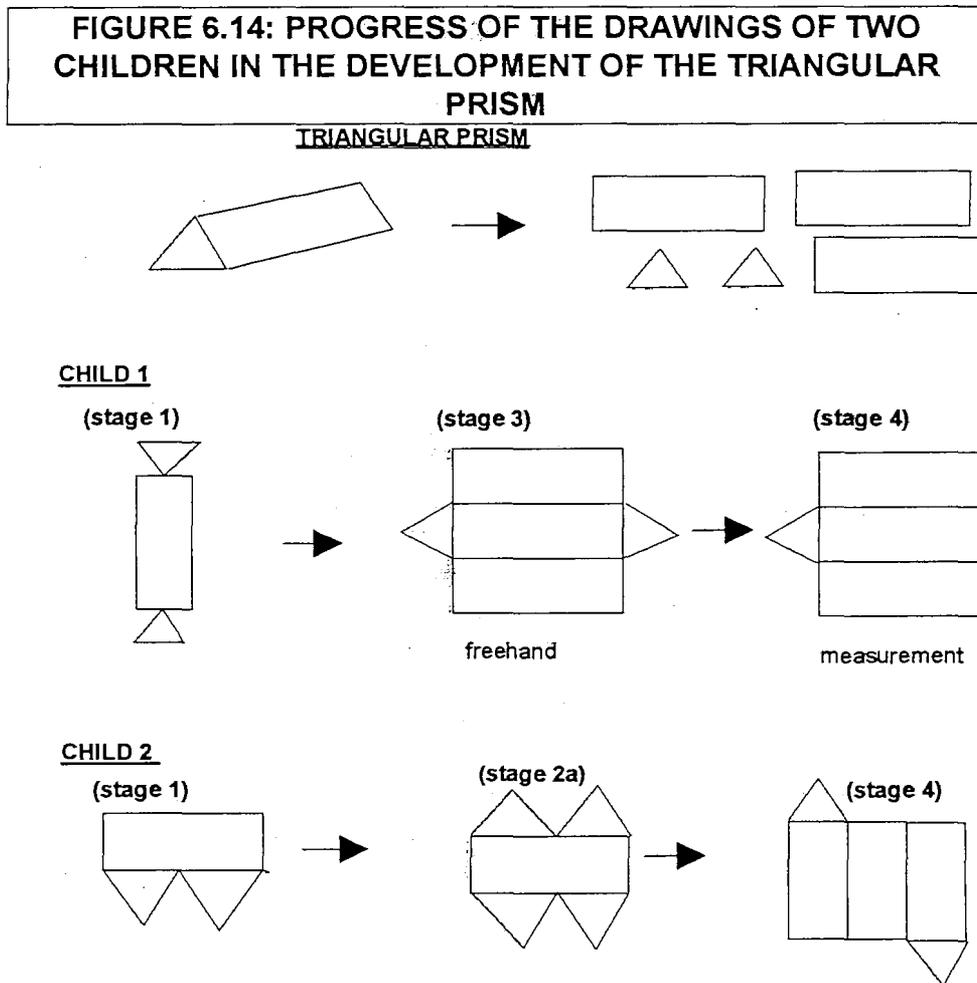
6.4.1.8 Drawings of the progress of individual children in the development stages for all the objects.

6.4.1.8.1 Cube



Child 1, in figure 6.3, started out with a stage (2a) version and proceeded to add one square to get a stage (4) version. This child did not realise that the solution was reached. This child proceeded to go through a succession of stage (2a) and stage (2b) versions ending in step 9 at the same version of stage (2a), that she started with. This time, however, it was realised that adding one block to the picture would lead to the final solution. It was only then that the child proceeded to reassemble the net. Child 2, figure 6.12, started out with the correct number of squares but with the wrong orientation. The child then drew another stage (2) drawing before the final version. Child 3, in figure 6.12, started out with the incorrect number of squares and drew two stage (2a) versions, with the second version clearly more advanced than the first. This child then skipped the (2b) stage by adding the last square for a final version of stage (4).

6.4.1.8.3 Triangular prism 1



Both child 1 and child 2 in figure 6.14 started with a stage 1 drawings. Child 1 skipped the stage (2) drawings and drew a stage (3) followed by a final version. Child 2 proceeded to draw the stage (2a) drawing but skipped stages (2b) and (3) drawings to give the final version.

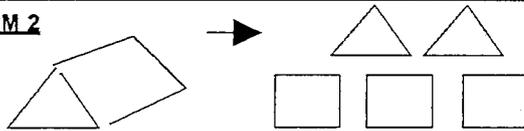
6.4.1.8.4 Triangular pyramid (Tetrahedron)

Child 1 in figure 6.15 started with a stage (2a) drawing and proceeded to make a stage (2b) drawing, presenting it as the final solution. Child 2 started with a stage (2b) drawing, fell back into a stage (2a) drawing and ended with another (2b) version that

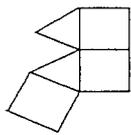
FIGURE 6.17: PROGRESS OF THE DRAWINGS OF FOUR CHILDREN IN THE DEVELOPMENT OF THE TRIANGULAR PRISM

TRIANGULAR PRISM 2

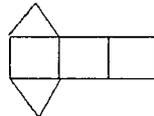
CHILD 1



(stage 2b)

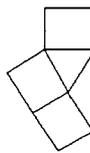


(stage 4)

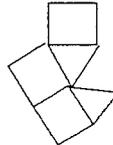


CHILD 2

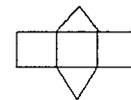
(stage 2a)



(stage 2b)

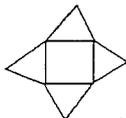


(stage 4)

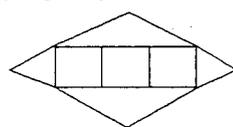


CHILD 3

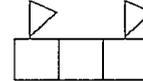
(stage 2a)



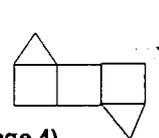
(stage 2a)



(stage 2b)

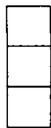


(stage 4)

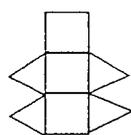


CHILD 4

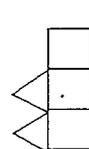
(stage 2a)



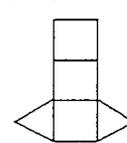
(stage 2a)



(stage 2b)



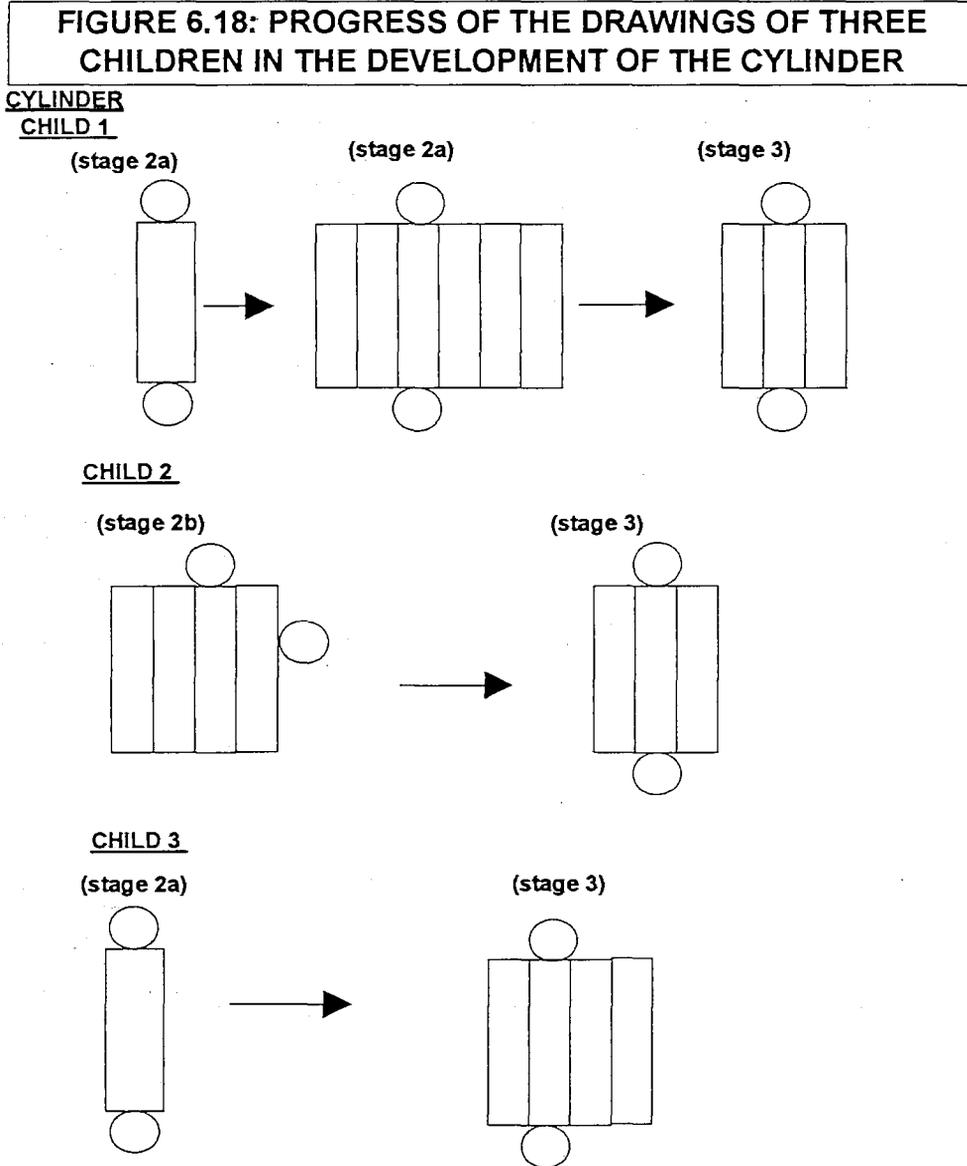
(stage 4)



The square-based pyramid was done just before the triangular prism and there were several children who showed the tendency to confuse the two shapes in their representations. The child then proceeded with a stage (2a) version, followed by a stage (2b) version and finally a stage (4) version as the final solution. Child 4 started

with a stage (2a) version, followed by another stage (2a) and then a stage (2b) and finally a stage (4) version as the final solution.

6.4.1.8.7 Cylinder



Child 1, in figure 6.18, started with a stage (2a) version, followed by another (2a) version and finally ending with a stage (3) drawing as the final version. Child 2 started with a stage (2b) drawing, followed by a stage (3) drawing as the final version. Child 3 started with a stage (2a) version and proceeded to give a stage (3) drawing as the final version.

6.4.2 Discussion

6.4. 2.1 Task

The order in which the objects were presented should be kept in mind when looking at the analysis of the difficulty of the different objects. The cube and the rectangular prism (cuboid) were given at the beginning of the experiment and the children worked through quite elaborate activities as illustrated in table 6.1. Other researchers have found contradictory results concerning the order of difficulty for the different solids. Bourgeois (1986:223) reported that other researchers also found that children have more difficulty with the tetrahedron than with the cube. Piaget and Inhelder (1956:286) found that children discover the nets of the cylinder and cone before that of the cube. They suggest that a possible reason for this might be the fact that the cylindrical and conical surfaces are not flat but curved, with the result that the curvature itself tends to suggest the action of unrolling them. Mitchelmore (1978:238) found that younger children had more difficulty with the cube and cuboid than with the pyramids. The results from the researcher's experiment indicate that the children found the cube and the cuboid (rectangular prism) more difficult than any of the other objects.

The contradiction in the above-mentioned findings may in part be attributed to the task and the experimental background of the activities. There is a great danger in merely interpreting drawing results *per se*, without taking cognisance of the context of an experiment.

To illustrate this point the findings of the foldout of the square-based pyramid is illustrated in figure 6.21 (4ii-vi). Woodward and Brown (1994:454) reported that their fifth grade children produced the solutions (in the shaded area) when they were asked to draw the nets of the square-based pyramid. What is interesting, though, was that none of the children in Woodward's & Brown's study came up with the final result

which the children in the author's study came up with. What is important here is the type of task that was put to children.

In Woodward and Brown's (1994:458) experiment, the children found the different nets only after they had assembled a given net which corresponded to the one the researcher's children ended with. In other words, Woodward's children were given a net that corresponded to the stage (4)-net of the square-based pyramid that the children in the author's experiment ended with! Once Woodward's children had assembled their nets into pyramids they were asked to unfold them into as many different combinations of nets as possible. These results are represented in fig 6.21 (4ii-vi).

The children in Woodward and Brown's (1994:454) experiment produced more versions as well as more difficult versions in comparison with the nets produced by the children that took part in the researcher's experiment. What is important, though, is that in Woodward and Brown's experiment the solution process that was induced by the experimental set-up was mainly mechanical. Woodward and Brown's children simply took the solids and unfolded them. Because of the nature of the materials that were used (polydrons) it was not possible to end with a net that could not be reassembled again. Their solution strategies were therefore partly embodied by the nature of the materials and the task.

The above was in stark contrast to the way in which the materials in the researcher's experiment were used. Even though use was also made of polydrons, the task that was put to the children left it to the children to make a choice between physical or mental-visual manipulation of the materials. They were not given a solid which they merely had to unfold and then take note of the different combinations that were possible. They were given a solid and asked to construct a net for the solid without taking it apart by first making a scale drawing or net. After the children were sure it was correct, without taking the object apart, they were to cut it out and reassemble it to see if it fitted. In other words, their only way of "testing" their results before they cut them out was to

“visualise” what they might look like. The total context against which the problems were put to the two groups (Woodward and Brown’s and the researcher’s) were very different and it would not be plausible to draw a comparison between their resulting nets.

One consideration throughout the experiment of the researcher, was that the materials and the way in which the task was set allowed children to work at their specific level when solving a problem. The important question that needs to be answered at the end of this exercise should not be how many different nets the children got, but how they set about in solving the problem and what they learnt from it, on the one hand, and what the teacher learnt from it on the other. It is these important issues that need to be addressed when it comes to deciding on the type of tasks that should be given to young children in order to enhance their spatial skills.

6.4.2.2 Object

From the different stages that the children drew for the objects, it followed that most children mastered the number and shape of the components before they mastered the actual correct orientation of the individual components (compare figures 6.3, 6.12, 6.13, 6.14, 6.15, 6.16, 6.17 and 6.18).

For the entire group the objects in order of increasing difficulty were: cylinder and triangular prism 1, square based pyramid, triangular prism 2, tetrahedron, rectangular prism and finally the cube (figure 6.11).

6.4.2.2.1 Cube

The objects that were used for the cube were solid wooden cubes. The majority of the children in this experiment found the cube to be the most difficult object. At first they had difficulty selecting the correct number of components even though they could

quickly “tell” one another that six squares were needed. After they had mastered the fact that six identical squares were needed, they had considerable difficulty finding the correct orientation for the parts. The fact that all the parts were similar gave the children no specific starting point and many nets were designed which did not match a cube when it was reassembled.

From figure 6.5 it is clear that all the children had difficulty during the first period with the cube as object. Most of the children produced nets that fell into the first two stages (stages (1) and (2a)). By the end of the third and fourth periods there were still a few children in the bottom group that were producing nets that fell into stage (2b).

The majority of the children drew solutions for the cube that are illustrated by figure 6.3 (4i and 4ii). Only the children in the top group discovered the other three solutions. Bourgeois (1986:228) mentioned that not all the nets of the cube present the same difficulty for young children. He also mentioned that children have more difficulty assembling the net of a tetrahedron than of a cube. Piaget and Inhelder (1956:275) found that children have difficulty imagining the lateral surfaces of solids before the age of 8 or 9. They also found that children discover the nets for the cylinder and the cone before those of the cube and the tetrahedron. Bourgeois (1986:223) mentions that some researchers have found the easiest solids to be the cube and the tetrahedron, whereas solids with curved surfaces and the trapezoidal solids were the most difficult. Mitchelmore (1978:238) is further of the opinion that the factors in operation for the drawing of solids in perspective may also influence children’s drawings of nets of these solids.

Bourgeois (1986:228) found that the degree of difficulty of a net is a function of both the form of the solid and the arrangement of the parts in the net. He found that children had more success associating nets with polyhedra when the solids contained triangular regions, and they also had more success when those regions were distributed around

the base of the solid in the net. The children who took part in his experiments found the nets of the cubes the most difficult.

The fact that children drew the correct net for the cube did not always automatically indicate to the children that they had reached the solution. In the case of child 1 (figure 6.3) the correct net had already been drawn in step two, but it took the child seven steps further to realise that step two was the correct result. This is a powerful example of the importance of the process in the development of these objects. This same situation was not only witnessed with the cube, but also with the other objects, although to a lesser extent.

6.4.2.2.2 Rectangular prism (cuboid)

The objects used for these activities were a variety of boxes which the children had brought from their homes and which were used as packaging materials for various household products. This was the only object where use was made of everyday articles because the majority of the packaging materials for products come in containers that are shaped like rectangular prisms. It is the ideal that all the objects should come from the child's world.

The object for the rectangular prism (cuboid) differed from the cube in that although it also consisted of six pieces, the pieces are not all of the same shape. The results for the cuboid by the children in the bottom and middle group were very similar to the first period for the cube object (compare figures 6.5 and 6.7). The children indicated that they had difficulty with the new object and that they did not know what to do. This was in stark contrast with the progress of the top group. Three of the children in the top group could design the correct net during the first period without having to go through any of the activities set out in table 6.1. By the second period all the children in the top group were able to construct the correct nets and they were even able to make variations of the objects. The children in the middle and bottom group took five

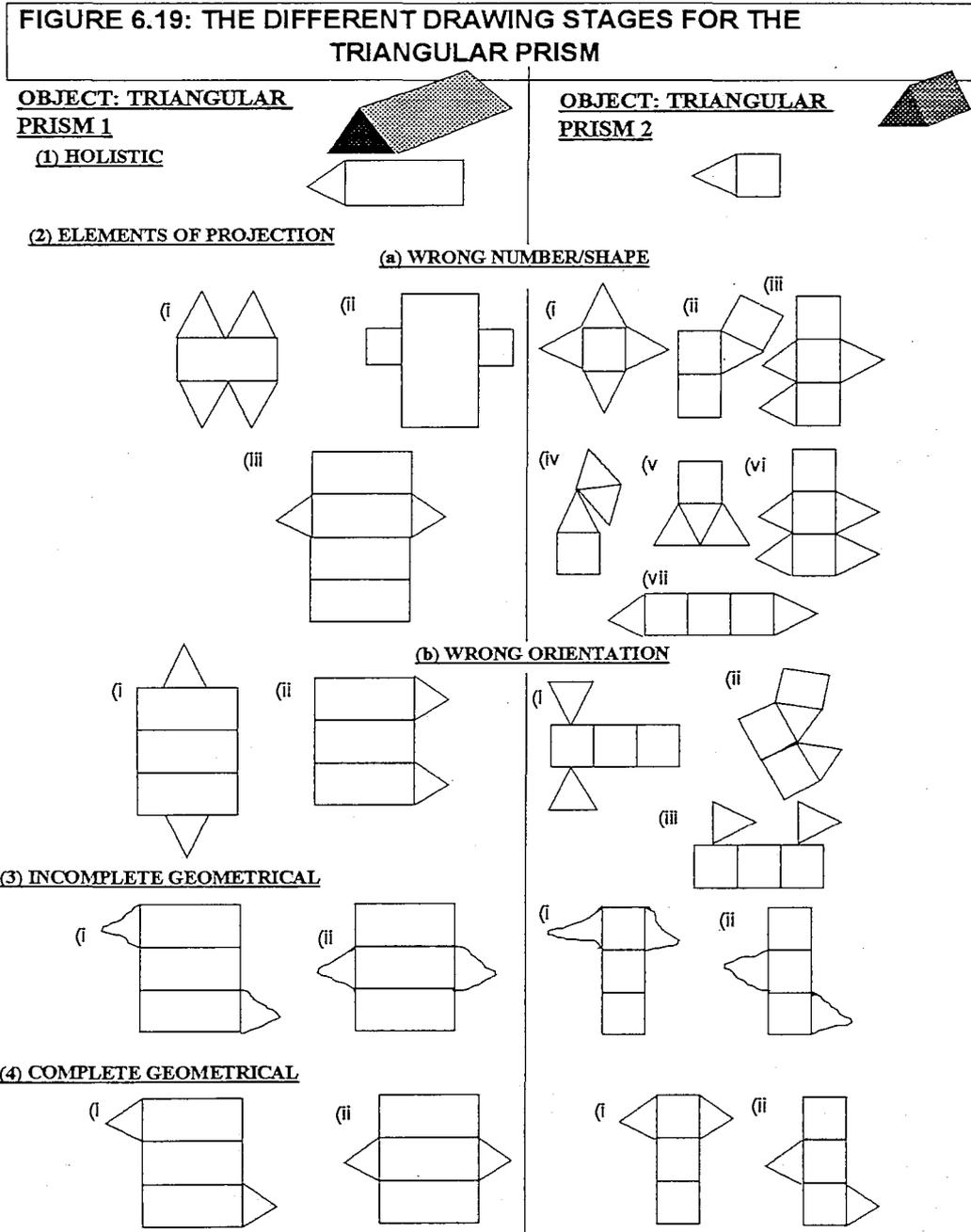
periods and they had to work through all the activities in Table 6. 1 before they could assemble the correct nets for the rectangular prisms.

An interesting solution for the cuboid that was actually incorrect, but went through as correct, was done by quite a few of the children in all the groups. Compare figure 6.6 (2bi) with (4I); when the nets were folded up they both gave the same final results but on close inspection the different individual pieces did not correspond with the original object. This situation resulted in a heated discussion among the children about the importance of the position and orientation of the different pieces in the net. They reached consensus amongst themselves that the former could not be seen as the correct solution of the problem.

It was during these activities that it became clear that it took the children in the top group less time to solve problems of this kind. Another interesting situation that occurred was that the majority of the children in the top group preferred a “mental-visual” strategy to a “mechanical” strategy for designing their nets.

6.4.2.2.3 Triangular prism

A very important feature that kept occurring after the objects for the cube and prism were completed, was that the children persisted in searching for six parts in the designs (drawings) of their nets (figure 6.19 (2aiii) and fig 6.20 (2aiv) and fig 6.21 (2aii) and fig 6.22 (2aii)). This points to the fact that the order in which the objects are being done has an influence on the activities which follow. Mariotti (1995: 110-11) calls this effect the role that prototypes play. She implies that the mental image of the object seems to be assimilated to form a standard image. This standard image does not seem to correspond to the real object. She has found that children have this confusion when working with regular tetrahedra and square-based pyramids (Mariotti, 1995:112).



Two different triangular prisms were given to the children as represented in figure 6.19. In the first case, triangular prism 1 consisted of two triangles and three rectangles. Triangular prism 2 consisted of two triangles and three squares (figure 6.19). Both the bottom and the top groups indicated that they found triangular prism 2 far more difficult than triangular prism 1. This was not the same for the middle group who found both these prisms to be of approximately the same difficulty (compare figure 6.9).

It should be mentioned, though, that the top group was only presented with the second triangular prism after the cylinder. The reason for this is that they moved through all the different objects at such a pace that by the time the other children had finished the first triangular prism, the top group had already finished the cylinder. It was during the construction activities of the first triangular prism, that one of the children in the middle group stumbled onto the other version of the triangular prism and it was then decided to include it as one of the objects.

The majority of the children that had difficulty with the second triangular prism indicated that the “pointedness” of the overall figure led them to be confused by the number of triangles that were needed. In many cases, children, especially those in the bottom group, designed the net for the triangular prism 2, which is illustrated in figure 6.19 (2ai).

This was also the first object for which the children produced incomplete geometrical representations. (Children from all the groups did this.) They indicated that they had difficulty in recognising the correct orientation of the triangle when it was in a three-dimensional representation and showing its equivalent in a two-dimensional representation, even though the triangles were all equilateral. Children that ended with stages 3(i) and (ii) in figure 6.19, were the ones that had previously drawn stages 2b (i) and (iii). In other words, they did not use the object to draw in the triangles, but they added the triangles freehand.

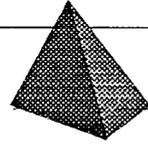
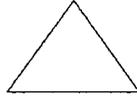
6.4.2.2.4 Tetrahedron

Most of the children found the tetrahedron the most difficult of all the objects that had triangular pieces (compare figure 6.11).

FIGURE 6.20: DRAWING STAGES IN THE DEVELOPMENT OF THE TETRAHEDRON

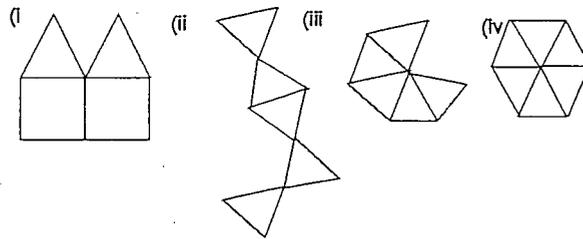
OBJECT: TETRAHEDRON

(1) HOLISTIC

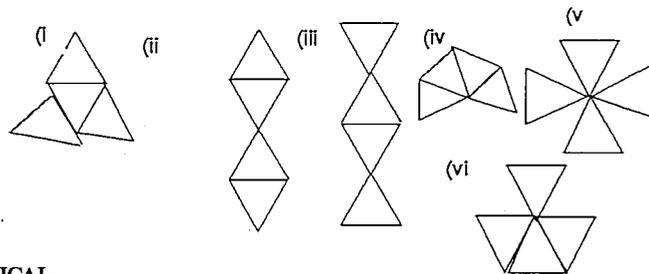


(2) ELEMENTS OF PROJECTION

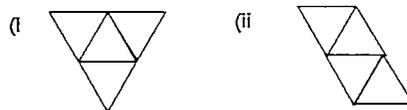
(a) WRONG NUMBER/SHAPE



(b) WRONG ORIENTATION



(4) COMPLETE GEOMETRICAL



Not all the children immediately realised that they only needed triangles to construct the net of a tetrahedron. Some of them also drew squares fig 6.20 (2ai). Object 2b (iv) was the net that most of the children, who employed a pure mechanical solution, drew. During the solution of the tetrahedron many children, who followed a mechanical solution strategy, lost track of the actual triangle that they were working with. This led to a situation where they came up with picture (2biv) in figure 6.20 From the graphs of the progress for the different children's objects (figures 6.8, 6.9 and 6.10) it follows

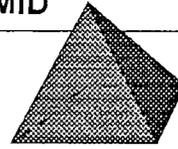
that more than half of the children reasoned that version 2biv, was the final net for the tetrahedron. It was only after they had cut it out and reassembled it that they found it to be incorrect.

6.4.2.2.5 Square-based pyramid

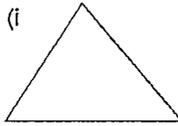
The majority of the children found the square-based pyramid fairly easy (figures 6.8-6.10). The issue of the six individual components (compare figure 6.21 2aii) were still in some of the children's representations (two children in the bottom group) even though it did not appear to be as frequent as at the beginning of the experiment. Another interesting problem that many of the children exhibited was their inability to find the right orientation for the different triangles. Many children (in all three ability groups) were not capable of finding the correct orientation for the four triangles by utilising the object. This resulted in quite a few incomplete geometrical solutions both as final productions and as intermediate steps on the way to the final production.

FIGURE 6.21: DRAWING STAGES IN THE DEVELOPMENT OF THE SQUARE-BASED PYRAMID

OBJECT: SQUARE BASED PYRAMID

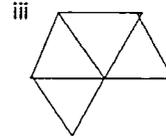
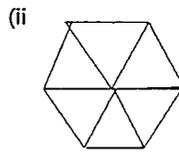
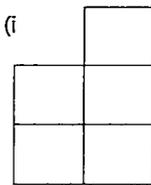


(1) HOLISTIC

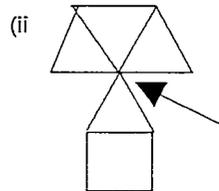
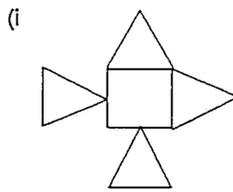


(2) ELEMENTS OF PROJECTION

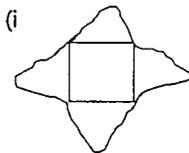
(a) WRONG NUMBER/SHAPE



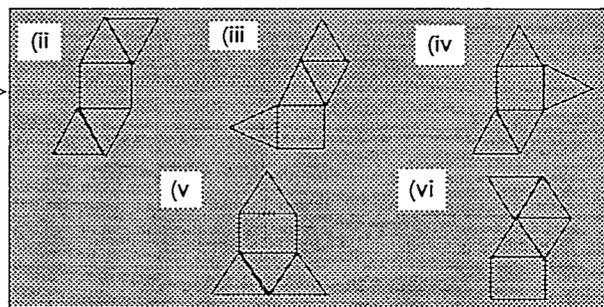
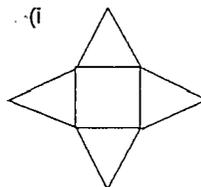
(b) WRONG ORIENTATION



(3) INCOMPLETE GEOMETRICAL



(4) COMPLETE GEOMETRICAL



The square-based pyramid (figure 6.21) was one of the objects for which only one final result was reached by all the children in the researcher's experiment. On inquiry all the children explained that result (4i) in figure 6.21 is reached through "cutting"

the three-dimensional shape along the sides of the triangle. Another interesting result, given by one of the children, is (2bii) which also resulted in a correct net. The children argued though that it was actually two loose pieces, and according to the definition which was negotiated at the beginning of the experiment, one is not allowed to have such a small area of intersection as indicated by the arrow, because then it is not a net anymore. The child (who repeated grade one) who reached this solution made use of a mechanical strategy for most of his other nets.

6.4.2.2.6 Cylinder

From the cumulative results (compare figure 6.11) it followed that the majority of the children found the nets for the cylinder relatively easy in comparison with the majority of the other objects. This is in contrast with the report of Bourgeois (1986:223), that the foldout shapes of solids with curved surfaces were the most difficult. It corresponds with the findings of Piaget and Inhelder (1956:286) though.

FIGURE 6.22: DRAWING STAGES IN THE DEVELOPMENT OF THE CYLINDER

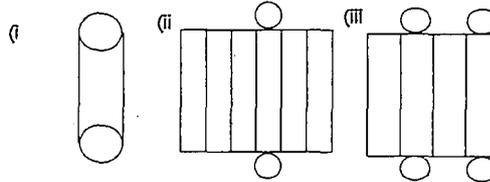
OBJECT: CYLINDER

(1) HOLISTIC

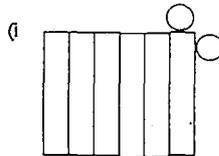


(2) ELEMENTS OF PROJECTION

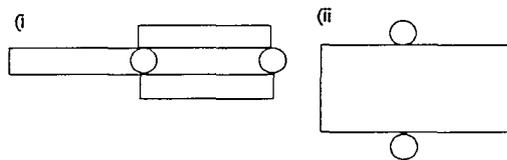
(a) WRONG NUMBER/SHAPE



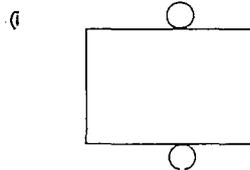
(b) WRONG ORIENTATION



(3) INCOMPLETE GEOMETRICAL



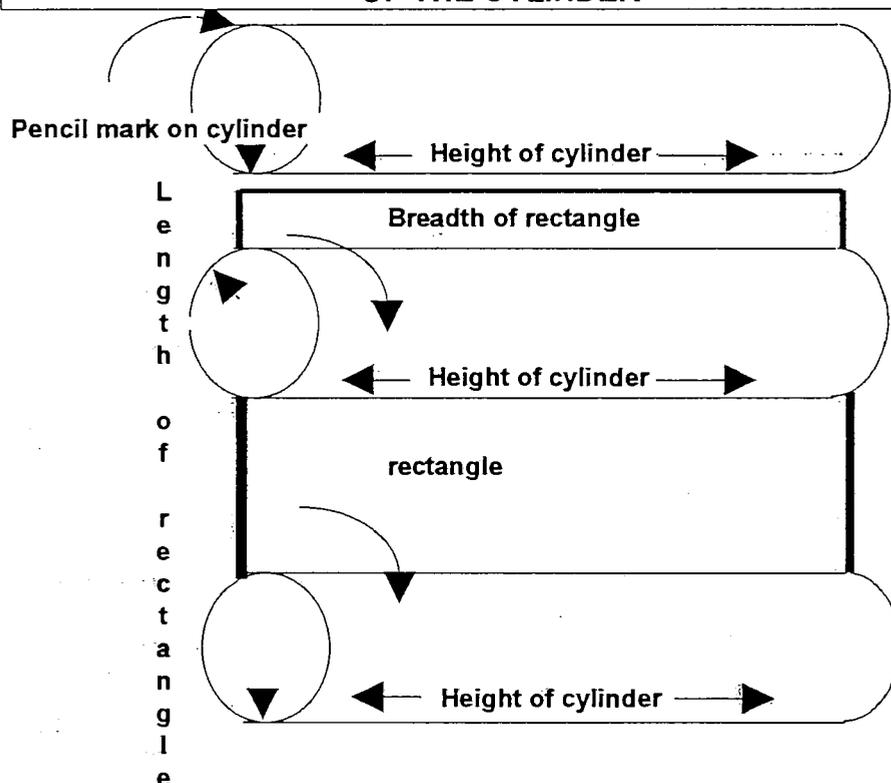
(4) COMPLETE GEOMETRICAL



The notion that all solids are constructed from six individual pieces was still a consideration for a few of the children (figure 6.22 2a(ii) and 2b(i)). There was also confusion as to the number of circles that was needed. Most of the final objects could be classified as incomplete geometrical in the sense that the children did not determine the length of the rectangle correctly. They estimated it and after they had cut it out and folded it up, they simply cut off the excesses. If the rectangle was too short they went back to their drawings and added the extra length that was needed, in a trial and error fashion. Almost all the children realised that the breadth of the rectangle is the same as

the height of the cylinder, but very few children realised that the circumference of the circle was the length of the rectangle.

FIGURE 6.23: A METHOD TO DETERMINE THE DIMENSIONS OF THE RECTANGLE, THAT COMPRISES THE CURVED SURFACE OF THE CYLINDER



Most of the children who calculated the circumference correctly, simply took their papers and wrapped them around the cylinders and marked the paper with a pencil where the two pieces of the rectangle met.

One child (middle group) had a very original way of determining the length (figure 6.23). This child took the object of the cylinder and drew a line along the height of the triangle in order to get the length of the rectangle. He then drew two parallel lines that were perpendicular onto the previously drawn length. He drew a pencil line on the physical object of the cylinder. He put the object with the position of the line on top of the corner at the place where the right angle of the rectangle was. He rolled the cylinder until it stopped, with the line on the object in exactly the same position as

when he started. He made a mark and then drew the other length perpendicular onto the point where the line on the object had ended. The final line for the opposite breadth was then drawn parallel to the original one. He finally drew two circles on the sides.

The biggest difference between the top group of children and the other two groups was found when working with the cylinder. The top group had already started with the cylinder while the other two groups were still busy with the triangular objects. Not one of the children in the top group ended with a complete geometrical object of the cylinder. They also had difficulty when they started with their drawings and a number of them indicated that it was not possible to draw the curved part. This was in stark contrast with the other two groups who tackled the cylinder with the same attitude as all the other objects, and they were not disturbed by the curved part.

During the activities and discussion of their drawings it became clear that the children in the top group, who had mostly employed “mental-visual” strategies to solve the previous objects, had difficulty using this strategy for the cylinder. This seemed less of a problem for the other two groups who had spent more time on the “mechanical” strategies. They simply went about it in a “mechanical” way for the construction of the cylinder as well. This once again stressed the importance of the process through which children should move in their problem solving. The top group, because of their quick understanding, moved through the objects on a “higher level” (mental-visual strategies rather than mechanical strategies) than the other children, until they reached another type of object with a curved surface.

6.4.2.3 Dimension and viewpoint

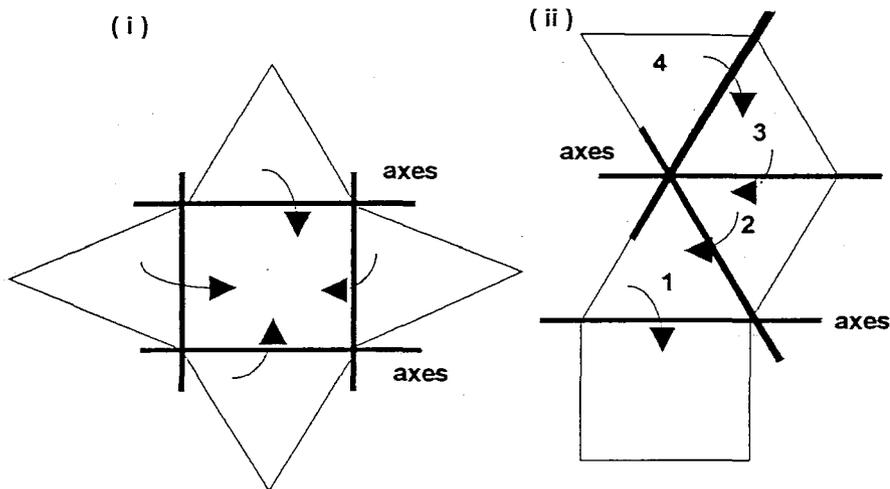
When children have to represent (draw) a solid in a two-dimensional way, they are forced to discover the properties of the spatial figures they are working with and they

are then confronted with the laws which govern such a spatial figure in a two-dimensional level (Potari & Spiliotopoulos, 1992:46).

The type of drawings (foldout drawings) that were made cannot be rated in the same category as those that are made for three-dimensional objects. The children did not have to find a way to overcome the depth-dimension on the paper as is the case with three-dimensional objects. The depth dimension problem had to be dealt with before they did the illustrations on paper. In other words, they had to decide what the axis of rotation was in three dimensions and then had to draw it in such a way that it still operated along that same axis after the drawing was folded.

It is clear from figure 6.24 that there is a difference between the processes of constructing a net for the square-based pyramid as illustrated in figure 6.24(i) and figure 6.24(ii). From these studies it followed that all the children started by first drawing the square and then followed this by adding the individual triangles. It did not matter which triangle was drawn first because it could not influence the orientation of any of the other triangles. Even so, some of the children still had difficulty, even with the orientation of the individual triangles. This is different for figure 6.24(ii) because there is a specific order that should be followed when drawing the triangles, because the position and orientation of one influences that of the next one. This implies that the child, whilst drawing one triangle, must keep in mind the orientation of the previous one as well as the next triangle. Another difference between figures 6.24(i) and (ii) is that the depth rotation is only along the x-axes and z-axes in the case of (i), while for figure (ii), rotation along the y-axis is also necessary. These differences might attribute to the finding that the children preferred constructing the net which is illustrated in figure 6.24(i).

FIGURE 6.24: TWO DIFFERENT WAYS OF DRAWING AND CONSTRUCTING THE NET OF THE SQUARE-BASED PYRAMID



Mitchelmore (1980:84) argues that the same stages found in the drawings of solids can be found in the drawings of the foldouts of these solids. These stages for the solids are characterised by the different views that they represent (orthogonal, orthoscopic, and a view from a single viewpoint). Potari and Spiliotopoulos (1992:39) divided these drawings into five different stages on the grounds of the suggestions made by Mitchelmore. One of Potari and Spiliotopoulos' (1992:46) findings concerning the dimension of the drawings, was that the very young children, (9 years) in their experiment, drew nets of which the dimensions were smaller than those of the solids. They found that the older children (11 years) actually made use of the solids to measure and calculate the dimensions. This is in stark contrast to most of the work done by the children in the experiment of the researcher. The only instance where the children did not utilise the physical object *per se*, was during the early stages of the development of the cylinder. In all the other cases where this phenomenon presented itself, was with the incomplete geometrical presentations where only parts of the o' were not done on scale (figure 6.19 3(i) and 3(ii), 6.21 3(i), 6.22 3(i) and 3(ii)).

6.5 Constructions

Throughout the actual construction of these shapes children gradually become aware that spatial figures are built up from two-dimensional shapes. Although these activities imply that children do lots of physical cutting and drawing, they also indicate that these make quite a demand on their mental capacities (Potari & Spiliotopoulous, 1992:38 and Parzysz, 1988:79).

6.5.1 Results

Information on the construction activities was gathered through analysing the video recordings of the actual classroom periods. The objects for the cube, rectangular prism and the cylinders were readily available in the sense that these shapes are found in the form of packaging materials in South Africa. It was far more difficult, though, to get hold of an example of the square-based pyramid, triangular prisms and the tetrahedron. The researcher made use of the educational construction toy called “polydrons” to construct the latter-mentioned objects. The advantage of these materials is that, because of their specific design, they are very suitable for making nets (Woodward & Brown, 1994:451). The construction activities included all the cutting pasting and building activities in which the children engaged. The results of these are represented in figures 6.3, 6.4, 6.6, 6.12, 6.13, 6.14, 6.15, 6.16, 6.17, 6.18, 6.19, 6.20, 6.21, 6.22, 6.23 and 6.24.

6.5.2 Discussion

6.5.2.1 Task

The construction activities were an integral part of all the activities in which the children were involved. It was also the one medium which was the final “judge” of their results. From a problem-centred point of view it is the ideal situation where there

is an built-in factor in the situation which excludes the teacher as the judge of the success of the solution.

In this experiment the task was of such nature that the children had final products in front of them in the form of the solids and they had to construct the equivalent of those objects by utilising the materials that were available to them. They were not allowed to take the objects apart but they were allowed to use the objects to take the necessary measurements. After they had measured, drawn, and cut out their nets, they had to reassemble them again and then compare them with the original objects. They also compared their objects with those of their friends. Very little, if any, intervention was necessary for the teacher in order to judge the success of the outcome.

6.5.2.2 Object

During the construction activities it became clear that some of the objects were more difficult to reconstruct than others, such as the cube and the rectangular prism (compare figure 6.11 and the discussion under 6.4.2).

6.5.2.3 Dimension and viewpoint

The construction activities involved the constant switch between two and three dimensions. Pallascio *et al* (1993:8) point to the importance of geometry teaching where activities should be alternated between activities which are analytic (where one observes shape; two-dimensional) and activities which are more synthetic in nature (where the construction of scale objects play a key role; three-dimensional).

These construction activities differed from the activities that were done with the blocks. During the activities with the blocks, constructions were made with three-dimensional objects. In the case of the solids, constructions were made with two-dimensional objects (see figure 6.1). The other important feature of these activities is

that a learning situation is created where the children can experience the physical transition from three to two dimensions and back.

6.6 Conclusion

The aim of this chapter was to investigate the continuation of the development of the different skills (visual, verbal, tactile and mental) but on a “higher level”. The nature of the original problem that was put to the children in this part of the experiment was more difficult than the nature of the tasks that were put to the children in the blocks experiment. The construction of the nets of different solids implies that the children had to co-ordinate “mental” operations with the physical analysis of the *individual parts* (edges, vertices and faces). During the experiment with the blocks the materials were used and manipulated in their *holistic* forms without taking them apart.

This chapter is a continuation of the description of how young children develop their different spatial skills through the utilisation of different media.

Only one task, namely to design a “net” for the different objects, was intended. The fact that the researcher could not make use of the term “net” because it had not been part of the children’s “geometry” vocabulary at the time, complicated the task considerably. The situation that manifested itself was the transition from the use of the terminology in the everyday use to a subject specific use. Apart from the fact that this in itself is a difficult transformation in one’s own language, quite a number of the children had to do this in their second language. This was only a problem, though, with the first and to a lesser extent with the second object. It was not a problem with any of the other objects. This might be an indication that the children had a better understanding of what was expected from them or it was an indication that they started to understand the concept of making a “net”, or both.

The influence of the *order* in which the different objects were presented to the children, as well as the *way* in which the task was formulated, can be seen in the drawings that the children made. During the first few periods the researcher stressed the fact that the cube consists of six individual parts. The result of this was that quite a few children still divided the rectangular part of the cylinder into six different parts. The difference in the way in which the task is formulated, is highlighted by the comparison of the different drawings of the nets of the square-based pyramid of the children in Woodward and Brown's experiment and the researcher's experiment.

The construction activity once again acted as the final "judge" of the success with which the children interpreted the task. It was very clear from the construction activities whether or not the children understood what was expected from them.

The different objects that were used implied that the children and the researcher/teacher should start using specific terminology to indicate different objects as well as different individual parts of the objects. The complex situation of distinguishing between linguistic competence versus conceptual competence was once again highlighted. Another factor that appeared was that the children could verbally recall the correct information but on closer inspection, through the construction medium for example, it became clear that there was little insight. Even though they *said* that six pieces were needed for the cube they *took* only four pieces when they selected their materials to construct the cube!

The drawing results of the different objects showed several important trends. The first trend was the fact that the majority of the children had difficulty progressing even beyond stages (1) and (2a) for the cube and the rectangular prism after the first period. In other words, the majority of the children had difficulty coping with the *number* of individual components of the two objects. This is in stark contrast with the other objects (triangular prisms 1 and 2, tetrahedron, square-based pyramid and cylinder) where the majority of the children had few problems with the *number* of individual

parts but the *orientation* of the parts was the actual problem. Secondly the process through which the children moved was not always hierarchical, namely from stage (1), (2a), (2b), (3), through to stage(4). Some children moved into higher stages and moved back again before they moved on. Some children skipped intermediate stages. The general trend, though, was that children's results always reflected a higher stage as the final result in comparison with the stage that they started with. Thirdly the "curvedness", "pointedness" or "straightness" of the surfaces of the objects seemed to influence many of the drawings that the children made.

Through the construction activities the children indicated that some objects were more complex to construct than others. Not only did some of the children experience difficulty when working with different types of objects but also between two objects that could be classified under the same name namely triangular prisms. The triangular prism, which was constructed of three rectangles and two triangles, was more difficult to construct than the triangular prism that consisted of three squares and two triangles.

The children had reached consensus on the use of the different deictic terms, and the dimensional problems that they had to overcome were highlighted by the activities in the other two media.

The types of drawings that were made as part of the solutions were different from the normal representations of three-dimensional objects in a two-dimensional medium. The children had to "visualise" or "mentally" imagine what the dimensions of the foldout would amount to after the drawing was cut out. In other words, they had to overcome the dimension problem by keeping in mind the different axes along which the individual parts had to move in order to result in a correct foldout. It implied that the children had to move to and fro between the dimensions. The children employed two definite solution strategies namely a "mechanical" strategy and a "mental-visual" strategy in their endeavour to overcome the dimensional problem.

The constant switch between the two dimensions was also visible during the construction activities. The construction activities with these materials differed from activities of the previous chapter in that the different objects were taken apart and put together. This implied that materials *per se* implied a dimensional change namely from a three-dimensional cube to six congruent two-dimensional squares.

In the next chapter the objective will be to highlight the fact that the geometry knowledge of the children was investigated by starting in the real worlds of the children. This was followed by a gradual buildup of insight through the utilisation of a mathematical/spatial approach which culminated in some form of mental activity or “visualisation”. At the same time the spatial sense of the children was developed in preparation for geometry teaching later on.

CHAPTER 7

DEDUCTIONS RECOMMENDATIONS AND CONCLUSIONS

7.1 Introduction

South Africa, like many countries in the world, has a history of a lack of adequate geometry training in primary school. The introduction of a problem-centred approach at the beginning of the 1990s, which only focused on the number component, serves as an indication of this bias (compare 1.2). The fact that there is a general tendency all over the world to change the kind of geometry that is needed for the primary school can be summarised by the words of the research report by the NCRMSE (1994:1): “Visual reasoning is central to mathematics. It is an integral part of mathematical and scientific inquiry.”

Geometry also has another aspect that it shares with the broader field of mathematics, namely its potential use for modelling situations. Because of this aspect, geometry should (those who seek a reformed school mathematics curriculum claim) be integrated into the mathematics courses children study throughout their K-12 years. The NCTM Standards (1989) and other reform documents call for increased content in geometry and spatial reasoning across grades K-4, 5-8, and 9-12. The isolation of geometric content to a single high school course and the emphasis within this course on proofs contrasts with international practices, which give geometry and spatial reasoning more central roles in K-12 curricula.”

From the literature it has become clear that a major paradigmatic shift has been made concerning the research approach that is needed to answer current research questions regarding the teaching and learning of mathematics (compare 3.4.6). According to Schoenfeld (1994:708), the reason for this was, on the one hand, the limitations of the established methods and perspectives on teaching and learning, and on the other hand the opening of new avenues. One such avenue was the renewed interest in the study of

the mind. The study of the mind was once again seen to be possible as well as scientific. The study of the mind has opened at least two strands of investigation, firstly the phenomena of problemsolving with the ensuing issues of metacognition and beliefs and secondly the relevant research methods and perspectives.

According to Schoenfeld (1994:708), Piagetian studies which have been marginalised for years began to flourish again, and with the renewed interest in constructivism came a host of research methods. Schoenfeld continues by mentioning the increasing interest in anthropological studies along with the methodological tools that produced such studies as being of significant value. As the field came to recognise the complexity of the issues it faced, it also came to recognise the value of multiple perspectives and approaches. In the words of Schoenfeld (1994:708), “educational researchers are finding themselves emerging from a methodological straightjacket into a Pandora’s box of opportunities and problems”.

Verschaffel (1993:12) states that when a researcher uses a new method it is important that certain criteria should be met. The criteria that he considers important are, according to the prescriptions of Schoenfeld (in Verschaffel, 1993:12):

- A clear description of the objective with the method should be stated (compare 1.3).
- Reasons for the choice of a new research method (compare 2.10).
- A detailed description of the methods in order to enable anyone else to go ahead and use the method (compare 5.2, 5.3, 5.4, and 6.2)
- An adequate description of the concrete data set to enable researchers to (a) check their results against other authors (compare 5.5.1; 5.6.1, 5.7.1, 5.8.1, 6.3.1, 6.4.1 and 6.5.1) and (b) to check if the results and interpretations that were found by the researcher, correspond with the results and interpretations of other authors (compare 5.5.2, 5.6.2, 5.7.2, 5.8.2, 6.3.2, 6.4.2 and 6.5.2).

- A critical discussion of the method during which the strong and the weak points of the method are highlighted, as well as commenting on the scientific validity of the method.

During a research method where the major question is to describe the conditions under which learning takes place, the focal point is not the improvement of the learning/teaching situation *per se* but the “how” of the situation. According to Gravemeijer (1994:450), in developmental research knowledge gain is the main concern. The focus is on building theory, explicating implicit theories. He states that at the start the theory consists of a global framework and eventually concretises into local theories. The global theories that were used as starting points for this research were taken from a wide range of approaches (compare chapters 2, 3 and 4).

Throughout all the research activities it was expected of the children to express themselves in different media. The objective with this was to utilise the different media (language, drawing and construction) to develop specific skills. The four different skills that were focused on were: verbal, visual, tactile and mental. Furthermore the researcher introduced different variables into the situation the so-called instructional variables. The three major instructional variables that are described in this research approach are the nature of the task presented to the children, the type of object used as stimulus materials and the dimension and viewpoint in which these materials and situations were presented.

It is considered that children learn at least some things better from concrete objects and from visual representations of experience than they do from language (Olson, 1974:16). Yet the media should also be regarded as a variable because it is not known what the structure of the media, that is, their informational potential, would be when working in this specific area with young children.

Olson (1974:17) summarises the difference in skill development in different media by saying: "Media converge as to knowledge conveyed, but they diverge as to the skills they assume and develop. Instructional media cannot simply be chosen in terms of their ability to convey certain kinds of content, but must also be chosen in terms of their ability to develop the processing skills that make up such an important part of intelligence."

The variables and conditions under which the spatial development was investigated, with the final focus on the development of the different skills, are discussed under three main headings namely language, drawing and construction.

7.1.1 Language

Van Hiele (1981:7) is of the opinion that language has its own inherent structure that can lead to acceleration or deceleration of the development of a subject. He elaborates on the function of language when he states that language does not only function as a medium of communication between individuals but also allows the individual to think. Some knowledge concept acquisition takes place through the use of language and others are called up through language symbols (Van Hiele, 1981:45). This research also emphasised this dual role of language (compare figure 6.2 and section 6.3.2.1).

Cox (1986:155) mentions that the importance of studying the child's acquisition of deictic terms is that it provides an opportunity to tap the child's understanding of the changing roles of conversational participants and the shifting referents to which the terms relate. This shift in referents was also noticeable in this research in that some children used different deictic expressions for the same physical positions in space (compare figures 5.10 and 5.11). These terms are not learnt by a simple association between a word and an object, nor are they influenced directly by the frequency with which they hear the terms in adult speech around them (Cox, 1986:156). By the age of about 2 or two and a half they have mastered the terms *I/me* and *you* to refer to the

speaker and the addressee, respectively and have done so with minimal error. Other terms such as the locative contrasts *here* and *there* and *in front* and *behind* are worked out later. Cox (1986:111) has also found that the occurrence or non-occurrence of the naming of the object seems to be particularly important, as is the way that the scene is described. This phenomenon was also illustrated through this research in that the children originally did not pay attention to the use of the correct terminology but as their understanding increased, they started focussing on the specific subject terminology (compare figure 6.2, section 5.5.1 and 5.5.2.3).

In the medium of language the researcher manipulated the development of the different skills through the introduction of the instructional variables namely: task, object and dimension, and viewpoint of the stimulus materials. During the activities with the blocks different instructions were given and a number of tasks were introduced by the teacher. The children responded through the use of the different deictic terms during their verbal discussions. They were expected to use their natural language correctly. The focus at this stage was the use of natural language for the development and expression of spatial phenomena. This had to be established before the transitions to subject specific knowledge was possible. In the bottom group this was quite an issue for the majority of the children who were second language learners (compare section 6.3.2.2).

Van Hiele (1986:83) focuses on another important aspect of language use especially during the introduction of new materials. He states that at the visual level it is possible to see certain things without being able to explain why they look that way. The discussions that follow, as well as the understanding of that which was seen, need the language that belongs to the next level of development. During the work with the solids the children were required to make transitions from the use of natural language: "A fish is caught in a net" to the use of those same words but in a spatial context: "The net of the cube consists of six similar squares linked together." The ease with which children were able to switch from the use of the language in the natural form to the

subject-specific form gave indications of the expansion of their verbal skills. Van Hiele (1986:86) once again points to the important matter that language is not necessary for a reaction to observation, but it is still useful because by the mention of a word, parts of a structure or concept can be called up.

The children made use of the different deictic terms in allocating specific parts to objects. This led to a classification of objects as being canonical or non-canonical, which in turn indicated the extent to which children found some objects more difficult to describe or comprehend than others. In the work with the blocks the children found a need to indicate specific parts of the soma cube, (top, in front etc.), where in the activities with the solids the nature of the task was such that there was no need to identify the top or bottom of an object. This again pointed to the importance of the way a task is presented to children. This illustrated that the type of task can be manipulated to enhance specific skills. It also gave the teacher insight into the extent to which the children's verbal skills were developed.

Evidence in the research of Cox (1986:138) has indicated that children first relate deictic terms to their own position in space. Some researchers have assumed that this self-centred beginnings necessarily imply that the child is egocentric. However, as Cox argues, with regard to young children's use of their own position in space, taking themselves as reference points do not necessarily imply that the children are egocentric, in the sense that they are "locked in" and unaware of others. It is simply a useful and stable reference point to adopt. After all, children have to start somewhere, and self, being stable, seems a sensible choice. The point is, in order to use the terms correctly to refer to themselves, the children must first have understood how they are used by other people. They then have to reverse the roles and apply them to themselves. Thus any child who has mastered the shifter properties of *I* and *you* and *me* and *you* in ordinary conversation has achieved some degree of de-centering.

In the case of the blockwork the children mainly allocated these terms with regard to their own bodies or the environment as reference point. The use of the different deictic terms in the correct way was negotiated by the children during the blockwork to such an extent that there was very little sign of this problem during the work with the solids.

7.1.2 Drawing/writing

In the medium of drawing the child moves into a two-dimensional situation which is complicated by the restrictions of the medium to represent a third dimension. Research has shown that the deductions to be made from a pictorial or written representation by a child should not be done without considering the total context in which the activity takes place.

During the work with the blocks the children were directed, through the task, to draw the blocks without special reference to the purpose. The main instruction was to "Draw the structure that is in front of you". In the case of the blocks the task was relatively simple in the sense that the three-dimensional structure had to be represented in two dimensions. It was very different when it came to the task with the solids because the drawing was a transformation of the three-dimensional structure. This implied that in the activity with the blocks the visual and drawing skills were enhanced whereas in the drawings of the solids the structure that was in front of the children had to be mentally manipulated before it could be drawn. This activity did not only call on the utilisation of visual and drawing skills, but also on the mental skills.

The objects that were drawn during the activities with the blocks were "static" representations of the physical features. The objects that had to be drawn during the activity with the solids had a "dynamic" component. These solid objects had to be drawn in such a way that the physical objects could be formed after they had been manipulated through cutting and folding. In both activities the differences in the types of objects that had to be represented proved to influence the way in which the objects

were drawn. In the case of soma cubes 5, 6 and 7 the children made use of different strategies to represent the depth dimension in comparison to somas 1 to 4. Furthermore, the extent of contrast or similarity between objects in a scene has also been shown to influence the kind of pictures that children draw (Cox, 1986:84). This was also evident in the drawings that the children made of the mirror images of the soma cubes (compare figure 5.20). In the case of the drawings of the solids the overall view of the objects, the number of different parts that the object consisted of, as well as the orientation of the different parts with regard to one another, seemed to influence the drawings that the children made (compare figure 6.11).

During the drawings of the blocks the children had to overcome the problem of depth representation on paper, while they had to overcome the problem of depth representation for the solids before they drew them on the paper. Once again this implied that the children had to make use of a mental activity for the drawings of the solids. In the case of the blocks the children were free to make their own choices as to the view from which they drew the objects. This resulted in a multitude of representations for the same object. The children also had no way to evaluate the "correctness" of their block drawings and many different drawing versions of the same objects were acceptable to them.

This was very different for the drawings of the solids. The children were forced by the nature of the task as well as the medium of drawing to establish the role of the view before they started with the drawing. After the drawings had been finished the final judgement lay in the reassembly of the cut-out of the drawing. Another prominent difference between the nature of the drawings of the blocks and the solids lies in the movement between the dimensions. In the case of the block-drawings there is a movement from the three-dimensional physical representation to the two-dimensional representation on paper. In the case of the drawings of the solids there is a movement from the three-dimensional representation via a mental manipulation to the two-

dimensional representation on paper and back to a three-dimensional evaluation in the final assembly phase.

Cox (1986:84) comes to the conclusion that it is possible, then, for young children to draw them as they appear. She states that it is generally much more difficult for them to show what objects look like. So, for example, children will draw two faces of a cube if they can see only two, but they have great difficulty in getting the angles right so that the drawing reflects the shape projected to the eye. This was also the case with the drawings of the children in this experiment. It turns out that this is a problematic task even for adults.

There is also the problem of overcoming one's own knowledge about objects in order to draw them as they appear. So, if it is known that a cube is made up of rectangular surfaces, it is difficult to suspend this knowledge and draw shapes which are radically different. Unless we have artistic training to overcome these biases, even adults have difficulty in drawing a visually realistic view of the scene. So the ability to draw something from our own point of view turns out to be quite a struggle, and not something that we naturally do.

The communication of the different pictorial and physical structures through the use of other forms of written codes (apart from drawings) proved to rely on yet other skills that were not yet well developed at the time of the experiment (compare 5.7).

7.1.3 Constructions

The tasks when doing construction work with the blocks, were simple in that the children were required to "copy" what was presented to them. The success of the outcome of their constructions was immediately evident from the original presentations. This was different in the case of the solids because there was a "time lapse". The children had to first go through a design phase (drawing and cutting)

before they could be sure of the success of their constructions. The type of task that was put to the children in order to construct the solids also depended on their understanding of the problem. In this situation it also implied that the children had to understand the use of words like “nets” in the natural language context to “nets” in the geometry context. In other words, the task for the development of solids was more difficult for the children than the task in the case of the blockwork.

The type of object that was used during the blockwork could be freely manipulated during the construction process. It was evident, though, that objects that differed slightly in orientation like the number 6 and 7 soma cubes, which are the mirror images of one another, presented problems. Even though the children were able to handle the physical objects they were still confused about the actual issue of similarity. This was once again an indication that another skill apart from the sensory-motor skills needs to be developed. The construction work, with the solids as objects, was complicated because the children had to make mental or visual images of what the final constructions would look like after their drawing phases. Once again more than the tactile or sensory-motor skills were needed to construct the correct solids.

During the block building work, constructions were made while utilising pictures (2-D) as well as physical constructions (3-D). This implied that the children had to either work from two dimensions to three dimensions or from three dimensions to three dimensions. In the case of the solids the children worked only from three dimensions via two dimensions and in some cases a mental activity to three dimensions. The movement between the different dimensions in the case of the solids once again implied that several different skills were called upon, apart from the sensory motor skills (tactile), in order to successfully execute the activities.

7.2 Deductions.

The following deductions can be made from this research:

7.2.1. The execution medium (language, drawing, writing and construction)

The medium in which the work is done plays an important role in both the development of the skills as well as to indicate some form of competence with a skill (compare 4.5, 5.5, 5.6, 5.7, 5.8, 6.3, 6.4, 6.5).

7.2.2 The context

The context within which the task is presented is influenced by the cultural background of the children (compare 4.2.1.1, chapter 5 and chapter 6).

7.2.3 The order of the activities

The order in which the activities are presented to children has an influence on the way in which they execute the task (compare 4.2.3 chapter 5 and chapter 6).

7.2.4 The type of task

The type of task that is given to the children can be influenced by the movement of the object, as well as the movement of the viewer that is implied through the setting of the question (compare 4.2.2.1, 4.2.2.2, chapter 5 and chapter 6).

7.2.5 The type of object

The type of object that is used in the experiments can influence the difficulty of the task (compare 4.3, chapter 5 and chapter 6).

7.2.6 The dimension of the stimulus materials

The dimension of the stimulus materials can and should preferably be varied between a two-dimensional representation (picture or writing) and a three-dimensional representation (physical structure), or an undefined representation (verbal instruction or mental manipulation) in order to ensure that quite a range of skills is developed (compare 4.4, chapter 5 and chapter 6).

7.2.7 The type of mathematical knowledge (physical, social and logico-mathematical)

The type of mathematical knowledge that is involved in the development of spatial knowledge influences the role of the teacher as facilitator (compare 2.6.3, 2.6.5 and figure 6.2).

7.3 Recommendations

The following recommendations are made as a result of the findings in this experiment:

7.3.1 Aims

It is important that the aims and objectives for the teaching of geometry in the primary school phase are understood and accepted by everyone involved in teaching and designing the geometry materials.

7.3.2 Contents

The question of “what” should be taught is important. The inevitable issue of emphasising the “breadth” or the “depth” in the teaching of geometry comes to the fore. Identifying a core curriculum against the diverse cultural background of the South African population should be very carefully considered, while taking into consideration all the variables that might influence the design. There is a widespread agreement that the teaching of geometry should reflect the actual and potential needs of society. It is therefore important that these needs should be very clearly addressed and reflected in the teaching materials. In order to accomplish this, careful consideration should be given to the knowledge systems of the different cultural groups.

7.3.3 Methods

The question of how and through which methods geometry should be taught is very closely linked to (1) the aims of teaching geometry; (2) the current teaching methodology that is followed when teaching the number component of mathematics; and (3) the different knowledge systems that the different cultural groups might use as their frame of reference. Currently the problem-centred approach is gaining grounds in many of the classrooms. It should be remembered, however, that there are still many schools where this approach is not part of the teaching philosophy. Until teachers’ beliefs about teaching and the way in which children learn are not addressed, it would be futile to expect that geometry could be successfully taught in primary school.

7.3.4 Teaching aids

The use of teaching aids is important. It should be remembered that the majority of the learners are not in a financial position to acquire even some of the basic materials (cardboard, sticky tape, wooden blocks, protractors, etc.). In the light of the findings of

the importance of the construction component these materials are important and means should be found to supply them on a large scale.

7.3.5 Assessment

The way of assessment and the evaluation of pupils strongly influence teaching and learning strategies. In the case of geometry, teaching the different skills is not only reflected in paper and pencil products, but also through the medium of language and construction activities. Research has shown that these are very important parameters to indicate progress of the spatial sense of young children. More research needs to be done in order to describe the assessment techniques that are needed to evaluate these different skills. Another important aspect of assessment is the question whether only the end product should be evaluated or the process also. The research findings of this experiment have clearly shown that both are very important.

7.3.6 Teacher preparation

It is crucial that efficient teacher preparations should be considered regarding both disciplinary competence and educational, epistemological, technological and social aspects. South Africa has a legacy of poorly trained teachers with regard to geometry knowledge because of the absence of geometry as a subject in primary school, a restricted version of geometry at secondary school and very little geometry at teacher training colleges, apart from the standard Euclidean approach in some cases. This implies that the preparation should be done at pre-service as well as in-service training courses.

7.3.7 Evaluation of long-term effects

All along, the success of an efficient teaching/learning process of a curricular and/or methodological reform or innovation for a certain school system has been evaluated on

the basis of only a short period of observation of its outcomes. Moreover, there are no comparative studies on the possible side effects of a change of content or methods. It is therefore important that longitudinal research studies need to be conducted to follow these changes.

7.3.8 Researchers

The issue of having a multi-cultural society which needs to be taught inevitably asks for a “multi-cultural” research corps. In developmental research the role of the researcher is tightly interwoven not only with the role of the teacher but also with the cultural baggage of the community that is studied. One of the very important components of the development of the spatial knowledge of the children proved to be their use and understanding of language. It is consequently of great importance that the researcher is familiar with the home language of the children under investigation. This implies that an active canvassing for these researchers needs to be made with all the consequences of such an endeavour kept in mind.

7.3.9 Implementation

The type of research approach that is advocated throughout this experiment leads to a cyclic situation where parts of the materials are actually implemented as part of the design and development phase. It is proposed that this research approach be developed through a networking situation where the research starts out on a small scale and slowly develops in order to be implemented on a grand scale.

7.4 Conclusion

Three important considerations should finally be taken notice of when conclusions are made about pursuing such a daunting research task into the spatial knowledge of young children.

7.4.1 The cultural reason

With its almost complete lack of consideration and knowledge of the black child's authentic world view, the current school curriculum in South Africa is scarcely suitable to address their needs. The result is that children at some point might have to choose between their native system of knowledge and the Western alternative. Most children might never be able to choose and come to live in a divided world. The alternative is to redesign the curriculum with due consideration to the influence of the different worldviews of all the different inhabitants of South Africa (Western and Indigenous). This could not only have far-reaching consequences for South Africa but could also act as a model for many African countries which have up to now depended heavily on their inherited curricula of the past colonial governments.

7.4.2 The epistemological reason

We can all learn something of importance from the different visions of the world. It is fascinating and theoretically rewarding to work out alternatives to the historically and culturally specific outlook that predominates nowadays. The challenge of bridging the gap between different views is a most rewarding and important task for any scientist who claims to work towards universal truths.

7.4.3 The evolutionary reason

Through a systematic superimposition of the world view of one particular group on the systems of thought and action all over the world, a tremendous uniformization is taking place. In the past inputs from other civilisations and cultures into the Western pool of thought have proved significant and at some points even crucial, while the orderly communication and interactions of Western ideas have proved beneficial to other cultures at some point. Pinxten *et al* (1983:174) mention the special case of some Chinese concepts via Leibniz's transformation of modern mathematics and physics.

This systematic extinction of other systems of knowledge that is currently taking place is dramatically reduced and ends up being identical with the Western pool of knowledge. The risk we take on a world-wide scale and the impoverishment that can be witnessed from this is frightening. It is the view of Pinxten *et al* (1983:174) that as long as science cannot pretend to have valid answers to all basic questions, it is foolish to exterminate all other so-called primitive, pre-scientific, or otherwise foreign approaches to world questions.

Mutual inspiration and interaction at a fundamental level of epistemology and ontology will probably always be rewarding and illuminating in all instances. If we remain observers, objective scholars of another society, we will never enter into its essence. Peat (1994:16) is of the opinion that if we approach it in a spirit of humility, respect, enquiry, and openness it becomes possible for a change of consciousness to occur.

All these different above-mentioned variables contribute to the complexity of developing, discovering and describing the spatial competence of young children. The forgoing discussions should be seen as the first step towards initiating a long walk into investigating the world of spatial knowledge of the young child, with the objective of understanding the complex issues that comprise these entire worlds.

ANNEXURES

**ANNEXURE A: A LETTER FROM THE TRANSVAAL EDUCATION
DEPARTMENT GIVING THE RESEARCHER THE PERMISSION TO DO
THE RESEARCH IN THIS PARTICULAR SCHOOL**



DEPARTEMENT VAN ONDERWYS EN KULTUUR
DEPARTMENT OF EDUCATION AND CULTURE

ADMINISTRASIE: VOLKSRAAD
ADMINISTRATION: HOUSE OF ASSEMBLY

TRANSVAALSE ONDERWYSDEPARTEMENT
TRANSVAAL EDUCATION DEPARTMENT
ONDERWYSNAVORSINGSBURO

Navrae: Dr. M.M.E. Mattheus
Enquiries:
Verw.:
Ref.: TOA 9-7-1/4/93
Tel.: (012) 317-4062

TOD-gebou
TED Building
Privaatsak X76
Private Bag X76
PRETORIA 0001

15 April 1993

Mev. H.M. van Niekerk
Essenhoutstraat 38
PHALABORWA
1390

WISKUNDEPROGRAM VIR GRAAD i-LEERLINGE TE FAUNA PARK PRIMARY SCHOOL

U skrywes van 1993-01-11, 1993-02-01 en 1993-03-31, asook 'n telefoniese gesprek op 1993-03-05 het betrekking.

Die Transvaalse Onderwysdepartement kan ongelukkig nie aan u toestemming verleen om Graad i-leerlinge van Fauna Park Primary School tydens skoolure by u ondersoek te betrek nie, aangesien daar geoordeel word dat dit die leerlinge se formele onderrigprogram sal ontwig en dat die professionele ruimte van die betrokke onderwyser versteur sal word. Toestemming word egter aan u verleen om leerlinge, met die goedkeuring van hulle ouers, na die amptelike skoolure by u ondersoek te betrek.

Die toestemming is aan die volgende voorwaardes onderworpe:

- . U moet self die samewerking van die betrokke skoolhoof en voorsitter van die bestuursliggaam verkry.

- . U moet hierdie brief aan die skoolhoof en voorsitter van die bestuursliggaam toon as bewys dat u die Departement se toestemming verkry het om die ondersoek na amptelike skoolure uit te voer, maar u mag dit nie gebruik om hulle samewerking te probeer afdwing nie.
- . U moet die skriftelike toestemming van die ouers/voogde verkry om hulle kinders by u ondersoek te betrek.
- . Die name van die leerlinge en die skool mag nie in u navorsing vermeld word nie.

Verwys asseblief in alle toekomstige korrespondensie met die Transvaalse Onderwysdepartement in verband met hierdie aangeleentheid na die TOA-verwysingsnommer soos op die brief aangedui.

Sterkte met u navorsing.

A. leuter
 namens UITVOERENDE DIREKTEUR: ONDERWYS



Rig korrespondensie aan die Uitvoerende Direkteur, Transvaalse Onderwysdepartement en meld verwysingsnommer
 Address correspondence to the Executive Director, Transvaal Education Department and quote reference number

✦ TRANSOND/TRANSED
 FAX: (012) 317-4038

**ANNEXURE B: A COPY OF THE LETTER TO THE PARENTS OF THE
INDIVIDUAL CHILDREN, ASKING FOR PERMISSION TO WORK WITH
THEIR CHILDREN**

Dear Parents,

I am busy with the development of the investigation of the mathematical knowledge of grade one children by looking at their spatial development abilities. (This will develop their skills to do geometry in later years.) I am currently busy developing this program so that it complements the existing number development program that the children are following in school. This project is supported by the Transvaal Education Department.

I would therefore like to ask you as the parents of (name of the child)..... permission to keep him/her at school for one day of the week for the rest of the year until one o' clock. I need this information by the beginning of May so that the project can start.

If you need more information you can phone me at 3793.

Yours faithfully
Retha van Niekerk
28/4/93

I..... (parent)
of.....(child) give you permission to involve my child in this
development project.

ANNEXURE C: PROGRESS OF THE INDIVIDUAL CHILDREN AT THE END OF THE YEAR AS EVALUATED BY THEIR CLASSROOM TEACHER

TOP GROUP			
Child	Mathematics	English	First Language
1	2	2	English
2	2	2	English
3	1	3	English
4	1	2	Malawian
5	1	2	Northern Sotho
6	2	2	Asian
MIDDLE GROUP			
1	3	3	English
2	2	3	Northern Sotho
3	3	3	English
4	3	2	Northern Sotho
5	3	3	English
6	3	3	Xhosa
7	3	2	Afrikaans
BOTTOM GROUP			
1	3	3	Shangaan
2	4	4	Afrikaans
3	3	3	Zulu
4	3	3	English
5	3	3	Northerns Sotho
6	3	3	Shangaan
7	4	4	Polish
8	4	4	Northern Sotho

(The grading is from excellent (1) to weak (5))

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