Chapter 7

Summary and Conclusions

This study focused on the evolution of pulsar wind nebulae (PWNe), with two specific aims in mind. The first of these was to simulate the morphological evolution of a composite supernova remnant evolving into a homogeneous interstellar medium (ISM) using a hydrodynamic model. As this model cannot be used to calculate the evolution of the energy spectra of the particles in the nebula, the second aim of this thesis was to develop a spatially dependent particle evolution model based on the Fokker-Planck transport equation. The main results found from these two types of models are summarised below.

7.1 The hydrodynamic model

For this part of the study an extended hydrodynamic model that includes the magnetic field in a kinematic fashion was used. This implies that the effect of the fluid on the magnetic field is taken into account, but that any counter-effect of the magnetic field on the fluid is neglected. This approach naturally has some limitations, as it requires that the ratio of electromagnetic to particle energy σ be sufficiently small. As demonstrated by *Vorster et al.* (2013a), this requirement is met when $\sigma < 0.01$. Although small, this value of σ is reasonable within the framework of PWNe (see, e.g., *De Jager and Djannati-Ataï*, 2009), and the kinematic approach does not necessarily represent a severe limitation of the model.

The hydrodynamic simulations presented in Chapter 3 focused specifically on the effect of the pulsar's initial luminosity and spin-down rate, the supernova ejecta mass, and the density of the ISM on the evolution of a spherically-symmetric composite remnant. These simulations placed an emphasis on the evolution of the PWN, and in particular the evolution of the pulsar wind termination shock, the outer boundary of the PWN, and the average magnetic field in the nebula. Although previous (magneto)hydrodynamic models, such as those presented by *Blondin et al.* (2001), *Van der Swaluw et al.* (2001b), *Bucciantini* (2002), and *Volpi et al.* (2008), have been used to model aspects of the evolution of composite remnants, this is the first (known) time that such a comprehensive study has been presented. A large part of the work presented in this chapter has also been published in *Vorster et al.* (2013a).

From the hydrodynamic simulations the three evolutionary phases of the outer boundary of the PWN, as predicted by, e.g., *Van der Swaluw et al.* (2001b) and *Bucciantini et al.* (2003), could be identified: (1) an initial expansion phase; (2) a phase where the nebula is compressed by the reverse shock of the supernova remnant (SNR); and (3) a second, unsteady expansion phase. The hydrodynamic simulations further showed that either a larger initial luminosity or a larger ISM density can notably reduce the time required by the reverse shock to reach the outer boundary of the PWN. When the reverse shock interacts with the PWN, the nebular compression can be increased by either decreasing the initial luminosity, or by decreasing the spin-down rate. One notable result that follows from these simulations is that the nebula does not necessarily have to be compressed by a significant amount. While the present study did not specifically focus on the ratio of the positions of the SNR forward shock to the PWN outer boundary, the evolution of this quantity can also be derived from the simulations. As argued by *Van der Swaluw and Wu* (2001c), this ratio can be useful to infer the initial spin-period of pulsars located in composite remnants.

A similar evolutionary trend, when compared to the evolution of the outer boundary, was also found for the termination shock. The radius of the termination shock initially increases, while the compression of the nebula causes the radius of the shock to decrease. However, in contrast to the outer boundary of the PWN, the termination shock radius does not expand again when the nebula enters the second expansion phase.

As magnetic energy is conserved in the model, the evolution of the average magnetic field $\bar{B}$ in the nebula is dependent on the evolution of the outer boundary, and can therefore also be divided into three phases: (1) before the interaction with the reverse shock, the average field decreases as $\bar{B} \propto t^{-1.1} - t^{-1.5}$; (2) during the compression phase $\bar{B}$ increases, with this increase determined by the level of PWN compression; and (3), $\bar{B}$ again decreases as a function of time when the nebula has entered the second expansion phase.

Apart from providing a quantitative calculation of the effects that various parameters have on the morphological evolution of composite remnants, these hydrodynamic simulations may also help to elucidate the nature of some of the unidentified TeV sources that have been observed by the H.E.S.S. experiment (*Aharonian et al.*, 2008). The defining characteristic of these sources is that they lack a clear synchrotron counterpart, rendering their identification difficult. To explain these sources, *De Jager* (2008) proposed that a rapid expansion of the PWN will cause the magnetic field in the nebula to evolve to a very small value, resulting in these sources being undetectable at synchrotron frequencies. However, due to the longer lifetimes of the electrons responsible for the very high energy gamma-ray emission, these ancient PWNe may still be visible at TeV energies. The hydrodynamic simulations show that such a rapid expansion is indeed possible. Furthermore, the simulations also show that for the scenarios where the nebula is not significantly compressed, the magnetic field strength remains at a low level.

Lastly, it follows from the simulations that the velocity decreases as $V \propto r^{-2}$, while the magnetic field increases as $B \propto r$. This result is valid for time scales before and after the interaction with the reverse shock. These profiles are of importance to spatially dependent spectral evolution models, such as those discussed in Chapters 5 and 6. It is, however, emphasised that these profiles are only valid for scenarios where $\sigma < 0.01$, as larger σ scenarios will have different profiles.

7.2 The particle evolution models

The evolution of the particle energy spectrum in the PWN is generally calculated using spatially independent models (e.g., *Zhang et al.*, 2008; *Gelfand et al.*, 2009), with only a handful of spatially dependent models developed thus far. Although useful, these latter models either neglect diffusion (see, e.g., *Schöck et al.*, 2010), or were developed for a specific PWN (see, e.g., *Hinton et al.*, 2011; *Van Etten and Romani*, 2011). Furthermore, the models that do include diffusion were never used to study the effect of this process on the evolution of the particle spectrum. The primary aim of the second part of the study is therefore to develop a comprehensive spatially dependent model, with an emphasis on investigating the role of diffusion in spectral evolution.

7.2.1 The spatially independent model

Although not necessarily one of the focal points of this study, Chapter 4 first presented a spatially independent particle evolution model, and illustrated how such model can be of use. One notable characteristic of this model is that a broken power law is used as the source spectrum for the electrons injected into the PWN at the termination shock. In contrast to previous PWN models of a similar nature (see, e.g., *Zhang et al.*, 2008; *Tanaka and Takahara*, 2010), the source spectrum in the model of Chapter 4 allows for the possibility of a discontinuity in intensity at the transition between the low and high-energy components. This model was applied to the young ($t_{\rm age} \sim 1\,{\rm kyr}$) nebula G21.5-0.9, where it was found that a discontinuous source spectrum leads to a better agreement between the model and multi-wavelength data, in contrast to a continuous one. A similar conclusion has also been drawn by *De Jager et al.* (2008b) from their modelling of the evolved ($t_{\rm age} \sim 11\,{\rm kyr}$) nebula Vela X. As a discontinuous spectrum is required for both G21.5-0.9 and Vela X, the discrepancy between the two components cannot be an artefact of PWN evolution.

A requirement of the discontinuous spectrum is that a particle conversion efficiency must be specified for both the low- ($\eta_{\rm R}$) and high-energy ($\eta_{\rm X}$) components, with a ratio of $\eta_{\rm R}/\eta_{\rm X} \sim 5.7$ derived for G21.5-0.9. As discussed in Section 2.3.3, *Spitkovsky* (2008) found that the acceleration of the particles at the termination shock leads to a Maxwellian energy spectrum with a non-thermal tail. Such a spectrum would naturally explain the ratio $\eta_{\rm R}/\eta_{\rm X} > 1$. Furthermore, hydrodynamic and magnetohydrodynamic models such as those presented by, e.g., *Blondin* *et al.* (2001), *Volpi et al.* (2008), as well as the model presented in Chapter 3, have been very successful in simulating the morphological evolution of PWNe. In all these models the assumption that the particle energy spectrum is described by a Maxwellian distribution is inherent. If a non-thermal component is present, these models require that the morphological evolution be driven by the (low-energy) Maxwellian particle spectrum, which in essence implies that $\eta_{\mathrm{R}}/\eta_{\mathrm{X}} > 1$.

Based on these considerations, the data for G21.5-0.9 were also modelled using a Maxwellian source spectrum with a non-thermal tail. It was found that this spectrum predicted a synchrotron radio spectrum that is harder than the one observed. However, as discussed in Section 2.3.3, the results of *Sironi and Spitkovsky* (2011) indicate that magnetic reconnection at the termination shock can accelerate particles, leading to a modification of the Maxwellian that would produce a softer radio synchrotron spectrum.

One requirement of the spatially independent model is that the fraction of the pulsar's spin-down luminosity converted into electromagnetic and particle energy has to be specified. The model can therefore also be used to estimate the value of σ, with a value of $\sigma \sim 0.12$ derived for G21.5–0.9.

This model was also applied to the unidentified TeV sources HESS J1427–608 (*Aharonian et al.*, 2008) and HESS J1507–622 (*Acero et al.*, 2011). Parameter sets were derived that are reasonable within a PWN framework, thereby strengthening the argument that these unidentified sources may be identified as evolved PWNe.

7.2.2 The spherically-symmetric model

Having presented a spatially independent model, the focus of Chapter 5 subsequently shifted to the development of a spatially dependent model. For this purpose, a Fokker-Planck transport equation was used to describe the spatial and temporal evolution of the particle spectrum. This chapter focused on the solution of the transport equation for a spherically-symmetric system where particles are transported by convection and diffusion, while simultaneously suffering energy losses due to adiabatic cooling, synchrotron radiation, and inverse Compton scattering. In order to obtain a better understanding of the results, as well as to test the model, the transport equation was first solved time-independently.

For these time-independent simulations, the transport equation was solved using the *Crank-Nicolson* finite-difference scheme. It was found that in a system where diffusion is neglected, synchrotron losses lead to the familiar cut-off in the spectrum at high energies. On the other hand, in the absence of synchrotron losses, diffusion leads to a spectrum that is softer than the source spectrum, with this softening directly related to the momentum dependence of the diffusion coefficient. For both of these scenarios it was found that the results could be divided into a low- and high-energy regime. In the high-energy regime, diffusion/synchrotron losses dominate, while the low-energy regime in both scenarios is dominated by convection and adiabatic losses. Although adiabatic losses reduce the intensity of the spectra, this process does not lead to an evolution of the spectral index. The above-mentioned results are similar to those found by, e.g., *Jokipii and Higdon* (1979), *Lerche and Schlickeiser* (1981), and *Atoyan et al.* (1995), thereby indicating that the transport equation can be used to describe the evolution of the particle spectra.

The fact that the low-energy regime is only subject to convection and adiabatic cooling has some implications for the hydrodynamic simulations. As discussed previously, these simulations per definition assume that the particles responsible for the morphological evolution have a Maxwellian spectrum. If the evolution is driven by a low-energy particle component, then diffusion and synchrotron losses will not lead to an evolution of the Maxwellian spectrum, and the hydrodynamic models will still be valid.

The last of the steady-state scenarios focused on the evolution of the spectrum in a system where all of the above-mentioned transport and energy loss processes were taken into account. These simulations show that diffusion leads to a significant moderation of the synchrotron cut-off. The result is a spectrum that is harder than one would expect, with the synchrotron cut-off only appearing at the highest energies. This result can also be used to explain the spatial evolution of the X-ray synchrotron spectra that have been observed for a number of PWNe (see, e.g., *Gaensler et al.*, 1999; *Mangano et al.*, 2005; *Schöck et al.*, 2010). A comparison between the predicted indices and those derived from observations show that the model predictions are compatible with the data. To place this result into a clearer context, one need only refer to the models of *Vorster* (2010) and *Schöck et al.* (2010). These authors tried to predict the spatial evolution of the X-ray indices using a model that neglected diffusion, and had considerable difficulty in fitting the data from the inner region of Vela X and MSH 15-52, respectively.

Chapter 5 also investigated the influence of the magnetic field profile on the evolution of the spectra. Two types of scenarios were simulated, where the first had a radially decreasing magnetic field, and the second a radially increasing magnetic field. It was found that both scenarios lead to the same qualitative evolution of the spectra.

As discussed previously, the hydrodynamic simulations showed that the outer boundary of the PWN is continually evolving. It is therefore necessary to include this dynamical process into the transport model, as it may have an influence on the spectral evolution. In order to do this, the transport equation was solved time-dependently using the *Douglas* Alternating Direction Implicit (ADI) scheme. While an evolving boundary has a quantitative effect on the evolution of the spectra, it was found that this is limited. More importantly, the simulations showed that the spectra are not qualitatively affected.

7.2.3 The axisymmetric model

An inherent limitation of the spherically-symmetric transport model is that the particles can only diffuse in the radial direction. However, in a more realistic system diffusion is determined by the geometry of the magnetic field, and particles can also diffuse in both the polar and azimuthal directions. Furthermore, the geometry can also lead to the additional transport processes of gradient and curvature drift.

In the last part of this study (Chapter 6) the evolution of the particle spectrum was investigated for a system where radial and polar diffusion, as well as drift, can occur. An investigation of this nature requires an additional spatial dimension, and the Fokker-Planck transport equation was therefore solved numerically in an axisymmetric geometry using the *Douglas* ADI scheme. The magnetic field used for the simulations was the Archimedean spiral derived in Section 2.2.2. As discussed in Section 2.2.3, the dipole nature of the pulsar's magnetic field also leads to the presence of a neutral sheet that separates the magnetic field lines that are directed in opposite directions in the two hemispheres. The simulations therefore also included drift motion along this neutral sheet.

Apart from investigating the role of drift, these simulations also investigated the effect of the particle's electric charge q, as well as the orientation A of the pulsar's rotation and magnetic axes relative to each other (see Section 6.1) on the evolution of the spectra. The aim was not to present a detailed study, but rather an initial investigation into the possible effects that drift and polar diffusion could have on spectral evolution. In order to obtain a better indication of how drift affects spectral evolution, synchrotron losses were neglected. As far as is known, this is the first time that such an investigation has been done for a central source system.

For the first set of simulations, the tilt angle of the neutral sheet, i.e. the angle between the rotational and magnetic axes of the pulsar, was chosen to be $\alpha = 0°$. For these simulations it is possible to distinguish between a $qA > 0$ and $qA < 0$ scenario. In the former scenario particles drift from the poles towards the equator, and out along the neutral sheet, and vice versa in the latter scenario. From the simulations it was found that diffusion in the polar direction, as well as gradient and curvature drift in both the radial and polar direction, have a negligible effect on spectral evolution. Neutral sheet drift does, however, lead to the development of a "trough"-like feature in the $qA < 0$ spectra (see Figure 6.7). This feature is visible at all radial distances and polar angles, but is most prominent in the equatorial plane where the neutral sheet is located. For the $qA > 0$ spectra a prominent "peak"-like structure develops in the equatorial plane, but in contrast to the $qA < 0$ spectra, this feature is not visible at other polar angles.

Due to the fact that one typically has $\alpha \neq 0°$ for pulsars, the simulations were repeated for a tilt angle of $\alpha = 30°$. It was found that the trough-like feature in the $qA < 0$ spectra is diminished, while the peak-like structure in the $qA > 0$ spectra disappears completely. Following this result, one might therefore conclude that drift will not play an important role in spectral evolution. However, drift may still be important when synchrotron losses are taken into account, particularly if the magnetic field has an angular dependence. As shown in Figure 6.4, particles with a specific electric charge will drift to regions with a strong magnetic field, while the oppositely charged particles will drift into a region with a weaker magnetic field. The result is that the synchrotron loss rate for electrons and positrons will differ, potentially leading to large differences in their spectra.

7.2.4 Future research

With the models presented in this study, a number of future research possibilities are available:

- The hydrodynamic model can be used to simulate the evolution of individual PWNe, with the idea that this will lead to a better understanding of these sources. Note that this model can also be used to calculate the evolution of a PWN in an inhomogeneous medium, as demonstrated by *Vorster et al.* (2013a). This modelling can be extended by coupling the hydrodynamic simulations with one of the particle evolution models presented in this study.

- The spatially independent particle evolution model presented in Chapter 4 can be applied to a large sample of known PWNe. This will allow one to derive a statistically significant set of parameters that may be unique to PWNe. Additionally, this model can also be applied to additional unidentified TeV sources.

- The transport model presented in Chapter 5 can be used to fit the spatially dependent X-ray spectra observed from a number of PWNe, thereby allowing one to derive the magnetic field profile, and subsequently the value of σ. As far as is known, this would represent only the second time that a model has been used to rigorously derive the value of σ.

- The transport model can be extended by including Klein-Nishina effects. This may lead to additional, and unexpected, spectral features.

- The axisymmetric model can be expanded by including synchrotron losses.

- Current PWN particle evolution models generally assume a purely radial flow and an azimuthal magnetic field. The axisymmetric transport model can be used to calculate the spectral evolution more realistically by including complicated flow and magnetic fields.

- It has been suggested that the positron excess observed at energies above 10 GeV by PAMELA (*Adriani et al.*, 2009), Fermi-LAT (*Ackermann et al.*, 2012), and more recently by AMS-02 (*Aguilar et al.*, 2013), could be caused by positrons that have escaped from nearby PWNe (see, e.g., *Büsching et al.*, 2008; *Yüksel et al.*, 2009; *Linden and Profumo*, 2013). The spatially dependent transport models can thus be used to calculate the spectrum of leptons that will escape from PWNe.

- As the Fokker-Planck transport equation does not distinguish between leptons and hadrons, the spatially dependent transport models can also be applied to hadronic scenarios.

Pulsar wind nebulae remain an active area of interest, and the research presented in this study can be used to advance the understanding of these sources. Furthermore, the nature of the hydrodynamic and transport models also allows one to extend these models to other systems such as globular clusters.