School effectiveness and effective Mathematics teaching: towards a model of improved learner outcomes

KG Pule

orcid.org / 0000-0002-9438-6826

Thesis accepted in fulfilment of the requirements for the degree Doctor of Philosophy in Mathematics Education at the North-West University

Promoter: Prof HD Nieuwoudt

Graduation: May 2020

Student number: 12543853
DECLARATION

I, K. GILBERT PULE declare that the thesis:

School effectiveness and effective Mathematics teaching: towards a model of improved learner outcomes

is my own effort and that all the sources used or quoted have been acknowledged and indicated by means of complete references. This study was not submitted by me at any university for a degree or examination.

K.G.Pule

2019-09-12
DEDICATION

This piece of research study is devoted to my family and all selfless mathematics teachers in the department of education, especially in Mahikeng area.
ACKNOWLEDGEMENTS

I wish to thank God Almighty in the name of Jesus Christ for granting me courage, perseverance, industriousness and wisdom to complete this outstanding work.

My sincere appreciation, acknowledgements and gratitude flow to the following individuals:

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ABSTRACT

The purpose of this study was to investigate the possible determinants of quality mathematics teaching which could mark those schools and teachers effective in mathematics teaching among selected Mahikeng secondary schools of the North-West Province. Moreover, the intention was to develop a model towards improved learner outcomes in mathematics which can possibly be a diagnostic tool for better performance. The literature study was carried out on relevant theories, outcomes of previous studies involving similar issues and empirical inquiry tailed.

The researcher used a sequential explanatory strategy i.e. mixed-methods, starting with quantitative method followed by qualitative method in a case-study paradigm. A structured questionnaire was used in the quantitative phase, in which 12 and 360 survey instruments were issued to mathematics teachers and learners respectively, of which 12 and 321 responses were received SPSS 23 was used to analyse quantitative responses. Descriptive statistics such as the mean, standard deviation, variance and frequency distributions were used to describe the demographic characteristics of the study respondents. These statistics were also used to describe and identify ‘the possible determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in secondary schools’.

A Pearson’s moment correlation coefficient was conducted to measure the relationship between the factors identified and were presented as a correlation matrix. The Pearson coefficient revealed a mixture of negative and positive insignificant relationships among constructs identified i.e. teacher attributes and learner conditions, with school conditions within them. It is evident that there are high correlations (in excess of 0.3), looking at the correlation matrix. It is also evident that the correlation matrix is not unitary providing a strong relation between the teacher, learner and school attributes. The performance of one attribute in one way or another has a certain influence on the other attribute. The p-value of most of the attributes is less than 0.01 and 0.05 levels of significance, confirming the interrelations between the attributes. Consequently, it shows that there is an association among the attributes. This further endorses the viability of the multiple relationships between these attributes.

The second part, the qualitative phase, used semi-structured interviews with 12 mathematics teachers and 12 mathematics learners in focus groups, who also took part in the quantitative phase. The observations were made in grade 12 mathematics classes. Furthermore, the
documents analysis was done to confirm all other data collection instruments. The qualitative data were analysed descriptively. The process involved clustering the responses into categories, coding the responses through application of in vivo coding allowing the themes to emerge.

The findings supported the quantitative findings. It was revealed that teachers generally have a variety of challenges in different schools that affect effective mathematics teaching. These factors included, amongst others, unresponsive professional teacher development, lack of support by stakeholders, learner indiscipline, challenges in learner assessments and promotions, learner age cohort, underachieving learners, lack of safety and security, school location, congested work schedules, overcrowding and overload as well as poor leadership styles. The findings further indicated that a lack of effectiveness in mathematics teaching results in demoralisation of both teachers and learners, which affects the learner outcomes in mathematics, suggesting a relationship between effective mathematics teaching and learner outcomes in mathematics. The study concludes that effectiveness may usher in improved learner outcomes in mathematics and performance may show the way to effectiveness.

The study developed a model towards improved learner outcomes in mathematics (tiLOM); which may be used as a diagnostic tool for effective mathematics teaching and improved learner performance. The identified attributes/conditions in the model supplement each other for the success of this intervention model.

*Key words for Indexing:* Mathematics teaching; effective mathematics teaching; school effectiveness; meaningful mathematics learning; improved learner outcomes; case study.
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ACRONYMS AND ABBREVIATIONS

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<tbody>
<tr>
<td>ABET</td>
<td>Adult Basic Education and Training</td>
</tr>
<tr>
<td>ANA</td>
<td>Annual National Assessment</td>
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<td>ANOVA</td>
<td>Analysis of Variance</td>
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<td>CAPS</td>
<td>Curriculum and Assessment Policy Statement</td>
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<td>CCK</td>
<td>Common Content Knowledge</td>
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<td>DBE</td>
<td>Department of Basic Education</td>
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<td>DoE</td>
<td>Department of Education</td>
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<td>ELRC</td>
<td>Educators Labour Relations Council</td>
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<td>FET</td>
<td>Further Education and Training</td>
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<td>GER</td>
<td>Gender Enrolment Ratio</td>
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<td>HoD</td>
<td>Head of Department (school based)</td>
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<td>LTSM</td>
<td>Learner Teacher Support Material</td>
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<td>KCS</td>
<td>Knowledge of Content and Students</td>
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<td>MDG</td>
<td>Millennium Development Goal</td>
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<td>MLA</td>
<td>Monitoring Learner Assessment</td>
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<td>NCS</td>
<td>National Curriculum Statement</td>
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<td>NMM</td>
<td>Ngaka Modiri Molema</td>
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<tr>
<td>NWU</td>
<td>North West University</td>
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<tr>
<td>OBE</td>
<td>Outcome-based Education</td>
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<tr>
<td>OECD</td>
<td>Organisation for Economic Co-operation and Development</td>
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<tr>
<td>PCK</td>
<td>Pedagogical Content Knowledge</td>
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<tr>
<td>PGCE</td>
<td>Post Graduate Certificate in Education</td>
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<td>PL 1</td>
<td>Post Level 1</td>
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<td>PSF</td>
<td>Professional Support Forum</td>
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<td>PTD</td>
<td>Professional Teacher Development</td>
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<tr>
<td>QLTC</td>
<td>Quality Learning and Teaching Campaign</td>
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<td>Acronym</td>
<td>Description</td>
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<td>RNCS</td>
<td>Revised National Curriculum Statement</td>
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<td>SADTU</td>
<td>South African Democratic Teachers Union</td>
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<td>SASA</td>
<td>South African School Act</td>
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<td>SCK</td>
<td>Specialised Content Knowledge</td>
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<td>SGB</td>
<td>School Governing Body</td>
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<td>SIP</td>
<td>School Improvement Plan</td>
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<tr>
<td>SKAV</td>
<td>Skills, Knowledge, Attitude and Values</td>
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<td>SMC</td>
<td>Subject Matter Content</td>
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<td>SMK</td>
<td>Subject Matter Knowledge</td>
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<td>SMT</td>
<td>School Management Team</td>
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<td>SPSS</td>
<td>Statistical Programme for the Social Science</td>
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<td>SONA</td>
<td>State of the Nation Address</td>
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<td>StatsSA</td>
<td>Statistics South Africa</td>
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<tr>
<td>TIMSS</td>
<td>Trends in International Mathematics and Science Study</td>
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<td>UNESCO</td>
<td>United Nations Educational, Scientific and Cultural Organisations</td>
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CHAPTER 1

INTRODUCTION TO THE STUDY

1.1. INTRODUCTION

International studies have established that teachers have an influence on their learners’ school and life-long accomplishments (Chetty, Friedman & Rockoff, 2014). Pragmatic indication from studies done on teachers’ views on effectiveness measures (Kyriakides, Demetriou & Charalambous, 2006; Teddlie & Reynolds, 2000) have shown the necessity to use various, not just distinct, assessment methods, as the source for the assessment of school effectiveness and effective teaching.

The Australian Association of Mathematics Teachers (2006) states that “effective schools are only effective to the extent that they have effective teachers”. Further, they indicated that there are some efforts to ascertain teachers’ characteristics like their certification, experience and educational levels that might be correlated with school effectiveness. Local districts and schools like the selected ones under this study toil hard to achieve the agreed set targets or goals of learner attainment, especially in mathematics (Johnson, 2012).

A learner’s attainment in mathematics is influenced by various factors, with effective teaching as one of them (Pretorius 2013). Teachers have to play a great role as they have significant impact on what their learners do, and to assist learners overcome challenges in performing well in mathematics (Rice, 2003). Thus, teachers need to be more effective in teaching mathematics, as studies continue to confirm the significance of their role (OECD, 2005; Barber & Moursesh, 2007).

Studies further show that content knowledge of mathematics is very important for the teacher to deliver effective teaching (Ball, Hill & Bass, 2005; Kreber, 2002) and therefore a central component of effective mathematics teaching. Thus, adverse learner outcomes in mathematics could possibly be associated with teachers’ inadequate content knowledge, leading to less effective teaching.

There is a perception that the quality of education has declined in the Republic of South Africa (RSA) (Christie, Butler & Potterton, 2007, Spaull, 2013), especially in mathematics (Department
of Basic Education (DBE), 2013). There is consistent underperformance and lack of interest of learners in mathematics, which may be associated with ineffective teaching (Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya & Makwakwa, 2009; Ogbonnaya, 2010).

The underperformance of South African Grade 12 mathematics learners shows that Further Education and Training (FET) learners are not effectively prepared when they leave school with the mathematics knowledge that they require to be dynamic contestants in an exceedingly technological culture (Mohlala, 2015). Makoelle (2012) maintains that how effective schools attain particular goals though the nature of effective teaching generally remains a closed book. This study investigated the interaction between the learner conditions, effective mathematics teaching (teacher conditions) and school conditions that enable the effective teaching of mathematics.

The Department of Education (DoE) has since focused on ways of increasing effectiveness in schools and teaching by applying a whole-school evaluation approach to address effectiveness and improvement (DoE, 2003). The ministerial report on ‘Schools at work’ puts forward that Grade 12 results offer a barometer or gauge of the effectiveness of the schooling system with a focus on learners and teachers (Christie et al., 2007:8).

Christie et al. (2007) posit that the Grade 12 learner outcomes are a public gauge of regular performance, that is, those outcomes are used as a measurement of school effectiveness and effective teaching. Christie et al. (2007) maintain that schools which achieve above the average mark, are effective and operational schools. However, Jansen and Taylor (2003) aver that the habit of integrated methodical thinking style would lay open to debate whether Grade 12 examinations are a sufficient pointer to school effectiveness or whether there should be an enquiry into additional aspects in the school as a whole.

1.2. STUDY BACKGROUND

The apartheid legacy carries on influencing the way in which school effectiveness is conceptualised in South Africa (DoE, 2009). The educational ups and downs saw the system altered from a pre-1994 disapproving line to a post-1994 progressive line (Mazibuko, 2007). That legacy is still prevalent in schools regardless of the political changes since 1994 (Jansen &
Sayed, 2001). It carries on defining how effective schools are and the quality of education they deliver. Botha (2004) indicates that the absence of a culture of teaching and learning at numerous formerly underprivileged schools still produces ineffective teaching and poor learner outcomes.

The poor mathematical learner outcomes in many African countries (Howie & Plomp, 2002; Ogbonnaya, 2007) make it necessary that learner attainment in mathematics on the African continent is improved. In the midst of other factors, the challenge of poor learner outcomes in mathematics has been accredited to unfortunate strategies used in the teaching and learning of mathematics, poor infrastructure in various schools and absence of proficient mathematics teachers (Onwu, 1999; Spreen & Vally, 2006; Stols, Kriek & Ogbonnaya, 2008).

In RSA, the condition is very serious; there is considerable evidence of poor learner outcomes in mathematics (Howie & Plomp, 2002; Ogbonnaya, 2010; Wessels & Nieuwoudt, 2010), and consistently poor outcomes in the international assessments such as TIMMS (Spaull, 2013; Areff, 2015). According to Spaull (2013), there are numerous policies that the DBE has launched to address some of the main grounds for underperformance in the education system. This include: the current workbook programme, the Action Plan 2014: Towards the realisation of schooling 2025, the Action Plan to 2019; towards the realisation of schooling 2030, the Curriculum Assessment Policy Statement (CAPS), as well as the carrying out of the annual national assessment (ANA) are all initiatives in the correct direction (Spaull, 2013). However, there are some challenges, as Spaull further posits that there is an amount of work which needs to be done if we are to improve the practices of teaching and learning in various classrooms in South African schools.

Atagana et al. (2009) indicate that numerous learners find various areas in mathematics very challenging to learn. For example, these authors reported that 46% of learners in grades 10-12 testified that they found learning trigonometry challenging. Moore (2009) added that most learners who tended to experience serious challenges with trigonometry had problems with trigonometric functions. Trigonometry is an essential area in a mathematics field that finds various applications in mathematics-related career paths like architecture, economics, and statistics as well as various subdivisions of engineering (Weber, 2009). Yet, there is a standing challenge to develop an accurate tool/model to gauge the subject matter knowledge and effective teaching of teachers (Ogbonnaya & Mogari, 2011). Heritage and Vendliski (2006) argue that
there is a lack of reliable and valid instruments to precisely gauge the subject matter knowledge of teachers and their effectiveness in teaching, owing to the insubstantial nature of knowledge and the ambiguity of effective teaching.

Various studies indicate that differences in learner outcomes in mathematics are due to the dynamics of the teachers (Darling-Hammond, 2000; Rice, 2003; Ingvarson, Beavis, Bishop, Peck & Elsworth, 2004; Mogari, Kriek, Stols & Ogbonnaya, 2009; Spaull, 2013). Pretorius (2013) contends that teacher dynamics are the most significant determinants of effective mathematics teaching. As a result, teacher dynamics/variables can possibly offer an elucidation for poor learner outcomes in mathematics in RSA. An attempt will be made to formulate the problem statement below.

1.3. STATEMENT OF THE PROBLEM

The general poor learner outcomes in mathematics throughout South Africa has been a serious concern for quite some time (DBE, 2016). Studies by Howie and Plomp (2002), Ogbonnaya (2010) as well as Wessels and Nieuwoudt (2010) provide considerable evidence about the problem of poor learner outcomes in mathematics and emphasise the serious nature of this condition. DBE (2017) indicates that poor learner performance at NSC level remains a critical factor in the provision of quality learning and teaching. Although learner performance in most subjects shows an improvement, learner performance remains low in Mathematics and Physical Science (DBE, 2017). The failure rate in mathematics in the National Senior Certificate examination continues to show that there are some limitations in teaching and learning of mathematics in RSA schools (South African Democratic Teachers Union [SADTU], 2016).

The Action Plan 2014 identified the learner, the teacher and the school as key focus areas for better schooling. Additionally, in teacher effectiveness studies, Cohen and Hill (2000); Rice (2003) and Pretorius (2013) made an effort to find the effect of inputs such as resources and learner contextual features upon the school outputs, while Chetty et al. (2014) argue excellent, effective teachers are critical to learners’ success.

National and international reports in mathematics have established that RSA learner outcomes in mathematics are the lowest in the world (Areff, 2015, TIMMS, 2015). There is an urgent
necessity to marshal the reduction or remove the pitfalls that led to mathematics outcomes reaching rock bottom in RSA schools. Most scholars, like Spaull (2013) and Pretorius (2013), direct the concern to ineffective teaching. At the same time, Preedy (1993) argues that “effectiveness cannot be static but must be continually reassessed for each school in its own particular circumstances”. However, little is acknowledged about the changing and challenging circumstances at the teachers’ workplace and how these affect learners’ performance in mathematics.

The aim of the study was to investigate the possible determinants of quality mathematics teaching which could mark those schools and teachers as effective in mathematics teaching among selected Mahikeng secondary schools of the North-West Province. Moreover, the intention was to develop a model towards improved learner outcomes in mathematics which can possibly be a diagnostic tool for better performance.

1.4. PURPOSE OF THE STUDY

The study sought to investigate the determinants of school effectiveness and effective mathematics teaching in the changing and challenging conditions towards the development of a framework of improved learner performance. Human beings are complex, and understanding their behaviour requires a great deal of knowledge and skill. Initial effectiveness studies were grounded primarily upon quantitative methods in the educational production processes (Marzano, 2000). Those studies endeavoured to determine the effect of inputs like resources and learner contextual features on school outputs. This study will possibly assist teachers (the researcher included) to be effective mathematics teachers and hence achieve better learner outcomes. This study will further contribute to the existing knowledge/literature on school effectiveness and effective mathematics teaching, and hence a comprehensive model to improve learner outcomes in mathematics can be developed.

The findings of the study may contribute towards laying a foundation for policy growth and thus could be utilised as a source of support to teachers in improving the learner outcomes in mathematics. The study can therefore serve as diagnostic tool in changing and challenging teacher workplace conditions which may be utilised to inform relevant decision-making bodies.
in developing and implementing appropriate interventions to improve teaching and learning of mathematics in basic education.

1.5. RESEARCH QUESTIONS AND OBJECTIVES

The study seeks to answer the following main research question:

What are the possible determinants of effective mathematics teaching in the selected secondary schools?

According to Leedy (1993), to make the study problem more adaptable, the researcher may split the problem into sub-problems. Determining these sub-problems would eventually resolve the problem; consequently, the focus of the research problem under inquiry was best expressed by asserting the following sub-problems:

- What factors/conditions of teachers facilitate effective mathematics teaching?
- What learner conditions enable effective mathematics teaching?
- What school conditions enable effective mathematics teaching?
- How do the views of teachers, learners and heads of the department support their perspectives on school effectiveness and effective mathematics teaching towards the development of an improved learner outcomes model?

Research Objectives

The intent of this study was to investigate the possible determinants of quality mathematics teaching which could make schools and teachers effective in mathematics teaching in secondary schools. The secondary objectives are to:

- Identify and describe possible factors/conditions of teachers which facilitate effective mathematics teaching.
- Identify and describe possible learner conditions which enable effective mathematics teaching.
• Identify and describe possible school conditions which enable effective mathematics teaching.
• To develop a model from teacher, learner and school conditions/factors which lead towards improved learner performance.

These aims were addressed thematically through the literature review, findings and discussions in the main study.

1.6. RESEARCH DESIGN

The study pursued an explanatory sequential mixed-methods design in a case-study paradigm. Creswell (2015) describes a mixed-methods research design as “an approach to research in the social, behavioural, and health sciences in which the investigator gathers both quantitative (closed-ended) and qualitative (open-ended) data, integrates the two, and then draws interpretations based on the combined strengths of both sets of data to understand research problems”. The researcher in this study used an explanatory sequential research design. The use of both quantitative and qualitative designs consolidates the study and as a result consolidates the internal validity of the technique which crystallises the necessary ratio of this research (McMillan & Schumacher, 2010).

The researcher used the quantitative survey instrument or questionnaires in the first phase of the study. The quantitative data mainly came from mathematics learners’ and teachers’ questionnaires. The quality of classroom activities and practices is of fundamental significance in defining the learners’ chances to learn and, as a result, learning outcomes. Therefore, all the factors associated with school effectiveness and effective mathematics teaching in the questionnaires is independent variables while the learner outcomes are dependent variables. To confirm that units in the questionnaire are reliable, the Cronbach Alpha was computed on the questionnaire that made use of the Likert scale. The questionnaire was pilot tested with Grade 12 mathematical literacy learners from another school which did not participate in the project. The teacher questionnaire was piloted with accounting and physical science teachers as they had studied mathematics at their first year university level.
The second phase of the research took a form of the qualitative approach. The qualitative data was obtained mainly from the interviews conducted with learners, teachers and mathematics heads of the departments from the two selected schools. The researcher employed semi-structured interviews to collect data from two head of departments [HoDs] of mathematics from each school, and mathematics teachers (10 mathematics teachers). Grade 12 mathematics learners were interviewed according to their performance, grouped as eight (8) girls and eight (8) boys i.e. 16 learners in all. Performance is “associated with quantity of output, timeliness of output, presence/attendance on the job, efficiency of the work completed [and] effectiveness of work completed” (Mathis & Jackson, 2009:324) The performance in this case means the learners whose marks were not adjusted, and the learners whose marks were adjusted or did not pass mathematics in the previous examination. One Grade 12 mathematics class per school was observed i.e. 2 classes in all were observed. All semi-structured interviews, focus group interviews and observations were audio-recorded. School documentary analysis was done, which included. among others, school results analysis, the school’s functionality, resources, teaching and learning, management, employee wellness and safety. Correct procedures were followed to guarantee trustworthiness throughout the qualitative stage of the study.

1.7. THEORETICAL FRAMEWORK

The theoretical framework is the foundational theory that is used to provide a perspective upon which the study is based (Niss, 2006). Theories benefit the understanding of social approaches, performance, relations, behaviours as well as the obligations of teachers at the place of work in realisation of institutional products or outcomes. According to Henning, van Rensberg and Smit (2005), a theoretical framework is a view on which the researcher locates his or her work. It assists with the design of the expectations around the research and how it links with the world. It is similar to a lens through which a scholar interprets the world as well as angles of his or her research. It replicates the standpoint embraced by the researcher and thus frames the work, assigning and enabling dialogue between the literature and the study.

The study drew on some of the following theories that have been previously used in school effectiveness studies. The study is framed by two theoretical resources that explain the different aspects of the study. Complexity theory is recruited to assess school effectiveness whereas
constructivist theory is drawn to gain insight into effective mathematics teaching. A detailed literature overview in Chapter 2 presents a more critical outline of the theories as they are intended to be used in the study.

1.7.1. Complexity theory

Complexity theory is based on the interaction of contextual conditions as determinants of effectiveness in educational organisations. Complexity theory shows a concern in the informal institutions, and features of randomness in the collaborations of group members and the occurrence of new forms of behaviour, which could be dysfunctional or functional (Scheerens, 2015). Cilliers (1998) argues that connectedness needs a distributed knowledge coordination; knowledge is not basically positioned in a grasp and control axis (e.g. a departmental head’s office or principal’s office); rather it flows through the system, and communication as well as collaboration are significant components of complexity theory.

Teacher effectiveness and learner outcomes are not linear, closed processes but open, recursive processes developing from and intertwined in diverse, often commonly constitutive factors in the connection between:

- The institutional character, (in)efficiencies, disciplinary domains as well as departmental and institutional resource allocation and planning;
- Teachers and learners’ habits, characters and life-worlds;
- Macro-collective factors influencing teachers, learners and parents including the school and its networks;
- The unfolding of individual, compromised, challenging as well as changing teaching and learning journey as contextualised in the two selected secondary schools.

These constitutive factors were discussed at some length in the chapters.

1.7.2. Constructivist theory

There is a substantial amount of literature (Prawat & Floden, 1994; Larochelle & Bednarz, 1998; Jonassen, Myers & McKillop, 1996; Morrison & Collins, 1996; Jonassen, 1991) that identifies
constructivism as a learning theory centred on the notion that knowledge is dynamically constructed by the learner. Constructivist theories are grounded in the certainty that learners construct their theoretical understanding and specific knowledge from their own actions (Fleury, 1998). The part of the teacher is to create a mathematical environment to allow learners to build this mathematical knowledge. This environment would afford learners’ chances to link their understanding, use resources, make assumptions and experiment with their thinking in the direction of constructing mathematical knowledge. An attempt was made to use this medley of theories to explain findings, building towards the development of a model of improved performance in mathematics in the next Chapters.

1.8. DELIMITATIONS

This case study was done in the two selected secondary schools of Ngaka Modiri Molema [NMM] district in the North-West province. The study was undertaken in challenging circumstances, such as school, teachers and learner conditions with pockets of poor learner performance in mathematics of secondary schools in NMM district. These circumstances cannot be generalised to other districts. The findings are therefore specific to selected secondary schools in the NMM district and cannot be compared to other schools or districts.

It is assumed that the participants (learners, teachers and mathematics HoDs) truthfully and accurately responded to the survey and interview questions based on their personal experience and that they answered honestly to the best of their individual abilities. The study distress could be done over a period of time to measure variation and stability of effective mathematics teaching towards improving learner outcomes in mathematics in NMM district.

1.9. THESIS OUTLINE AND CHAPTER SUMMARY

The study is presented across six chapters as follow:

1.9.1. Thesis Outline

Chapter 1: Introduction and overview
It contains an introduction and background of the problem, statement of the problem, purpose of the study, significance of the study, research question with secondary questions, research design, theoretical-conceptual framework, delimitations, definition of terms and chapter summary.

**Chapter 2: Theoretical framework**

Theoretical framework is the foundational theories which are used to offer a viewpoint upon which the research is centred. This study is centred on complexity and constructivist theories.

**Chapter 3: Conceptual framework**

An elucidation on the concepts of school effectiveness and effective mathematics teaching was done in this chapter. It further elaborates on the previous studies on the characteristics and factors that influence school effectiveness and effective mathematics teaching. Particular consideration was given to the South African context.

**Chapter 4: Methodology**

The chapter deals with how the empirical investigation was conducted. It covers research designs and methodologies, research paradigm, instrumentation, validity and reliability of the research instrument, measures to ensure trustworthiness, data collection and processing.

**Chapter 5: Empirical results**

The results of this study are presented. This chapter covered data analysis and interpretation. It is the most comprehensive of the entire study and also contains a built–in literature analysis to support the study’s thesis. It covered the analysis of questionnaires of both learners and teachers, documents, semi-structured and focus group interviews questions conducted in gathering data for this study.

**Chapter 6: Conclusion and Recommendations**

The results of this study are presented. The main findings of this study which emanate from the literature study and empirical data are summarised. The proposed model is drawn in this chapter to answer question four of the study. Recommendations for further research flowing from these findings were made in this chapter. Finally, limitations of the study are discussed.
1.9.2. Summary

This chapter provided the introduction, background and motivation of the research, problem statement with sub-problems and aims, overview of research design, delimitations and thesis outline. The next chapter provides a review of literature of theories on school effectiveness and effective mathematics teaching.
CHAPTER 2

THEORETICAL FRAMEWORK

2.1. INTRODUCTION

The theoretical framework is the foundational theory that is used to provide a perspective upon which the study is based (Niss, 2006). Theories benefit the understanding of social approaches, performance, relations, behaviours as well as the obligations of teachers at the place of work in realisation of institutional products or outcomes.

According to Henning, van Rensberg and Smit (2005), a theoretical framework is a view on which the researcher locates his or her work. It assists with the design of the expectations around the research and how it links with the world. It is similar to a lens through which a scholar interprets the world as well as angles of his or her research. It replicates the standpoint embraced by the researcher and thus frames the work, assigning and enabling dialogue between the literature and the study. The study draws on some of the following theories that have been previously used in school effectiveness studies. The study is framed by two theoretical resources that explain the different aspects of the study. Complexity theory is recruited for school effectiveness whereas constructivist theory is drawn on to gain insight into effective mathematics teaching.

2.2. COMPLEXITY THEORY

The theory of complexity is not a theory of cognition, learning and memory, as such; complexity is a wide-ranging theory regarding the evolution, in addition to the functioning, of deviating systems that may be useful in various areas like economics, evolution, immunology as well as cognition, learning and memory (Hase & Kenyon, 2013). There are new concepts and vocabulary which are essential to understand the critical aspects of a complexity view (Doolittle, 2014). Basic complexity theory concepts include, amongst others, agents, adaptation, complex or complexity adaptive systems, fitness, development, hierarchy, non-linearity, internal models,
schemas, regularity and randomness, self-organization, selection and selection pressures, systems as well as system dynamics.

Complexity theory is a theory of change, evolution and adaptation, commonly for the benefits of survival, and frequently through a blend of competition and cooperation (Battram, 1999). It breaks with the open cause-and-effect representations, linear probability, atomistic, analytically-fragmented and reductionist approach to comprehending phenomena, interchanging them with organic, holistic and non-linear approaches, in which associations within interrelated networks are the order of the day (Cilliers, 1998).

In complexity theory, an organism, well-defined, senses and reacts to its surroundings; in this manner varying its environment, which modifies the organism again, so that the organism reacts to it again, thus proactively varying its environment. The procedure in iterating the situation, yields continuous and dynamic change regularly (Cilliers, 1998). Further, one cannot reflect on the organism devoid of considering its environment; the highlight is on mutual, relational performance and holism rather than on independence, uniqueness and solipsism. The whole is bigger than the sum of its parts, and these parts network in dynamic, diverse ways, so producing new validities, new groups and new associations.

Institutions, educational systems and practices show various elements of complex adaptive systems, being emergent and dynamic, at times changeable, non-linear establishments operating in unpredictable and varying external environments (Stacey, 1996). Complexity theory delineates ‘the basics’ of education, far from controlling and a controlled subject-based education and on the way to a discovered, emergent, constructivist and inter-disciplinary curriculum, and a confirmation of freedom as a sine qua non of education (Doll, 1993:46). Complexity theory channels us in a path opposite to the carefully stated, over-determined, orderly, old-fashioned, externally authorised and structured prescriptions of the DBE for the aims, content, teaching and assessment of learning.

It tries to enlighten why complex or system-wide activities emerge from the collaboration amongst ‘large collections of simpler components’ (Kernick, 2006). There is an extensive discrepancy in the way this theory is referred to and used (Geyer, 2012), but we can classify six
key themes concerning how complex systems work and how we ought to study them. Complexity theory is mostly advocated as a method new to science in which we classify and then describe systems or processes which are not in order, and unstable, required to yield collective rules around behaviour and outcomes. When used entirely in the sciences, it is pronounced as an innovative break from the ‘reductionist’ method of science and the pattern of order (Geyer & Rihani, 2010). Cairny (2012) classifies six (6) main themes concerning how complex systems work and how we have to study them:

- A complex system cannot be explained merely by breaking it down into its component parts because those parts are interdependent: elements interact with each other, share information and combine to produce systemic behaviour.

- The behaviour of complex systems is difficult (or impossible) to predict. They exhibit ‘non-linear’ dynamics produced by feedback loops in which some forms of energy or action are dampened (negative feedback) while others are amplified (positive feedback). Small actions can have large effects and large actions can have small effects.

- Complex systems are particularly sensitive to initial conditions that produce a long-term momentum or ‘path dependence’.

- They exhibit emergence, or behaviour that evolves from the interaction between elements at a local level rather than central direction. This makes the system difficult to control (and focuses our attention on the rules of interaction and the extent to which they are adhered).

- They may contain strange attractors or demonstrate extended regularities of behaviour which are liable to change radically (Bovaird, 2008; Geyer & Rihani, 2010). They may therefore exhibit periods of punctuated equilibria in which long periods of stability are interrupted by short bursts of change.

- The various problems that complexity theory seeks to address include predicting climate change, earthquakes, the spread of disease among populations, the processing of DNA
within the body, how the brain works, the growth of computer technology and artificial intelligence, school results or performance, learner outcomes and the behaviour of social and political systems, which can only be solved by interdisciplinary scientific groups (Mitchell, 2009).

Complexity theory displays a concern with the natural institutions and elements of randomness in the cooperation of group members and the manifestation of new customs of behaviour, which could be inefficient or practical (Scheerens, 2015). Cilliers (1998) maintains that connectedness necessitates a synchronisation of distributed knowledge; knowledge is not mainly situated in an axis like a principal’s office or departmental head’s office; rather it cascades through the system with communication as well as collaboration which are the significant elements of complexity theory. In this manner, complexity theory is included as a candid exposition of effective mathematics teaching and learner outcomes.

Effective mathematics teaching and learner outcomes are not linear, closed processes but open, recursive processes developing from and intertwined in diverse, often commonly constitutive factors in the connection between:

- The institutional character, (in)efficiencies, disciplinary domains as well as departmental and institutional mathematics resource allocation and planning;
- Mathematics teachers and learners’ habits, characters and life-worlds;
- Macro-collective factors influencing teachers, learners and parents, including the school and its networks;

and

- The unfolding of an individual, compromised, challenging as well as changing mathematics teaching and learning journey as contextualised in the two selected secondary schools.
2.2.1. Criticism of complexity theory

Complexity theory has its strong points and weaknesses, and is open to criticism like any other theory. Some concerns and challenges are:

- Stacey, Griffin and Shaw (2002) stress that complexity theory has to be used realistically, not as a loose metaphor, as it appears in the case of some management literature. (The researcher believes these concepts are useful for assisting SMTs to see and envisage circumstances in a different way).

- Various scholars mention that Complexity Theory is theoretically interesting, but give a difficult impression to apply on a daily basis. However, the researcher argues that complexity theory creates the circumstance for the need to incorporate various forms of knowledge in the educational system and gives some perceptions about the progressions through which various forms of knowledge are generated and interrelate.

- To what extent is it a theory appropriate to human systems; to what extent is it a way of perceiving human systems through (for example) a biological metaphor?

- Passion for complexity perceptions can lead to a diverging, ‘two-valued logic’, elevating this ‘new paradigm’ of thinking and removing everything associated with the ‘old paradigm’ (frequently considered as Newtonian).

- Does relating Complexity Theory to the sphere of human experience preserve a kind of physical science ‘imperialism’ relative to knowledge?

- Is Complexity Theory just a newer, if slightly fuzzier, type of positivism?

2.3. CONSTRUCTIVIST THEORY

Constructivism puts prominence on learner abilities and interests. This theory of learning is viewed as an interior process where the learner builds meaning by processing fresh information
as well as knowledge to integrate and grow previously assimilated knowledge and skill (Krahenbuhl, 2016). It is generally the approach that learners construct their own knowledge from interpreting their experiences. It is also considered as a theory of teaching, as it approaches education in a different way to those regarded by traditional teaching approaches. Constructivism has its resilient hold in mathematics education and stresses learning in complex situations, discovery learning, and learning in social contexts. It also segments positions from common rationalism and two other movements, which have vibrantly influenced the recent schools of education.

In accordance with Jaworski (1994), radical constructivism has its underpinning in two principles: knowledge is not submissively received but is dynamically built up by the cognising subject, and the purpose of thought is adaptive and does not assist the discovery of ontological reality but the organisation of the experiential world. Radical constructivism distinguishes learning as a dynamic process in which learners attempt to find solutions to problems that show up as they take part in the mathematical practices of the classroom. It underscores the role of specific features on what is to be observed and identified (Carr, 2006).

Fosnot (1996:20) gives a more articulate and comprehensive definition: “Learning from this perspective is viewed as a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights, constructing new representations and models of reality as a human meaning-making venture with culturally developed tools and symbols, and further negotiating such meaning through cooperative social activity, discourse, and debate”. This theory designates knowledge as internally constructed, temporary, developmental, non-objective, culturally and socially intermediated (Fosnot, 1998).

Educational observations for radical constructivism take in the interpretations of learners’ opinions as a whole, i.e. of their inclusive experiential world, the challenging nature of unabridged mathematical knowledge and not just the learner’s personal knowledge, as well as the instability of all research practices (Sriraman & English, 2010). Fosnot (1996) avows that knowledge cannot be seen as a precise representation of certainty, but the set of activities tried by someone that are evidenced to be valid in his/her experience.
Piaget’s theory is correlated to children’s activities and emphasises that interaction between environment and the individual is a strategic issue for the growth of cognitive structures. To be precise, knowledge is constructed. Based on this conjecture, Piaget’s assessment leads to some pedagogic concerns in teaching as well as learning in the classroom. On the basis of Piaget’s opinion, diverse pedagogic actions in the classroom to improve the cognitive structures of a learner. He recommends that to sort out thinking schemes, teachers ought to give learners problems to solve. That would permit them to reflect more reflexively and determine the solutions by themselves. Reflexive reasoning constitutes the first principle of learner-centred education, as in this kind of education learners turn out to be more critical thinkers and are dynamic in the construction of knowledge. Thought-provoking situations arouse cognitive schemes so that learners can learn to notice/observe, relate/equate, define/label, make/synthesise, and clarify/simplify situations. This can only be attained when teachers practise active methods, exclusively those that give emphasis to problem solution, manipulation of objects, experiment, and group work where learners can exchange ideas. That would arouse the growth of mental schemes of learners. These learning conditions are linked to learner-centred teaching, and are the result of the cognitive development theory of Piaget.

It is this recipe of learner independence and all-inclusive perspective that has propelled constructivism to the front position of learning mathematics as well as science (Doolittle, 2014). Learner autonomy is the notion that learners are dynamic participants in the learning procedure and in due course in charge of their own learning. This all-inclusive viewpoint is a non-reductionist method that gives emphasis to learning in the environment. The incorporation of learner autonomy as well as all-inclusive perspectives puts constructivism as the combination of beliefs and psychology (Doolittle, 2014).

The basic issue in this belief and psychological nexus is the part of epistemology; that is, what is the kind of knowledge and how does the knower come to know it (see Doolittle & Hicks, 2003; Ernst, 1995). From this branch, von Glasersfeld (1996:4) and Doolittle and Hicks (2003:74) mention the pillars of constructivist epistemology as:

- Knowledge construction is an individually and socially active process.
• This active process of constructing knowledge is adaptive in that the end result is to make one’s thoughts and behaviours more effective relative to achieving one’s goals.

• Understanding of one’s experience is a function of individual and social interpretation of one’s experience.

These pillars, though illuminating, permit for great unpredictability in what is characteristically so-called ‘constructivism’ (Phillips, 1995; Prawat, 1996). Moshman (1982) came around to describe this unpredictability through the field of constructivism. Moshman (1982:372) described the pillars of this field as endogenous, exogenous, dialectical constructivism, and what would more characteristically be currently termed trivial constructivism and radical constructivism.

Trivial constructivism highlights the outer nature of knowledge (Doolittle, 2014). Knowledge is perceived as the internalisation and reconstruction of exterior actuality. Knowledge acquirement or learning is the process of building precise internal representations of external constructions in the physical world. This opinion takes for granted that reality is recognisable. Trivial constructivism is commonly inaccurately related with information processing and its constituent processes, containing procedural, schemata, declarative knowledge with propositional-networks (Derry, 1996). Some characteristics of knowable reality is constructed by the learner. This knowledge is segmented into disconnected sub-skills by the teacher who then conveys this knowledge to the learner. An effective teaching/learning experience results when the learner, after this transmission of knowledge, has constructed a correct image of the original, understandable knowledge (Doolittle, 2014).

Trivial constructivism denotes one extreme of the constructivist field, whereas radical constructivism signifies the other extreme. Radical constructivism highlights the inner nature of knowledge and is constructed on the theoretic footing of Piaget (1973, 1977). Knowledge is constructed from exterior understandings and prior mental constructions. Learning or knowledge attainment is the renewal and restructuring of childhood knowledge constructions in the light of new practises. As a result, knowledge is not a precise depiction of exterior reality, but rather it is
an inside comprehensible and corresponding collection of processes and constructions that are responsible for adaptive behaviour. Concentrating on the learner, the trivial constructivism approach to mathematics depends on discovery learning, learning in authentic or complex conditions, learning in social circumstances and doubt of empirical evaluations and problem grounded learning, inquiry learning as well as problem solving (Casas, 2011).

Casas (2011) maintains that all these sorts of learning differ; the core principle of learning is through involvement in the environment that consists of:

i. Active learning

ii. real-world and meaningful challenges,

iii. the idea of ownership, choices and responsibility,

iv. opportunity to solve problems, answer questions or address real needs, and

v. opportunity for the learners to feel empowered.

In spite of various accomplishments that discovery learning conveys to learners in terms of getting a desired construct, this method of learning has been at times subject to question. For example, Anderson et al. (1998) designate the interference of time consumed in obtaining a construct when using discovery-learning teaching. They argue that since learning only takes place after the construct has been discovered, the time needed to execute a task is long, or the search is fruitless, and this may impact on the individual’s motivation.

Radical constructivists also avow that learning of any subject, such as mathematics, should take place in the framework of complex problems. Complex learning backgrounds comprise learning in which learners are obliged to answer complex problems. This is centred on the constructivist statement that complex and sincere learning tasks arouse interest, creativeness and higher-order thinking, and that the real world does not provide enough conditions that can steer learners to deal with complex circumstances. Consequently, it is of great significance that learners drill
complex problems. Complex problems allow learners to make a selection of what kind of learning they should follow. For instance, if learners are shown diverse ways of determining a solution they are likely to achieve the goals of cutting-edge knowledge acquisition. Vygotsky (1978) emphasises that higher order thinking can simply be accomplished through social collaboration.

The notion that complex learning circumstances may advance high-order-thinking steered radical constructivists to endorse that learning should take place in the setting of complex problems. Anderson et al. (1998) recognise the presence of two problems with this method. First, if a learner is experiencing challenges with a lot of the components, he/she could be confused by the process pressures of a complex assignment. Furthermore, in the circumstances in which all the components of assignment are understood by the learner, she/he will be wasting valuable time working through all those components that are at present understood in order to get to those that still require additional practice. In contrast, it is accepted that part of the workout is often more effective when a portion of the component is independent.

Social constructivism lies somewhere between the spread of understandable reality of the trivial constructivists, and the explanation of individually practicable reality of the radical constructivists. Social constructivism highlights the collaborative nature of knowledge (Doolittle, 2014). Knowledge is the effect of the collaboration between the learner and the environment, as well as other learners.

Learning or knowledge acquisition is the process of constructing interior models or depictions of exterior structures as clarified through and inspired by one’s principles, values, prior experiences, and language, built on collaborations with others, direct teaching, and modelling (Doolittle, 2014). This opinion presumes that “reality” is not understandable. This lies as cornerstone of challenging and changing conditions which affect learner outcomes in mathematics teaching in various schools.

Cognitive growth is based on a learner’s ability to learn informally relevant tools (like teaspoon, bicycle, spade, and calculators) and culturally established signs (writing, language, and number systems) through collaborations with other learners and teachers who socialise them into their
culture (Vygotsky, 1978). These culturally facilitated activities offer social skills that are internalised and which later turn out to be part of the learner’s mental functioning. Consequently, knowledge is an outcome of social experience, promoted by one’s socio-cultural background, and resulting in an improved representation of understanding.

Learners network with knowledge in a socio-cultural setting. This exterior social experience results in the construction of interior mental structures that are influenced by the existence of social, cultural, contextual, and activity-based aspects (Doolittle, 2014). A learner does not obtain a precise representation of this knowledge but rather an individual interpretation of the exterior knowledge. The feasibility of this newly constructed knowledge will be grounded on the learner’s prior knowledge and the effect of the cultural, social, contextual, and activity-based aspects.

Doolittle (2014) avers that constructivism laid various theories of learning, including anchored instruction, situated cognition, cooperative learning, inquiry and problem-based learning, exploratory learning, reciprocal teaching, cognitive apprenticeships, generative learning, and information processing. Nonetheless, from constructivist theories as well as the constructivist models, Doolittle and Hicks (in Doolittle, 2014) have developed the subsequent principles of learning:

- The construction of knowledge and the making of meaning are individual and social active processes.
- The construction of knowledge involves social mediation within cultural contexts.
- The construction of knowledge is fostered by authentic and real-world environments.
- The construction of knowledge takes place within the framework of the learner’s prior knowledge and experience.
- The construction of knowledge is integrated more deeply by engaging in multiple perspectives and representations of content, skills, and social realms.
The construction of knowledge is fostered by students becoming self-regulated, self-mediated, and self-aware.

These principles incorporate the essence of constructivism, that is, learning as the adaptive and self-organised construction of knowledge that is a function of both one’s past knowledge and skill, and one’s existing socio-cultural action. This viewpoint on learning replicates the complexity of learning as linking adaptation, history, self-organisation, and interaction (Doolittle, 2014).

Constructivist theories are grounded on the certainty that learners construct their theoretical understanding and specific knowledge by their own action (Krahenbuhl, 2016). The role of the teacher is to create a mathematical environment to allow learners to build this mathematical knowledge. This environment would afford learners opportunities to link their understanding, use resources, make assumptions and test their thinking in the course of constructing knowledge of mathematics and being active on a social basis.

The constructivist view of learning is grounded on the conjecture that learners have personal substantial experiences, developed in their own surroundings, which may function as a base to advantage them to understand new concepts. As a result, learning functions as a self-calibrator of engagements that arise amongst individual interior models and the new dimensions, letting new ideas have an effect on the base of social experience. Consequently, teaching should permit learners to advance specific questions and construct their own concepts and approaches. In contrast, concepts are communicated and imparted out of learners’ experiences in a teacher-centred approach.

Challenging the notion of traditional teaching that concepts and signs/symbols can be learned out of the setting, constructivists proclaim that learning may take place in an atmosphere in which learners can contextualise their personal experiences, making them expressive. Hence, in the course of learning, learners may be able to question, construct their own configurations, strategies, concepts and models (Fostnot, 1998). The classroom may be appreciated as a part of the society where the community of learners take part in discussion and reflection of what they
have learned. These thoughts are opposite to the old teaching, where teachers take an autocratic stance and always facilitate the learners’ learning activities.

Fosnot (1998) contends that the teaching of mathematics in a constructivism setting may steer learners to take part in:

1) solving noteworthy problems;

2) assessing and probing for appropriate solutions; and

3) constructing their specific algorithm and formulas.

2.3.1. Problem solving in constructivism

Constructivist strategies in mathematics also put emphasis on problem solving as one of the forms of learning. One way of working with the nature of problem solving is to analyse what a problem is all about. Robertson (2005) maintains that a problem comes up when one is challenged with something she or he does not know how to do. That implies that if one already knows the procedure on finding a solution to the problem then there is a way. Problem-solving can be regarded as an order of cognitive actions focused on a goal. These actions embrace the unknown and the known techniques. The known techniques result from personal experiences, and the unknown arise from the circumstances. The National Council of Teachers of Mathematics (2007) delineates problem-solving in mathematics as a “process of interpreting a situation mathematically, which usually involves several interactive cycles of expressing, testing and revising mathematical interpretations”.

Problem-solving techniques call for an open instruction in the finding/solving of mathematical problems and they uses investigative and metacognitive approaches such as comprehending the problem, creating/developing a plan, executing the plan, and reflecting. Problem-solving techniques share some deductions with constructivists; to be precise

- the importance of the problem as a way of discovering challenge;
the significance of problem interpretation;

the impact of clarifying strategies that are usually implicit but effective; and

the worth of reflecting on one’s solution route.

Carson (2005) delineated the hypothetical and epistemological suppositions one must stick to if accepting constructivist teaching:

- “reality is dependent on the perceiver, and thus constructed

- reason or logic is not the only means of understanding reality, but one of many; and

- knowledge of truth is subjective and relative to the individual or community”.

And when it is discovered, it is plain that learner achievement through such a vantage point will not be tied to mastery of content knowledge but reasonably on physical engagement with the problem through which an independent understanding of the problem is socially constructed (Krahenbuhl, 2016).

2.3.2. Criticism of constructivism

Constructivism has its strong points and weaknesses, and is open to criticism like any other theory. The fast and slow learners differ on how they learn and often teaching slow learners to adopt strategies used by expert teachers creates problems for learning. Learners need extensive opportunities to develop background knowledge and scaffolds to even remotely engage in these “expert” skills in a way that contributes positively to their development. The constructivist pedagogical approach frames the classroom in a way that makes objective learning and growth more difficult, as novice learners are treated as experts and fast learners as merely skilled. Learners lack the experience and knowledge to apply subject-content skills effectively; they have limited space in their mind to grapple with issues, and furthermore, their constructed meaning may or may not correspond with reality. Learners may construct meaning that does not
correspond with reality going directly against a major purpose of learning - to correctly know that which corresponds with reality and that which does not, hence equipping students with the breadth of background knowledge needed for their improved outcomes in mathematics. Some learners can make incorrect and untrue discoveries which may lead to misconceptions stored in long-term memory that are even more difficult to rectify.

The constructivist assumptions of construction of knowledge have significant implications for the nature of learning and instruction. The reality is diverse, and education should take into account such multiple diversity, allowing learners to interpret their own world. The argument can be defended analogously to the way Ayer (1946) defends the Principle of Verification. The role of the teacher is to focus on learner-centred teaching and learning and guide learners’ apprenticeship, helping them to interpret the world. Furthermore, constructivists have close ties with his views that “education is dialogue” and that “students’ views constitute education” (Beckett, 2013:50). It sends an exponential rippling effect though educational circles as it has left a legacy of impact that is equalled by few others (Beckett, 2013:50).

2.4. COMPLEX CONSTRUCTIVISM PRINCIPLES

Complex constructivism principles are grounded on the mutual fundamental beliefs of both complexity theory as well as constructivism theory (Doolittle, 2014). These principles offer a concrete foundation for accepting the nature of learning in a complicated, culturally and dynamic related world.

Learning encompasses learner adaptation to the environment. The notion that an organism adjusts its actions to more effective networks within its environment is mutual to both a constructivist and complexity perspective. For constructivists, this adaptation includes the creation of new mental arrangements, and the adjustment of existing mental arrangements, to facilitate learners in interacting importantly and effectively in their physical and sociocultural environments. “Learning occurs as [students] make sense of instructional events by using their existing cognitive structures to interpret environmental stimuli. It also occurs as they modify and elaborate their knowledge structures through a process of adaptation to the environment” Eisenhart and Broko (1991:142).
This concept of constructing and modifying internal models in order to adapt to an environment is mirrored in complexity theory. Martin (1999:263), in delineating the essential characteristics of a complex system, advocated that “as a rule, these systems are adaptive; changes in their internal states occur in response to the environment”. Undeniably, Waldrop (1992:146) linked the gap: “in fact, you can think of internal models as the building blocks of behaviour. And like any other building blocks, they can be tested, refined, and rearranged as the system gains experience”.

Learning includes the dynamic construction of knowledge by learners. There are various kinds of constructivism, though their merging theoretical view is the belief in a dynamic learner, a learner that eagerly constructs knowledge from understanding. This construction practice is responsible for learners internalising their values and constructing sense of their environment. This deed of construction relates equally well from knowing basic mathematics facts to understanding cultural patterns. Knowing and constructing the sense of an environment, as a complex, means the active search for consistencies. These consistencies replicate the knowledge of the environment and permit means to effectively familiarise them. Consequently, from a complex constructivist viewpoint, learners construct interior mental representations by actively searching for consistencies in their experience.

Learning comprises the self-organisation of knowledge and experience into interior representations. Constructivism and complexity theories indicate that the organisation of learners’ interior representations is a process that is done exclusively by the learner or agent. Specifically, the organisation of a learner’s or agent’s knowledge or interior model is not enacted on the learner or agent by either an interior or exterior source. This does not disaffirm the effect of the environment and society; rather, society and the environment have a secondary impact on self-organisation, as a stimulus for adaptation. Consequently, self-organisation accounts for the unique or subjective nature of knowledge and “the view that learning is both a process of self-organization and a process of enculturation that occurs while participating in cultural practices, frequently while interacting with others” (Cobb, 1989:41).
Learning involves the rise of interior representations as a natural concern of a learner’s experience. The complex constructivist views the idea that interior representations are not “actively constructed,” but rather, interior representations are “naturally emerging.” This avowal is not as negating to a constructivist perception as it may at first seem. Learners’ active construction includes the active search for consistencies in their experience and these consistencies represent knowledge.

Nonetheless, the interior representations that provide the relational organisation of this knowledge appear as a natural result of knowledge acquirement. Hence, the knowledge is dynamically constructed while the organisation (school) is naturally developing. Moreover, these naturally developing interior representations provide the non-linear base for the concept that the whole, the interior representations, is more than the sum of the quantities of the knowledge. The ascending levels of the hierarchy of complexity demonstrate emergent properties at each level which appear to be non-predictable from the properties of the component parts.

Learning is a function of learner interaction as well as existing interior representations. In constructivism, this complex constructivist standard links the social interactionist perception of the socio-cultural constructivists with the subjective constructivist views of the radical constructivists. Yackel (1995) provided a provision for this standard of linking the specific and the social by avoiding this contrast and putting forward an “emergent perspective” to constructivism that incorporates these two views. Finally, the complex constructivist approach mirrors the link amongst the mathematics teachers’ and learners’ conditions as well as school conditions in the selected secondary schools.

2.5. SUMMARY AND PROJECTION FOR THE NEXT CHAPTER

This chapter provided the elucidation of complexity and constructivist theories in brief from various scholars’ perspectives. The criticism of both theories was laid in this study though criticism of constructivist theory was from various scholars like Ernest (1991), Davis (1972), Kline (1980) and many others. Complex constructivism was brought in to cement both theories. Complex constructivism provides a new perspective on learning; a perspective that emphasizes both the active, self-organizing construction of knowledge, and the adaptive nature of those
constructions. It combines and addresses the common concerns of complexity and constructivist theories.

The next chapter starts from the general theory about school effectiveness and then narrows to mathematics teaching and learning specifically, merging it with relevant theories of mathematics teaching and learning. The main focus and the core (essence) of the argumentation in the next chapter is on mathematics teaching for improved mathematics learning in schools to meet expectations in the current era.
3.1. INTRODUCTION

In chapter one, the researcher presented and discussed the introduction to the study, that is, the background, motivation for the research, statement of the research question, aim and objectives of the study, assumptions of the researcher, the purpose and significance of the study. The research design and methodology, theoretical framework of the study, ethical considerations, delimitations and limitations of the study, and demarcation of the study were also discussed in this chapter.

This chapter provides an account of the literature reviewed on school effectiveness and effective mathematics teaching. The purpose of the literature review is to outline the determinants that contribute towards school effectiveness and effective mathematics teaching as researched internationally and within South Africa. Leedy and Omrod (2010) view review of literature as an organised, critical analysis and summary of existing literature which is in line with the research topic. It involves an in-depth reading of relevant selection of literature at such as articles, books, dissertations, magazines, reported news events in the newspapers, and opinions made on issues under investigation.

3.2. CONCEPT OF SCHOOL EFFECTIVENESS

The concept of effectiveness denotes an organisation achieving its explicit objectives (Beare et al., 1989). The concept of school effectiveness can mean diverse things, and this has steered an inclusive deliberation about the thought (Mortimore, 2000). School effectiveness has been seen from the input-output viewpoint; from the viewpoint of schools in which learners advance more than might be anticipated from the nature of its intake (Reynolds & Teddlie, 2000); and progression in learner attainment (Yatsko, Lake, Bowen & Cooley, 2015).

In accordance with Sun, Creemers and De Jong (2007), studies of school effectiveness have two dimensions: firstly, to identify features of effective schools, and secondly to identify dissimilarities between education results in these schools. The choice and usage of measures of
results has been open to deliberation in numerous ranges of educational investigations (Sun *et al.*, 2007). One of the major issues of effective schools is their influence on the results of learners. In this respect, Bennet, Crawford and Cartwright (2003) explain an effective school is “a school in which students progress further than might be expected”.

Reynolds and Teddlie (2000) and Hopkins and Reynolds (2001) argue that a school is effective if its processes result in evident positive outcomes amongst its learners constantly over time. This indicates that the effectiveness of a school is more reliant on its ‘processes’ and evaluated by its ‘outcomes’ than on its ‘intake’. A school is considered effective if the outcome of its actions surpasses its aims, hence Potberg (2014) is of the opinion that an effective school is the one which upholds high levels of learner success for all learners in the school. An effective school is therefore a school that can succeed or surpass its academic aims; therefore school effectiveness is the capability of a school to exceed its aims. The aims set should be reflective of learners’ academic capability.

Scheerens (2000) postulates school effectiveness as the level at which schools accomplish their goals in contrast to other schools, while Botha (2010:606) portrays school effectiveness being when “a school accomplishes its objectives”. Mortimore (1993) professes school effectiveness as a paradigm constructed on the management as well as organisation of schools. Conversely, Harris (2003) asserts the political landscape of school effectiveness by remarking that governments regulate how schools should function as a value-for-money notion because a significant investment may have been moved into the education financial plan. Nonetheless, the control of the government opinion is tested by aspects of leadership and the functions of parents and community in the management of the school.

Hajnal, Walker and Sackey (1998) aver that school effectiveness could designate how well the school is managed by the principal, how much the community and parents are hands-on in its undertakings, and how broadly the school is acknowledged. School effectiveness came into existence as a product of disparity in the society which ignited a change concerning education for all (Scheerens, 2015). School effectiveness, as used in this study, refers to all contextual variables at school level which enhance effectiveness in mathematics teaching such as administration, governance, school leadership, learning, teaching, assessment processes,
learners’ motivation and commitment, parents’ participation as well various stakeholders’ involvement at large, which collectively lead to academic excellence of learner.

The next section looks at a concise outline of literature that focuses on the concepts of school effectiveness and teacher effectiveness in mathematics. It describes the features of effective schools and teachers, and identifies determinants that may influence the effectiveness of secondary schools in mathematics in NMM district.

3.3. CHARACTERISTICS OF EFFECTIVE SCHOOLS

The common characteristics of effective schools are picked up from the observation of developed and developing countries. Researchers such as Edmonds (1979) and Lezotte (1991) maintain that effective schools have a number of common characteristics. Makoele (2011) lists the following characteristics of effective schools that are pertinent to the South African setting: vision, mission and expectations; teaching and learning; management and leadership; assessment of learners; school-home relations, and relations with other schools. Shannon and Bylsma (2007) have demarcated nine characteristics that were more common in effective schools and they agree with numerous scholars such as Levine and Lezotte (1990); Reynolds and Teddlie (2000). The nine characteristics as outlined by Shannon and Bylsma (2007) are:

- a clear and shared focus
- effective school leadership
- curriculum, instruction and assessment aligned with standards
- supportive learning environment
- high standards and expectations for all students
- frequent monitoring of learning and teaching
- focused professional development
- high levels of collaboration and communication
- high level of family and community involvement.

No particular characteristic leads to school effectiveness. Various studies classify arrangements of five or more of these characteristics. This study has revised and polished the nine characteristics in order to separate and categorise corresponding elements or mutual qualities.
Table 3.1 lists six revised characteristics utilised as a context for effective schools in this study and which are reflected in detail beneath:

Table 3.1 The characteristics of an effective school

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Features</th>
</tr>
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<tbody>
<tr>
<td>Effective teaching and improvement</td>
<td>Teacher effects in the classroom</td>
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<td></td>
<td>Quality of teaching</td>
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<td></td>
<td>Teachers’ learning opportunities</td>
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<tr>
<td></td>
<td>Purpose to improve learner outcomes</td>
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<td></td>
<td>Ensure suitably qualified teachers</td>
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<tr>
<td>High expectations and standards</td>
<td>Focus on personnel</td>
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<td></td>
<td>Focus on learners</td>
</tr>
<tr>
<td></td>
<td>Active role of stakeholders</td>
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<td></td>
<td>Responsibility for achieving success</td>
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<tr>
<td>Supportive learning environment</td>
<td>Link between learning outcomes and classroom practices</td>
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<tr>
<td></td>
<td>School climate</td>
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<tr>
<td></td>
<td>Emphasis on academic performance</td>
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<tr>
<td>Effective monitoring and leadership</td>
<td>Transformational leadership</td>
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<tr>
<td></td>
<td>Relationship of followers and leaders</td>
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<td></td>
<td>Instructional leadership as an approach</td>
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<td></td>
<td>Studying and refining processes</td>
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<tr>
<td></td>
<td>Ensure feedback provided</td>
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<tr>
<td>Clear and shared focus</td>
<td>Create a shared vision</td>
</tr>
<tr>
<td></td>
<td>Core purpose of the organisation</td>
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<td></td>
<td>Unite all purposes</td>
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<td>Communication and group effort</td>
<td>Effective partnership</td>
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<td></td>
<td>Joint efforts</td>
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<td></td>
<td>Communication is fundamental</td>
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<td></td>
<td>Parental involvement</td>
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<td></td>
<td>Consideration of family circumstances</td>
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</tbody>
</table>
3.3.1. Effective teaching and improvement

Consideration in schools on school effectiveness study has lessened: the classroom is getting more prominence than in the past (Creemers & Reezigt, 1996). A number of studies similar to those on effective schools simply narrate that schools are effective if there are effective teachers in a school (Wright, Horn & Sanders, 1997). Wright et al. (1997) hold that teaching has an undeviating relationship with learner outcomes. The study findings point out that there is much deviation in schools suggesting that the value of teaching as well as learning is the most significant element responsible for the discrepancy in learner outcomes in Grade 12. It can be deduced that such results show that learning occurs in classrooms, although it is a process: there may be dissimilarities in the improvement made by learners in various classes in a single school, but teachers do make a critical difference in the classroom. This issue is maintained by Christie et al. (2007), who show that teachers’ impact in the classroom defines the learners’ outcomes. However, specific teachers in schools have an effective influence on the achievement of the learners hence they maintain that what transpires in the classroom adds to the effectiveness of the school.

The Ministerial Task Team Report (DBE, 2013) endorses effective teaching as a vital aim to be accomplished. DBE tries to intensify and develop the ability of personnel to convey effective teaching which will probably improve learning and achieve better learner outcomes. Effective teaching can succeed through teachers’ development and regular mathematics subject matter workshops with new strategies. Teacher development includes a comprehensive scope of learning prospects.

Shannon and Bylsma (2007) affirm there is increasing consent that constant, job-related professional development is paramount in moving teaching to advance learner knowledge. Hawley and Valli (1999) posit that conservative personnel development approaches for creating essential improvement in taking orders have been ineffective: they concur with the assumption of diverse ways to expedite professional learning. The assessment of the effectiveness of professional growth stands in relation to its influence on pupil learning and development of teaching expertise. It does not purely reveal the attitudes of member fulfilment.
Effective mathematics teacher development replicates the following common subjects in accordance with Lieberman and Miller (2000):

- The significance of a clear link between teacher and pupil learning;
- Supportive professional relationship and mutual responsibility, with time as well as opportunity for dialogue, dual action and criticism;
- Relate to teaching and assessment practices;
- Boost the development of a corporate language over written and oral communication;
- Make and use organised tools and customs to guide dialogue; and
- Use real-life measures of teaching as a foundation of professional development.

In South Africa, The National Policy Framework for Teacher Education and Development (2007) is an endeavour to address the necessity for properly trained teachers. The policy framework emphasises dual sub-systems: early professional education of teachers as well as continuing professional teacher development (CPTD). Steyn (2008) suggests that the policy framework is a transformation model intended to change the quality of teaching in the RSA. Its realisation cannot be definite except in its potential to steer the consideration of teachers’ professional development which may lead to effective mathematics teaching.

There are diverse aspects to be reflected and linked to effective mathematics teaching. The significance of a designed didactic method of teaching has been validated by school effectiveness studies. In a designed teaching method, a mathematics lesson is presented to give learners a preview of what is anticipated. The subject matter to be learnt is offered in coherent stages with clear, in-depth clarifications as well as dynamic mathematics learner input for each stage. It is anticipated that during the course of the mathematics lesson, a mathematics educator checks for comprehension and understanding by probing and giving activities where necessary. An effective teacher should conclude his/her lesson with a reflection of what was done in the lesson. Time management of a mathematics lesson supports effective teaching i.e. beginning and concluding a lesson on time; prior lesson planning as well as adjusting lessons to the suitable level of the learners (Reynolds & Teddlie, 2000).
3.3.2. High expectations and standards

Mortimore (1993) and Moore (1998) affirm that learners have a tendency to achieve at a level consistent with the educator’s expectation. Mathematics learners tend towards better performance, and are more capable, when teachers establish high expectations as well as making learners accountable for accomplishing better outcomes by stimulating them. Mortimore (1999) elucidates that these results are denoted as a self-fulfilling prophecy. Increasing pupil learning necessitates that learners and their educators have confidence in their capability to attain high educational standards. The ability to change beliefs is often associated with changing actions. In accordance with Fullan in Saphier (2005:105) ‘we can act our way to new beliefs’. Saphier (2005) considers that all pupils can carry out demanding academic efforts at extraordinary standards: educators who integrate this trust in their preparations are realistic about the goals which they as well as their pupils can reach.

An essential element of effective school mathematics teaching is the degree to which mathematics learners sense that they are expected to learn and the magnitude to which, and means by which, those expectations are strengthened. Mohiemang (2008) postulates that expectations can be either negative or positive. Mathematics educators have different expectations and highlight them in different ways. For instance, educators are inclined to give low performers closed or convergent tasks, though open, divergent assignments are earmarked for pupils with high capacity. Moore (1998) states that educators communicate expectations to learners tacitly: low performers are less involved in questions and afforded less time to respond to questions. This is an unexpressed signal that low achievers in mathematics are not expected to respond to question. Moreover, low performers are assessed more commonly for incorrect answers and admired less often for precise answers. Academically, the contrary should apply; low mathematics performers should be cheered and acknowledged to boost their confidence hence leading to better achievement in mathematics.

Harris (2001) maintains that undesirable expectations intensify poor achievement in schools in any learning area, but especially in mathematics. Low prospects can lower success by discouraging a pupil. Mathematics learners begin to adopt their educator’s undesirable
expectations which suppress the pupil’s self-assurance and inspiration. Muijs and Reynolds (2001) advocate habits of overcoming this difficulty:

- Educators should recall that all pupils can learn, and this should be cascaded to learners;
- Educators should make sure that all pupils have the chance to take part in classroom undertakings;
- Educators should be mindful of prospective dissimilarities in their reply to pupils; and
- Educators should take into cognisance how they allocate rewards and reprimand pupils.

High expectations link with a more dynamic role for mathematics educators in assisting pupils to learn mathematics. Low expectations place mathematics pupils in a submissive attitude to teaching. Mohiemang (2008) elucidates that great expectations have more effect once they are part of school values that puts expectations on everybody in the school. If the principal has unlimited expectations from mathematics educators, this positive assertiveness crosses over to the SMT and the entire school. Sammons (1999) refer to the state of affairs where a school has great expectations for their pupils: educators offer rationally inspiring content for pupils in all lessons. Great expectations unaided, though, can do little to increase mathematics performance. It is more effective as soon as shared with a solid emphasis on academics; teacher effectiveness; a favourable learning atmosphere; as well as the monitoring of improvement. These are the controllable and viable features of effective school mathematics teaching.

### 3.3.3. Supportive learning atmosphere

Shannon and Bylsma (2007) delineate a reassuring learning atmosphere as a school culture of rational expectations concerning conduct; constant and impartial use of rules as well as regulations and a compassionate open relationship amongst teachers and learners. They further argue that in a supportive learning atmosphere, children are cherished. Their tradition and upbringing are appreciated as resources. This gratefulness leads to shared esteem and conviction at the centre of a supportive mathematics learning atmosphere.

Reynolds and Teddlie (2000) suggest that in a supportive learning atmosphere, emphasis on learning embraces academic significance and full usage of existing learning time which have been revealed to be the basic components of effective schools. An academic significance is
perceived through aspects such as assignments, which are not merely a means of scrutinising mathematics pupils’ comprehension but in addition take full advantage of mathematics learning time.

There are numerous fields in school effectiveness studies that are appropriate in building and upholding supportive mathematics learning atmospheres:

- Edmonds (1979) conveyed that effective schools studies delivered features of a secure and systematic school atmosphere that is logically flexible, discreet, fair, and favourable to the instructional job.
- Studies on small classes and schools define tailored learning settings that raise learners to fit in and chances to take part energetically in the school (see Molnar et al., 1999; Fidler, 2001).
- De Frondeville (2009) advocates that classroom and teaching representations involve pupils emotionally, academically and on a social basis.
- Gay (2000) highlights the significance of teachers identifying the background of their learners from all contexts.

Hopkins and Reynolds (2001) uphold that effective schools as well as effective educators follow and pay thoughtful attention to the association between classroom practices and pupils’ learning outcomes. Those kinds of mathematics teachers dedicate more time to educational actions and offer mathematics learners who come across problems the assistance they require to flourish in mathematics.

3.3.4. Effective monitoring and leadership

Burns (1978) describes leadership as the inspiration of individuals to accomplish goals described conjointly by followers and leaders over the practice of formal, administrative, spiritual and other means. The goals mentioned signify the ideals and drives which could be inferred as the desires and opportunities of both followers and leaders. It can further be assumed from Burns’s
(1978) explanation that effective leadership is reliant on the relationship between followers and leaders. Hallinger (2007) and Stewart (2006) contend that learner outcomes are significantly impacted by the SMT and educators by the approach by which leadership is applied. They further point out that two theoretical models appeared which emphasis educational leadership viz., instructional as well as transformational leadership.

Leithwood, Jantzi and Steinbach (1999) describe instructional leadership as a method that gives emphasis to the performance of educators as they participate in tasks directly impacting on the improvement of learners. The obligation of the principal to improve teaching and learning undertakings began in the 1980s as a area of significance after research on effective schools and was named instructional leadership. Principals need to make prior arrangements for mathematics class visits to avoid surprises: they are not automatically specialists on the subject of mathematics. This marks instructional leadership as a challenge in secondary schools. As a result, principals habitually detach themselves from class visits. Nonetheless, it is mandatory that monitoring is done to make sure that effective mathematics teaching and learning takes place.

Schmoker (1996) generally describes monitoring as an analysis of what is done in contrast to the outcomes being achieved. He elucidates that monitoring calls for systematic studying and refining of routines that “directly contribute to the designated results” (Schmoker: 1996:7). Southworth (2004) posits that monitoring includes classroom visits, general observations of teachers and timeous feedback. The significance of monitoring is to afford feedback to mathematics teachers, learners and other interested parties who are liable for making sure that effective mathematics teaching as well as learning takes place. Leadership is resilient once it is informed by facts which various levels of leadership use. Shannon and Bylsma (2007) elucidate that monitoring teaching as well as learning necessitates paying attention to pupil learning, school effectiveness and classroom processes. Their findings support that mathematics learning is monitored by pursuing a diversity of assessment outcomes while mathematics teaching is monitored by SMT and educator appraisal programmes.

The theoretical ideal of transformational leadership is the input of others in the exercise of leadership. Leadership is reliant on others as well as the relationship they receive. Transformational leadership is related to that of distributive leadership. They elucidate that the notion of distributive leadership concedes and advocates leadership that is existent in the
establishment. Elmore (2000) maintains that leadership cannot be an individual task assigned to leaders since the responsibility of teaching and learning is multifactorial. His findings support that a number of developments should emanate from those directly accountable for mathematics instruction and not from the supervision of instruction. Leadership should be distributed and centred on particular benefits, dispositions, expertise, facts and title role.

Leithwood, Louis, Anderson and Wahlstrom (2004) suggest that effective leaders nurture a continuum of accepted abilities and talents in an establishment. Once mathematics educators lead, principals spread out their specific ability and thus mathematics educator leadership advantages schools and ultimately mathematics classrooms. Mathematics teacher leadership can be perceived as a task of inspecting class attendance, effecting departmental as well as school policies, monitoring pupil improvement and effecting action where required.

Leithwood and Jantzi (2000) spell out transformational leadership in schools with the following six arguments:

- Structuring the school vision and goals;
- Providing logical motivation;
- Contributing improved support;
- Signifying professional practices and principles;
- Establishing beliefs in high performance; and
- Increasing structures to nurture input in decision-making.

Harris (2003) acknowledges that these elements stimulate teacher collaboration, intensify passion and magnify teacher effectiveness. Schools advance mathematics learners’ achievement if they integrate and keep such advancements.

3.3.5. Clear and mutual vision

A critical element of an effective school mathematics teaching is the identification of its core purpose as an organisation. Newmann, Smith, Allensworth and Byrk (2001) emphasise that an effective system has a strong programme; practices within the system are consistent and tightly connected. A sustained and precise effort is expected to advance mathematics learner outcomes more than uneven, clumsy structures. A clear and shared vision is tantamount to the
accomplishment of healthy school mathematics values. Schein (1985) describes school culture as elementary expectations and principles mutually accepted by stakeholders of the school, which drive a basic view of it and its environment.

School mathematics culture can be described in the framework and background of the school, its inner practices and the sense of main purpose by which personnel control and organise their operational setting. These descriptions suggest that having a clear and shared school mathematics vision is an organic role of energies in the specific structure.

Sammons, Hillman, and Mortimore (1995) consider that a collective vision and goal afford unity of intent, reliability of the practice, cooperation and collegiality in the the staff. One of the important steps in order to improve school mathematics outcomes is to create a clear and shared vision of learning mathematics. Rosenholtz (1987) clarifies that a shared effort in a school affords a way and significance for teacher cooperation: it generates an improved conviction with orientation to teaching practice. This helps in supporting mathematics activities for a school to become effective in mathematics teaching.

Shannon and Bylsma (2007) posit that a clear as well as mutually determined vision produces an idea that captures the thoughts and eagerness of members of an establishment and particular goals. Effectiveness thus necessitates a collective process of evaluating information with respect to specific mathematics teaching goals in the school: this procedure is arranged amongst SMT and all stakeholders.

3.3.6. Communication and group effort/collaboration

In education, collaboration is envisioned to “promote the most effective teaching possible for the greatest number of students” (Pugach & Johnson, 1995:178). Shannon and Bylsma (2007) support the idea that mathematics teacher collaboration is a range of shared activities in a school improvement plans. “Collaboration is the process of shared creation: two or more individuals with complementary skills interacting to create a shared understanding that none had previously possessed or could have come to on their own. Collaboration creates a shared meaning about a process, a product, or an event. In this sense, there is nothing routine about it. Something is there that wasn’t there before. Collaboration can occur by mail, over the phone lines, and in person.
But the true medium of collaboration is other people. Real innovation comes from the social matrix... [and] is a relationship with a dynamic fundamentally different from ordinary communication” (Schrage, 1990:40–41).

Johnson (2012) explains collaboration in the following four ways:

- supportive effort in the midst of educators;
- collective involvement in school governance;
- working together of business and schools for monetary maintenance and
- partnership amongst schools and public agencies to deliver social services.

Pupils, parents, teachers, support staff, business, universities and the society at large have a joint obligation for the education of learners. Obligations may embrace demonstrating the significance of mathematics education, displaying support and inspiring learners to learn mathematics. Strong communication amongst mathematics educators through all grades and with support staff is indispensable with good collaboration.

Rosenholtz (1987) highlights the significance of collaboration for educator effectiveness and learner outcomes. Barott and Raybould (1998) state collaboration calls for interdependence and could be seen as a loss of independence and choice. Schools that usually benchmark pursue and share learning, frequently form a close-knit society of constant survey and shared development.

The precise kind of constant, organised mathematics educator collaboration expands the quality of teaching and learning of mathematics with better professional drive in nearly all settings.

It is clear that collaboration amongst stakeholders leads to effective school mathematics teaching and learning. In effective schools, consistent and flawless communication is essential: mathematics learning is improved when schools, communities and families share objectives, show reciprocated respect and reliance, and connect as a team to support the welfare of learners. This suggests that parents and society should play a part in school mathematics-learning activities at various levels like governance, implementation and development.

Research has revealed that family participation is a crucial element in a learner’s improved school performance: the relationship between family and school relates to financial, cultural and educational backgrounds, and amongst learners of all ages (Henderson & Mapp, 2002). Learners
benefit from improved learner achievement, improved attendance in classrooms, conduct and social expertise. Constantino (2005) supports that children perform better in mathematics when their parents inspire and support them in their school mathematics activities.

Epstein (1995) advanced a framework for school, parents and public partnerships which incorporated six forms of envelopment for effective interdependence: communication, decision-making, learning at home, volunteering, parenting and community group effort. She further points out that participation by family and society does not automatically necessitate them devoting time in the school yard. Wagner (2002:147) maintains that the primary step leading to effective communication is distinguishing the meaning of “listening to the public and creating dialogue”. Schools and societies must link together to raise social capital which is required to assist learners and families accomplish the shared challenges. It is imperative for parents and schools to communicate on a regular basis, and clearly, about information linked to mathematics learner accomplishment.

Constantino (2005) contends that schools ought to enlighten parents about values and how they relate to the curriculum, aims and objectives, assessment methods, disciplinary codes, school programmes, and learner progress. He further puts forward numerous ways on effecting and improving communication *viz.* conferences, handbooks, email, home visits, hotlines, newsletters, open days, and voice mail. Personalised interactions as well as mobile calls are most effective for mutual understanding and communication to come to pass.

In the framework of this study, which is built on effective school mathematics teaching, a strong relationship with parents can possibly enhance learner outcomes in mathematics. Darling-Hammond (1997) endorses that mutual understanding eases cultural differences that form competing psychological motivation for learners. In divergent societies such as those under consideration in this study, family involvement includes family circumstances, choices, validity of culture and values, and unequivocally lay emphasis on the significance of family support of pupils’ learning. Henderson and Mapp (2002) maintain that all families can, and habitually do have a progressive effect on their kids’ learning of mathematics. The researcher believes that each learner in any school needs a good parental support to enhance their positivity towards their schoolwork.
3.4. THE SCHOOL EFFECTIVENESS IN THE SOUTH AFRICAN CONTEXT

The concept of school effectiveness is generally attached to learner attainment in South Africa (DoE, 2016). The deliberation on school effectiveness relates mostly to Grade 12 mathematics outcomes in secondary schools. Schools with good Grade 12 mathematics results are mostly presumed to be more effective than schools with lower mathematics outcomes. The South African understanding of school effectiveness relies on output processes and in general mathematics learner attainment. This is in agreement with Scheerens’ (2015) correlation of a system: school effectiveness points out that the input has an influence through the process and the subsequent output of the education system.

Latest studies in South African education on school effectiveness include the Ministerial Task Team Report on the implementation of Mathematics, Science and Technology (DBE, 2013), the National Effectiveness Study (Taylor, 2011), the Ministerial Report on Schools that Work (Christie et al., 2007) and the study on Standard -Based Accountability in South Africa (Taylor, 2009). The DBE (2013) report on Mathematics, Science and Technology affirms that the direst significance in South African education has to do with teachers and teaching-related matters. Taylor (2011) states that solid inner accountability should be focused in a harmonised effort on teaching and learning by the school management team, teachers and parents. He postulates that systems which boost performance in South African schools are curriculum planning, systematic use of assessment to focus on teaching and learning, time management, procurement and retrieval of textbooks.

It is evident from public spending on education in South Africa that the government is committed to an improvement in outcomes nationally and that they wish to establish equality and equity within the schooling system. Investment in education in the post-1994 period has resulted in improved access to education in South Africa generally, but has not yet provided the desired improvement in outcomes (DBE, 2010). The DBE (2010) argues that, despite the high financial investment in education, South Africa is still performing poorly in national and international studies of learner achievement. Attempts to meet concerns are reflected by the allocation of 20% of the country’s budget to education, which is the largest sector allocation of the budget (SSA, 2013).
Issues of quality are well demonstrated by South Africa’s performance in national and comparative international tests. Tests referred to are amongst others:

- Trends in International Mathematics and Science Study (TIMSS) test on mathematics and science proficiency (2011); The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) III study administered in 2007
- The United Nations Educational, Scientific and Cultural Organization (UNESCO) Monitoring Learner Assessment (MLA) tests.
- The Grade 3, 6 and 9 Systemic Evaluations
- The Annual National Assessment (ANA) test (2012)

SSA (2013) reports that South Africa achieves below international standards as measured through TIMSS, even though President Zuma indicated that there is considerable improvement in mathematics and science in his 2017 State of the Nation Address (SONA, 2017). According to Christie et al., (2007), such tests show a worrying bifurcation: the best results are achieved by historically privileged schools. There is a significant gap between these and previously disadvantaged schools.

In line with the second Millennium Development Goal (MDG), there is a desire by the Department of Education to intensify educational improvement initiatives in order to ensure effective schooling for the citizens of South Africa.

3.5. THE CONCEPT OF A TEACHER AND EFFECTIVE TEACHING

Education Law and Policy Handbook (1999) avers that a teacher is any individual who educates, or teaches, or trains the general public, or delivers professional education amenities, as well as therapy and educational psychology services at any community secondary school, further education and training institutions, adult basic education centres, and who is employed in any post-educator establishment.

The term teacher is used in this study and can synonymously be used with educator, which embraces the school principal, deputy principal as well as departmental heads. An educator is a classroom practitioner who deals with prescribed instructions to learners and whose professional
actions are to impart knowledge, attitudes as well as skills that are specified to learners registered in the school’s educational programme (Van Amelsvoort, Hendricks & Scheerens, 2000).

For the purpose of this study, a mathematics teacher is any individual who is formally employed by the School Governing Body (SGB) or the DBE to teach or educate and/or basically provide curricular and extra-curricular knowledge, attitudes, skills as well as values of mathematics to learners in a secondary school, being either post level 1, or head of the department (HOD), or deputy principal, or principal.

“We define effective teaching as that which leads to improved student achievement using outcomes that matter to their future success. Defining effective teaching is not easy. The research keeps coming back to this critical point: student progress is the yardstick by which teacher quality should be assessed. Ultimately, for a judgement about whether teaching is effective, to be seen as trustworthy, it must be checked against the progress being made by students” (Coe, Aloisi, Higgins & Major, 2014:2). In this study effective teaching is teaching that helps to achieve the teaching goals or teaching that produces desired students’ academic results.

3.6. THE CONCEPT OF TEACHER EFFECTIVENESS IN MATHEMATICS

Teacher effectiveness in mathematics was equated to effective mathematics teaching in this study. Effective teaching is unquestionably the utmost imperative goal in school mathematics teaching. In accordance with Stanford (2001), teacher effectiveness is the point at which a teacher accomplishes expected sound effects upon learners i.e. teacher effectiveness is how well and how significantly learners accomplish and exhibit commitment as well as flexibility in the face of difficulty. Generally, in relation to mathematics teaching, the paramount exercise is naturally alleged to be a teaching approach that upholds continuous learner understanding and achieves the anticipated learner outcomes (Stanford, 2001) in relation to effective mathematics teaching. Thus, “effective teachers must possess the knowledge and skills needed to attain the goals, and must be able to use that knowledge and those skills appropriately if these goals are to be achieved” (Anderson, 2004:22).

Effective mathematics teaching is not merely aligned with pupil-centredness but also with the educational accomplishments that will best convey some anticipated learning (Kyriacou,
Some instructional features are linked with effective mathematics teaching. In accordance with Protheroe (2007), the features of effective teaching should embrace learners who are dynamically involved in doing mathematics, who work on thought-provoking problems, create interdisciplinary links, converse mathematically through sharing concepts and routine manipulatives. Lovitt and Clarke (2011:1) identified the subsequent elements of what they called a “rich and balanced mathematics lesson”:

- it draws on an array of essential mathematical content;
- it is pleasing for learners;
- all learners are able to have a start, as it provides for a range of stages of accepting;
- it can be effectively carried out using a choice of approaches or methods;
- it affords an extent of choice or frankness, leading to a logic of learner ownership;
- it includes learners actively in their particular learning;
- it illustrates the approach in which mathematics can help to make sense of the world;
- it creates an appropriate as well as effective use of technology;
- it permits learners to display networks they are capable to make amongst the ideas they have learned;
- it attracts the attention of learners to main features of mathematical activity; and
- it assists teachers to resolve what specific assistance learners may need in appropriate content areas, or ways in which learners might be extended.

An analysis of the research by Fajet, Bello, Leftwich, Mesler and Shaver (2005:718) around teachers’ views of ‘good’ teaching indicate that they fall into two kinds viz, professional competence as well as affective qualities. With reference to professional competence they found the following:

- sufficient content knowledge;
- the ability to clearly convey their knowledge;
- the ability to spark interest;
- good classroom management skills;
- being fair;
- being well organised;
• encouragement of learners;
• the use of hands-on activities.

Some of the affective qualities of proficient teachers embrace the point that they are caring, enthusiastic, kind, optimistic, patient and supportive.

The use of learners’ hands-on activities is commonly connected with learner-centredness as well as effective teaching. Furner, Yahya and Duffy (2005) embrace various learning theories, with Piaget’s findings; back the notion that learners should be dynamic in the classroom. Teachers use tangible materials as a means to include learners (Furner et al., 2005). Mutodi and Ngirande (2014) explored the views of 30 mathematics teachers in Limpopo province on the use of tangible materials in constructing mathematical meaning. The findings were that 96.7% of the teachers held that the use of tangible material “bridge(s) the gap that separates how mathematics is taught and how mathematics is learned” (Mutodi & Ngirande, 2014:449). The imperative concern, though, is whether these activities with hands-on material leads to mathematical knowledge as well as effective mathematics teaching.

Effective mathematics teachers have a comprehensive array of particular teaching approaches available to them. Defining the finest plans for a lesson is a significant characteristic of an educators’ resourceful role in the classroom and every teacher has his/her own ideals to decide which classroom teaching approaches will be most effective. Researches have shown that there are various elements of an effective mathematics teacher and many of these elements are those that include learners. Posamentier and Stepelman (1999) conveyed that effective mathematics teachers cultivate their learners’ progressive assertiveness by;

- being thoughtful to their learners’ frame of mind,
- cherishing every single learner’s input,
- distinguishing learners’ quest for achievement,
- including learners in their specific learning,
- making mathematics stimulating and thought-provoking.

Seah (2007) argues that effective teaching and learning can be a role of collaborations amongst teachers and their learners, amongst learners, and among the class and its setting. As a result, effective teaching is reflected by effective learning. Effective mathematics teachers are mostly
goal-driven through various approaches like learner-discovery as well teacher-directed approaches in carrying out mathematics teachings, thereby making some mathematics teachers more effective than others (Seah, 2007). Ingvarson et al. (2004) identified four major elements that have an effect on the effectiveness of learners’ results in mathematics, viz;

- the teacher’s enabling condition i.e. experiences and professional developments of teachers;
- the school’s enabling conditions i.e. conditions in the school where the learners are situated;
- the practice of the teacher i.e. actual duty a teacher does in their respective classroom and
- the capacity of the teacher i.e. beliefs, knowledge and understanding of the teacher.

3.7. ELEMENTS OF EFFECTIVE MATHEMATICS TEACHING

Teacher’s beliefs and self-evaluation, teacher content knowledge, teacher pedagogical content knowledge (PCK), teacher knowledge of error analysis, teaching effectively, learner conditions, professional mathematics teacher development, the culture of the mathematics classroom, teaching for understanding, and school conditions are some of the elements of effective mathematics teaching discussed below:

3.7.1. Teacher’s beliefs and self-evaluation

A teacher’s belief is his/her personal opinion, viewpoint or construct that can provide an understanding of the teacher’s practice (Pajares, 1992). Therefore, a teacher’s belief about teaching and learning mathematics is the teacher’s viewpoint (philosophy) about the teaching and learning of mathematics. It is an expression of the teacher’s values and thoughts about mathematics teaching and learning. In other words, it describes an overall system that guides the teacher’s teaching decisions and classroom instructional behaviours. Beliefs are personal principles that an individual constructs from experiences and employs, often unconsciously, to interpret new experiences and information to guide action. The beliefs of teachers are dynamic in nature, undergoing change and restructuring as individuals evaluate their beliefs against their experiences in various institutions as much as in the two selected schools.
Mathematics teachers’ beliefs play a central role in their teaching (Handal & Herrington, 2003); they are stable sources of teachers’ reference in their teaching. They are built up over time, and are related to teachers’ theories of mathematics, the nature of mathematics teaching practices, their roles as teachers, and their relationships with their students. Teachers’ beliefs about teaching and learning are formulated and reformulated as they go through the stages of teacher development. The beliefs they hold are the basis for their personal knowledge about teaching and learning. Each teacher’s belief has a strong influence on his/her planning, instructional decisions and classroom behaviours. This most probably explains how and why different teachers may have different reasons for selecting a particular content, place different emphasis on the same content and have different styles of teaching.

Carter and Norwood (1997) categorized teacher beliefs about teaching and learning mathematics into three: belief about the nature of mathematics, belief about the nature of mathematics teaching, and belief about the process of learning mathematics. These three categories form the basis of mathematical philosophy in the system of beliefs toward teaching and learning mathematics.

A teacher’s belief about the nature of mathematics is his or her conception of the nature of mathematics as a whole. Such views form the basis of the philosophy of mathematics, although some teacher’s views may not have been elaborated into fully articulated philosophies. The teacher’s belief about the nature of mathematics is not necessarily a consciously held view; rather it may be an implicitly held philosophy (Ernest, 1989). A teacher’s belief about the teaching of mathematics is the teacher’s conception of the type and variety of teaching roles, actions and classroom activities associated with the teaching of mathematics.

A teachers’ belief about learning of mathematics is the teacher’s mental models or views of learning mathematics. This is closely related to the teacher’s belief about mathematics teaching. It consists of the teacher's perception of the process of learning mathematics, the behaviours and mental activities the students need to be involved in to learn mathematics, and what constitute correct and ideal learning activities (Ernest, 1989).

Research suggests that teacher’s beliefs relate to their classroom practice (Stipek, Givvin, Salmon & MacGyvers, 2001). The beliefs teachers hold about teaching and learning will
determine the teachers’ implementation or not of curriculum requirements. If teachers’ beliefs do not match curriculum demands, it is likely that they will not work in line with the requirements. Teachers’ beliefs therefore can make them to be either obstacles to or conveyers of curriculum requirements. He noted that teachers’ knowledge, beliefs and attitude explain the core of their instructional practices and hence their teaching effectiveness. Stipek et al (2001) acknowledged that, there are filters through which teachers make sense of the curriculum and influence the classroom instructional behaviour they display.

A teacher’s beliefs are influenced by the teachers’ background, subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Research by Beswick (2008) and De Leon-Carillo (2007) have documented how teachers’ beliefs about teaching and learning are influenced by the teachers’ training, experience, and professional development (both as learners as well as teachers). Beswick (2008) posits that the relationship between teachers’ beliefs and their classroom practices, though complex and indirect, is an influential and critical contributing factor to what teachers do in their classrooms.

From the above, it can be concluded that a teacher’s belief has an effect on his/her classroom instructional activities. A teacher’s belief about the nature of mathematics is the source of the teacher’s beliefs about the teaching and learning, as well as self-evaluation in mathematics.

In self-evaluation, a teacher conveys self-assessment of his/her observations. It is a procedure that empowers teachers to reflect and evaluate their own teachings in review. Therefore, the teachers evaluate their performances and rate them. A teacher’s self-evaluation is an important source of a confirmation of his/her effective teaching and it validates the teacher’s knowledge about teaching and observed effectiveness in the classroom (Darling-Hammond, 2010). One leading concern about the use of teacher’ self-rating to gauge effectiveness is the potential of the teachers’ subjective estimate of their effective teaching.

Conversely, a teacher’s self-rating of his/her effective teaching can provide support for what the teacher does in the classroom and can present a picture of the teacher’s teaching that is unobtainable from any other source. Also it gives a reasonable impression that a teacher’s self-assessment of his/her teaching should amount to something in the teaching effectiveness equations (Berk, 2005). An effective self-evaluation mechanism should be designed to try to find
teacher’s information in the areas of lesson planning and presentation, classroom methodologies, prospective teacher communication and knowledge of content in the selected secondary schools of this study.

3.7.2. Teacher content knowledge

Even and Tirosh (1995:2) posited that “the mid 1980’s marked a change in conceptions of the teacher’s role in promoting learning; which now came to include setting mathematical goals and creating classroom environments to pursue them; helping students understand the subject-matter by representing it in appropriate ways; asking questions, suggesting activities and conducting discussions”. Correspondingly in South Africa, the CAPS document of mathematics specifies that the teaching of mathematics ought to support learners to grow deep content understanding in order to mark a sense of mathematics and gain particular knowledge as well as expertise essential for the use of mathematics to solve physical, social, technological and mathematical problems. These roles call for deep subject matter knowledge (SMK) on the part of the teacher.

Teachers’ SMK has been afforded diverse conceptualisations over the preceding years. A few decades back, teachers’ SMK was explained quantitatively, for example in relation to the academic qualification, the number of courses taken at college or university, teachers’ test scores on certification tests, etc. (Even & Tirosh, 1995). In more recent years, teachers’ SMK has been defined more qualitatively with stresses on how the facts, ideas and values are connected and organised, as well as cognitive processes and understanding of facts, thoughts and main beliefs. Conceptualisation of teacher SMK in terms of the nature and scope of SMK have also received more consideration (Even & Tirosh, 1995).

The subject matter of any area of research embraces the concepts, definitions, topics, facts, procedures, organising structures, influences, representations, truths, reasons, as well as connections in and outside the area of research (Davis, 2003). Further, Davis avows that SMK is knowledge that is explicit to a set topic or content area. Leinhardt and Smith (1985:247) with reference to mathematics abstracted mathematical SMK as the knowledge of “concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curricular presentations”.

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Some researchers have theorised teachers’ SMK as incorporating knowledge and understanding of facts, ideas, and main beliefs and the means by which they are organised, as well as knowledge of the methods to create truth in the discipline (Even, 1990; Kennedy, 1990). This presupposes three features of SMK: knowledge of the organisation of the content, knowledge of the content of the subject, and knowledge of the methods of inquiry used in the subject. Kennedy (1990) presented details of each of the three features of teachers’ SMK as follows:

- Knowledge of the methods of inquiry includes knowledge of assumptions, rules of evidence, or forms of argument that are or can be employed in the discipline.
- Knowledge of the organization of the content refers to knowledge of the relationships among facts and ideas which students of the discipline have developed. The facts and ideas may not be important in their discrete isolated forms but are made important through the relationships that exist among them. The relationships among the facts and ideas form a body of knowledge such that the significance of any idea or fact is ascertained by its apparent relation to other ideas and facts.
- Knowledge of the content of the subject includes knowledge of the facts, concepts, principles or laws that have been gathered over years of inquiry into the subject.

Shulman (1986) makes a distinction between two types of understanding of SMK that teachers must have. These are ‘knowing that’ as well as ‘knowing why’. ‘Knowing that’ is the greatest elementary level of SMK. It contains declarative knowledge of concepts, algorithms, procedures and rules associated with particular mathematical topics in the school curriculum. ‘Knowing that’ is significant as a source of adequate PCK. ‘Knowing why’ is knowledge which relates to the primary meaning as well as understanding of why matters appear as they are. Although “knowing that” is unquestionably essential, it is not sufficient; “knowing why” leads to enhanced pedagogical conclusions (Even & Tirosh, 1995:3).

Shulman (1986:9) argues that “teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing and how it relates to other propositions”. Drawing from Shulman’s understanding of teacher SMK, the researcher argues that SMK incorporates the comprehension of the intellectual material plus the principles of SMK itself.
In the same way, Schwab (1978) contends that SMK is more than knowledge of concepts as well as facts. Schwab theorised SMK to involve knowledge of the fundamental and the syntactic structures of the subject matter. Knowledge of fundamental structures denotes knowledge of concepts, facts and the relationships among the concepts as well as facts in the subject domain. This knowledge of the fundamental structures is what Shulman stated as ‘knowing that’.

Knowledge of the syntactic structures is knowledge of validity principles for the guidelines of a subject domain (Ogbonnaya & Mogari, 2011). That is the ‘knowing why’ in Shulman’s conceptualisation. Teachers’ knowledge of the fundamental structures and knowledge of the syntactic structures of the subject domain are equally required for effective teaching.

3.7.3. Teacher pedagogical content knowledge (PCK)

The concept of PCK was advanced by Shulman (1986) in which he suggested the link between teacher content knowledge and pedagogy. He claimed that PCK is a combination of content knowledge as well as pedagogical knowledge that supports the change of the SMK into pedagogically advantageous practices. It is the content knowledge that transacts with teaching. Shulman regarded PCK to be different from the subject knowledge of the topic practised and the general pedagogical knowledge common to teachers through various subjects.

Although Shulman’s original notion of PCK has been criticised and revised by numerous scholars, the uniqueness of SMK in PCK has been accepted by various researchers (Van Driel, Verloop & De Vos, 1998). In the middle of PCK is the change of SMK in teaching. To various scholars, SMK is an undeniable footing for the improvement of PCK (Rohaan, Taconis & Jochems, 2010). PCK and SMK are of utmost importance for effective teaching, for strong and useful pedagogical content knowledge cannot be built on a shaky content foundation and hence it is also discussed in section 3.8 and depicted in Table 3.1.

3.7.4. Teacher knowledge of error analysis

Teachers encounter errors in the mathematics classroom almost every day. Teachers are dynamically carrying out formative assessment the minute they respond to learners’ errors in their classrooms, throughout the lesson (Black & William, 2006). Answering learners’ errors is a focused activity of formative assessment, which depends on teachers’ profound knowledge of
content, and necessitates the teacher’s professional ruling on how to reply to learners’ requirements when teaching that content. Working with learners’ errors diagnostically in perspective indicates that the “cognitive architecture” (Hugo, 2015:81) of teachers’ mathematics knowledge is solid and that their knowledge is kept in a system of “networked schemas” in their long-term memory, arranged to be carefully chosen economically, e.g. in a system of representations, principles as well as other symbolic forms.

This knowledge is “tailored to the work teachers do with curriculum materials, instruction and students” (Ball et al., 2005:16). These researches hold that teachers need expert knowledge of what they teach, an expansive wisdom of assorted approaches of teaching and, most notably, the means of clarifying and demonstrating the content they teach, with the understanding to convey it to learners of a particular age and cognitive level of growth. Shulman (1986:8) maintains that it is a “special amalgam [blend] of content and pedagogy that is uniquely the province of teachers, their own special form of professional understands”. Numerous theorists have followed Shulman’s original impression and advanced diverse sets of teachers’ knowledge of mathematics for teaching.

Explaining and prolonging Shulman’s work, Ball et al. (2008) elucidate that teachers’ knowledge entails six primary domains. Domains one and two explain the speciality of SMK (common content knowledge and specialised content knowledge). The knowledge of what is seen as a right solution, considering the age and cognitive growth of learners, is encompassed in these two domains.

The subsequent four domains explain the speciality of PCK from the perception of learners, teaching and curriculum (Ball et al., 2008). Two of the four speak of teaching SMK from the perception of curriculum loads (knowledge of curriculum, content and horizontal content knowledge). The last two discuss mediating content in the view of what learners of a particular age are able to know about the idea being taught and misconceptions arising during learning (knowledge of content as well as learning and knowledge of content and teaching). Teachers’ responsiveness to errors and their investigative activities about learners’ thinking in relative to learner errors are contained in these domains, though in a specific order.
Ball *et al.* (2008) put emphases that teachers’ responsiveness to errors and their investigative activities around learners’ thinking shaped on the first two specific domains: identifying incorrect answers is common content knowledge (CCK), while sizing up the nature of an error, particularly an unaccustomed error, naturally necessitates quickness in thinking around numbers, thoughtfulness to patterns, and malleable thinking about the meanings that are unique to specialised content knowledge (SCK). Acquaintance with common errors as well as determining which of some errors learners are most likely to make are examples of knowledge of content and students (KCS).

The inter-dependence among these domains has firm effects for the hope that teachers should toil diagnostically with pupils’ errors. Research on teaching dealing with learners’ errors endorses that teachers’ intellectual bearing is necessary for the improvement of remediation of error, without which teachers purely re-teach without the mathematical foundation of the error or its metacognitive structure (Brodie, 2001; Gagatsis & Kyriakides, 2000; Prediger, 2010).

According to Ball *et al.* (2008), teachers must judge if there is a pattern in learner errors. Teachers also must size up whether a non-standard approach would work in general. When teachers size up a learner’s error or the basis of its production, they are working diagnostically with the subject matter being imparted, for which they employ diverse “networked schemas of knowledge” of particular features of error analysis consistent to the concept or the technique they teach.

Recruiting different aspects of error analysis places simultaneous cognitive and pedagogical demands on teachers and thus makes for a challenging form of PCK. Building on the work of cognitive load theorists such as Sweller, Kirschner and Clark (2007:117) shows that, because the capacity of working memory is limited, structuring one’s knowledge along “networked schemas” and not by “tiny elements [of information] at a time” is essential. Conceptual network schemas (connections between concepts) are developed through systematic formal learning and are stored in long-term memory. They include tiny and contingent elements but because these elements are ordered in a schema, over time the conceptual frame stored in long-term memory becomes the organising tool for processing new contingent elements (Sweller *et al.*, 2007).
This is the nub of the challenge in diagnostic work of formative assessment; many aspects of a lesson can be planned and extensive preparation can be done for every lesson, but learners’ errors can raise unanticipated questions, for which teachers cannot necessarily prepare. In these situations, teachers need to make quick decisions as to how to conduct their general pedagogy in the course of a lesson and how to attend to learners’ errors specifically. Sapire et al. (2016) aver that the cognitive load of these situations, which can only be inferred, consists of the work of synthesising and making decisions about aspects drawn from curriculum knowledge (what of the actual content knowledge to focus on), curriculum coverage (how to deal with pressures such as lack of knowledge or time constraints) and pedagogical knowledge (what to anticipate in learner’s response, how to listen to learners and how and when to respond to learners).

The pedagogical load of focusing the response to a learner’s error on the germane load and controlling the extraneous load and the cognitive load attached to it are managed with more ease by teachers whose mathematics knowledge is strong, since their knowledge of mathematical errors and misconceptions is structured in networked schemas. Teachers with weak mathematics knowledge, however, experience a high extraneous cognitive load in recognising and interpreting errors, thinking about them and responding to them in the context of the engagement. These teachers may be unable to fully grasp the learner’s position. They may be hesitant or even unable to adapt their own knowledge in order to respond appropriately and are more likely to avoid dealing with errors, as might be the case with teachers in the study.

3.7.5. Teaching effectively

Teaching effectiveness or effective teaching is an elusive term amenable to several meanings and interpretations depending on the context of where it is used and who uses it. Anderson (2004) maintains that “effective teachers are those who achieve the goals which they set for themselves or which they have set for them by others”. Teaching entails the application of skills and the carrying out of appropriate activities that enable learners to develop and ultimately exhibit the expected learning behaviours. The purpose of teaching is to promote learning and the major role of a teacher is to facilitate learning.

There is an indication that there is a strong connection between effective teaching as well as learners’ academic success (Darling-Hammond in Rice, 2003). This suggests that teacher quality
is a significant factor that impacts on learning since decisions that teachers take around their teaching can also assist or inhibit learners’ learning. Coe et al. (2008) perceived that even though a combination of research studies illustrates that some teachers are a bit more effective in contributing to their learners’ learning than others, it has been a test for any research to thoroughly clarify the substantial difference in teachers’ abilities that are responsible for the inconsistency in their teaching effectiveness.

Tsang and Rowland (2005) disputed that for a teacher to be effective, he/she needs to have mastery of the essential syntactic structures of the matter which qualifies him/her to discharge the subject’s content in a way that creates meaning to the learners. Such a teacher can be seen to comprehend the subject matter and is moreover capable to present it to learners in the techniques that create a base of knowledge that the learners can build on. An effective teacher has a solid knowledge base of the subject matter content (SMC) and furthermore has a range of pedagogical approaches that he/she can choose from in order to make the lesson understandable to the learners. In a simpler form, an effective teacher needs to have an all-inclusive understanding of the subject content and a dominant pedagogical demonstration of the subject. This is in agreement with the key goal of teacher education that includes the corrective education over which SMC and instructional knowledge can be assimilated.

Bransford, Brown and Cocking (2004:188) avow that for teaching to be effective, teachers are required “to have a deep understanding of the subject matter and its structure, as well as an equally thorough understanding of the kinds of teaching activities that help students understand the subject matter in order to be capable of asking probing questions”. This suggests that mathematics teachers require in-depth understanding of the subject matter of mathematics to know how to address the topics in their classes.

Anderson (2004) posits that effective teachers are those teachers who by correct use of their list of knowledge and skills attain the teaching goals enforced on them by the established order or the goals they recognised for themselves. This implies that an effective teacher expedites the realisation of the curriculum goals in his/her class. In the same way, the South African revised National Curriculum Statement for mathematics anticipated, amongst other things, that the teaching of mathematics can aid the learners to ascertain that mathematics is an innovative part of human activity, cultivate conceptual thought in order to mark the logic of mathematics, and
attain the particular knowledge and abilities crucial for the use of mathematics on physical, shared and mathematical problems. This implies that effective teaching of mathematics will coordinate the realisation of the vision.

These interpretations display that some teacher features are necessary for effective teaching. The features are corresponding, consistent and can be clustered under knowledge of the subject; lesson preparation, organisation and presentation; effective student assessment and communication with the students. These features work collectively to benefit the teacher to achieve the curriculum learning goals (Ogbonnaya & Mogari, 2011). Effective teachers must hold the knowledge as well as skills required to accomplish the curriculum goals (standards) and must similarly practice the knowledge and skills correctly in order to realise the goals.

Goe et al. (2008:8) avow that “teacher effectiveness consists of the following:

- Effective teachers have high expectations for all students and help students learn, as measured by value-added or other test-based growth measures, or by alternative measures.
- Effective teachers contribute to positive academic, attitudinal, and social outcomes for students such as regular attendance, on-time promotion to next grade, on-time graduation, self-efficacy, and cooperative behaviour.
- Effective teachers use diverse resources to plan and structure engaging learning opportunities; monitor student progress formatively, adapting instructions as needed; and evaluate learning using multiple sources of evidence.
- Effective teachers contribute to the development of classrooms and schools that value diversity and civic-mindedness.
- Effective teachers collaborate with other teachers, administrators, parents, particularly for the success of students with special needs and those at high risk for failure”.

3.7.6. Teaching for understanding

Teaching for understanding is steering learners on the way to being capable to do different thought-provoking things with a particular mathematics topic such as finding evidence in tasks, applying, generalising, explaining and representations of mathematical concepts. The Fourth
Industrial Revolution is dependent on the capacity to think effectively with precision and power, to collaborate, to solve current and future problems, to communicate evidently with effect, to partake in civil affairs, and is dependent on the possession of an ethical core, then the schools must do their work in innovative ways. School mathematics assessment system should determine whether these goals have been achieved or are achievable.

Mathematics teachers understand that changes in the learner outcomes should be supported by parallel changes in instruction and curriculum. Unfortunately, most teachers in RSA are caught in the midst of changing curricula e.g. OBE, NCS/CAPS, etc. for which they may not have been professionally prepared. Most teachers were taught in classrooms where the role of a learner was to perform mathematical calculations using particular algorithms, to memorise data/information, and were evaluated on their ability to repeat these tasks or remember specific facts. Nonetheless, the situation still prevails in most mathematics classrooms throughout RSA.

Talbert, McLaughlin and Rowan (1993) contend that American schools have been reasonably effective in engendering basic-skills accomplishment but have not achieved well in stimulating learners’ success in tasks differently described as problem-solving, higher-order thinking, critical analysis or flexible comprehension of mathematical subject matter knowledge linked with teaching for understanding. Teaching for understanding strategies are deeply rooted in the new beliefs about teaching and learning, for example, belief system which support mathematics-as-rule-learning as opposed to mathematics-as-thinking-and-reasoning are basically different. Teaching for understanding is unlikely to take place if the teacher’s belief has not changed.

Perkins and Blythe (1994:5-6) argue that “understanding is a subtler matter, which goes beyond knowing, … understanding is being able to carry out a variety of performances that show one’s understanding of a topic and, at the same time, advance it…”. Teaching for understanding necessitates the conceptual frames of a learner in mathematics to be known so that strategies which yield change and progression can be established. The key principle in teaching for understanding is sense-making rather than annual teaching plan coverage, of which there is an extensive mathematics annual teaching plan in all grades in RSA. The rigidly imposed departmental “pace-setter” practice to cover the curriculum at all costs, in effect defies the goal of teaching mathematics for understanding.
There are four main concepts for understanding-orientated classrooms, according to Perkins and Blythe (1994) viz; generative topics, understanding goals, understanding performances and ongoing assessment. This background undertakes that teachers have a deep knowledge of mathematics as it echoes constructivist beliefs around cognition and knowledge. A different indicator of constructivist beliefs around cognition and knowledge bring into being being inquiry teaching.

Inquiry teaching of mathematics is seen as to fit with constructivist view of knowledge and learning. The inquiry demands activity, offer challenges to stimulate mathematical thinking and create opportunities for reflection on mathematical understanding (Cobb et al., 1990; Jaworski, 1994). It leads to development of conceptual, relational and principled understandings of mathematics. These suggest that through inquiry-teaching, learners can go beyond the use and application of algorithms and rules, develop understanding of general relationship in mathematics, and deal with problematic aspects of the abstract and problem-solving.

A social constructivist perspective sees discussion, negotiation and argumentation in inquiry and investigation practices to underpin knowledge growth in mathematics and its teaching (Wood, 1999). In contrast, inquiry is seen more like a tool for individual cognitive development but not explicitly addressing social structures and development more broadly. Furthermore, the social norms and processes of social enculturation are more powerful influences on learning than cognitive stimulus central to constructivist theory.

Nonetheless, inquiry is a tool that promotes critical alignment with modes of practice and corresponding development practice. Therefore, teachers and learners engage with each other in different degrees of various forms of inquiry practice. In constructivist terms, inquiry teaching can be viewed to stimulate accommodation of meanings central to individual growth.

From the above discussions, inquiry teaching and learning can be addressed in three forms viz,

- Inquiry in mathematics: learners in school learning mathematics through exploration in tasks and problem solving in classrooms; teachers using inquiry as a tool to promote students’ learning of mathematics.
• Inquiry in teaching mathematics: teachers using inquiry to explore the design and implementation of tasks, problems and activity in classrooms; educators using inquiry as a tool to capacitate teachers to develop teaching.

• Inquiry in research which results in developing the teaching of mathematics: teachers researching the processes of using inquiry in mathematics and in teaching mathematics.

Wiggins (1993) argue that it is inadequate to embrace ground-breaking teaching techniques if the work does not permit or expect learners to use their minds soundly or if the work has no value or meaning to learners beyond accomplishing success in school mathematics. It should lean towards sustaining hard work, and it should cultivate high order thinking as well as problem-solving in mathematics.

3.7.7. Learner conditions

Learner factors denote what the learners bring to the class. It embraces the nature of the societal upbringing of the learners - their attitudes, goals, interests and proficiency level, motivation, their prior knowledge, beliefs and dispositions they carry into class with them (Gallagher, 2004). These can inspire learners’ classroom relations and consequently impact on the teachers’ effectiveness. Teaching involves the procedure of learners’ scores in a standardised test to quantify the effect of teachers’ teaching on the learners. For some time, learners’ academic performance in both external and internal examinations had been utilised to assess teachers’ effectiveness (Gallagher, 2004). Unfortunately, grade 12 learner attainments have been used as a yardstick to assess school effectiveness and teacher’s effectiveness in recent times throughout South Africa.

An effective teacher in accordance with Goe et al. (2008), is one that yields anticipated learners’ academic outcomes. Learners’ examination scores are habitually used as a measure of educational output for the reason that learners’ test scores have been revealed to be certainly associated with their secondary school pass rate, future job prospects as well as adult earnings (Currie, 2001; Goe et al., 2008). Teachers have a major influence on learners’ academic success for it is teachers that are in due course answerable for carrying out educational policy as well as
curriculum intents into learning prospects for the learners. Learners’ average scores can well reflect the outcome of a teacher’s effective teaching. As a result, an accepted measure of teacher effectiveness might artificially be the typical attainment test scores of his/her learners. Their investigation pointed out that learners’ test scores can be used as a benchmark for teacher effectiveness.

Learners’ appraisal of teachers is one of the most commonly used means for assessing teacher teaching effectiveness (Currie, 2001). The use of learners’ rating of teachers’ teaching as a way of assessing effective teaching holds that the most suitable principle for determining effective teaching is the extent of learners’ learning that happens in class. In accordance with research, there are regularly high correlations amongst learners’ assessments of the amount learned in a subject matter and their overall assessments of the teacher (Theall & Franklin in Aiken, Hawley & Vanjani, 2007).

The reliability and validity of learners’ evaluation as a degree of teaching performance have been largely sustained by researchers (Greenwald, 1996). The review of literature by Prebble et al. (2004) on the bearing of learner valuation of teaching on teaching value confirms that learners’ valuations of teaching are amongst the most accessible and reliable pointers of teacher’s effective teaching. Learners are the most probable basis to assess the extent to which teaching is informative, productive and meaningful. Zabaleta (2007) specified that student evaluations of teaching become a measure of the assessment structure in higher education, and outcomes from learner assessments of effective teaching have been employed to make significant findings in higher education.

3.7.8. Professional mathematics teacher development

The literature casts a wide net for what might be incorporated as professional teacher development (PTD) (Faulkner & Cain, 2013). The term “professional teacher development” means an all-inclusive, intensive, and sustained approach to improving teachers’ effectiveness in increasing learner achievement (Faulkner & Cain, 2013:116). The meaning distinguishes that teacher development can be delivered in numerous ways, extending from the formal to the informal. It can be accessible through exterior knowledge in the form of courses, workshops or formal criterion programmes, through partnership amongst schools or teachers through schools
or in the schools in which teachers are employed. Lastly, improvement can be offered through mentoring, shared planning and teaching, and the sharing of good practices.

In this way, the absence of acceptable PTD prospects for some teachers is not an obstacle to learners’ academic success in mathematics. It was conveyed that knowledgeable teachers do not have sufficient opportunities to advance their knowledge and talents, and that in-service training prospects for teachers are rated poorly (US Department of Education, 2000). The statement cites the subsequent problems about the value of teachers’ in-service training: it remains basically short-term as well as non-shared, it is often not linked to the teachers’ desires and to the challenges encountered by learners, and it only provides preparation opportunities that take place commonly for less than eight hours. Education reformers are paying substantial consideration to the part that effective PTD can offer in improving teaching of mathematics. There seems to be consent on the key elements of high-quality professional development; they are:

- “deepening subject matter knowledge specifically for teaching, including understanding how students learn and the specific difficulties they may encounter in mastering key concepts;
- allowing sufficient time for significant learning;
- focusing on what teachers are being asked to do and building on what teachers already know and are able to do;
- actively engaging educators, rather than just listening to a lecture or watching a demonstration;
- encouraging educators to learn together, enabling them to support each other in using what they have learned” (National Academy of Education, 2009:6).

3.7.9. The culture of the mathematics classroom

Various researches on the culture of the mathematics classroom have established that the learner’s role and activities hinge mainly on the interpretation of the mathematics syllabus by the teacher (Ingvarson et al, 2004; Mecer, 2006). Nickson (1992:110) resolved that “the linearity and formality associated with most teaching of mathematics from published schemes or textbooks tend to produce a passive acceptance of mathematics in the abstract, with little connection being made by pupils between their work and real life. Pupils accept the visibility of mathematics in
terms of a ‘right or wrong’ nature, and their main concerns seem to be with the quantity of mathematics done and its correctness…. When beliefs about mathematics differ and where views of mathematics as socially constructed knowledge prevail, pupils take on quite a different role. The messages they receive are that they are expected to contribute their own ideas, to try their own solutions, and even to challenge the teacher ….”. This linearity and formality is followed through annual teaching plans in RSA or even in other countries. Whether it is too rigid or flexible remains a question to be answered.

Teachers with “an integrated, conceptual understanding” of mathematics incline to organise their classrooms as well as learning activities that inspire learners to take part and work together with the abstract features of mathematics (Nickson, 1992:110). Moreover, the depth of the mathematics taught relates highly with the depth of the mathematical knowledge of teachers. Courageous and compassionate teachers are more effective than critical teachers (Ingvarson et al., 2004).

Teachers maintain learner commitment in doing mathematics at a higher degree if they

a) ask students consistently to provide meaningful explanations of their work and reasoning,
b) support proactively the student’s activity,
c) select appropriate tasks for the student,
d) push students consistently to make meaningful connections, and
e) do not reduce the complexity/cognitive demands of the task (Henningsen & Stein, 1997:526).

Learner commitment in mathematical activities drops if teachers

a) remove the challenging aspects of the tasks,
b) shift the students’ focus from understanding to either the correctness or completeness of an answer, or
c) do not allow an appropriate amount of time for students to complete the task (Henningsen & Stein, 1997:528).

A crucial role of the teacher is to structure “a pervasive norm in the classroom that helping one’s peers to learn is not a marginal activity but is a central element of students’ roles” (Slavin, 1989: 66)
In a review of 80 studies on groups in mathematics classrooms, Davidson (1985) resolved that learners working in small groups considerably outscored learners working independently in more than 40 percent of the research. Learners working independently in mathematics classroom achieved better in only two of the studies (and Davidson put forward that these studies were defective in design).

Learners working on mathematics problems in small groups display cognitive behaviours and processes that are basically parallel to those of knowledgeable mathematical problem-solvers (Artz & Armour-Thomas, 1992). When learners are in secondary schools, the research data is not pure, as these learners show stronger specific drives, interact generally in more complex means, and habitually are defensive or uncomfortable around their knowledge as well as learning in mathematics (Steen, 1999).

The research results on the influence of cooperative learning in mathematics classrooms are reasonably reliable (Leiken & Zaslavasky, 1999):

- Learners with different ability levels become more involved in task-related interactions.
- Learners’ attitudes toward school and mathematics become more positive.
- Learners often improve their problem-solving abilities.
- Learners develop better mathematical understanding.
- The effects on learners’ mathematics achievement have been positive, negative, and neutral.

Learners working in cooperative groups outperform individuals competing against each other. Teachers can make the most of mathematical learning in a small group setting by engaging learners in learning activities that encourage “questioning, elaboration, explanation, and other verbalizations in which they can express their ideas and through which the group members can give and receive feedback” (Slavin, 1989:8).

Learners solving mathematical problems in small groups raise three features that improve the individual learner’s cognitive (re)organisation of mathematics:

- The learner experiences “challenge and disbelief” on the part of the other members of the group, which forces them to examine their own beliefs and strategies closely.
The group collectively provides background information, skills, and connections that a single learner may not have or understand.

The learner might internalise some of the group’s problem-solving approaches and make them part of their personal approach (Noddings, 1985:58).

Teachers trying to form and sustain mathematical dialogue among learners must create a setting in which learners build an individualised relationship with mathematics (D’Ambrosio, 1995). Three main elements need to be in this setting:

- Learners need to engage in authentic mathematical inquiries.
- Learners must act like mathematicians as they explore ideas and concepts.
- Learners need to negotiate the meanings of, and the connections among, these mathematical ideas with other learners in the class (D’Ambrosio, 1995:770).

Numerous factors influence learner engagement to a degree essential to ensure quality mathematics. The key factors are: high-quality tasks that build on learners’ prior knowledge of mathematics; an effective platform on the teachers’ part; a suitable amount of time to engage in the mathematics; teacher as well as learner modelling of high performance activities; and a sustained determination by the teacher to inquire about clarifications and sense.

Teachers must do more than asking questions in a mathematics classroom, as the cognitive level of the questions which are asked is very important. Nonetheless, the research regarding the questioning style of teachers is quite consistent:

- 80 percentages of the questions asked by mathematics teachers were at a low cognitive level (Suydam, 1985:6).
- There were about five times more interactions at low cognitive levels than at high cognitive levels during each school day (Hart, 1989:243).
- Low cognitive level interactions occurred about 5.3 times more often than high cognitive level interactions.
- Of an average of 64.1 interactions in a 50-minute class period, 50.3 were low-level cognitive interactions, 1.0 involved high-level cognitive interactions, and the remaining interactions were not related to mathematics.
Learners are less likely to correct their specific errors because of either reluctance or an incompetence to search for errors. Most learners are “just too thankful to have an answer, any answer, to even dare to investigate further” (Hart, 1989:243).

3.7.10. School conditions

School conditions includes the immediate working conditions in the school, like the provision of teaching resources, the professional work community and also the educational policies, the curriculum, time allocated for teaching and any other legislation under which the school is operating. Teacher effectiveness can be affected by the conditions within the school (Ingvarson et al., 2004). The quality of teaching is in turn affected by a wide variety of conditions at the school level. Workplace conditions can exert a powerful influence over the quality of teaching in two main ways:

- when they help to attract and retain quality people into the profession; and
- when they energise teachers and reward their accomplishments (Darling-Hammond, 2000).

Schools differ in their capacity to ensure that all mathematics teachers are well qualified in mathematics and with specific training in the teaching of mathematics. The study by Peterson, McArthy and Elmore (1996) doubted the capacity of innovative management structures to advantage classroom practice. They further posited that teaching is the most crucial factor in effectiveness, whilst school structures can offer opportunities and the structures do not directly add to this teaching.

Leadership

Hill, Rowe, Holmes-Smith and Russell (1996) have made known, in the Victorian Quality Schools Project, that 26.5% of in-school variant in learner achievement was due to leadership backing for teachers’ practices. They remarked that: “leadership support is overwhelmingly important in establishing a positive working environment for teachers … [it] has significant direct and/or indirect effects on all measured aspects of teachers’ affect and perceptions of their work environment” (Hill et al., 1996:578). Despite the fact that leadership is central in schools, it is perhaps limited in the level to which it can impact teaching practices in classrooms due to the
fact that teachers largely do their individual work in the classroom, away from the undeviating control of the school management.

*Program coherence*

Program coherence is a quantity of integration of the various components in the school as an establishment. Newman, King and Youngs (2000) argue the idea of ‘program coherence’ as a measure of school capacity. They avowed that it is “the extent to which the school’s programs for student and staff learning are coordinated, focused on clear learning goals, and sustained over a period of time”. Teacher professional development is associated with the attainment of the school’s objectives for improved learner outcomes.

*Resources*

A meta-analysis of sixty primary research studies investigating a variety of school contributions and their influence on learner achievement was piloted by Greenwald et al (1996). The amounts spent on learners were found to be intensely correlated to learner achievement, as was also small schools or else small classrooms.

*School characteristics*

Identifying demographic factors of schools is important to understand which features influence variation in teacher effectiveness. These factors may consist of the language background of the learners, the socio-economic standing of the school catchment zone, the ratio of indigenous learners, and the geographic locality of the school (Potberg, 2014). The researcher summarised the discussed elements of effective mathematics teaching in Table 3.2 below as conceptualised in the study.
Table 3.2 Elements of effective mathematics teaching

<table>
<thead>
<tr>
<th>Elements</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher’s beliefs and self-evaluation</td>
<td>Personal opinions, viewpoint or construct; values and thoughts; instructional behaviours and self-assessment and self-rating.</td>
</tr>
<tr>
<td>Teacher content knowledge</td>
<td>Deep content understanding; knowledge of facts, ideas and methods; knowing ‘that’ and ‘why’ and diverse conceptualisation.</td>
</tr>
<tr>
<td>Teacher pedagogical content knowledge (PCK)</td>
<td>Integrated mathematics instructions; knowledge of learners and their behaviour; knowledge of curriculum and educational contexts.</td>
</tr>
<tr>
<td>Teacher knowledge of error analysis</td>
<td>Acknowledging learner errors; clarification and demonstration of content; knowledge of right solutions; responsiveness to errors and investigative activities.</td>
</tr>
<tr>
<td>Teaching effectively</td>
<td>Creation of meaning to learners; all-inclusive understanding of a subject; dominant pedagogical demonstration; in-depth understanding of subject matter and realisation of curriculum goals; high expectations for all learners; positive outcomes (e.g. academic, social, etc.); use of diverse resources and effective teacher collaboration.</td>
</tr>
<tr>
<td>Teaching for understanding</td>
<td>Generative topics; understanding goals; guidelines; ongoing assessment; performances of understanding</td>
</tr>
<tr>
<td>Learner conditions</td>
<td>Learner attitude and motivation prior knowledge and proficiency level; beliefs and dispositions</td>
</tr>
<tr>
<td>Professional mathematics teacher development</td>
<td>All-inclusive and intensive improvement; mentoring and sustained approach and shared good practices.</td>
</tr>
<tr>
<td>The culture of the mathematics classroom</td>
<td>Proper organisation and inspiring activities; consistent meaningful explanations; mathematical dialogue and high-quality task; exploring ideas and concepts.</td>
</tr>
<tr>
<td>School conditions</td>
<td>Leadership; programme coherence; resources and school characteristics.</td>
</tr>
</tbody>
</table>
3.8. MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

Mathematical knowledge for teaching (MKT) entails more than knowing mathematics for oneself. Knowing classroom practice means knowing what is to be taught and to plan, conduct, and assess effective lessons on that mathematical context. Teachers certainly need to be able to understand mathematical concepts correctly and perform procedures accurately (Ball, Bass, Hill, Sleep, Phelps & Thames, 2006). They must be able to understand the conceptual foundations of that mathematical knowledge. They must understand mathematics in ways that allow them to explain and unpack ideas in ways not needed in ordinary adult life. The mathematical sensibilities they hold matter in guiding their decisions and interpretations of learners’ mathematical effort.

Recent research in the Fourth Industrial Revolution context focuses on MKT as the intersection of three knowledges:

- Content knowledge
- Pedagogical content knowledge
- Technological content knowledge

This overarching knowledge construct as the intersection and interconnection of Technology, Pedagogy and Content Knowledge is known as TPACK; the total knowledge package for teaching subject matter content with technology (Niess, 2005; Mishra & Koehler, 2006). Fundamentally, these researchers defined TPACK as the knowledge that teachers need to teach with and about technology in their assigned subject areas and grade levels. The knowledge of incorporating technology in teaching mathematics as the knowledge and beliefs teachers demonstrate are consistent with Figure 3.1.

CCK and SCK as well as horizon content knowledge are subject matter knowledge whereas KCS, KCT and curricular knowledge are pedagogical content knowledge (Ball et al., 2006). CCK denotes mathematical knowledge and skills held by any well-educated adult e.g. 99-35. SCK denotes mathematical knowledge and skills used by teachers in their job but not ordinarily possessed by well-educated adults e.g. knowing different procedures of calculating 99-35. Horizon content knowledge is an awareness of how mathematical topics are related over the
span/area of mathematics included in the curriculum e.g. a lower grade teacher needs to know the relationship of the mathematics he/she teaches to the one in the succeeding grade(s). It clearly suggests that sampled teachers in this study should be well-versed with preceeding and succeeding grades’ content.

KCS relates to knowledge about both mathematics and students e.g. recognizing why students might give the answer 63 to 75-18. KCT relates to knowledge about mathematics and teaching e.g. knowing instructional advantages of different representations of 75-18. Curricular knowledge is the wide range of programmes designed for the teaching of mathematics and topics at a certain level, various LTSM and a set of features that are indicators and contra-indicators for the use of particular curricular or LTSM in certain conditions. Shulman (1986) also suggested two other dimensions which fall under curricular knowledge viz. lateral and vertical curricular knowledge.

Lateral curricular knowledge refers to the knowledge of mathematics curriculum being imparted to the curriculum that learners are learning in other classes i.e. integration of subjects. Vertical curricular knowledge embraces understanding of the topics and issues that have been and will be taught in mathematics during the preceding and succeeding years in the school and LTSM that represent them. PCK explores knowledge about mathematical explanations, tasks and activities, styles of teaching and learning as demonstrated by the activities chosen or explanations given (Ball et al., 2006). SMK is the knowledge about mathematical structures, body of concepts, facts, skills, definitions, methods of justification which offer results on the way teachers hold that knowledge (Hill et al., 2007).

From the above argument, do teachers have TPACK or not? Niess, Sadri and Lee (2007) proposed a developmental model for TPACK based on Rogers’ (1995) five-stage process by which a person makes a decision to adopt or reject a new innovation.

- Recognising knowledge – teachers are able to use technology and recognise the alignment of the technology with mathematics content, yet are not willing to integrate technology into teaching mathematics in classrooms.
- Accepting persuasion - teachers may attempt to engage their learners in learning mathematics with appropriate technology as part of the process of determining if they
have a favourable or unfavourable disposition toward incorporating technology in their classrooms.

- Adapting decision – teachers engage their learners in teaching and learning activities of mathematics with an appropriate technology.
- Exploring implementation – teachers actively integrate teaching and learning mathematics with an appropriate technology.
- Advancing confirmation – teachers evaluate the results of the decision to integrate teaching and learning with an appropriate technology.

The key challenge is for mathematics teacher to design, implement and evaluate new teacher preparation programmes that provide experiences that support the development of the knowledge, skills and dispositions in TPACK for teaching mathematics. How mathematics teachers learn, with whom they learn, and the context in which they learn are key to what they learn.

Primarily, mathematical knowledge for teaching (MKT) is entailed in the four domains/dimensions as maintained by Hill, Ball and Schilling (2008) in the Figure 3.1.
3.9. IDENTIFICATION OF MATHEMATICS DIFFICULTIES IN THE LEARNERS

Croark, Mehaffie and Greenberg (2007) and Joiner (1978) have evidently documented that the prompt identification of learners who experience challenges in learning is of critical significance in enabling such children not only to make more progress but also to become participating supporters of society. It is imperative to identify learners who experience problems to learning as quickly as possible so that they can be attended to, to eliminate or minimise learning challenges. Nonetheless, these learners stand an enhanced chance of success if the challenges can be addressed.

Steele (2004) further backs the view that timely identification of learning mathematics challenges offers a foundation for future learning as well as academic success practices for learners at risk. He also asserts that if learners who experienced challenges to learning are recognised in early childhood, they will have a better chance to advance to their actual potential.
These learners will have a better chance of not developing secondary challenges, like frustration as well as anxiety.

Learners having secondary challenges may drop out of school, exhibit behavioural problems and develop bigger academic deficiencies. Furthermore, early identification of learning challenges may increase educational prospects and outcomes for all learners with complex necessities or certain patterns of challenge in learning in the normal school system. Though on a daily basis teachers are confronted with learners who fail or drop out of school; this does not mean that these learners are not capable of doing something. They may also experience challenges in spatial arrangement when copying or writing. The learner’s letters or numbers may look distorted or rotated (Pierangelo, 1994).

*Mathematics: a universal challenge for learners*

The challenge of poor performance in mathematics is not only felt in South Africa, it is universal (Reddy, 2003). Australians (Van Kraayenoord & Elkins, 1998) tried to address the challenge by identifying certain factors that contribute to poor mathematics performance, *viz.*: teaching method (whole class teaching); failure to use knowledge associated with mathematics; language; lack of flexibility; beliefs; and quality of educator-child interaction. Brophy and Good (1986) also affirmed that poor performance in mathematics has always been linked with whole class teaching.

The opinion of this researcher is in agreement with the above-mentioned scholars. Learners do not learn in the same way when teaching the whole class at the same time. Learners who grasp quicker may benefit, while learners who do not grasp quickly will be disadvantaged. Brophy and Good (1986) maintain that in different cases, poor mathematics performance has also been related with whole class teaching. The teacher who uses the whole class teaching process may not be capable of interacting with all the learners at the same time. In that manner, challenges experienced by some learners are not timeously detected and remedied (Brophy & Good, 1986).

*Other factors*

A number of other factors could contribute to the poor performance of learners in mathematics. Brophy & Good (1986) pointed out that a number of factors impact on learners’ academic
performance. One such factor is the social pressure put on the learner by fellow learners in the school surroundings. Van der Westhuizen, Mosoge, Nieuwoudt & Steyn (2002) avow that lack of learner commitment and discipline is directly associated with poor learner performance. Learners with poor behaviour (such as paying no attention to instructions by the teacher, failing to do and/or finish work given, displaying disrespect to the teacher) are inclined to spend more time being reproached, or else outside the classroom. As a consequence, the contact time of authentic teaching as well as learning is lessened (Van der Westhuizen et al, 2002). Inadequate LTSM is also another issue that can impact on poor performance in schools.

Mathematics remains abstract without any LTSM to boost the understanding and use of knowledge (Makgato & Mji, 2006). There are still South African schools which are inadequately supplied with resources (e.g. textbooks); as a result, in some schools learners are not given a textbook to take home.

Various misconceptions about mathematics teaching

Schunk (2004) avows that the content area of mathematics is a good ground for cognitive as well as constructive research. He further complements that topics that have been investigated include how learners build mathematical knowledge, how specialists contrast with beginners, and which techniques are more effective. Nonetheless, learners and grown-ups often construct procedures to solve mathematical problems; nonetheless, the errors are unplanned but rather methodical mistakes. Methodical mistakes reveal the constructivist suppositions that learners form methods based on analysis of their experiences. Most learners make mistakes especially in operation signs e.g. a common mistake in subtraction is to subtract the smaller number from the bigger number in each column, irrespective of direction, as follows:

\[ x - y = 7 \] \[ 2x + 3y = -4 \]

\[ (ii) - (i) : x - 4y = 3 \]

Methodical mistakes develop when learners come across new problems and inaccurately generalise productions, particularly when they are unclear what to do. They usually modify the instructions to suit the new problem (Schunk, 2004). Brown and Burton (1978) similarly left a
gap on how teachers teach mathematics. This is a teacher challenge in teaching mathematics effectively and is precisely what the researcher in this study wishes to investigate.

*Language issues in the teaching of mathematics*

Naude, Pretorius and Vandeyar (2002) maintain that many Grade 1 learners enter South African schools having different academic and learning challenges that might happen as a result of narrow language proficiency. A learner with narrow language proficiency may carry learning and understanding at a slower rate. This elucidates why Grade 3 learners who receive tuition in a language other than their home language find it difficult to comprehend numerical concepts. It indicates that they also struggle to be experts in the medium of instruction itself as language and thought are interlinked. Narrow language proficiency leads to learning challenges. A learner has to be proficient in expressive and accessible languages to command understanding and put forward academic tasks as well as mathematics (Naude *et al.*, 2002). Learners should also be capable to commit what they learn to memory as well as be able to replicate it when required. It is therefore clear that lack of language proficiency would be a barrier for learners, especially in mathematics.

Mercer (2006) postulates that the trend of mathematics teachers and policy makers to put emphasis on the distinction between the mathematics subject language and casual talk can hamper the process of inducting learners into mathematics practices. The researcher believes group activities provide valuable opportunities for learners to construct answers for themselves through the discourse which would not be established in whole class teaching. This way encourages learners to participate positively in getting solutions to problems and at the same time utilising language to converse. In so doing, learners are able to comprehend better and assess mathematics exercises as routine problems rather than something simply associated with the school environment as well as a special language.

Vygotsky (1978) put emphasis on the impact of language as a cultural and psychological means (Mercer, 2006). He further acknowledged that the social contribution in problem-solving activities created an imperative factor for personal development. He asserts that inter-mental (social) activity, facilitated through language, can stimulate intra-mental (personal) intellectual development. Using language and illustrations with which the learner can relate and those that
comes from his/her close environment, will bring about better understanding and support the learner to relate the practice in his/her own words. The learners’ capability will also progress.

There are two means of collaboration through which the verbal language can be correlated to the learning of school mathematics. The first is teacher-led interaction with learners; the teacher directs the learners in their progress and understanding, which can be essential in the learners’ orientation into discourses, linked with certain knowledge domains (Alexander, 2000). “It concerns more subtle aspects of interaction such as the extent to which teachers elicit children’s own ideas about the work they are engaged in, make clear to them to discuss errors and misunderstandings and engage them in extended sequences of dialogue about such matters” (Mercer, 2006:509). “Dialogic” strategies, according to Mercer (2006:510), achieved better learning outcomes.

Alexander (2004) advocates that dialogic teaching is a way that uses the power of talk to inspire and expand the learners’ thoughts and improve their learning with understanding. It takes in both the teacher and the learners, and relates to teaching through the curriculum. It is a method that is based on the principles of cognition, reciprocity, cognition and observation. Dialogical teaching, therefore, requires learners to be actively involved in doing and discussing or explaining what the lesson is about. This strategy as such, can be very helpful in the teaching of mathematics where learners have to handle actual objects and clarify what they see, and also learn the concepts.

**Dialogic teaching is realised by certain characteristics of classroom interaction, such as:**

- questions are structured so as to raise thoughtful answers.
- answers incite further questions and are seen as the building blocks of dialogue.
- individual teacher-learner and learner-learner exchanges are attached to coherent lines of investigation rather than left deserted and disconnected (Alexander, 2004). In this way, the learners experience the learning process as a cooperative means.

Peer group interaction is the second context in which spoken language can be associated with learning of mathematics. Learners come to be involved in interactions that are more symmetrical than those of teacher-learner discourse and have diverse kinds of opportunities for expressing
reasoned opinions and describing procedures when working in groups or pairs. A learner does not merely learn mathematics, but social interaction too. Teachers can help learners to gain appropriate knowledge of numerical techniques, terms, concepts and processes. They can also assist learners to learn how to use language to work effectively and to cooperatively enquire, reason, reflect on information, share and exchange ideas and to make united decisions. This kind of guidance is not usually offered.

Some learners suffer from dyscalculia, as asserted by Vaidya (2008). This is mostly due to poor understanding of the number concept and the number system as compared to their age group. Such learners experience challenges in counting, learning concepts of time, direction learning, and recollecting facts, arrangement of past and future happenings, and giving and accepting change. They are further unable to use rules and guidelines to build on acknowledged facts. For instance, they may know that 3+1= 4, but would fail to deduce that 3- (-1) = 4 and confuse 7- 4 = 3 with -4 - 7= -11. Such learners are commonly believed to have difficulties in the learning of mathematics. Devoid of identification and remediation, these learners would be unable to be numerically efficient. Most of the challenges seen in mathematics are as a result of underdevelopment of mathematics language, especially where the medium of instruction, i.e. English, is a second additional language or not a home language, as in the selected schools in this study.

The syntax, terminology, and the translation from English to mathematics language, and from mathematics language to English need to be openly and purposefully taught (Vaidya, 2008). Subsequently, mathematics language can pose challenges for learners. For a teacher to get through to his/her learners, he/she should be knowledgeable about the mathematics language. An added challenge is that some mathematical terms like “rhombus, sine, cosine, perimeter, etc.” are not found in daily conversations (Vaidya, 2008).

*Teacher development for addressing mathematics*

Middleton and Goepfert (2002) advocate that the ways and means that are used to capacitate teachers need to be reconsidered. In a traditional mathematics class, the teacher starts with correcting home-works, followed by the introduction of new concepts, predominantly with the teachers as lecturers, and the learners paying attention, sometimes taking notes. At the
conclusion of the lesson, learners are given or referred to a textbook for practice questions, and then in the final 15 to 20 minutes, learners continue doing a set of questions from the textbook on their own. This channel of learning does not stimulate the learners. Middleton and Goepfert (2002), Swars (2004) and Crawford and Witte (1999) opined this notion that active learning in an interesting context is the basis on which constructivist teachers structure their teaching stratagems and classroom environment, such as:

- In these classrooms, learners are more likely to play a part in hands-on activities than to pay attention to the teacher.
- They would rather discuss their solution approaches with other learners than ask the teacher to give them the right answer.
- They would rather work cooperatively in small groups as they figure out and reformulate their ideas than mutely practising mathematics procedures.
- Desks are not lined up, but learners are set in groups as these inspire active learning and learner interaction.

To add to the ideals of the scholars, each teacher should do a reflection of the lesson taught to gauge its success so as to improve his/her teaching hence better learning for improved outcomes.

**3.11. SUMMARY AND CONCLUSION**

This chapter presented a literature review of school effectiveness and effective mathematics teaching in general. The chapter addressed the question of what school effectiveness and effective mathematics teaching are, and defined related concepts. Furthermore, six elements of effective schools were discussed in detail including, amongst others, effective teaching and improvement, high expectations and standards, supportive learning environment, effective monitoring and leadership, clear and shared focus/vision, and communication and group effort.

This chapter also discussed elements of effective mathematics teaching with reference to teacher’s beliefs and self-evaluation, teacher content knowledge, PCK, knowledge of error analysis, teaching effectively, learner conditions, professional mathematics teacher development, culture of mathematics classroom, and school conditions. The researcher further looked at
literature on the identification of mathematics difficulties in the learners. These are some possible determinants which this study focuses on.

The next chapter presents how the data was collected as well as the research design. The data collection instruments and procedure were discussed. The chapter also included a discussion of the ethical issues of the study.
4.1. INTRODUCTION

In the previous chapters 2 and 3, literature relevant to the research was reviewed. Chapter 4 offers awareness into the research design and the research method that will be utilized in gathering and analysing the data to answer the research questions. The research paradigm, sampling, site selection, the participants, research methodology, data analysis and ethical issues are discussed at length. The outline of the data collection is also provided diagrammatically.

4.2. RESEARCH PARADIGM

The research is a case study on school effectiveness and effective mathematics teaching in two selected secondary schools in NMM district. It is towards the development of a model which may assist in the improvement of learner performance in mathematics. Case studies are constructed to richly describe, explain, or assess and evaluate a phenomenon e.g., event, person, program (Gall, Borg & Gall, 1996). However, a case study is an in-depth investigation of a specific research problem rather than a comprehensive statistical survey or comprehensive comparative investigation (Yin, 2003; Creswell & Creswell, 2018).

The features of case-study investigation as expressed by Robson (2002) precisely sum up the researcher’s reasons to decide on the case-study approach for this research:

- It is an approach rather than a technique.
- It is comprehensive of research in an all-encompassing sense.
- It is dependent on realistic evidence.
- It is specific about a particular case.
- It centres on the occurrence in context.
- It triangulates data from various sources.

A researcher using a case study as paradigm can apply different methodologies and rely on various sources to investigate a research problem (Greenhalgh, 2015; Stake, 1995).
Nieuwenshuis (2010) asserts that it is constructed on naturalistic methodologies that strive for understanding the phenomenon in a setting. The settings in this study are the two selected secondary schools in NNM district of the North West Province. The reader must never confuse this study as qualitative but rather get it clear that case study as explained above is used as a paradigm.

Pring (2004:48) noted that “the distinctions within the so-called paradigms are often as significant as the distinctions between them”. This study used both quantitative and qualitative approaches called mixed methods research. The end result of mixed methods research is findings that may be more dependable and provide a more complete explanation of the research problem than either method alone could provide.

4.3. RESEARCH DESIGN

Yin (2003) avers that a research design is a blueprint or a comprehensive strategy of the method in which one anticipates guiding an enquiry. According to Yin, (2003) a design is used to organise the enquiry and to illustrate how all of the key parts of the research venture (the samples or groups, measures, programmes and methods of the project), combine in an effort to address the main research questions. McMillan and Schumacher (2006) confirm that the goal of a sound research design is to provide findings that are deemed to be credible. Maree (2010) further maintains that a research design is an outlook which moves from the simple theoretic conjecture by insisting on the range of assistants, information accumulating processes to be made use of and information enquiry in the picture.

Nieuwenshuis (2010) explains that the selection of research design is constructed on the suppositions of the scholar, skills as well as observations and moreover, it impacts the mode in which information is composed. The researcher concurs with these definitions and goals of research design as addressed in the literature above and for this reason, selected the mixed methods study research design.

Creswell (2015:2) describes a mixed methods research design as “an approach to research in the social, behavioural, and health sciences in which the investigator gathers both quantitative (closed-ended) and qualitative (open-ended) data, integrates the two, and then draws interpretations based on the combined strengths of both sets of data to understand research
problems”. The quantitative data will come mainly from mathematics learners as well as teachers, and the qualitative data from the learners, teachers and mathematics HODs. The combined methods offer an enhanced understanding of the problem more than from any of the data single-handedly.

The researcher in this study used one of the basic mixed methods design namely, the explanatory sequential design (Creswell, 2015). There are three kinds of the basic mixed methods design. The three methods are the convergent design, explanatory design and exploratory design. The intent of the explanatory sequential design is to start with a quantitative phase followed by qualitative phase to elucidate quantitative outcomes. Creswell (2015:38) indicates that a researcher needs to follow these procedures in order to “conduct an explanatory sequential design:

- Collect and analyse quantitative data in the first phase.
- Examine the results of the quantitative analysis to determine (a) what results will need further exploration in the second, qualitative phase and (b) what questions to ask participants in this qualitative phase.
- Conduct qualitative data collection and analysis in a second phase to help explain the quantitative results.
- Draw inferences about how the qualitative results help to explain the quantitative results”.

Figure 4.1 offers a simple diagram of the actions in the two-strand explanatory sequential design.

**Figure 4.1 Explanatory sequential designs (Adapted from Creswell, 2015:39)**

<table>
<thead>
<tr>
<th>Phase 1</th>
<th>Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitative Data Collection and Analysis</td>
<td>Explained/confirmed by</td>
</tr>
<tr>
<td></td>
<td>Qualitative Data Collection and Analysis</td>
</tr>
<tr>
<td></td>
<td>Conclusions Drawn</td>
</tr>
</tbody>
</table>
The quantitative approach follows positivist logic, is fact-finding, verifiable, scientific or systematic, and conventional (McMillan & Schumacher, 2010). Quantitative results produce statistical significance, assurance intervals, and outcome sizes and offer the broad results of the study (Creswell, 2015). However, often it is not known how the findings occurred when results are obtained. Therefore, the qualitative phase is used to assist in explaining or confirming the quantitative phase outcomes.

Denzin and Lincoln as cited by Ospina (2004) assert that qualitative research includes naturalistic and interpretive methodology. McMillan and Schumacher (1993:479) maintain that qualitative research is “primarily an inductive process of organizing data into categories and identifying patterns (relationships) among categories”. Qualitative studies aim to provide illumination and understanding of complex psychosocial issues and are most useful for answering humanistic ‘why?’ and ‘how?’ questions (Marshall, 1996:522).

The acknowledged methods of data collection used in qualitative research are focus groups, documents analysis, in-depth interviews, uninterrupted observation, bulletin boards, and ethnographic participation/observation. By differentiation, De Vos (2006) circumscribes research method in the qualitative frame of reference as the unqualified transaction that encompasses the conceptualising of the disagreements in documenting the narrative review. The researcher believes that both sides of the coin are covered by the mixed methods design.

According to Shulze (Brown, 2004), a mixed-methods design has the capacity of generating sufficient, intricate and astute knowledge. The use of both designs consolidated the study and as a result consolidated the internal validity of the technique which crystallised the necessary proportion of this research (McMillan & Schumacher, 2010). This means that the researcher started with the quantitative approach followed by qualitative study as shown in Figure 4.1 above.

The fieldwork outlined below based on Figure 4.1, goes towards addressing the gap between theory and practice.
Phase 1

A survey instrument was given to all Grade 11 and 12 mathematics learners in the selected secondary schools, depending on the size of the school. All mathematics teachers (12) including the mathematics HoDs were given survey instrument in the selected schools.

Phase 2

The researcher used only one Grade 12 mathematics class observation in the selected schools. The focus group was conducted on eight mathematics learners in each selected school. All mathematics teachers (N=10) in the selected secondary schools were interviewed. SMT members (two heads of department: HoD: Mathematics) were interviewed after the learners and teachers. The researcher completed the fieldwork with the analysis of each school’s documents.

4.4. METHODOLOGY

Methodology brings up a coherent cluster of means that complement each other and that are adequately organised to convey information and findings (Leedy & Omrod, 2010). The argument in the subsequent section is grounded on selected sampling, site selection, participants’ selection, data analysis, researcher’s role and data collection strategies like questionnaires, semi-structured interviews, focus group interviews, observations as well as document analysis.

4.4.1. Mixed methods sampling

Choosing a study sample is an important step for the researcher in any research project, since it is rarely practical, efficient or ethical to study whole populations (Marshall, 1996). Creswell (2015:82) avers that “mixed methods sampling procedures should follow the particular mixed methods design. The researcher needs to be aware of design-specific issues”. He further states that random sampling is used for the quantitative phase whereas purposeful sampling is used in the qualitative phase (p.79). Therefore, the qualitative sample is the subset of the quantitative sample and because the qualitative data collection involves obtaining information from fewer participants in the quantitative sample, the sizes of the two samples are not equal. The results from the quantitative phase assisted to inform the questions asked in the qualitative sample and
so participants from the qualitative sample were individuals who were capable of answering the qualitative questions (Creswell, 2015).

As stated by Hardon, Hodgkin, Fresle (2004), in qualitative studies, researchers aim to identify information-rich cases or informants. Information-rich cases are those from which one can draw information about issues of central importance to the purpose of the research. The term ‘purposeful sampling’ is used when such people are selected. For the purpose of this study, purposeful sampling method was used to draw rich information from participants in the qualitative phase.

Marshall (1996:523) points out that although random sampling is “well defined and rigorous and provides the best opportunities to generalize the results to the population, it is not suited for qualitative research because it is not the most effective way of developing an understanding of complex issues relating to human behaviour”. He recommends the use of random sampling in the quantitative phase as opposed to purposeful sampling in the qualitative phase, where the “researcher actively selects the most productive sample to answer the research question”. This method of sampling according to Patton (1990) is very strong in quality assurance.

*Sample size adequacy results*

The success of any quantitative study is among others influenced by the size of the sample. Determining the appropriate sample size is very crucial in a quantitative study. Nayak (2010) further notes that neither a big nor a small sample size facilitates the success of the study but the most important aspect is adequacy. The sample size is an important feature of any empirical study in which the goal is to make inferences about the population from a sample i.e. given a constant precision requirement.

**4.4.2. Site selection**

According to McMillan and Schumacher (2006), site selection to trace participants involved in a certain event, is ideal when the investigation focus is on multifaceted procedures. They consider that a flawless definition of the norms for site selection is necessary and that it should be linked, and fitting to the research problem carefully chosen.
There are around 84 secondary schools in NMM district and only two schools were chosen as the setting. The two selected schools include a highly effective school and a less effective school on the basis of Grade 12 results. (NB.: In RSA, schools with more than 80% pass rate in Grade 12 are considered as highly effective and less effective ones have less than 50% pass rate). For the highly effective schools, Grade 12 results were used to select a school that has been performing consistently well and improving (upward curve or consistent at 100%) from 2012 to 2016 academic years. And for the less effective school one with a downward or fluctuating curve with an average of 50% and below was selected. The reader must note that the intent was not to compare these two selected secondary schools but to identify ‘the possible determinants of quality mathematics teaching which could mark those schools and teachers effective in mathematics teaching in secondary schools and hence draw a model.

The researcher used a quantitative method in the first phase of the data collection as illustrated in Figure 4.1. The research approach was a descriptive survey. Data was collected from a sample of the population relevant to the study.

The second part of the study was the qualitative phase. Basically, it means that the study was done in real life conditions and not in an experimental location. Creswell (2008) posits that qualitative researchers study concepts in their reasonable settings, striving to create understanding of, or make sense of phenomena the way people interpret them. This phase of the research involved all participants as shown below.

4.4.3. Participant selection

The participants were single-mindedly selected for this study due to the distinct diverse factors viz. proximity to the researcher, historical background i.e. one school is in rural area and the other is a former model C school in town and with diverse learner enrolments. Within the selected secondary schools, the study participants for this research were Grade 11 and 12 mathematics learners, all mathematics teachers (10) teachers in the selected schools and mathematics HoDs (2). Mathematics HoDs were selected as participants as they were subject specialists at school level and middle managers. It is believed that they are in a position to clarify most matters concerning the school conditions and even matters pertaining to their mathematics teachers. In total 12 (including HoDs) teachers participated.
The researcher used all Grade 11 and 12 mathematics learners in the two selected secondary schools from the district. The selection of schools was to enable the researcher to study a smaller sample of learners. The possible number of participants was 200 mathematics learners in Grade 11 and 160 Grade 12 mathematics learners totalling, 360 learners. All of the learners took part in the survey, but only one Grade 12 class per school was selected for observations and eight (8) Grade 12 mathematics learners (boys and girls) for focus group interviews. These grade 12 learners were chosen because they were on the exit level, hence excluding grade 11 in the qualitative phase.

4.4.4. The data collection strategies

This section provides an overview of strategies used in collecting data in this study.

4.4.4.1 Questionnaire

The questionnaire technique is one of the most used research instruments in educational studies due to its various benefits. It can be managed in the absence of the researcher for an extensive sample, and it offers numerical data from which variables can be associated. These characteristics make the questionnaire suitable for use by an educational researcher, as in this study as part of data collection instrument, especially when it comes to investigating some conditions in teaching and learning including perceptions, beliefs, attitudes, and points of view.

Questionnaires have been used to study teachers’ and learners’ beliefs as well as self-concept, self-esteem, self-beliefs. Adnan, Zakaria and Maati (2012), and Adnan and Zakaria (2010), used a mathematical beliefs questionnaire to measure teachers’ constructivism and traditional beliefs, as well as beliefs about teaching mathematics. Other studies also used questionnaires to measure teachers’ beliefs about mathematics teaching (see Waldeana & Abraham, 2013; Wilks & Brand, 2004; Haciomeroglu, 2013; Aslan, 2013; Zerpa, Kajander & Berneveld, 2009) and teachers’ perceptions of self-efficacy (Evans, 2008). All these scholars conducted their studies using a questionnaire to generate data.

A questionnaire was used to collect data for the quantitative phase of this study. Participants (learners and teachers) were given the assurance that their responses were anonymous; thus, they may be more truthful than they would be in a personal interview, especially when addressing
sensitive or controversial issues (Leedy & Ormrod 2010:189). The questionnaire was made up of two sections. Section B was based on teacher and learner conditions, with those school conditions embedded in each questionnaire which may enable effective mathematics teaching. Section A gave teachers’ and learners’ biographical details. Questionnaires comprised of closed questions in which participants were expected to mark with X to indicate their choices.

In order to encourage honest responses to a somewhat sensitive subject, students were assured that their anonymity would be observed and that the results of the study would be used for research purposes only. For example, the participants were requested to respond to questions by marking with X to indicate their preferred answer on the four-point Likert scale in section B as follows:

1. strongly disagree
2. disagree
3. agree
4. strongly agree

The instruments being evaluated in this study were items using Optical Character Recognition (OCR) based on teacher, learner and school conditions questionnaire. The researcher captured the collected data manually in Microsoft Excel. Presently, the OCR technology enables the questionnaire (with the same items and sections) to be scanned and directly exported to Microsoft Excel. This data-capturing approach improves accuracy and it is time-efficient.

The questionnaire was used to elicit insights on school effectiveness and effective mathematics teaching with special reference to teacher, learner and school conditions. One of the assumptions of factor analysis concerns the type of variables analysed. Field (2013) suggest the use of interval scaled variables. A Likert scale is assumed to be interval scaled, according to Ratray and Jones (2005), although the item scores are discreet values. Coussement, Demoulin and Charry (2011) support this view and vouch that previous studies showed that such a practise does not necessarily produce unreliable results.

Generally, effective mathematics teaching was the dependent variables. The dependent variables were continuous data as theoretically they have an infinite number of values in a continuum. The determinants were the independent variables. McMillan and Schumacher (2010:55) point out that
independent variables are ‘predictors’ (antecedent) and dependent variables act as ‘criterion’ (predicted) variables in non-experimental research, such as in this study.

4.4.4.2. The reliability and validity of questionnaire

Leedy and Ormrod (2010) suggest that face validity is the measure to which the researcher assumes the mechanism monitors what it is conceived to measure. Face validity is a biased or personal view of how adequately an investigation represents what it was to render. The units of the questionnaire were formulated in such a way that they test the aspects that they are planned to measure.

Content validity is the measure to which an assessment tool is an illustrative specimen of the theme being assessed (Leedy & Omrod, 2010). De Vos (2006) asserts that for the questionnaire to have content validity, a literature review overlaying the majority of the theory about the research question must be carried out. Subsequently, representativeness was displayed by the questionnaire.

This study used Cronbach’s Apha to estimate internal consistency reliability of the questionnaires (Cronbach, 2004). The first step for testing the reliability of the items of the questionnaire was to compute the Cronbach’s Alpha based on the formula adopted from Rao and Sinharay (2006). Reliability of a survey instrument is the measure to which the statistical accumulation technique produces undeviating verdicts or results if reproduced by different researchers (Leedy & Ormrod, 2010). To confirm that units in the questionnaire are reliable, Cronbach’s Alpha was computed on the questionnaire that used the Likert scale. Subsequently, reliability established the determinants of effective mathematics teaching in NMM district secondary schools. It is usually a recommended criterion in cases such as in this investigation for the acceptance of reliability using Cronbach’s Alpha being 0.7 or beyond (Cronbach, 2004).

4.4.4.3. Piloting the survey instrument

A pilot study offered the researcher a chance to assess and improve the interview process in advance of any research data for the core study being collected. The process of pilot testing is to make certain that all errors and pitfalls within the questionnaire are identified and put right before data collection starts. A swift pilot survey is an outstanding procedure to confirm the
viability of the research (Leedy & Ormrod, 2010). The questionnaire was pilot tested with Grade 12 mathematical literacy learners from another school which did not participate in the project. The teacher questionnaire was piloted with accounting and physical science teachers as they studied mathematics at first year university level. The aspects were clarified as:

- Are the questions easy to follow?
- Are the questions relevant to what the research aims to accomplish?
- Is there a good flow of questions in the various sections of the questionnaire?
- How long does it take to answer the questionnaire?

The edited version of the questionnaire was completed and circulated after improvements were made concerning layout, spell checking, serial order as well as validity of questions (Collis & Hussey, 2009; Frazer & Lawley, 2000). These mathematics learners were acquainted with the study purpose and were not involved in the final study.

4.4.4.4. The semi-structured interviews

The researcher employed semi-structured interviews to collect data from the SMT, specifically the two mathematics HoDs, and 10 mathematics teachers. An interview is a two-way dialogue in which the investigator questions the participants’ matters to collect data in a bid to know more regarding views, ideologies, viewpoints as well as attitudes of the participants (Maree, 2010). The intention of interviews was to view the world through the eyes of the participants (all mathematics teachers and mathematics HoDs in the two selected schools) as they can be valuable sources of evidence when used in an approved manner.

De Vos (2006) maintains that semi-structured interview is an interview planned about areas of certain interest while sanctioning significant flexibility in choice as well as depth. Maree (2010) affirms that a “semi-structured interview is commonly used to corroborate data emerging from other sources and the main role of the researcher is to identify new emerging lines of inquiry that are directly related to the phenomenon being studied to probe and explore”. In this case, interview data was used to corroborate the quantitative phase. De Vos (2006) asserts that the value of data attained throughout the interview is principally reliant on the skills of the interviewer.
Semi-structured interviews are likely to work well when the interviewer has pre-identified a number of features he/she wants to specially address. The interviewer can resolve in advance what parts to cover, however he/she is open to unexpected facts from the interviewee. This can be important if restricted time is offered for each interview and the interviewer desires to make certain that the key issues are covered (Hancock, 1998).

According to Bryman (cited in Denscombe, 1998), interviews have both advantages and disadvantages. The advantages include the following:

- Interviews aid the memory of the researcher when properly transcribed.
- The data can be thoroughly examined.
- Repeated analyses of data are possible.
- Data can be scrutinised by other researchers.
- The research value (bias) can be checked.
- Data could be used several times.
- Interviews are useful when the participants cannot be observed directly.
- Interviews generate historical information.
- They permit the researcher to control the pace and style of questioning.

The interviews were a more personal form of this research than the questionnaires. The researcher had the opportunity to probe or ask follow-up questions which emanated from the quantitative phase of this study. The interviews were generally easier for the respondents, as the researcher sought to determine the determinants of quality mathematics teaching which could mark schools effective in teaching mathematics and hence draw a model as a diagnostic tool.

The disadvantages of interviews, according to Bryman (cited in Denscombe, 1998), are the following:

- Indirect and unnecessary information could be given.
- They take place at designated places that are not natural settings.
- The researcher’s presence could trigger bias.
- People are not always the same in articulating their points clearly.
- Interviews are time-consuming.
Though time consuming, the semi-structured interviews comprise a combination of closed-ended and open-ended questions in order to cover objectively particular themes. Kielmann, Cataldo and Seeley (2011) advocate that in a semi-structured interview, the interviewer deals with a topic guide that is a roughly structured checklist of topics he or she wants to cover. They continue by suggesting that this guide embraces some questions that are more structured. As a rule, these have a tendency to be followed by less structured ‘probes’ which is a method of following on a topic in order to produce more information. The interviewees answered liberally to questions based on qualitative phase of this study. These questions are asked in the order specified in the guide. Supplementary questions were made to get more information around the understanding, training as well as putting into practice of the policy and effects for teaching and learning.

4.4.4.5. Focus group interviews

A focus group interview is a qualitative method in which researchers interactively question a group of participants in order to test theory-driven hypotheses. Carey, cited in McLafferty (2004) delineates focus group interviews as “using a semi-structured group session, moderated by a group leader, held in an informal setting, with the purpose of collecting information on a designated topic”. For this study, the researcher brought together a small number of participants (8 learners) in each selected secondary school, as suggested by Hancock (1998), to discuss the topic of interest.

The focus-group interview technique is used in a research inquiry because it has the following known advantages (Babbie, 2001):

- It is a socially oriented research method capturing real-life data in a social environment.
- It is flexible.
- It has high face validity.
- It produces speedy results.
- The costs are low.

This was one of the most effective tools to get information from the mathematics learners. The researcher was able to discover how different learners think about effective mathematics teaching. Learners added their own dimensions to deepen understanding and explain statistical data.
However, according to Babbie (2001), focus-group interviews have the following disadvantages:

- Focus groups afford the researcher less control than individual interviews.
- Data are difficult to analyse.
- Moderators require special skills.
- The interview situation must be conducive to discussions.
- It is difficult to constitute groups.
- Differences between groups can be a problem.

The group size was kept small, so that its participants did not feel intimidated but could express opinions freely. Two focus group interviews were conducted. The two groups were comprised of individuals who had something in common, the Grade 12 learners. Rabiee (2004) accepts as true that rich data can only be produced by the use of consistent groups so that the characters in the group are equipped to take part fully in the dialogue. Each group interview lasted in the region of an hour, depending on the number of questions as well as the number of participants. Even though the strategic questions were pre-planned, the interviews were relaxed, with questions rolling from prior reactions where possible.

Grade 12 mathematics learners were interviewed in accordance with their potential; in other words, low and high achieving learners were selected to make focus groups, hence the most productive sample to answer the research questions (Marshall, 1996) and/or information-rich cases or informants (Hardon et al., 2004). The researcher asked for the 2016 schedule from the selected schools to select the above-mentioned groups in the selected schools. It was a group of eight (8) girls and eight (8) boys (i.e 4 girls and 4 boys in each selected school). Focus group interview was a mixture of unstructured questions, group discussion and semi-structured questions.

The key drive of the focus group investigation was to draw upon participants’ attitudes, beliefs, experiences, feelings, as well as reactions. The interviewer (researcher) and interviewees (learners) worked together harmoniously in a trustful environment. The researcher built familiarity and rapport with the learners so as to put them at ease during the interviews; hence
the interviews followed the observation period. It is believed that the number of days that the researcher spent in the schools doing observation built familiarity (Rabiee, 2004).

4.4.4.6. Observations

The observation strategy is used as the main data-gathering technique in qualitative study because researchers are fascinated by the ways in which individuals generally make sense of or attach a significance to the world round them (De Vos, 2001). This technique is discreet and needs interaction with participants. The researcher was able to observe particular information like the behaviour of learners, which is not possible to detect during interviews (Denzin and Lincoln, 1998). The stages of observation formed a funnel, gradually lessening and directing the researcher’s consideration deeper into the components of the situation that appeared as ideally and/or empirically necessary until these components merged into the authentic determinants contributing to poor learner outcomes in mathematics (Denzin & Lincoln, 1998).

One Grade 12 mathematics class in the selected secondary schools was observed for two weeks. The elements which were observed were:

- Demographic details
- time management,
- classroom climate and practice,
- teaching and learning in the classroom
- subject content knowledge, pedagogical content knowledge
- equivocative
- evaluative skills
- reflective
- error analysis
- general conduct around the school hence the school conditions.

The observation schedule based on the above-mentioned elements was then analysed. During the process of observation, the researcher had to knowingly remain alert of his own biases and presumptions, and how they may influence what was observed and recognised.
4.4.4.7. Document analysis

Document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge (Denzin & Lincoln, 1998). It is in fact, used to get information that might not be accessible during the interviews and/or from questionnaires. Document analysis is often used in combination with other data collection instruments as a means of triangulation i.e. “the combination of methodologies in the study of the same phenomenon” (Denzin & Lincoln, 1998:91).

Creswell (2015) mentions the following advantages of documents analysis:

- Language and words used by the participants could be learned.
- Data that were thoughtfully designed over a period of time could be analysed.
- Documents serve as written evidence.
- Data could be analysed at a convenient time.

Cresswell (2003) and Denscombe (1998), remark the following disadvantages of document analysis:

- Documents may not be readily available to the public.
- The search for documents might be a difficult process.
- They might need transcribing and scanning for computer use.
- Some documents may be incomplete and some documents may not be authentic.

Documentary analysis was preferred to supplement other data-collection methods and to triangulate the outcomes of the data collected. An analytic contrast among interview data, focus-group data, as well as documentary-analysis data offered a clear, all-inclusive picture of the investigation phenomenon in the study.

Field documents that were perused and analysed included the following:

- annual teaching plans
- lesson preparation
- class and personal timetables
- assessment plans
• school results analysis
• management and its functionality
• resources
• attendance registers
• mark sheets
• quarterly and half-yearly schedules

4.4.5. Data analysis

Bogdan and Biklen (2003) view data analysis as a process of systematically searching and arranging the interview scripts, field notes and other materials which are accumulated by the researcher to increase his/her understanding of them and enables the researcher to present that which was discovered by others. Ary, Jacobs, and Razavieh, (2006) assert that data analysis take in attempts to follow the phenomenon under study, produces information and illuminates relationships, speculates about how, as well as why, the relationships look like they do, and connect up the new knowledge with what is by now known. The route of data collection was not a conclusion in itself. The statistical figures, themes and concepts that are implanted within the comprehensive data collected should be drawn out (Fontana & Frey, 2005).

4.4.5.1. Quantitative data analysis

The following statistics were used as part of the quantitative methodology as they were applied in the data analysis. Descriptive statistics as well as inferential statistics are usually applied to analyse the quantitative data (McMillan & Wergin, 2010). Descriptive statistics such as the mean, standard deviation, variance and frequency distributions were used to describe the demographic characteristics of the study respondents. Pearson’s correlation coefficients were used to establish whether there was a relationship between the teacher and learner conditions as well as school effectiveness and effective mathematics teaching in NMM district secondary schools.

Exploratory factor analysis was used to determine the extent to which learners agree or disagree with the statements suggested by the study. Hinkin (2009) emphasises that the use of a large sample in factor analyses assists in obtaining stable standard errors and also assures that factor
loadings are accurate reflections of the actual population. This view is emphasised by Costello and Osborne (2005), Field (2013) and Tabachnick and Fidel (2007). The recommended criteria to determine the sample size by Comrey and Lee (1992), as cited in Tabachnick and Fidel (2007), are as follows; 50 is very poor, 100 is poor, 200 is fair, 300 is good, 500 is very good and at least 1000 is excellent. Tabachnick and Fidel (2007) advise that due to the reduced reliability of correlation coefficients between the variables as a result of small samples, the sample size acceptable for a reliable underlying construct should range from 200 to 400. In this study, 330 participants (learners) returned the completed questionnaire. The model showing linear combination of factors is shown below in Figure 4.2.

Figure 4.2 Model showing the linear combination of factors (Montshioa & Moroke, 2014:356).

\[
y_1 = \mu_1 + \lambda_{11} f_1 + \lambda_{12} f_2 + \ldots + \lambda_{1m} f_m + \epsilon_1 \\
y_2 = \mu_2 + \lambda_{21} f_1 + \lambda_{22} f_2 + \ldots + \lambda_{2m} f_m + \epsilon_2 \\
y_p = \mu_p + \lambda_{p1} f_1 + \lambda_{p2} f_2 + \ldots + \lambda_{pm} f_m + \epsilon_p
\]

(1)

The models in (1) for the \( p \) variables can be combined in the single matrix expression

\[
y = \mu + \Lambda f + \epsilon
\]

(2)

Where

\[
y = (y_1, y_2, \ldots, y_p), \quad \mu = (\mu_1, \mu_2, \ldots, \mu_p), \quad f = (f_1, f_2, \ldots, f_m), \quad \epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_p)
\]

and

\[
\Lambda = \begin{bmatrix}
\lambda_{11} & \lambda_{12} & \ldots & \lambda_{1m} \\
\lambda_{21} & \lambda_{22} & \ldots & \lambda_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{p1} & \lambda_{p2} & \ldots & \lambda_{pm}
\end{bmatrix}
\]

Firstly the data was investigated for adequateness since factor analysis relies heavily on a large sample size. Having said that, the study used the Kaiser-Meyer-Olkin (KMO) method as a measure of sample adequacy prior to implementing the exploratory factor analysis method.
Figure 4.3 Statistical equation used to describe the KMO test (Montshioa and Moroke, 2014:355).

\[
KMO = \frac{\sum (correlations)^2}{\sum (correlations)^2 + \sum (partial\ correlations)^2} = \frac{\sum \sum r_{ij}^2}{\sum \sum r_{ij}^2 + \sum \sum a_{ij}^2}
\]

Where:
\( r_{ij} \) = Pearson correlation between items \( i \) and \( j \)
\( a_{ij} \) = partial correlation coefficient between items \( i \) and \( j \)

For factor analysis to be suitable, the Bartlett’s test of Sphericity should be less than <0.05. The formula that describes the Bartlett’s test is shown in Figure 4.4 as follows:

Figure 4.4 Formula for Bartlett’s test (Kaiser, 1974).

\[
\chi^2 = \left[ (n-1) - \frac{1}{6} \left( 2p + 1 + \frac{2}{p} \right) \right] \ln S + p \ln \left( \frac{1}{p} \right) \sum \lambda_i
\]

Where:
\( p \) = number of variables
\( k \) = number of factors
\( \lambda_i \) = \( i \)th eigen value of the sample covariance matrix
\( df = (p-1)(p-2)/2 \)

Literature dictates that the KMO value should be in excess of 0.5 but no more than 1 for the appropriateness of factor analysis. Different authors have different views about a pattern of correlation matrix. Field (2005) opines that a value closer to 1 is an indication that the pattern of correlations is reasonably compact; implying that factor analysis would yield diverse and consistent factors.
This study referred to Kaiser’s (1974) rule of thumb to help decide on the suitability of the sample used. The KMO ranges as follows: 1 to 0.9 is characterised as excellent, 0.8 and 0.89 as meritorious, 0.7 to 0.79 as middling, 0.6 to 0.69 as mediocre, 0.5 to 0.59 as poor and 0 to 0.49s as unacceptable. The Kaiser-Meyer-Olkin is used in conjunction with the Bartlett’s test of sphericity. Bartlett’s test was used to check the factorability of the variables used in the analysis. The null hypothesis tested is that the inter-correlation matrix comes from a population of an identity matrix and the non-zero correlations. The hypothesis is rejected at least the 5% level of significance or less to confirm that the data will yield diverse factors.

Henson and Roberts (2006) aver that each factor is well-defined by a number of items; sample sizes can in fact be comparatively small. In this contrast, 12 mathematics teachers (including 2 HoDs) participated in the questionnaire and there was a 100% return rate. The teacher questionnaire has 9 factors and 92 items. Guadagnoli and Velicer (1988) argue that solutions with correlation coefficients >0.80 require smaller sample sizes while Sapnas and Zeller (2002) maintained that even 50 items may be sufficient for factor analysis. Therefore, the number of items using 12 mathematics teachers as the sample was adequate for factor analysis hence getting the stronger data.

The nature of data determined the adequacy of sample size consistent with findings of former studies by Fabrigar et al. (1999) and MacCallum et al. (1999). Universally, the stronger the data, the less the sample size can be for accurate data analysis. The stronger the data in the factor analysis relates to homogeneously high communalities devoid of cross-loadings, and numerous variables loading strongly on respective factors (Costello & Osborne, 2005).

Reliability analysis

The study adopted a commonly accepted rule of thumb to describe internal consistency. This rule was also suggested by Kline (1999:42) and Cronbach and Shavelson (2004:394) and is applied as follows: $\alpha \geq 0.9$ is excellent, $0.8 \leq \alpha < 0.9$ is good, $0.7 \leq \alpha < 0.8$ is acceptable, $0.6 \leq \alpha < 0.7$ is questionable, $0.5 \leq \alpha < 0.6$ is poor and $\alpha < 0.5$ is unacceptable. Due to the sample used in this study, a benchmark of 0.7 was referred to as a measure of internal consistency of the instrument used. The calculated Cronbach’s alpha from the 41 items of 321 mathematics learners responses was 0.869 which is almost 0.9.
Cronbach and Shavelson (2004) suggested that using just the alpha coefficient to conclude reliability may be flawed - though a valuable measure, the standard error of the KMO might be enhanced and there are correspondingly other measurements to acknowledge when exploring reliability. However, Cronbach and Shavelson (2004) mention that the other statistics are also of use: estimated variances calculated above can be useful for identifying where a test may be having problems. In the case of teacher responses, the estimated variances of each attribute are greater than 3, which indicated strong internal consistency. For instance, if a solid residual restricts the alpha calculation (i.e., lowers it), then the predictor may deduced that there is an exceedingly large relation amongst the respondents and the test (i.e., respondents are being scored in diverse ways). The p-value of most of the attributes is less than 0.01 and 0.05 levels of significance. For a test that illustrates no confirmation of internal consistency, a complete range of statistics at this point can be a valuable diagnostic tool which supplements the data acquired when equated to running objective alpha coefficient.

4.4.5.2. Qualitative data analysis

The main exercise throughout the qualitative data analysis was to recognise familiar themes in the respondents’ accounts of their exposures. Subsequent to translating the interviews, the researcher basically used the steps below (Creswell, 2013):

- Categorising and contrasting units: The researcher arranged a manuscript of the interviews. The researcher cautiously read through every manuscript to identify relevant units which may be regulated and positioned in groups and sub-groups.
- Combining groups and their features: Once groups are classified, distinctions were done. They were categorised and combined within the groups which most describe their characteristics.
- Delimiting the formation: Groups were assessed continuously so as to classify groups that can be clustered together to limit the number of groups. Emerging themes were recognised.
- Codes were used for ordering the data. Hancock (1998:17) calls this coding (labelling) and categorising “content analysis”. She defines “content analysis” as “a procedure for the categorisation of verbal or behavioural data, for purposes of classification,
summarisation and tabulation”. This author carries on by signifying two levels of analysis which were used by the researcher in this study:

1. Descriptive account of the data: this is what was truly said with sub-meaning and no rules made about it
2. Higher level of analysis is interpretative: it is focused on the importance of an answer and what was concluded or implied. (1998:17).

The list of topics/questions in the interview guide worked as a primary set of codes. The researcher further read through the extended notes of the interviews and transcripts of tapes to complement this list of codes. Where unforeseen topics appeared, codes for these topics were included in the analysis. The interpretation of the findings was described in a narrative form which was validated by direct quotes from the participants. The accepted data analysis process channelled the researcher to draw empirical conclusions as well as recommendations.

Independent coder: The independent coder can be the researcher’s supervisor or headmaster. Unrefined data was forwarded to the independent coder who confirmed the groups and themes. The ensuing analysis in the next chapters considered each set of data in relation to the question. The findings are offered relative to the themes that steered the collection of data; that is:

- teacher factors/conditions
- learner conditions
- school conditions

which were finally used to develop a model of improved learner outcomes.

4.4.5.3. Content analysis

Leedy and Ormrod (2010:144) define content analysis as “a detailed examination of the contents of a particular body of material for the purpose of identifying patterns, themes, or biases.” It is clear from this definition that there is a procedure followed when studying and analysing content. Similarly, Holsti (1969:14) defines it as “any technique for making inferences by objectively and systematically identifying specified characteristics of messages.”

Furthermore, the same scholars regard content analysis as a systematic study of a qualitative data whose purpose is to identify and recapitulate message content. It can, therefore, be used when
studying diaries, journals, survey open-ended questions, focus group discussions and interviews, texts or any form of communication (Leedy & Ormrod, 2010). In this study, data from the quantitative and qualitative phases were processed and systematically analysed.

4.4.5.4. Strategies for rigour

A number of strategies for rigour (triangulation, negative case analysis, peer debriefing and support, and auditing) were used in order to reduce potential bias and enhance confidence in interpretation of research findings (Padgett, 2012). First, multiple kinds of triangulation were used, including theoretical triangulation, methodological triangulation, data triangulation, interdisciplinary triangulation, and analytic triangulation (Denzin, 1978; Padgett, 2012).

Theoretical triangulation involves the use of multiple theories or conceptual framework to interpret data (Denzin, 1978). Methodological triangulation involves the use of multiple methods to study the given topic (Denzin, 1978), and in this case, both quantitative and qualitative methods were used to shed light on the study aims. The study also employed the data triangulation by relying on multiple sources like questionnaires, semi-structured interviews, focus group, observations and document analysis.

Drawing upon insights from multiple disciplines in a single study constitutes interdisciplinary triangulation (Janesick, 2000). Accordingly, an interdisciplinary perspective drove the design, conduct, and analysis of this study. Finally, the study benefited from analytic triangulation, which involves the use of multiple coders (Padgett, 2012). Robust findings were bolstered through a data analysis process that allowed for identified codes, analytic categories, and interpretations of these data (van Dongen et al., 2013). Another strategy that was employed to reduce potential bias and provide fresh sources of insight for the study was a peer debriefing and support group (Lincoln & Guba, 1985; Padgett, 2012).

Finally, the conduct and analysis of this study were documented through an audit trail in the spirit of transparency, and to enhance reproducibility (Padgett, 2012). It is in the spirit of that protocol for this study, part of the study is in the book proceedings of ISTE (2018), part of this study was published in accredited journals viz Pythagoras and The Journal of Educational
Studies (Powell et al, 2013). This serves to hold this author accountable to the stated aims and methods of this study, and demands that deviations from the published protocol are published.

4.4.5.5. Diagrams of procedures

A diagram of procedures is a figure utilised to express the techniques used in a mixed methods design (Creswell, 2015). It consists of information around the data collection and analysis as well as the interpretation of the research (Creswell, 2015). However, these diagrams were supportive as visual diagrams pulling altogether the components of the research hence the overview of procedures to assist readers to understand. The figure 4.5 below depicts the argument.

Figure: 4.5. An explanatory sequential mixed methods design of the study (After Creswell, 2015:60)

In conclusion, the sequential explanatory design, research questions, aims of the collection and methods of data analysis are mostly outlined in this chapter by the researcher.
4.4.5.6 Trustworthiness

‘Reliability’ and ‘validity’ in qualitative research is generally described as ‘trustworthiness’ (Leedy & Ormrod, 2010). In qualitative research, the researcher is the prime data accumulation tool. The researcher watched and spotted what was required to be established. Afterwards, the focal point was on the merit of the approaches practised by the researcher. The subsequent procedures guaranteed trustworthiness throughout the qualitative stage of the study (McMillan & Schumacher, 2010):

- Extended data acquisition session: The data was assembled over an ample interval of time if there were no constraints. This provided the researcher the chance for repetitive data analysis, contrast and endorsement to process data.
- Interviewees’ language: An understandable language was utilised by the researcher to warrant maximal understanding on the role of the participants, whose home language is primarily Setswana in the district. Even though interviews were handled in English, respondents were permitted to use Setswana if viable and the researcher interpreted it to make questions clear. The data was written in English which is the interviewees’ language of learning as well as teaching in schools.
- Practical research: Interviews were done in the regular site of the participants’ work i.e. schools, and were around 10 to 25 minutes long. The school background shows the authenticity of their job experiences more precisely.
- Controlled subjectivity: Research is biased. It revealed a variety of the researcher’s individual curiosity, benefits, proficiencies, anticipations, intends and aspirations. In this study, the researcher kept responsibly his track for subjectivity as well as biases.
- Literal version: It was produced from transcripts. Straight extracts were utilised to show the logic, desires, emotions and opinions of the participants.
- Limited-inference descriptors: It signified the exact representation used by the researcher as opposed to the complex analytical language mostly used by researchers.
- Electronic data recording: A tape/voice recorder was employed in the interviews sessions.
### 4.4.5.7 Researcher’s role

The researcher requested access to the schools through proper channels as outlined in the ethical considerations. The researcher never influenced any process of the study e.g. survey, observations nor interviews. The researcher issued the survey instruments to participants (mathematics teachers, all Grade 11 mathematics learners [200 learners] as well as Grade 12 mathematics learners [160 learners] in April 2017 and collected them after seven (7) school days. Subsequent to the collection of the survey from teachers and learners, the researcher conducted the focus group (16 Grade 12 mathematics learners) interviews to fortify/enrich the data received from the survey.

The researcher did the observations in May 2017 for two weeks. N.B.: The observations were done on only one Grade 12 mathematics class per school and the rationale for this was that the researcher would get more relevant data as the senior mathematics students of the selected secondary schools. The last roles of the researcher at the two selected schools were to interview all mathematics teachers (10) and 2 mathematics HoDs (12 teachers in all). The researcher also did the documentary analysis as well at the end of the data analysis to triangulate and corroborate the study as indicated in 4.4.4.7.

### 4.5. ETHICAL CONSIDERATIONS

Denscombe (1998) point out that social research should be ethical in that it respects the dignity and rights of the respondents/participants, avoids harm to the participants, and runs with honesty as well as integrity. Ethical considerations refer to the trust relationship between the participants and the researcher. It deals with what is reasonable conduct for the researcher to carry out (Makhanya, 2006). Ethics refers to the use of common rules and main beliefs as well as co-opting the moral standards of the researcher (Mauther, Birch, Jessop & Miller, 2002). It further denotes consultations about what is reflected as suitable or reasonable activity in the exercise of social investigation. The concentration was on the ethical dimensions of the researcher, approaches and philosophical skills including awareness in carrying out information flow. Maree (2010) signifies that the critical feature to observe when carrying out a study is the point of
confidentiality of the outcomes, the results of the study as well as the safety of the participants’
details.

Leedy and Ormrod (2010) elucidate that informed consent speaks to the communication of every
piece of information as precisely as possible regarding the research to the participants, so that
they can effect an informed judgement whether they desire to be involved or not. The
participants of this study were 10 mathematics teachers and 2 mathematics HoDs i.e. 12 adults
who are able to give consent directly; learners who participated, received consent letters to
forward to their respective parents/guardians. Furthermore, Cohen et al. (2008) maintain that the
researcher should first get consent from learners’ guardians or parents during the study as well as
from the learners themselves. The researcher provided all the details about the purpose,
procedures and benefits of the research to the participants prior to starting with the research.

The researcher was also granted permission by the NWU Ethics Committee through the
supervisor for permission to conduct research in NMM secondary schools (see Appendix H). The
researcher further contacted the NMM education district office as well as sub-district managers
of the schools and officially requested permission to involve mathematics teachers, learners and
their HoDs in the research.

Participation in this study was strictly voluntary with the participants having the freedom or
option to withdraw at any time. The participants in the study were given the assurance that all the
information provided by them would be held in strict confidence. The participants were
requested not to include their names, addresses or the names of their schools so that traceable
details of the participants would not be recognised. The focus in educational study is mostly on
studying human beings and there should be reasonable accountability to appreciate the values
and convictions of participants in the research project.

4.6. SIGNIFICANCE OF THE STUDY

Human beings are complex, and understanding their behaviour requires a great deal of
knowledge and skill. Initial effectiveness studies were grounded primarily upon quantitative
methods in the educational production purposes (Marzano, 2000). Those studies endeavoured to
determine the effect of inputs like resources and learner contextual features on school outputs.
This study will possibly assist teachers (the researcher included) to be effective mathematics teachers and hence produce better learner outcomes. This study will further contribute to the existing knowledge on school effectiveness and effective mathematics teaching in the North-West province and hence a comprehensive model to improve learner outcomes in mathematics can be developed.

The study can therefore serve as a diagnostic tool which may be utilised to inform relevant decision-making bodies in developing and implementing appropriate interventions to improve teaching and learning of mathematics in basic education.

**4.7. LIMITATION OF THE STUDY**

This study was done in the secondary schools of NMM district in the North-West province. The study was undertaken in the challenging circumstances of secondary schools in NMM district and these circumstances cannot be generalised to other schools nor districts. The findings are specific to selected secondary schools in the NMM district and cannot be compared to other schools or districts. The study could be done over a period of time to measure variation and stability of effective mathematics teaching towards improving learner outcomes in mathematics in NMM district.

**4.8. SUMMARY**

This chapter presented a picture of the mixed-methods research design as well as methodology i.e. explanatory sequential mixed-methods was applied in this research. The concept of the case study was included as a paradigm or prototype, not a method in this study. The intents, the research design and the research methods of this research were illuminated. The reader must not confuse the study as being comparative but the intent of this study was to ‘identify the possible determinants of quality mathematics teaching which could mark those schools and teachers effective in mathematics teaching in the two selected secondary schools’ as clearly outlined as objectives in chapter 1. All efforts to warrant trustworthiness and rigour of the research were examined and maintained, and ethical considerations were also carried out. The results and findings of the empirical analysis are presented and discussed in the subsequent chapter.
5.1. INTRODUCTION

The results of this study are presented in this chapter. It covers data analysis and interpretation. It presents the results of the data obtained from the questionnaires administered to learners and teachers; documents; classroom observations; semi-structured interviews with all mathematics teachers (12) in the selected schools and focus groups with 16 Grade 12 mathematics learners at the research site. The data was presented and analysed with a view of addressing the objectives and answering the questions outlined in Chapter 1.

Statistical Package for Social Science (SPSS) 23 was used to analyse quantitative responses. Descriptive statistics such as the mean, standard deviation, variance and frequency distributions were used to describe the demographic characteristics of the study respondents. These statistics were also used to describe and identify ‘the possible determinants of quality mathematics teaching which could mark those schools and teachers effective in mathematics teaching in secondary schools’. Inferential Pearson’s moment correlation coefficient was used to measure the relationship between the factors identified. The analysis starts with the quantitative data followed by qualitative data.

5.2. QUANTITATIVE PHASE

The researcher sourced help for the distribution of questionnaires to different schools in the area. The author did not rule out the possibilities of overstating or understating certain aspects in the responses. Consequently, all reasonable efforts were made to collect and capture data effectively without compromising any standards. As noted in the previous chapter, a total of 12 teachers and 321 learners participated. The teachers’ response rate was 100% while learners’ was 89.2%.

The descriptive statistics were used to describe the demographic characteristics of the respondents’ i.e. a small sample of teachers and an adequate sample of learners. These characteristics include gender, age group, home language and race of both teachers and learners. Other demographic characteristics are nature of employment, highest teaching qualification,
current position, working experience, average class size, school location, etc. Descriptive statistics results are presented below in Table 5.1 as well as in Table 5.2.

This section presents the response rate and the demographic profiles of the respondents.

5.2.1. Demographic factors

Demography is mentioned as the numbers, settlement and movement of the target group (Steyn, Steyn, de Waal & Wolhuter, 2002). According to Steyn et al. (2002), the numbers point to the numbers of learners and teachers, considering their age, gender as well as some special needs that the educational system has to serve. Steyn et al. (2002) state that the site of the target group is the place where learners and teachers live i.e. for example in this study learners and teachers reside in either an urban area or a village.

5.2.1.1. Demographic profile of teachers

All mathematics teachers in the selected schools responded to the completion of the questionnaire positively, i.e. all responses were received. The study started by analysing the demographic profile of the respondents, who are the teachers and learners. This data analysis was done separately as a different instrument was administered to each group. The quality of data used in quantitative studies is very important as it determines its acceptability (Mavetera & Moroke, 2014). Social characteristics of the respondents who were surveyed during the investigation of ‘the possible determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in secondary schools’ were analysed. It is important to understand the characteristics of teachers and learners who contributed towards developing guidelines in the form of recommendations which the researcher suggested to help towards improved learner performance in mathematics. The demographic profiles of teachers are summarised in Table 5.1 and those of learners in Table 5.2.

- Gender

The results in Table 5.1 revealed more female teachers (66.7%) compared to males (33.3%) who participated in this study. This was a reasonable distribution of respondents in terms of gender and is also confirmed by the 2011 and 2013 population census which reported that the national
gender staffing ratio (GER) was higher for females than for males. It could also mean that, for a range of reasons, females remain in the system longer than males.

- **Age group**
This study revealed that there were few young people in the field as the age group 20-30 years was 16.7%. The results revealed that the majority (33.3%) of teachers’ ages ranged from 41 to 50, followed by 31-40 years (25%) and 60 years and above (25%). From this analysis, it can be deduced that mathematics teachers in the selected schools are matured educators who have the ability to deal with learners and to perform their duties accordingly. This further suggests that the confidence level in mathematics teaching skills is higher, signifying better motivation and job performance for these teachers who have been there in the teaching fraternity. Oshagbemi (in Crossman & Harris, 2006:32) posited that older teachers cope better with job related matters since they have established more strategies in the teaching field.
Table 5.1. Teachers’ demographic profile

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category</th>
<th>Number</th>
<th>Percentages %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Female</td>
<td>8</td>
<td>66.7</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>4</td>
<td>33.3</td>
</tr>
<tr>
<td>Age group</td>
<td>20 - 30 years</td>
<td>2</td>
<td>16.7</td>
</tr>
<tr>
<td></td>
<td>31 - 40 years</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>41 - 50 years</td>
<td>4</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>60 and above</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td>Home language</td>
<td>Setswana</td>
<td>6</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>English</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>Afrikaans</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>Sesotho</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td>Race</td>
<td>African</td>
<td>10</td>
<td>83.3</td>
</tr>
<tr>
<td></td>
<td>Coloured</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>Indian</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td>Nature of employment</td>
<td>Contract</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>SGB Post</td>
<td>1</td>
<td>16.7</td>
</tr>
<tr>
<td></td>
<td>Permanent</td>
<td>10</td>
<td>75.0</td>
</tr>
<tr>
<td>Highest teaching qualification</td>
<td>Bachelor’s degree</td>
<td>5</td>
<td>41.7</td>
</tr>
<tr>
<td></td>
<td>ACE/PGCE</td>
<td>4</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>Honours degree</td>
<td>2</td>
<td>16.7</td>
</tr>
<tr>
<td></td>
<td>Master’ degree</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td>Current position</td>
<td>Educator</td>
<td>10</td>
<td>83.3</td>
</tr>
<tr>
<td></td>
<td>HOD</td>
<td>2</td>
<td>16.7</td>
</tr>
<tr>
<td>Working experience</td>
<td>Below 3 years</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>3 – 10 yrs</td>
<td>5</td>
<td>41.7</td>
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<tr>
<td></td>
<td>11 – 15 yrs</td>
<td>1</td>
<td>8.3</td>
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<tr>
<td></td>
<td>16 – 20 yrs</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>Above 20 yrs</td>
<td>4</td>
<td>33.3</td>
</tr>
</tbody>
</table>
• **Home language and race**

The aspect of language barriers is closely associated with different cultures. Most learners have different academic and learning challenges that might happen as a result of narrow language proficiency. Given the background of the selected schools, there are relatively more Setswana-speaking mathematics teachers (at 50%) which suggested that it was easier for teachers to code switch from English as a language of instruction to learners’ mother tongues for better understanding of some mathematical concepts. 25% of mathematics teachers are Sesotho-speaking and Sesotho is relatively understandable to Setswana-speaking learners and 8, 3% are each English and Afrikaans-speaking teachers. At least 83, 3% of the mathematics teachers in the selected schools are Africans whereas 8, 3% are each coloured and Indian. It suggested to the researcher that race and culture is not an issue of racial discrimination as there were no incidents reported up to so far in these schools.

• **Nature of employment**

Almost all mathematics teachers in the selected secondary schools were permanently employed, at 83%, and as a result, there seems to exist some form of stability in mathematics teaching and learning. There was a possibility that the 17% who were not permanent were retirees, as from 3.2.1.2, a quarter (25%) of the teachers in these selected schools were at least 60 years old. The possible association within the context of the study was that there is need for more committed mathematics teachers.

• **Teaching qualification**

The teachers’ qualifications give a good picture of what a qualified cohort of teachers must be made up of. Nonetheless, with regards to higher educational qualifications the study revealed a situation which is totally different from the North West Province’s general indicators. While Statistics South Africa (2012) reported in the 2011 population census that the North West Province has the lowest percentage of people with higher education qualifications, this study revealed that 100% of the respondents have higher education qualifications.

The teacher qualifications results indicate that of the 12 educators who participated in the study, 41.5% have a bachelor’s degree and 33.3% have PGCE/ACE as their highest qualifications. It is
also revealed that 16.7% and 8.3% have degrees at postgraduate levels (honours and master’s degrees) respectively.

Laczko-Kerr and Berliner (2002), in their study of the effect of teacher certification on learner achievement, created 109 matched pairs of unqualified and qualified teachers within the same school, district or similar districts. The study showed that certified teachers are more effective in their teaching than uncertified teachers. They further reported that learners in the classes of uncertified teachers would have had an additional 20% growth if they had been taught by the certified teachers.

As reported in 3.5, learners with fully certified teachers achieve more than learners with teachers who are not fully certified. Boyd et al (2006) also reported a similar finding in poor schools in New York City. Darling-Hammond (2009) also reported that evidence from recent research studies show that teacher certification is positively related to learners’ achievement; learner accomplishment is most improved when teachers are fully certified.

The findings in this study confirm that the Department of Basic Education employed more skilled and suitable teachers in these selected secondary schools. It is however inconclusive to know only that these mathematics teachers have higher qualifications whereas their exact areas of specialisation are unknown especially in the post-graduate category.

- **Working experience**

There is only one new entrant (teacher with minimal experience) in the two selected secondary schools, and five have experience from three to ten years, which suggests that they are relatively young teachers of mathematics who will stabilise or grow with the system. At least two have fair experience and four have a vast experience in the teaching of mathematics, which suggested that at least a few of them are nearing retirement or are in retirement as experience correlates with age. A teacher’s teaching experience is used in this study to refer to the number of years the teacher had taught the subject at school. Some studies were found in literature that used teaching experience of teachers to estimate teaching effectiveness of teachers. It is reported that teacher experience is associated with teaching effectiveness (Alexander & Fuller, 2005).
One possibility here is that as teachers progress in teaching they are likely to gain more knowledge of the subject and also of learners’ learning problems in the subject, and also how to help the students overcome the learning difficulties. These make the teacher likely to become more effective over the years of teaching. The report indicated that teachers’ teaching experience is related to their effectiveness in enhancing their learners’ success but the relationship was not linear; teachers that had fewer than five years of experience were less effective than those that had at least five years teaching experience, but the benefit of experience levelled off after five years.

- School location

There are nine mathematics teachers in the selected urban school whereas there are only five of them in the rural school. The results of StatsSA (2015) reveal that there are more teachers in urban schools than rural schools. This suggests that there are more mathematics learners in the urban school as compared to the rural one as dictated by the post provisioning model (PPM) of school teacher learner ratio. Furthermore, there might be some movements from the rural to the urban area due to effective teaching and better performance in mathematics, as this urban school has a good track record of performance in mathematics.

In conclusion, this study used the cohort of teachers who teach mathematics in the rural and urban parts of Mahikeng selected schools in the North West Province – RSA.

5.2.1.2. Demographic profile of learners

Out of the envisaged 360 participants, 330 (i.e. 89.2%) learners responded and returned documents.
Table 5.2. Learners’ demographic profile

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category</th>
<th>Number</th>
<th>Percentage %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Male</td>
<td>94</td>
<td>29.3</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>227</td>
<td>70.7</td>
</tr>
<tr>
<td>Age group</td>
<td>15 years</td>
<td>15</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>16 years</td>
<td>55</td>
<td>17.1</td>
</tr>
<tr>
<td></td>
<td>17 years</td>
<td>92</td>
<td>28.7</td>
</tr>
<tr>
<td></td>
<td>18 years</td>
<td>84</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td>19 years and above</td>
<td>75</td>
<td>23.4</td>
</tr>
<tr>
<td>Home language</td>
<td>Setswana</td>
<td>273</td>
<td>85.0</td>
</tr>
<tr>
<td></td>
<td>Afrikaans</td>
<td>19</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>Sesotho</td>
<td>13</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>16</td>
<td>5.0</td>
</tr>
<tr>
<td>Race</td>
<td>African</td>
<td>312</td>
<td>97.2</td>
</tr>
<tr>
<td></td>
<td>Coloured</td>
<td>9</td>
<td>2.7</td>
</tr>
<tr>
<td>Number of years in a grade</td>
<td>1st attempt</td>
<td>256</td>
<td>79.8</td>
</tr>
<tr>
<td></td>
<td>2nd attempt</td>
<td>56</td>
<td>17.4</td>
</tr>
<tr>
<td></td>
<td>3rd attempt</td>
<td>9</td>
<td>2.8</td>
</tr>
<tr>
<td>Number of learners in your class</td>
<td>31-40</td>
<td>134</td>
<td>41.8</td>
</tr>
<tr>
<td></td>
<td>41-45</td>
<td>88</td>
<td>27.4</td>
</tr>
<tr>
<td></td>
<td>46-50</td>
<td>47</td>
<td>14.6</td>
</tr>
<tr>
<td></td>
<td>Above 50</td>
<td>52</td>
<td>16.2</td>
</tr>
<tr>
<td>How would you rank yourself</td>
<td>Top 5</td>
<td>56</td>
<td>17.4</td>
</tr>
<tr>
<td></td>
<td>Top 10</td>
<td>53</td>
<td>16.5</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>137</td>
<td>42.7</td>
</tr>
<tr>
<td></td>
<td>Lower 10</td>
<td>60</td>
<td>18.7</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>15</td>
<td>4.7</td>
</tr>
</tbody>
</table>
• **Gender**
Table 5.2 depicts that there were more female learners, 227, who took mathematics at selected schools. This suggested that either those females in this group or cohort were more interested in doing mathematics than their male counterparts.

• **Age group**
Table 5.2 shows that there are more learners aged 17 years and 18 years who are doing mathematics, with 92 and 82 respectively. The numbers are followed by 75 (i.e. 28,2%) learners who are 19 years and above of age. In this 19 plus group, there are learners who happen to be repeating the grade, or who dropped out and came back, or who are progressed learners. There were 55 learners of 16 years of age, and only 15 were 15 year-olds doing mathematics in the selected schools.

• **Home language**
273 learners of the total 321 respondents speak Setswana as their mother tongue. This is most probable given the background of the selected schools. 5, 9% learners speak Afrikaans as their mother tongue. Only 4% speak Sesotho as mother tongue and 5% other languages as mother tongue.

• **Race**
The sampled schools are in a black African majority area with 97, 3% black learners in these two institutions doing mathematics. Only 2, 7% are coloured learners.

• **Number of years in a grade**
A total of 256 learners are doing these grades for the first time. 17, 4% learners are repeating for the first time. Only 9 learners i.e. 2, 7% are doing the grade for the third time and there are no fourth attempts.

• **Number of learners in class**
An average classroom is entitled to a maximum of 35 learners which means that a classroom with more than 40 learners is actually overcrowded. Table 5.2 depicts that the number of learners in most (58,2%) classes ranges from 41 to 50 plus which indicate classes are overcrowded. Most
schools do not adhere to the ratio of 1:35 teacher-learner ratio in secondary schools as stipulated in DBE policy. It is practically difficult for a teacher to attend to all the needs of learners in a class if there are too many learners. Only 41.8% of the classrooms are fairly occupied with numbers of around 31 to 40.

- **How would you rank yourself?**

In Table 5.2, 137 learners rank themselves in the middle of the performing group. 56 and 53 of them rank themselves in the top 5 and 10 respectively which suggest that there is a bit of competition amongst these mathematics learners. 75 learners rank themselves lower, which indicate either those learners may have a challenge in mathematics, or low self-esteem with regard to subjective evaluation of their own performance in mathematics.

In conclusion, this study used the cohort of learners in Grades 11 and 12 who were doing mathematics in their subject groupings from village (rural) and township (urban) schools around Mahikeng in the North West Province – RSA.

The succeeding section presents the major results of the study.

**5.3. Section B**

This section presents primary qualitative and quantitative results as well as the discussion thereof. The quantitative results are presented first, followed by the qualitative results.

**5.3.1. Quantitative analysis: teachers**

In this section, empirical results of the study are presented. According to Mavetera and Moroke (2014), quantitative studies use data of good quality and the data presented should be in an acceptable form. The results presented are in relation to the 9 attributes of mathematics teachers suggested by the researcher. Teachers were asked to rate the extent to which they agreed or disagreed with those attributes they identified themselves with. The results are summarised as the mean, standard deviation and variance.
• Analysed teachers’ perceptions about chosen attributes

The general perception of teachers about attributes impacting on effective mathematics teaching is positive as presented in Table 5.3 below.

Table 5.3 Variance of teachers’ perception about attributes impacting on effective mathematics teaching

<table>
<thead>
<tr>
<th>Factor</th>
<th>Variance</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setting correct mind-set</td>
<td>0.5272</td>
<td>5</td>
</tr>
<tr>
<td>Classroom management skills</td>
<td>0.6928</td>
<td>4</td>
</tr>
<tr>
<td>Creation of conducive classroom environment</td>
<td>0.6603</td>
<td>4</td>
</tr>
<tr>
<td>Use assessment data to improve teaching</td>
<td>0.8990</td>
<td>3</td>
</tr>
<tr>
<td>Realistic attributes</td>
<td>0.5125</td>
<td>6</td>
</tr>
<tr>
<td>Knowledge of error analysis</td>
<td>0.4343</td>
<td>10</td>
</tr>
<tr>
<td>Instructional capacity and content knowledge</td>
<td>0.3075</td>
<td>16</td>
</tr>
<tr>
<td>Professional development (PD)</td>
<td>0.4141</td>
<td>13</td>
</tr>
<tr>
<td>Teacher collaboration</td>
<td>0.5001</td>
<td>6</td>
</tr>
</tbody>
</table>

5.3.1.1. Mathematics teachers’ attributes

The researcher sought the views of teachers on the factors/conditions that facilitate effective mathematics teaching. Every effective teacher has his or her own teaching flair as well as philosophy. The researcher thinks teachers can effectively involve learners by handling teaching and learning as a collaborative process as well as by displaying to their learners that their thoughts and beliefs are valued.

It can certainly be a good motivation for learners to feel that they are working with their teacher to develop their understanding. The researcher noted from the reviewed literature that a good teacher should know his/her audience: what do they know? what do they expect? what is the goal? The researcher presented below the possible determinants which can mark effective mathematics teachers. The researcher only considered the usage of the mean in the interpretation of each attribute.
• Setting an appropriate mind-set

The responses in Table 5.4 mark the teachers’ perception about setting the correct mind set.

Table 5.4. Statistical teachers’ perception about setting the correct mind set

<table>
<thead>
<tr>
<th>Perception</th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I believe my learners can learn.</td>
<td>12</td>
<td>3.500</td>
<td>0.522</td>
<td>0.273</td>
</tr>
<tr>
<td>I have high expectations from my learners.</td>
<td>12</td>
<td>3.330</td>
<td>0.651</td>
<td>0.424</td>
</tr>
<tr>
<td>I give extra help to my learners.</td>
<td>12</td>
<td>3.500</td>
<td>0.674</td>
<td>0.455</td>
</tr>
<tr>
<td>I find ways to make school work interesting, relevant and easy.</td>
<td>12</td>
<td>3.330</td>
<td>0.651</td>
<td>0.424</td>
</tr>
<tr>
<td>I use diverse instructional strategies.</td>
<td>12</td>
<td>3.170</td>
<td>1.030</td>
<td>1.061</td>
</tr>
</tbody>
</table>

Table 5.4 presents a summary of teachers’ perceptions as far as setting a correct mind set is concerned. Upon rating their views about the suggested statements related to this attribute, the results in Table 5.4 show the overall mean scores as being higher than the median of 2.5. This seems that on average, teachers embrace the idea of setting a correct mind set as being acceptable.

Furthermore, learners tend towards better performance when teachers establish high expectations and make learners accountable for accomplishing better outcomes. In contrast, Mohiemang (2008) suggests that expectations can be either negative or positive. High expectations link with a more dynamic part for educators in assisting pupils to learn (the belief is that all pupils can learn). Mohiemang (2008) elucidates that great expectations are of more effect once they are part of school values that put high expectations on everybody in the school.

It should however be noted that there is a difference as far as the use of diverse instructional strategies is concerned rather than a single strategy. It appears according to the responses that some of the teachers prefer one or some of the instructional strategies over others available.

Seah (2007) maintains that effective mathematics teachers are mostly goal driven through various approaches like learner-discovery (constructivist approach) as well as teacher-directed approaches in carrying out mathematics teachings, thereby making some mathematics teachers more effective than others. Lastly, mathematics teachers should try to cultivate their learners’
progressive assertiveness by making mathematics stimulating and thought-provoking hence setting the correct mind-set in the learners.

- **Classroom management skills**

The responses given in Table 5.5 are perceived ideas of teachers concerning classroom management skills.

Table 5.5 Statistical analysis perceived ideas of teachers concerning classroom management skills

<table>
<thead>
<tr>
<th></th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I enforce rules fairly without favouritism.</td>
<td>12</td>
<td>3.250</td>
<td>0.866</td>
<td>0.750</td>
</tr>
<tr>
<td>I react to minor situations in class.</td>
<td>12</td>
<td>2.670</td>
<td>0.492</td>
<td>0.242</td>
</tr>
<tr>
<td>I allow learners to prevent others from learning.</td>
<td>12</td>
<td>1.580</td>
<td>0.793</td>
<td>0.629</td>
</tr>
<tr>
<td>I protect instructional time from interventions.</td>
<td>12</td>
<td>2.750</td>
<td>0.866</td>
<td>0.750</td>
</tr>
</tbody>
</table>

These mean scores are higher than the mid-point of the range, suggesting that teachers do impose stringent measures as far as classroom management skills are concerned. This is not a surprising finding judging from the age groups, maturity and the nature of teachers at these secondary schools. These classroom management skills help learners to realise what is expected of them, help them to focus their efforts on their books and show respect to their teachers. It is however worrisome to realise that some of the teachers are not as committed as others to an extent that they do allow learners to cause commotion in class which leads to loss of concentration by learners, unfinished classroom activities, noise-making, and probably fights between learners. Probably this is something that calls for attention by the head of the department or the principal. Time-on-task is an essential characteristic of instructional effectiveness of mathematics as it provides learners with a full opportunity on the path to better learning.
• **Creation of a conducive classroom environment**

Table 5.6 Statistical analysis of creation of a conducive environment

<table>
<thead>
<tr>
<th></th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I encourage mutual respect within a classroom.</td>
<td>12</td>
<td>3.500</td>
<td>0.522</td>
<td>0.273</td>
</tr>
<tr>
<td>I make a classroom a safe learning community.</td>
<td>12</td>
<td>2.920</td>
<td>0.793</td>
<td>0.629</td>
</tr>
<tr>
<td>I provide an inspirational learning environment.</td>
<td>12</td>
<td>3.170</td>
<td>0.718</td>
<td>0.515</td>
</tr>
<tr>
<td>All learners are at liberty to ask me questions concerning their subject mathematics</td>
<td>12</td>
<td>3.330</td>
<td>0.651</td>
<td>0.424</td>
</tr>
</tbody>
</table>

The classroom environment is the setting in which student learning takes place. It concerns the classroom’s physical setting, the social system, the atmosphere, and norms and values. However, as this environment often varies between subjects and teachers, even from class to class, it is not easy to identify domain-general indicators. All respondents are in agreement about the four items related to the creation of a conducive learning environment in mathematics classrooms.

• **Use of assessment data to improve teaching**

Table 5.7 Statistical analysis of the attribute of the use of assessment data to improve results

<table>
<thead>
<tr>
<th></th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I constantly assess my learners informally and formally.</td>
<td>12</td>
<td>3.250</td>
<td>0.866</td>
<td>0.750</td>
</tr>
<tr>
<td>I give feedback timeously to learners.</td>
<td>12</td>
<td>3.000</td>
<td>0.603</td>
<td>0.364</td>
</tr>
<tr>
<td>I usually reflect on my work and do interventions.</td>
<td>12</td>
<td>2.670</td>
<td>0.492</td>
<td>0.242</td>
</tr>
</tbody>
</table>

All respondents are in agreement about the three statements to use assessment data as an attribute for effective mathematics teaching leading to improved learner outcomes. It shows clearly that almost all teachers constantly assess their learners informally and formally. Respondents give feedback timeously to learners after assessing them. Respondents further indicate that they usually reflect on their work and do interventions.
• **Realistic attributes**

Table 5.8 Statistical analysis of teachers’ realistic attributes

<table>
<thead>
<tr>
<th>Realistic attributes</th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I understand that some learners will misbehave.</td>
<td>12</td>
<td>3.170</td>
<td>0.937</td>
<td>0.879</td>
</tr>
<tr>
<td>Learners reject my efforts to provide relations.</td>
<td>12</td>
<td>2.250</td>
<td>0.754</td>
<td>0.568</td>
</tr>
<tr>
<td>I focus on what I can do (teaching) best in an uncontrollable class.</td>
<td>12</td>
<td>3.000</td>
<td>0.739</td>
<td>0.545</td>
</tr>
<tr>
<td>All learners do their homework.</td>
<td>12</td>
<td>1.670</td>
<td>0.492</td>
<td>0.242</td>
</tr>
<tr>
<td>All learners submit their tasks timeously.</td>
<td>12</td>
<td>1.750</td>
<td>0.622</td>
<td>0.386</td>
</tr>
<tr>
<td>Pressure to cover large amounts of work forces me to move on quickly through the syllabus.</td>
<td>12</td>
<td>2.500</td>
<td>0.674</td>
<td>0.455</td>
</tr>
</tbody>
</table>

A mature basic repertoire can be used as a reference to monitor ways of learners’ thinking and acting. These teachers are very strong as they understand that some learners can misbehave in their respective classes/lessons and still manage to continue smoothly with their work. Learner discipline is generally a challenge in most schools. A few teachers disagree that learners submit their tasks timeously and that they do their homework. This suggests that there should be some kind of intervention done so that all learners can submit their work timeously.

Some teachers indicated that insufficient or unacceptable workloads are the obstacles to the productive role of teachers since they create a threat to efficient and effective assessment and immediate feedbacks. Teachers perceive the intensive and exhausting workload as a stressor and they do not possess sufficient time to accomplish the desired quality of teaching and learning. However, teachers in these selected schools are able to cover large amounts of syllabi/work without glossing over it irrespective of the pressure at work. Chisholm *et al.* (2005) found by using a closed as well as an open-ended survey instrument that approximately three in four educators experience that their workload has expanded a lot since 2000, and 75%, and in excess of 90% of educators respectively considered IQMS and continuous assessment stipulations, and a new curriculum raised their workload. Most educators undergo various, difficult and continuously changing demands in teaching and learning conditions due to the intolerable rise of
workload. Others are in agreement concerning the rest of the statements relating to realistic attributes.

- **Knowledge of error analysis**

  Table 5.9. Statistical analysis of the attribute of teacher knowledge of error analysis

<table>
<thead>
<tr>
<th>I am able to interpret learner activities and performance.</th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>3.170</td>
<td>0.835</td>
<td>0.697</td>
</tr>
<tr>
<td>I am able to explain the steps needed to get to the correct answer and the appropriate conceptual links between the steps.</td>
<td>12</td>
<td>3.420</td>
<td>0.793</td>
<td>0.629</td>
</tr>
<tr>
<td>I am able to unpack a mathematical procedure while probing learner’s errors.</td>
<td>12</td>
<td>3.080</td>
<td>0.900</td>
<td>0.811</td>
</tr>
<tr>
<td>I gloss over errors in the course of the lesson.</td>
<td>12</td>
<td>3.000</td>
<td>0.739</td>
<td>0.545</td>
</tr>
<tr>
<td>I am able to look on patterns of learners’ errors.</td>
<td>12</td>
<td>2.830</td>
<td>0.835</td>
<td>0.697</td>
</tr>
<tr>
<td>I am able to explain and provide a rationale for the way learners were reasoning when producing errors.</td>
<td>12</td>
<td>3.000</td>
<td>0.739</td>
<td>0.545</td>
</tr>
<tr>
<td>I have developed a repertoire of explanations with a view to address differences in a classroom.</td>
<td>12</td>
<td>3.000</td>
<td>0.603</td>
<td>0.364</td>
</tr>
<tr>
<td>I am able to identify the error around which the conversation is focused.</td>
<td>12</td>
<td>3.000</td>
<td>0.739</td>
<td>0.545</td>
</tr>
<tr>
<td>I am able to go beyond stating the actual error by using probing questions to try and follow the way a learner is reasoning about the error.</td>
<td>12</td>
<td>2.670</td>
<td>0.651</td>
<td>0.424</td>
</tr>
<tr>
<td>I enable learners to clarify their own thinking hence developing a deeper understanding of the mathematical concepts underpinning the errors.</td>
<td>12</td>
<td>2.830</td>
<td>0.718</td>
<td>0.515</td>
</tr>
</tbody>
</table>

The results in Table 5.9 revealed that the average scores on “knowledge of error analysis” are greater than their corresponding standard deviations. Results revealed that respondents are able to explain the steps needed to get to correct answers and appropriate conceptual links between the steps. Furthermore, the results revealed with a mean score of 3.17, that respondents are able to interpret learner activities and performance which is not in contrast with Sapire et al. (2016). These interpretations were done in either formal or informal assessment.
• **Instructional capacity and content knowledge**

Table 5.10  Statistical analysis of teacher’s attribute of instructional capacity and content knowledge

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I frequently evaluate learners to check their progress</td>
<td>12</td>
<td>2.920</td>
<td>0.793</td>
<td>0.629</td>
</tr>
<tr>
<td>I use a variety of instructional strategies appropriate to the topic.</td>
<td>12</td>
<td>2.830</td>
<td>0.718</td>
<td>0.515</td>
</tr>
<tr>
<td>I make adjustments in my teaching based on learner capabilities.</td>
<td>12</td>
<td>2.920</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>I seek information about my learners’ strengths and weaknesses.</td>
<td>12</td>
<td>2.670</td>
<td>0.651</td>
<td>0.424</td>
</tr>
<tr>
<td>I team up with parents to motivate my learners.</td>
<td>12</td>
<td>2.500</td>
<td>0.798</td>
<td>0.636</td>
</tr>
<tr>
<td>I frequently use various assessment data to adjust teaching.</td>
<td>12</td>
<td>3.000</td>
<td>0.426</td>
<td>0.182</td>
</tr>
<tr>
<td>I give learners individual feedback on their progress.</td>
<td>12</td>
<td>2.920</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>I have access to my learners’ standardised marks.</td>
<td>12</td>
<td>3.170</td>
<td>0.577</td>
<td>0.333</td>
</tr>
<tr>
<td>I demonstrate sound knowledge of the curriculum.</td>
<td>12</td>
<td>3.250</td>
<td>0.754</td>
<td>0.568</td>
</tr>
<tr>
<td>I use mathematical language in my lessons.</td>
<td>12</td>
<td>3.420</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>I clearly explain procedures for solving problems.</td>
<td>12</td>
<td>3.420</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>I plan my lessons in advance.</td>
<td>12</td>
<td>3.000</td>
<td>0.853</td>
<td>0.727</td>
</tr>
<tr>
<td>I create opportunities for learner participation in classroom problem-solving.</td>
<td>12</td>
<td>3.170</td>
<td>0.718</td>
<td>0.515</td>
</tr>
<tr>
<td>I support lessons with useful classwork and homework.</td>
<td>12</td>
<td>3.420</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>I usually give more than enough activities.</td>
<td>12</td>
<td>3.000</td>
<td>0.853</td>
<td>0.727</td>
</tr>
<tr>
<td>I use different resource materials.</td>
<td>12</td>
<td>2.920</td>
<td>0.793</td>
<td>0.629</td>
</tr>
</tbody>
</table>

All the average scores for instructional capacity and content knowledge are greater than the standard deviations confirming that teachers are in agreement as far as this attribute is concerned. It is clear that teachers have the ability to use the prescribed manuals and books and they also follow the correct protocols when teaching mathematics. It is also clear that these teachers have in-depth knowledge of mathematics and they have been capacitated with relevant skills in imparting knowledge to learners. The results further revealed that mathematics teachers in these selected schools do support their lessons with useful classwork and homework.
**Professional development (PD)**

Table 5.11 Statistical analysis of teacher’s attribute of professional development

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>My school has continuous professional development in</td>
<td>12</td>
<td>2.670</td>
<td>0.888</td>
<td>0.788</td>
</tr>
<tr>
<td>mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>My knowledge of mathematics content is improved by PD.</td>
<td>12</td>
<td>2.920</td>
<td>0.793</td>
<td>0.629</td>
</tr>
<tr>
<td>PD deepens my subject matter understanding.</td>
<td>12</td>
<td>3.000</td>
<td>0.853</td>
<td>0.727</td>
</tr>
<tr>
<td>I am able to identify how learners learn the content.</td>
<td>12</td>
<td>2.920</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>PD provides an adequate time to reflect on how to</td>
<td>12</td>
<td>2.750</td>
<td>0.866</td>
<td>0.750</td>
</tr>
<tr>
<td>improve my work.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am actively involved in meaningful discussions,</td>
<td>12</td>
<td>3.000</td>
<td>0.603</td>
<td>0.364</td>
</tr>
<tr>
<td>planning and practice.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I have the opportunity to observe and be observed in</td>
<td>12</td>
<td>2.750</td>
<td>0.754</td>
<td>0.568</td>
</tr>
<tr>
<td>using new concepts in a classroom.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am able to plan how to use new curriculum and</td>
<td>12</td>
<td>3.080</td>
<td>0.515</td>
<td>0.265</td>
</tr>
<tr>
<td>teaching methods in the classroom.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am able to review learners’ work.</td>
<td>12</td>
<td>2.920</td>
<td>0.515</td>
<td>0.265</td>
</tr>
<tr>
<td>PD relates to content standards and assessment.</td>
<td>12</td>
<td>2.750</td>
<td>0.754</td>
<td>0.568</td>
</tr>
<tr>
<td>PD builds on my previous knowledge.</td>
<td>12</td>
<td>2.83</td>
<td>0.718</td>
<td>0.515</td>
</tr>
<tr>
<td>PD encourages an integrated communal approach</td>
<td>12</td>
<td>2.75</td>
<td>0.866</td>
<td>0.750</td>
</tr>
<tr>
<td>Mathematics teachers in my school work together to</td>
<td>12</td>
<td>2.83</td>
<td>0.835</td>
<td>0.697</td>
</tr>
<tr>
<td>create a more dynamic and engaged learning community.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to responses in the table 5.11 above, all teachers are in accord as far as professional development is concerned. Professional development is paramount in moving teaching to advance learner knowledge. The assessment of the effectiveness of professional growth stands in relation to its influence on pupil learning and development of teaching expertise.

Professional development is the route which employers and professionals must follow if they want to keep professionals up to date, deepen their knowledge and sharpen their skills. The view of the respondents is that it is through continuous professional teacher development that teachers can keep up to date with what is happening in their field. All the scores are in excess of 2, marking agree and strongly agree. The results suggested a strong belief by teachers that this factor is included in models as one of the “determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in secondary schools”.

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Teacher collaboration

Table 5.12 Statistical analysis of teacher collaboration attribute

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number</th>
<th>Mean</th>
<th>Std. deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departmental teachers meet on lesson planning.</td>
<td>12</td>
<td>2.500</td>
<td>1.168</td>
<td>1.364</td>
</tr>
<tr>
<td>We discuss with each other on how to assist specific learners.</td>
<td>12</td>
<td>2.670</td>
<td>0.888</td>
<td>0.788</td>
</tr>
<tr>
<td>We meet to analyse and address learner results.</td>
<td>12</td>
<td>3.080</td>
<td>0.669</td>
<td>0.447</td>
</tr>
<tr>
<td>There is peer classroom teaching observations.</td>
<td>12</td>
<td>2.420</td>
<td>0.996</td>
<td>0.992</td>
</tr>
<tr>
<td>We engage in mentoring each other to improve.</td>
<td>12</td>
<td>2.580</td>
<td>0.996</td>
<td>0.992</td>
</tr>
<tr>
<td>We work collaboratively with subjects specialists.</td>
<td>12</td>
<td>3.080</td>
<td>0.900</td>
<td>0.8811</td>
</tr>
</tbody>
</table>

According to responses in the table 5.12 above, all teachers seem to embrace the idea of teacher collaboration. All the scores are in excess of 2, marking agree and strongly agree. The results suggested a strong belief by teachers that this factor is included in models as one of the “determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in secondary schools”.

5.3.1.2. Sub-scale correlation

The results presented here are presented as a correlation matrix. The correlation coefficients were calculated using Pearson’s moment correlation. Chen and Krauss (2004:67) suggested the Pearson correlation coefficient as a measure of the relationship between the attributes or factors.
Table 5.13. Correlation matrix of teacher attributes

**Keys**: γ - Pearson Correlation; A-setting correct mind-set; B-classroom management skills; C-creation of conducive classroom environment; D-use of assessment data to improve teaching; E-realistic attributes; F-knowledge of error analysis; G-instructional capacity and content knowledge; H-professional development; I-teacher collaboration

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>γ</td>
<td>1</td>
<td>0.596**</td>
<td>-0.63**</td>
<td>0.345*</td>
<td>-0.006</td>
<td>-0.272</td>
<td>0.477**</td>
<td>-0.133</td>
</tr>
<tr>
<td>B</td>
<td>γ</td>
<td>0.596**</td>
<td>1</td>
<td>-0.373*</td>
<td>0.500**</td>
<td>0.056</td>
<td>-0.372</td>
<td>0.444**</td>
<td>0.268</td>
</tr>
<tr>
<td>C</td>
<td>γ</td>
<td>-0.63**</td>
<td>-0.373*</td>
<td>1</td>
<td>-0.256</td>
<td>-0.075</td>
<td>0.269</td>
<td>-0.532*</td>
<td>0.000</td>
</tr>
<tr>
<td>D</td>
<td>γ</td>
<td>0.345*</td>
<td>0.500**</td>
<td>-0.256</td>
<td>1</td>
<td>0.114</td>
<td>0.211</td>
<td>0.325</td>
<td>0.003</td>
</tr>
<tr>
<td>E</td>
<td>γ</td>
<td>-0.006</td>
<td>0.056</td>
<td>-0.075</td>
<td>0.114</td>
<td>1</td>
<td>-0.331</td>
<td>-0.115</td>
<td>-0.095</td>
</tr>
<tr>
<td>F</td>
<td>γ</td>
<td>-0.272</td>
<td>-0.372</td>
<td>0.269</td>
<td>0.211</td>
<td>-0.331</td>
<td>1</td>
<td>-0.66**</td>
<td>-0.316</td>
</tr>
<tr>
<td>G</td>
<td>γ</td>
<td>0.477**</td>
<td>0.444*</td>
<td>-0.532*</td>
<td>0.325</td>
<td>-0.115</td>
<td>0.66**</td>
<td>1</td>
<td>0.060</td>
</tr>
<tr>
<td>H</td>
<td>γ</td>
<td>-0.133</td>
<td>0.268</td>
<td>0.000</td>
<td>0.003</td>
<td>-0.095</td>
<td>-0.316</td>
<td>0.060</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>γ</td>
<td>0.225</td>
<td>0.410*</td>
<td>0.007</td>
<td>0.400*</td>
<td>-0.144</td>
<td>0.017</td>
<td>0.059</td>
<td>-0.057</td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed).**

**. Correlation is significant at the 0.05 level (2-tailed).**

Firstly, the correlation matrix was studied with a view of optimising the target values or attributes according to their relevance. The results are clear according to the coefficient matrix associated with the nine variables. There is evidence of a mixture of negative and positive significant and insignificant relationships between the nine attributes. It is evident that there are high correlations (in excess of 0.3) looking at the correlation matrix. It is also evident that the correlation matrix is not unitary providing a strong relevance between the nine mathematics teaching attributes. This further endorsed the viability of the multiple relationships between these attributes.

For instance, classroom management, creation of a conducive learning classroom environment, use of assessment data to improve teaching, instructional capacity and content knowledge are among the nine factors which are found to be highly and strongly related with other attributes. The correlation coefficients of these factors are reported to be highly significant in some cases, i.e. in excess of 0.3 more than others. Other coefficients are lower but still are an indication of a type of relationship to some extent (Norusis, 1994). This could imply that the nine factors cannot be treated as individuals. The performance of one attribute in one way or another has a certain influence on the other attributes. The p-value of most of the attributes is less than 0.01 or 0.05 levels of significance, confirming the interrelations between the attributes. Consequently, none of
the factors is achievable without considering the others. This therefore implied that the nine (9) factors may be used as teacher conditions to develop a model towards improved learner outcomes in mathematics.

5.3.2. Quantitative analysis: Learners

Learner conditions factors refer to what the learners bring to the class. It embraces the nature of the social upbringing of the learners - their attitude, motivation, concerns and skill level, their past knowledge, aims, beliefs and characters they convey into class with them (Ahmad, 2008). These can impact learners’ classroom collaboration and thus influence the teachers’ teaching effectiveness. The researcher sought to seek the responses of learners on the factors/conditions that facilitate effective mathematics teaching.

Empirical results on the analysis of learners’ responses are discussed in this section. The learner data were primarily gathered to provide data to create indicators of effective mathematics teaching. The study was concerned to comprehend learner outcomes or learner attitudes concerning mathematics to the extent that these informed the study about the learners’ classroom teacher. Hence, learner data were only significant if their mathematics teacher had also given data for the research. In the same way, it was imperative to tie the learner data as well as teacher data to the school.

Table 5.14. KMO and Bartlett's Test

| Kaiser-Meyer-Olkin Measure of Sampling Adequacy | 0.830 |
| Bartlett’s Test of Sphericity   | Approx. Chi-Square | 3970.142 |
| Determinant of correlation matrix | 2.270E-6 |

According to the results presented, the sample used is meritorious as it ranges between 0.80 and 0.89 and the sphericity test is rejected implying that the sample used is adequate and the matrix is not singular. The fact that the determinant of a correlation matrix is not equal to zero also confirms the non-singularity of the factors. Factor analysis was found to be suitable as the test revealed a value of .000. Different factors are expected from the data.
Reliability analysis

With reference to the rule of thumb, the instrument used to collect data is good according to Kline (1999); and Cronbach and Shavelson (2004). This confirms the applicability of the suggested exploratory factor analysis for this study.

Table 5.15. Reliability Statistics with regard to learner questionnaire

<table>
<thead>
<tr>
<th>Cronbach’s Alpha</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.869</td>
<td>41</td>
</tr>
</tbody>
</table>

Presented in this section are the Factor Rotated results. Factor rotation simplifies the factor structure yet still allows the factors to be inter-related. Presented in Table 5.15 are only the significant loadings ranging between 0.3 and 1. Guadagnoli and Velicer (1988) recommended an interpretation of factors with four or more variables, each possessing loadings above 0.6 irrespective of the sample size used. On the other hand, Velicer and Fava (1998) recommend a minimum of 3 variables per factor as being critical. Since factor analysis is used in this study as an exploratory technique, no strict restrictions were imposed. The researcher was however cautious when interpreting the factors. A rule of thumb used in interpreting the correlation coefficients suggested by Norusis (1994) was adopted when interpreting the factors.

Table 5.16. Statistical analysis of learner conditions elements

<table>
<thead>
<tr>
<th>Factor</th>
<th>Variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-perception on mathematics</td>
<td>7.689</td>
</tr>
<tr>
<td>Attitudes concerning mathematics</td>
<td>2.945</td>
</tr>
<tr>
<td>Self-assessment</td>
<td>2.345</td>
</tr>
<tr>
<td>Independent learning</td>
<td>2.171</td>
</tr>
<tr>
<td>Affinity to teachers</td>
<td>1.835</td>
</tr>
<tr>
<td>Learning repertoire</td>
<td>1.308</td>
</tr>
<tr>
<td>Orientation to learning</td>
<td>1.200</td>
</tr>
<tr>
<td>Learner-teacher support material</td>
<td>1.170</td>
</tr>
<tr>
<td>Adjustment to school</td>
<td>1.143</td>
</tr>
<tr>
<td>Parental support</td>
<td>1.074</td>
</tr>
</tbody>
</table>
According to the results presented in Table 5.16, the general impression gathered is that respondents concur about the ten conditions suggested by the researcher. The correlation coefficients of the rotated factors are generally significant as a confirmation that learners either agreed or strongly agreed about these factors. There were a few respondents who were of a different view and that led to a lack of or poor convergence in some of the factors such as independent learning where learners disagreed on their participation in mathematics groupwork, and undertaking of independent mathematics research activities.

Some learners also had different opinions as far as learning repertoire is concerned. According to them, there is nothing interesting about mathematics lessons as their teachers fail to use innovative ways of teaching, which end up with them having dyscalculia. Among other things, learners complained about support structures given to them and lack of resources due to vandalism and theft. It should be noted that blank spaces on the table of factors are for statements which learners had disagreements on. The variances for each of the ten factors were calculated to differences in perspective about them. The factors were found to be different as expected but also less correlated after factor rotation was imposed. It is also evident that most of the learners’ responses coincided more on their self-perception on mathematics, affinity to teachers, and orientation to learning and parental support.

According to learners, the 10 attributes highlighted are of importance to their learning mathematics. Their responses confirm “Self-perception on mathematics” is one factor that schools should pay special attention to. The variance of this factor is greater than that of others. The three second most important factors highlighted by leaners are “attitudes toward mathematics”, “Self-assessment” and “Independent learning” with variances 2.945, 2.345 and 2.171 respectively. Other factors are important even though they received lower ratings in terms of the variances than others. This is a confirmation that the 10 factors could be used as learner conditions to improve learner performance in mathematics in the selected secondary schools.
In Table 5.17 below, empirical results of the article are presented to answer the second secondary question.

Table 5.17 Learner correlation matrix

**Keys:** $\gamma$ - Pearson Correlation; A-self-perception; B-attitudes concerning mathematics; C-self-assessment; D-independent learning; E-affinity; F-learning repertoire; G-orientation to learning; H-learner-teacher support material; I-adjustment to school; J-parental support.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\gamma$</td>
<td>1.00</td>
<td>-1.00</td>
<td>0.887</td>
<td>1.000</td>
<td>-2.41</td>
<td>-1.00</td>
<td>0.054</td>
<td>1.000</td>
<td>0.670</td>
</tr>
<tr>
<td>B</td>
<td>$\gamma$</td>
<td>-1.00</td>
<td>1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C</td>
<td>$\gamma$</td>
<td>0.887</td>
<td>-1.00</td>
<td>1.000</td>
<td>-0.32</td>
<td>-1.00</td>
<td>0.921</td>
<td>1.000</td>
<td>0.957</td>
<td>-0.53</td>
</tr>
<tr>
<td>D</td>
<td>$\gamma$</td>
<td>1.000</td>
<td>-1.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.255</td>
<td>0.265</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>$\gamma$</td>
<td>-0.24</td>
<td>-1.00</td>
<td>-0.32</td>
<td>1.000</td>
<td>-1.00</td>
<td>0.345</td>
<td>0.334</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>$\gamma$</td>
<td>-1.00</td>
<td>1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>$\gamma$</td>
<td>0.054</td>
<td>-1.00</td>
<td>0.921</td>
<td>1.000</td>
<td>0.308</td>
<td>-1.00</td>
<td>1.000</td>
<td>0.567</td>
<td>1.00</td>
</tr>
<tr>
<td>H</td>
<td>$\gamma$</td>
<td>1.000</td>
<td>-1.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>$\gamma$</td>
<td>0.670</td>
<td>-1.00</td>
<td>0.957</td>
<td>1.000</td>
<td>0.265</td>
<td>-1.00</td>
<td>0.746</td>
<td>1.000</td>
<td>-0.69</td>
</tr>
<tr>
<td>J</td>
<td>$\gamma$</td>
<td>-0.78</td>
<td>1.00</td>
<td>-0.53</td>
<td>-1.00</td>
<td>-0.39</td>
<td>1.000</td>
<td>-0.095</td>
<td>-1.00</td>
<td>-0.697</td>
</tr>
</tbody>
</table>

It should be noted that blank spaces on the table of factors are for statements which learners had disagreements on. These indicate that the elements are inter-related as it comes from the population of inter-related correlation matrix.

**5.3.3. Qualitative results**

Qualitative phase results are based on a different philosophical approach, which sees the individual and his or her world as so interconnected that essentially the one has no existence without the other. The researcher saw the social reality as unique; thus, the researcher only understood teachers and learners’ behaviour by focusing on the meanings that events have for the participants involved. The researcher looked not only at what participants did but also at how they thought and felt, and the researcher also attempted to understand their reality. The results of this qualitative phase are presented as a narrative report to understand the social reality experienced by the participants and mainly to complement the quantitative phase. Furthermore, because the researcher did not know in advance how naturally occurring events will unfold or what variables may be important, the researcher did not begin this study with hypotheses. The qualitative results are presented below.
The data was collected through semi-structured interviews with teachers and departmental heads, focus group interviews of learners and observations as well as through document analysis of the two selected schools. The data is presented under the themes that emerged during the interpretation and analysis. The researcher sought to seek participants’ perceptions about effective mathematics teaching and school effectiveness. The following section presents data collected through focus group interviews. The specific details about the date, composition and venue are presented in Table 5.18.

Table 5.18 Schedule of focus group interviews

<table>
<thead>
<tr>
<th>Location</th>
<th>Date</th>
<th>Time</th>
<th>Composition</th>
<th>Participants</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahikeng</td>
<td>7.05.2017</td>
<td>14:00-15:30</td>
<td>Learners from one of the selected schools (Group A)</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Mahikeng</td>
<td>8.05.2017</td>
<td>14:00-15:30</td>
<td>Learners from one of the selected schools (Group B)</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

The participants were enlisted from among the Grade 12 mathematics learners in the selected schools. The first focus group meeting comprised eight Grade 12 mathematics learners from the lower performing school. The second session with eight Grade 12 mathematics learners was in the constantly high performing school. The rule of data saturation as commended by Sagoe (2012) as well as Carlsen and Glenton (2011) steered this study. In the course of both sessions it was noticed that data saturation was reached.

The sessions lasted around 40 minutes of the scheduled 90 minutes and were conducted in English. The focus group interviews were done in May 2017 before the mid-year examinations, as indicated in Table 5.18 above. The location and the dates in the table are used to recognise the exact focus group.

A set of pre-approved questions were used to solicit perceptions into the attitudes, views and opinions on effective mathematics teaching and school effectiveness. The researcher expedited the process by posing a question at a certain point in time. Such data is presented under the categories that arose during the analysis and interpretation of the results. The words of the
respondents are rendered verbatim and set in italics. Given that most of the participants presented similar perceptions, the two focus groups interview results are offered below.

5.3.3.1. Learners’ views on school effectiveness and effective mathematics teaching

The key drive of the focus group data presentation and analysis was to collect information about experiences, beliefs and attitudes of learners in the mathematics class and to get some specific factors which impede or enhance their learning in mathematics. The identities of learners remained confidential. Learners were required to give their honest opinions without any fear of victimisation. Data collected was mainly to corroborate the quantitative phase i.e. only pre-identified features from the quantitative part were addressed and new themes also were covered.

Sub-theme 1: Self-perception on mathematics

A typical response from a learner in Focus Group 1 and 2 about self-perception is “…mathematics can be quite frustrating…mmm…..some kind of anxiety, especially when you see that you can do something but you don’t know how to do it, because in various circumstances …”

Learners experience mathematics anxiety for various reasons, such as fear about forgetting, worrying about obtaining poor results, afraid of their inability to solve a mathematics problem, distress that the mathematics problem is too difficult to do or to execute the mathematics principles correctly. Another concern includes limited time to complete mathematics assessments. Mathematics anxiety could also be a result of previous experiences in the mathematics classroom, parents’ influences and remembering of previous achievements in the subject.

Mathematics anxiety has certain consequences revealed in different forms, which may affect learners suffering from it. It manifests in different ways, such as memory loss during mathematics examinations, panic or total avoidance to execute mathematics related tasks. Thus, mathematics anxiety reduces mathematics performance regardless of the learner’s level of mathematics competence. Many learners fear and get frustrated when doing mathematics. Avoidance to do mathematics can be observed in terms of learners’ involvement and participation in mathematics lessons, time spent to study mathematics and regular submission of
homework. This learner anxiety affects participation in mathematics, which probably influences attitudes towards mathematics and avoidance of it.

Each learner is unique; nonetheless, certain elements impede the performance of a learner regardless of their uniqueness. Some of the respondents in both groups have negative perceptions about mathematics, such as difficult mathematical language, and end up frustrated when trying to solve mathematics problems. This perception led to many learners who were doing mathematics in Grade 11 to switch from mathematics to mathematical literacy in Grade 12. Some learners in group A and B indicated that their peers switched to mathematical literacy because they never passed mathematics in their secondary school examinations. Nonetheless, most of the learners felt strongly driven to learn mathematics, and believed that the mathematics they learn at school is useful to them and will also be useful to assist them to get work in the future.

*Sub-theme 2: Attitudes concerning mathematics*

A typical response from a learner in Focus Group 1 and 2 about attitudes concerning mathematics is “I am lazy and there is this thing in mathematics, so my problem is that, whenever I am lazy, I cannot practise and I am unable to obtain average marks ... ... the only reason I can give as to why I do not think that I am good right now in maths, is maybe attitude…”

Their negative attitude towards mathematics and lack of self-confidence possibly result in learner underperformance in mathematics. Many learners tend to avoid mathematics and do mathematical literacy. These numbers are confirmed by the ratio of mathematics learners to mathematical literacy in these selected schools, which is 1:3. Some respondents believe that mathematics takes a lot of their time hence they are too lazy to work on mathematical problems on their own. “I think that what contributes to my success in mathematics is having a positive mind”. Some respondents believe that a positive mind-set made them enjoy mathematics and hence achieve better performance in mathematics. “If you do something right in class they tell you that you think you better that you think you know everything that you better than them and that makes them to hate you ... silliness at the back so it’s not good... in class there’s very tension between the learners and the teacher most of the time they don’t do the work that he gives…”
Some respondents really had it tough in their classes, for there are some learners who have a negative attitude to mathematics and even the subject teachers. This challenge pulls down those learners who are trying hard to do the right thing in class i.e. positive participation during the teaching and learning process. Respondents believe that this kind of attitude impacts negatively on effective mathematics teaching in their classes. Respondents agree that most disruptive learners do not do various activities like homework, assignments, projects and even classwork and formal tests.

They argue that these disruptive learners usually indicate that they will be progressed to the next grade come the end of the year as has been done before. The interviewees suggested that older learners must not be allowed general admissions with the normal age cohort as is currently done and that the policy should be amended to cater for them. Participants argue that learners who have some learning challenges or special needs, and multiple repeaters with behavioural challenges should be placed in relevant learning institutions like adult basic education and training (ABET) centres, special schools or even TVET colleges.

Sub-theme 3: Self-assessment

A typical response from a learner in Focus Group 1 and 2 about self-assessment is “…in maths when I go home, I’m basically doing mathematics on my own and solve maths problems. My parents cannot really help me”. All the respondents indicated that they are responsible for their own learning in their own space when given work. Respondents maintain that it is worth trying hard in mathematics for they want to get very good mathematics results. Schunk (2004) agrees that learners learn better when they make mistakes in methods. Respondents argue that they are able to reflect and analyse errors committed when solving mathematical problems each time they work on their own, though a few of them discontinue working when they encounter challenges. Furthermore, respondents believe that they are able to explore mathematics learning and problem solving strategies through self-assessment.

“I enjoy doing group work and group work contributes to my success in maths so we have a positive relationship with our mathematics teacher and he assists us when having challenges, we motivate one another”. Most of the respondents value group work. They believe that it contributes positively to their success in mathematics. Respondents maintain that a hands-on
approach to their learning process, and thinking creatively as well as critically through the process of communication and stimulation with other learners, build on their existing knowledge through the route of constructivism.

Sub-theme 4: Independent learning

A typical response from a learner in Focus Group 1 and 2 about independent learning is “…I’m not a bright student, I have to work hard to get to understand properly in class, I have to do extra work on top of the time that I’m given with the teacher …. when I practise at home and when I put extra work on top of the work that I do with the teacher, I find maths very interesting and I enjoy it”. Most of the respondents believe that when they work in their own space, putting more effort, they find mathematics enjoyable. They argue that they have more time and are able to access mathematics resources needed in their learning. Respondents maintain that they are able to apply the acquired skills in class to solve problems independently.

“…what makes me improve is practice, constant practice, constant interaction with other learners and consultation with the teacher is the one that helps me to improve in my mathematics …..using extra materials, it helps me in improving my maths skills and my understanding”. All respondents agree that constant practice, constant group work and added consultation with their mathematics teachers assist them to improve their performance in mathematics. Furthermore, they have the view that the use of LTSM helps to improve their mathematical skills and understanding.

“I enjoy doing group work and group work contributes to my better performances in maths…” A high percentage of respondents have a view that group activities work for them and it assists them to undertake independent mathematics research activities.

Sub-theme 5: Affinity to teachers

A typical response from a learner in Focus Group 1 and 2 about affinity to teachers is “…this year I have a very devoted teacher who enjoys teaching and loves helping people I’ve seen that mathematics is not that hard…” Most of the respondents share the same sentiment in regard to their mathematics teacher including the response above. It shows clearly that they get on well with their mathematics teachers. The view that one respondent realised, that mathematics is not
difficult, they learn mathematics best when assisted by their teachers, is in contrast with constructivist pedagogy which keeps learners more actively engaged through “discovery”, “inquiry” and “collaboration” (Krahenbuhl, 2016:99). They believe their mathematics teachers are committed, and dedicated in teaching them, though the teacher-centred method is mostly used. Constructivist theorists hold the view that teachers should deal not only with learners having high abilities or high motivation for mathematics, but they should also look at the learners’ cognitive and affective dimensions.

“Our teacher is a good man he explains nicely in a good way as compared to other educators that I’ve been with in the past I understand him in a good level than I did with the other teachers .... sends question papers that we cannot print here at school ... gives us rewards which makes us feel more comfortable”. Most respondents felt strongly that their teacher knew them well; specially, that their mathematics teachers knew how to explain mathematics to them, were able to demonstrate to them how mathematics may benefit them in real life, and made them feel that they could learn mathematics. Some maintain that they do not make mathematics a rigid body of facts, rules to be learned by rote and practised by learners, as well as theories which they take as absolute. Constructivists believe that weight should be on designing activities which offer active knowledge, as an alternative to the conduction of traditional knowledge. Mathematics needs teachers to have a good understanding of it and to be able to exhibit content competence, concrete competence as well as reflexive competence.

Sub-theme 6: Learning repertoire

A typical response from a learner in Focus Group 1 and 2 about learning repertoire is “… presents a new topic she does not only give it to us in one method but she gives us alternative methods of doing it and that the other thing is that for instance is the linking is the linking of concepts or linking of topics in she tells us or she makes us aware that this topic may be in algebra it links in functions it links in calculus…”. Most of the respondents agree that they learn best when their mathematics teachers use different methods during the lessons. Moreover, they alleged that when their teachers use various methods for teaching mathematics, it assisted them to think for themselves in mathematics and helps to support everyone to do well in mathematics. There were only two negative worries that most learners said about their mathematics teachers. They argue that their teachers require using cooperative learning methods (e.g. group
investigation) in most instances. They believe that the concern will allow learners to learn from each other; improve communication skills; use prior knowledge to build on; as well as develop present knowledge. Subjects are no longer taught in seclusion, but rather there is an integration of subjects. No amount of knowledge subsists in isolation.

Sub-theme 7: Parental support

A typical response from a learner in Focus Group 1 and 2 about parental support is “…they cannot really help me because they do not understand what’s going on so I can say that no, they are not”. Learners tend to get frustrated and lose courage when they are unable to get the assistance they need at home and as a consequence their performance levels begin to drop. Respondents argue that the tangible contribution of parents in academic matters is not the main concern as they are mostly engaged in financial matters. Baker and Jones (2005) maintain that there is an association between low socio-economic status and poor performance in science in schools. Though data has suggested that it is not the socio-economic status per se but the dynamics associated with home incomes and contextual experiences that shapes the learners’ performance in mathematics.

“I remember when I had failed my Grade 11 she couldn’t help me with maths because she hadn’t done maths the way we are doing it now”. Research has revealed that parental monitoring leads to improved academic excellence due to the fact that parental attention assists learners to stay dedicated at school (Plunkett & Bamaca-Gomez, 2003; Ibanez et al., 2004). Grounded on the results of studies that have investigated the relationship between parental support and academic success, “parent involvement is positively related to expectations and importance of schooling” and by having a progressive attitude concerning education, a learner is more likely to excel.

5.3.3.2. Semi-structured interviews of teachers and HoD’s

All mathematics teachers in the two selected schools were interviewed individually. Most of the interviews lasted from 20 minutes with the longest lasting 28 minutes. All interviews were electronically recorded and transcribed verbatim. The interview questions were based around school effectiveness and effective mathematics teaching. These interviews were one-to-one but because most of the responses tended to be recurring, the researcher only quoted the most
encompassing responses and summarised the arguments i.e. in some instances words of some respondents were quoted verbatim and other responses with a similar meaning were summarised by the researcher. The researcher interviewed two (2) mathematics heads of department (HoD) and six mathematics teachers (PL 1).

Data presented and analysed about teachers’ views on school effectiveness and effective mathematics teaching was mainly to corroborate the quantitative phase i.e. only pre-identified features from the quantitative part were addressed. The researcher further resolved in advance which key issues to cover.

**Sub-theme 1: Effective mathematics teaching**

“I think what constitutes effective maths teaching; one... the teacher must be prepared to go to class well prepared and ....two...teacher must try as much as he can do to involve the learners in the activities...” (PL 1)

An overwhelming majority of the interviewees believed that effective mathematics teaching needs a teacher to be well prepared when going to class in any school. This is in agreement with Botha (2010) who argues that schools are effective if teachers are effective in a classroom with the involvement of learners. Another teacher has the opinion that “Effective teaching is constituted by making sure that all the learners become part of your learning entity, you must not exclude learners, you must always make sure that they are creative, they show their critical thinking by involving them in the lesson, for example when you just teach and not ask them questions, the teaching or learning will not be effective...”. (PL 1). This concurs with Doolittle’s (2014) notion that learners are dynamic participants in the learning procedure and in due course are in charge of their own learning. The argument will in turn make learners more critical thinkers and become more dynamic in the construction of knowledge. “I usually tell teachers to go extra mile, they should work as if that is their last day, they should go all out, because this is the future of your learners we are talking about, so, and it must be your, it must be inside you, you must not depend on extrinsic motivation 100%”. (HoDs) The HoDs of the selected schools anonymously agree that teachers should go the extra mile in their job even if there is no extra incentive for them. Their view is that teachers must ensure that the future of the children be protected by effective teaching resulting in better learner outcomes in mathematics. With the
adoption of a constructivist philosophy, mathematics teachers should now advocate more active
learning on the part of learners and a more facilitative role for teachers.

Sub-theme 2: Classroom management skills

“I always emphasise the arrival on time, time management towards the learners as well as me
as a teacher……and also encourage them to respect each other, not to respect the teacher....”
(PL 1). A majority of the teachers are of the view that timeous arrival in the classroom for a
lesson assists greatly in the management of the class. These teachers further indicate that
effective classroom leadership is reliant on association between teachers and learners. A
significant number of them agree that respect for each other helps to manage the class easily,
whereas others believe in laying the ground rules to the learners like “I always lay down the
ground rules when I first meet my class...”PL 1.

Sub-theme 3: Creation of appropriate mind-set and conducive learning classroom environment

“I start the lesson .... I motivate these kids that to perform to try to do their best. I always tell
them our background doesn’t make us....” (PL 1). Respondents have the opinion that those
learners who believe that they can learn mathematics perform better. A majority of the teachers
agree that they have high expectations of their learners. They further indicated they use diverse
instructional strategies in setting the appropriate mind-set. Respondents agree that the above
consults are possible if there is a conducive learning classroom atmosphere.

The respondents indicated that their classrooms were inclusive. All learners’ comments and
views were respected and valued, contributions from all learners were expected, all learners
engaged in a diverse sharing of opinions at some time, and the learners mutually shaped
understandings with the assistance from the teacher when required. The respondents said that
various elements are described or observed within three dissimilar areas that sustained the
creation of cultures of participation. These areas focused on classroom standards, classroom
techniques, and diversifying dialogues.

Respondents, especially from school A, argue that some school buildings are dilapidated. They
indicated that the departmental procurement moves at a snail’s pace to address these issues.
Respondents maintain that all these matters impact negatively on school effectiveness and
effective mathematics teaching. They indicated that they really need assistance to overcome all these challenges so that there can be at least some effectiveness in their schools.

Sub-theme 4: Use assessment data to improve teaching

“…every Friday we have declared ... for a maths day whereby we assess learners formally and then after assessing them we analyse the results and check. We do item analysis, we check in which topic or subtopic do the learners have some difficulties and then ... make a diagnostic report and then our educators use it in their lesson preparation just to reinforce their lesson preparation …” (HoD) The respondents agree that there must be a performance review on each assessment undertaken so as to close the gaps and hence improve their teaching of mathematics. An understanding of the factors which govern effectiveness can enable teachers to create more effective test questions and also regulate and standardise existing tests. This will probably enhance better outcomes in mathematics as the issue under study.

The respondents said the diagnostic report will assist in identifying challenges or threats to teaching and learning of mathematics in their schools. They further said it will assist defining strategic planning goals for school effectiveness and effective mathematics teaching. Finally, the analysis will benefit their schools in improving learner outcomes in mathematics.

Sub-theme 5: Realistic attributes

“We have got a serious challenge of learner-behaviour in our schools nowadays. They are rude, they are rowdy, and they are disrespectful towards the teachers” (HoD)

All the respondents maintain that there is a problem of discipline in their schools. Although there are excellent intervention policies to deal with discipline, interviewees indicate that it is very difficult to implement and handle these learners because they are so wayward. Respondents are concerned about the frequency and level of disruptive behaviour, cheekiness, disorderly conduct, laziness, uncooperativeness, and verbal abuse of learners by other learners which are the most important aspects of learner control. These participants believe that lack of learner control detrimentally affects school effectiveness and effective mathematics teaching as it affects their teaching and assessment processes. Respondents indicate that since the abolishment of corporal punishment as dictated by SASA 1996 Section 12(b), learners are so disrespectful and
compromise teachers’ effectiveness, since in some instances they spend more time trying to deal with ill-disciplined learners. Some respondents indicate that learner behaviour and frequent absenteeism contributes towards their ineffectiveness together with the policy of learner progression to the next grade.

“The learners lack commitment as far as their studies are concerned and even towards their schoolwork; they don’t show any commitment and responsibility” (PL 1). Respondents said that ill-disciplined learners lack commitment in doing their day-to-day mathematics tasks which affects teachers’ total focus on industrious learners and that ultimately impacts negatively in the overall school effectiveness and effective mathematics teaching. Learners with unbecoming conduct (such as disregarding all advice by the teacher, failing to do and/or finish tasks given, showing lack of respect to the teacher) have a tendency to spend more time being reproached or outside the classroom. Consequently, the contact time of genuine teaching as well as learning is lessened.

Sub-theme 6: Knowledge of error analysis

“Most of the time, item analysis is done at those formal tasks, therefore that’s only when we analyse. I only analyse learner errors during examinations at a point whereby we are doing the item analysis. Honestly we only check that error analysis at item analysis”. PL 1. All the respondents utter the same view that they are able to interpret learner activities and performance as they commonly do error analysis after formal tasks like examinations. This clearly shows that respondents do not do error analysis throughout the teaching and learning process to the fullest.

The respondents have the opinion that teachers are results-orientated throughout the year as there is continuous assessment on a daily basis but in reference to the analysis of errors, less is done. They consider teachers’ effectiveness in teaching and learning practices of mathematics lead to better results through the error analysis process but have little time to effectively do it hence sometimes they gloss over errors. The respondents agree that they are able to explain the steps needed to get to the correct answer and the appropriate conceptual links between the steps (Hugo, 2015) but their number one enemy remains the time for a satisfactory job.
Sub-theme 7: Instructional capacity and content knowledge

“…after introducing them to a certain topic, necessarily what I do first is to check their prior knowledge and then introduce the lesson ….and then we can discuss as a group .....I always become open and ask for help in case I need clarity regarding some topic or whatsoever ....”

Most respondents argued that they plan their lessons in advance and have a strong pedagogical content knowledge. Respondents further said that sometimes they meet to discuss challenging topics in order to have a common strategy to overcome the challenge. Respondents argue that they frequently evaluate their learners to check their progress using different methods appropriate to the topic. Respondents revealed that they seek information about their learners’ strengths as well as weaknesses and team up with parents to motivate their mathematics learners. Respondents emphasise that they give learners individual feedback timeously and use various assessment data to adjust teaching. Respondents say that they use mathematical language during their lessons presentations as they have sound knowledge of the subject content.

“….our learners are assessed regularly either formally or informally so I normally make sure that I prepare my assessment and most of the time I use previous question papers to assess them .... my learners are assessed properly according to CAPS”. Most respondents make the case that they assess their learners regularly as outlined in the CAPS document. Teachers are viewed as the basis of information for learners, which is distinctive of an academic belief. Incidentally, the curriculum lists teachers among many sources from which pupils need to access information. Nonetheless, the curriculum also embraces a learner-centred philosophy in those teachers must make “decisions regarding the [instructional] sequence” grounded on learners’ capabilities. This lays emphasis that teachers should “have a sound understanding of how to recognise and address barriers to learning, and how to plan for diversity” (DBE, 2010).This suggests that teachers should familiarise themselves with the curriculum as well as instruction in accordance with the needs and capabilities of learners.

Teachers are in authority for assessing learners by planning mathematics assessment tools which are developed to assess learners in all their abilities during the course of learning. Mathematics teachers are also expected to design and use evaluation tools using particular processes. Consequently, it looks as if teachers’ assessment in mathematics supports academics as well as social effectiveness beliefs. This is due to the fact that in these beliefs, the kind of assessment
tools is normative as well as criterion-referenced. Altogether, the role of mathematics teachers moves between learner-centred, scholar academic as well as social effectiveness principles.

Sub-theme 8: Professional development (PD)

“I’m not sure about other schools but in our schools there is that internal professional development for each and every department” (HoD). Half of the respondents shared the same view that there is PD in schools. The respondents further shared the sentiment that PD deepens their subject matter understanding and they are able solve problems with sound decisions.

“My school doesn’t show any professional development of mathematics teachers because we are aware of events like AMESA but in our school it’s a challenge for us to even attend that conference of mathematics teacher development” (PL 1). The other half maintains that their schools are doing nothing concerning PD for they are denied the opportunity to attend developmental conferences. This suggests that mathematics teachers in some schools are unable to create a more dynamic and engaged learning community. Though there are workshops on the new curriculum, being denied a chance to attend conferences may cause job dissatisfaction leading to ineffective mathematics teaching.

Analoudi (2007) avows that empowerment prepares employees (teachers) to make quality decisions, set their individual goals as well as solve problems. It is therefore imperative for schools to ensure that there is PD available for empowerment and capacity building of mathematics teachers which will possibly have a positive effect on the teaching of mathematics. PD is vital as it contains learning activities capable of improving individual performance through changes in knowledge, attitudes and skills. Respondents are concerned by lack of workshops, opportunities and development in their careers which will empower and sharpen their mathematics teaching skills so that they can perform their jobs effectively hence improve learner performance.

Sub-theme 9: Teacher collaboration

“All the stakeholders must play their role of effective communications, if we become effective communicators amongst ourselves then effective teaching can be achieved”(PL 1). Collaborations between learners and teachers play an important role to achieve effective teaching
of mathematics, which my work partially builds on. Otherwise participants allege that they have a jovial relationship with most of their colleagues and learners grounded on shared respect. Effective teachers collaborate with one another, parents as well as administrators mainly for the success of learners with special needs and those at risk.

“…..teachers of particular grades they meet together and share good practices with my help as the HoD….”(HoD). The respondents maintain that working together empowers and capacitates them fully as they happen to share good practices. The respondents believe that if there is a good collaboration amongst all parties, schools will run smoothly with guaranteed school effectiveness and effective mathematics teaching leading to improved learner outcomes in mathematics.

Sub-theme 10: Proper support

“Providing educators with resources, by providing support, support by stakeholders, different stakeholders, principals, the HoD’s you know, even the department themselves, itself, if they do support educators they show that they are really important” (PL 1). All the respondents, except one, feel that the support from all the stakeholders is insufficient. They indicate that guardians/parents do not assist them with their learners’ work at home and these are very serious problems which affect the effectiveness of teaching. This is in contrast with Meighan (1994) who maintains that there is a strong significant association between parental interest and encouragement of their learners at school and educational performance in academic tasks.

The academic problems include amongst others, no submission of tasks by learners which results in no marks for those tasks. They indicate that learners sometimes do not even bother to do their investigations or projects which end up leaving gaps in the subject mark sheets due to no work having been written. Respondents also indicated lack of support from the SMT due to non-compliance or reluctance of learners in doing their work timeously, and non-submission.

Respondents were concerned that they need more support in the intervention strategies rather than only having a policy, which does not speak to learners or is just a written article, of which learners do not understand its importance. Schools do buy some LTSM using section 21 allocations, and the department sometimes takes time to augment these due to the snail’s pace of delivery caused by some fraudulent suppliers. This issue greatly compromises school effectiveness and effective teaching, because these materials enhance better teaching and
learning of mathematics. LTSM have value for better teaching and learning as they maintain that constructivism is a process of adding new information to what is already known. The learners will not be able to add new information on their own if they do not have textbooks to refer to.

Sub-theme 11: Poor school leadership

“Autocratic principals, as an example, management which is found in the different schools, if principals of different school impose instruction or come up with their own ideology on how things must be done then teachers will not be satisfied. They impose things that teachers cannot perform on their daily basis obviously teachers will never be effective” (PL 1). Participants said that they are dissatisfied with a dictatorial kind of leadership in their schools. They said that education is a cooperative and collaborative system, but they are discontent about the way management runs schools. This is supported by Battram (1999) who emphasises cooperation, as a school is a complex system. They allege that management imposes binding instructions on them irrespective of the circumstances.

“…the results of mathematics are continuously dropping or there’s no any sign that shows improvement primarily because SMT in our school tends to treat for example Setswana and English and mathematics as the same subjects...”PL 1. The respondents indicate that sometimes SMT lack clear decision-making and vision for mathematics. They indicate that their SMT’s sometimes exploited their authority or demonstrate inappropriate diplomacy when they address concerns. The respondents are concerned that this kind of leadership drops teacher motivation which eventually affects school effectiveness and effective mathematics teaching. They indicated that this makes the life of teachers very difficult and they get so dissatisfied in their work place. They indicated that the situation creates disharmony as well as stress to colleagues. Consequently, the relationships amongst colleagues undergo a slow death. Respondents argue that poor teacher relationship in schools lead to gross insubordination and teachers neglecting work. Complexity theory displays a concern in schools/institutions about the cooperation of groups and the manifestation of new customs of behaviour, which leads to inefficiency. In contrast, some respondents contend that when there is a good relationship, mathematics teachers feel highly supported and satisfied; they perform excellently i.e. there is effective mathematics teaching and improved learner performance in mathematics.
Sub-theme 12: Overcrowding

“There are issues of overcrowding in schools due to population explosion. A teacher has more than enough learners in his class or in his or her classroom”. (HoD)

The national average of learner-educator ratio (LER) at ordinary schools in South Africa increased by a net 0.3% (SSA, 2015). Respondents are concerned about the overcrowding in the classes they teach. They argue that overcrowding leads to more workload as they are unable to handle so many learners at the same time in classes. They indicate that teaching and learning is not that effective due to high numbers in classes. They further indicated that they are unable to give slow learners some special attention. They believe overcrowding reduces their morale.

5.3.3.3. Observations schedule

The main issue in presenting and analysing the observations schedule data was to present observed authentic factors contributing to good and/or poor learner outcomes in mathematics from both schools.

- **Demographic details**: The range of mathematics teachers’ age is from above 20 to above 60 years as there is a pensioner paid by the SGB in school B. Almost all learners speak Setswana as their mother tongue. They are mostly from the communities around the schools. The majority of these learners are female. The average age of the learners I observed is 18 years and more than 90% of them never repeated a grade.

- **Time management**: School time is observed as set by the school governing body (SGB) and guided by ELRA (Education Labour Relations Act). All teachers sign the time register on their arrival at school and when the school goes out. There is a movement register when teachers need to attend some challenges and workshops during the course of the school day. All learners are expected to be in class throughout the day but many learners come late to school A rather than B in the morning. There are period registers to monitor class attendance of teachers and learners. Most learners go to their respective classrooms five minutes late after lunch-break. This matter appeared to be a challenge for teachers as well.
Classroom climate and communication: It is important to create an effective classroom climate in which learners feel safe and motivated to engage, collaborate and cooperate in the pursuit of mathematical understanding before the instruction starts. Though most teachers focused on what is to be taught, why it is taught and what mastery looks like, learners were immersed in a teacher-centred classroom environment. There was minimal (around 40% of the lesson) learner interaction with the teachers and the culture of questioning was less encouraged. Learners should be in a learner-centred environment which includes teachers who are aware that learners construct their own meanings, beginning with the beliefs, understandings, and cultural practices they bring to the classroom. With the adoption of a constructivist philosophy, mathematics teachers now advocate more active learning on the part of learners and a more facilitative role for teachers. A key component of most new instructional strategies is that students are expected to discuss mathematics with their peers and their teachers (Casas, 2011). This new emphasis upon mathematical communication is a challenge for teachers and students in classrooms elsewhere, which suggests a new avenue of research.

Teaching and learning in the classroom: The teachers were able to introduce the lessons by effectively activating relevant prior knowledge as well as showing clear links with new content chosen form CAPS. This is the cornerstone of Lovitt and Clarke (2011) who opine that such elements provide balanced and rich mathematics lesson. The lessons have been planned thoroughly. CAPS aims were broken down into clear lesson objectives. The teachers in school A appear to be effective in the teaching of mathematics which is in contrast with Brophy and Good (1986) who advocate that poor mathematics performance is linked to whole class teaching. Teaching and learning activities made good sense with the learning activities progressing from lower to higher levels which is not in contrast with DBE (2013) and Krahenbuhl (2016). Learner activities were meaningful and aligned with lesson objectives. Almost all lessons were around 70% teacher-centred activities as opposed to what is advocated by Doolittle and Hicks in Doolittle (2014) hence there is a need for more learner engagement with the content during teaching and learning in the classroom. At the same time, teachers should use the instructional design models for direct teaching/instruction and frame opportunities for discovery learning at
appropriate times. Teachers were always motivating their learners which enhanced learner affinity to them.

- **Subject content knowledge and pedagogical content knowledge**: The teachers from both selected schools demonstrated excellent content knowledge (CK) and the skills to teach specific content (PCK) by using mathematical explanations, models, technologies, activities, etc. whilst managing their classrooms effectively. The provision of LTSM makes lessons more fun for learners as well as teachers and assists them in comprehending certain mathematical concepts. It is therefore of the outmost importance for teachers to emphasise the use of LTSM in the endeavour to make the lessons better and to enhance their teaching. Learners were taught new ways of learning and working as advocated by constructivist philosophy and hence there was an increase in the learning repertoire.

- **Evaluative skills**: Assessment was well aligned with teaching and learning. For teachers to make a positive impact on teaching and learning in the classrooms, they should begin to steer away from a predominantly teacher-centred approach to a more learner-centred approach. Teacher-centred approach exemplifies the process of ‘one-way communication’ and, as such, has been criticised; yet some teachers continue to use it in schools. Some learners are unable to take the responsibility for their own learning hence there is no self-assessment. Some learners were capable of doing self-assessment tasks on the concept learned during the lesson. Some of the learners, especially in school B, were able to evaluate procedures followed in the problem-solving during the concept taught. According to the DBE (2010), “formal assessment provide teachers with a systematic way of evaluating how well students are progressing in a grade and a particular subject. Formal assessment tasks are recorded and used to determine whether students should be promoted to the next grade”. In other words, the purpose of assessment is to measure learner progress and to certify that learners have the skills that are developed in a particular grade, a feature of scholar academic and social efficiency ideologies. In this regard, there are two types of formal assessment, namely, continuous or school-based assessment as well as an internal end-of-year examination. As indicated above, continuous assessment is used to measure learners’ progress in attaining skills and
knowledge within a grade. Continuous assessment may also be used to diagnose learners’ abilities to facilitate further learning. End-of-year assessment is used to certify that learners have attained skills and knowledge required in the succeeding Grade. Both continuous and end-of-year assessments referred to above are typical of scholar academic and social efficiency ideologies

- **Reflective:** Professional development (PD) provided teachers adequate time to reflect on how to improve their work in both selected schools. Teachers were able to review learners’ work. PD encourages teachers to employ an integrated communal approach. Through PD, teachers were able to reflect on and identify how learners learn the content and raise their performance, which DBE and the Department of Higher Education and Training (2011) emphasise. Most learners in school B were constantly engulfed in the world of exploration of errors. Learners were able to respond to the questions assessing their understanding of the learned concepts in school B. In school A, very few learners were responding positively but their teachers were trying their utmost best. On average, learners reflected on the decisions they made in solving particular problems. The outlined issues will possibly lead to effective teaching and improvement (DBE, 2013).

- **Error analysis:** Some learners were able to identify mathematical errors committed when dealing with the concept taught. In some cases, learners were able to question the pros and cons in using a specific approach to solve problems. Some learners were able to identify incorrect ways and used alternative approaches to solve problems based on the concept taught, which characterised the constructivist philosophy. Most teachers were able to unpack mathematical procedures while probing learners’ errors. Some teachers were able to go beyond stating the actual error by using probing questions to try and follow the way learners’ reason about the error. Prompt addressing of learner errors is of critical importance in enabling such learners to minimise learning challenges and hence increase their chance of academic success.

- **General conduct around the school due to the school conditions:** Some learners are generally inattentive especially in school A. The management always attempts to address various learner and teacher problems which are generally accepted as strong elements of leadership, though most learners come late in school A. There is effective control and
management of the complex learning environment in school B. Communication and workplace conditions are generally good in the two selected schools, though there will always be skeletons of cabals in any institution. Some learners have a negative attitude towards mathematics and even their teachers, which possibly contributes to their poor outcomes in mathematics. The school expenditure on learners and/or teaching and learning is greatly associated with better learner outcomes which are not in contrast to the spending in school A, with less LTSM in school B. The researcher observed that mathematics teachers meet departmentally on a weekly basis regarding lesson planning. The matter is envisaged to promote effective teaching to all the learners hence better learner outcomes thorough teacher collaboration. More classes were overcrowded. This is a general concern as there are some classes with more than 50 learners.

5.3.3.4. Document analysis

A comprehensive documentary analysis was done to verify and substantiate the information collected during interviews and to strengthen the evidence about how the schools operate. The documentary analysis was done by perusing the documents for information that could support or refute the data collected through questionnaire, semi-structured interviews, focus-group discussions and observations.

The analyses below emphasise that an effective system has strong programmes, i.e. practices within the system are consistent and tightly connected. The complexity philosophy is drawn to frame this effective system in this study. The following section provides a record of the documents analysed and the supporting data culled from the documents for the following areas of school performance (these areas are those used by the schools to conduct their evaluation): annual teaching plan; lesson preparations; management and its functionality; assessment plans; teaching and learning; learner achievement and mark sheets.

- **Comparison of data from documentary analysis**

The documentary analysis results of the two schools were obtained by comparing the two schools in terms of the areas used to do the documentary analysis.
Analytic tables

An analytic table was drawn up for each of the areas subjected to documentary analysis. A comparison was made between highly effective and less effective schools to determine the aspects influencing school effectiveness.

- **School management and its functionality**

The functionality of a school refers to how successfully it operates to achieve the goals it has set for itself. Schools with effective management can be distinguished from their less effective counterparts by virtue of their high level of planning, organisation, leadership and control. The table therefore presents a comparative analysis of highly and less effective schools in this regard.

Table 5.19 School management and its functionality

<table>
<thead>
<tr>
<th>Less effective (A)</th>
<th>Highly effective school (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work is planned in advance but the implementation is difficult;</td>
<td>effort is well scheduled in advance;</td>
</tr>
<tr>
<td>committees dominated by teachers;</td>
<td>working groups are descriptive of teachers as well as parents;</td>
</tr>
<tr>
<td>average record-keeping structures;</td>
<td>good record-keeping systems in place;</td>
</tr>
<tr>
<td>teacher development is questionable;</td>
<td>teachers developed as well as mentored to expand performance;</td>
</tr>
<tr>
<td>communication is at times verbal;</td>
<td>worthy communication approaches;</td>
</tr>
<tr>
<td>orientation of new teachers a rare existence;</td>
<td>induction as well as motivation of teachers a main concern;</td>
</tr>
<tr>
<td>school-improvement procedures not well enunciated;</td>
<td>well-articulated school improvement plans;</td>
</tr>
<tr>
<td>meagre control of work, designation, decision-making;</td>
<td>work organised by HODs as well as management is dexterous in delegation; decision-making,</td>
</tr>
<tr>
<td>human relations are tolerable;</td>
<td>good organisation of interpersonal relations;</td>
</tr>
<tr>
<td>extracurricular events relatively intermittent and schools below par marketed;</td>
<td>extra curriculum well-planned as well as schools well-marketed;</td>
</tr>
<tr>
<td>learners are occasionally included within the learning process;</td>
<td>all learners incorporated in the learning practise;</td>
</tr>
<tr>
<td>performance of the school once in a while reviewed;</td>
<td>performance analyses conducted on a regular basis;</td>
</tr>
<tr>
<td>time is apportioned as a prerequisite but not used cost-effectively;</td>
<td>time is apportioned as a prerequisite but not used advantageously;</td>
</tr>
<tr>
<td>there are learner as well as teacher turnover period registers;</td>
<td>there are period registers to control learner turnover;</td>
</tr>
<tr>
<td>all teachers have their own time-tables;</td>
<td>all teachers have their own time-tables;</td>
</tr>
<tr>
<td>some staff associates are habitually not well versed with their obligations.</td>
<td>staff associates are usually not well versed with their obligations.</td>
</tr>
</tbody>
</table>

In briefing, almost all the findings in School B, where the intervention took place supported the characteristics of effective school. Though there are some similar elements present in School A,
the comparison school, it strongly suggests that the findings from highly effective school can be used as the school conditions which can mark schools effective as further supported by Table 2.1 in Chapter 2.

- **Teaching and learning**

Teaching and learning refers to how well teachers plan and facilitate teaching and learning. It also takes into account how teachers assess learners in order to achieve the curriculum objectives. It further includes learning or instructional approaches which determine the method for accomplishing mathematics learning objectives. They are also in the pre-instructional actions, data presentation, learner activities, testing, and follow-through. The strategies are generally tied to the needs as well as interests of learners to improve learning in mathematics through constructivist teaching like problem-solving.

Problem-solving ought to be integrated into lessons during the course of the year. Full integration of problem-solving in the classroom provides learners with “the chance to solidify and extend what they know and can stimulate mathematics learning” (National Council of Teachers of Mathematics, 2000:52). A true problem necessitates learners to use prior learnings in new techniques and perspectives. Problem-solving involves and builds depth of theoretical understanding and learner commitment.

The table below presents a comparative analysis of how teaching and learning differ at highly effective and less effective schools.

<table>
<thead>
<tr>
<th>Table 5.20 Comparative analysis of teaching and learning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>School A</strong></td>
</tr>
<tr>
<td>average levels of teaching as well as learning;</td>
</tr>
<tr>
<td>average planning as well as work often uncompleted;</td>
</tr>
<tr>
<td>some teachers do day-to-day lesson planning whereas</td>
</tr>
<tr>
<td>others weekly;</td>
</tr>
<tr>
<td>average record-keeping of teachers’ as well as learners’</td>
</tr>
<tr>
<td>work;</td>
</tr>
<tr>
<td>schools depend on comprehensively on summative</td>
</tr>
<tr>
<td>assessment as well as lecture procedures are used to</td>
</tr>
<tr>
<td>teach learners;</td>
</tr>
<tr>
<td>HoDs are from time to time unenthusiastic to give</td>
</tr>
<tr>
<td>upkeep to teachers;</td>
</tr>
<tr>
<td>There is improvement of teachers;</td>
</tr>
<tr>
<td>support by learning implementers commonly aligned on</td>
</tr>
<tr>
<td>subject supervision and less on subject content;</td>
</tr>
<tr>
<td>the school has sufficient LTSM.</td>
</tr>
</tbody>
</table>
It was in school B that what transpires in the classroom adds to the effectiveness of the school. However, some findings in school A are that poor performance in mathematics has continually been associated with whole class teaching.

- **Learner’s books**

The researcher requested teachers to provide the learners’ writing books. The aim was to find how learners perform in order to detect their level of competency. The researcher also wanted to see what problems the learners were encountering with regard to their work/activities. It was furthermore the researcher’s intention to observe how the teachers assessed the learners. The researcher wanted to see whether teachers give the learners the credit they deserve and hence motivate them, or whether they are too strict in their marking or give negative remarks that would demotivate the learners.

Generally, all participants gave the researcher samples of the children’s work. This helped the researcher to establish where learners encountered problems and what intervention could be undertaken. In both schools, the researcher observed that there are some learners who experience serious dyscalculia difficulties. Most of the work of the learners indicated some clear understanding. However, a few demonstrated severe mathematics difficulties.

- **Annual teaching plans**

The researcher requested the teachers to provide their work schedules. The rationale for doing this was to establish whether the teachers have work schedules, which resources they use when teaching mathematics and what approaches the teacher is going to use. The researcher wanted to discover whether the work schedule catered for individual learners or for groups and not only for the entire class as a unit.

This would also help the researcher to establish whether the work schedule included activities for various kinds of learners, not only for capable ones. The researcher wished also to look at the assessments envisaged so that he could compare them with how actual assessment was done by teachers. The researcher hoped to discover whether the teachers adhered to the content of the work schedule or deviated from it.
According to the teachers, the work schedule sometimes confuses them by for instance providing too many tasks dealing with different themes: for example, where learners are expected to learn various dissimilar topics in one week. This becomes a problem for the teachers as to what to concentrate on, and for the learners to master all these in a single week.

Generally, all participants possessed well-prepared work schedules. In each school the researcher realised that teachers were following different themes although they were using the same CAPS mathematics document. Most teachers give the same tasks to all learners irrespective of whether all the learners understand or not. The researcher believes that teachers should provide different tasks/activities in their work schedules as not all learners learn mathematics in the same manner.

- **Learner achievement and mark sheets**

The success rate at which learners achieve the curriculum outcomes is called learner achievement. The following table compares the achievement of learners in highly effective and less effective schools.

Table 5.21 Analysis of learner achievement and mark sheets

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower pass rate at all grades;</td>
<td>high pass rates in all grades;</td>
</tr>
<tr>
<td>proper records of learner attainment kept;</td>
<td>records of learner attainment well-maintained and imitate a continuous style to assessment;</td>
</tr>
<tr>
<td>assessment mostly summative and standards sustained;</td>
<td>assessment standards maintained;</td>
</tr>
<tr>
<td>learners are inspired and achievements less celebrated;</td>
<td>learners frequently motivated and their achievements celebrated.</td>
</tr>
</tbody>
</table>

To ensure assessments are valid and appropriate, teachers need to be knowledgeable about the diverse ways that pupils learn and master specific concepts. Teachers need to determine appropriate starting points for the learners, and then continue to assess individual and class progress as ideas, topics and/or skills develop. When learners are not progressing at the same rate, or there are variations of strengths and weakness, assessment for learning provides valuable data on which to base differentiation decisions. With this knowledge, teachers can provide their
learners with descriptive, ongoing, timely feedback during the learning process and design new mathematics tasks that challenge learners’ ideas and build new strong, deep understanding.

Teachers find that they need to differentiate instructions for all learners to succeed as they focus their assessment practices on the learning of mathematics. This means that teachers will plan some activities that are for all learners or groups of learners and some that are for individual learners. They will use multiple, adaptable resources for learners, including a wide variety of digital resources. They will design and support learning that provides learners with multiple, flexible ways to give expression to their emerging understandings. And they will design engaging differentiated tasks and activities based on their learners’ understandings, the outcomes in the programme of studies, the mathematical concepts they are addressing and what they know about how people learn. The above findings will probably mark schools and teachers as effective in mathematics teaching in secondary schools.

5.4. SUMMARY

The results of the study were presented in this chapter. It involved data presentation and analysis of both the quantitative and qualitative phases. This chapter covered data analysis and interpretation with some discussions. It also contained a built-in literature analysis to support the study’s thesis. It covered the analysis of questionnaires of both learners and teachers, documents, semi-structured and focus group interviews conducted in the gathering of data for this study. The following chapter will focus on the findings, conclusions and recommendations to the study.
6.1. INTRODUCTION

The study sought to identify possible determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in secondary schools. In this chapter, the researcher presents a critical discussion of the quantitative and qualitative findings presented in Chapter 5 in order to answer the research questions. This chapter is divided into themed sections starting with the discussion of biographical data and then moving on to possible teacher, learner and school conditions respectively which may influence effective mathematics teaching. The themes are formulated around the research questions guiding the study and have been consistently used for structuring in chapters 2, 3, 4 and 6 of the study.

There is much synergy between the quantitative and qualitative results to answer this question as the determinants appeared to complement each other. The participants provided what seem to be solid evidence and reasonable justifications to substantiate their views on school effectiveness and effective mathematics teaching. The qualitative data supported and informed the quantitative results given the explanatory nature of mixed methods in this study. Participants agreed that the complementary determinants will improve their effectiveness, insights, skills as well as knowledge for better performance in mathematics. Both participants and respondents also confirmed the point that the determinants have strengthened their knowledge of the theories, approaches and strategies which are seen or viewed as enablers of effective mathematics teaching in the classrooms.

6.2. ANALYSIS OF THE DEMOGRAPHIC DATA

There is a larger female teacher labour force in teaching as a profession (Statistics South Africa, 2015). It is also confirmed in the case of the participating schools by Table 5.1 in this study. Such evidence may suggest that female teachers are more likely to be passionate about teaching as a profession as prompted by their numbers in the selected schools. However, the findings of Crossman and Harris (2006:32) suggested that both men and women show the same degrees of
passion whereas Hickson and Oshagbemi (in Crossman & Harris, 2006) contend that in the UK, women display higher degrees of passion in teaching. Furthermore, the population census of 2011 conveyed that there are more female teachers in the North West province and South Africa (Statistics South Africa, 2012).

This study reveals that the teacher work force is constantly ageing as shown in Table 5.1. Similar studies found that 2015 youth labour statistics released by Statistics South Africa (2015) which revealed that there has been an increase in youth unemployment and participation in the economy since the 2008 economic recession. The results reveal that the majority of teachers’ ages ranged from 31 years to 60 years. This suggests that the educators in the selected schools comprise mature teachers who have the capacity to deal with learners as well as to perform their duties. This further puts forward that the confidence level in mathematics teaching skills is higher indicating better motivation as well as job performance, for these teachers in the teaching fraternity. Oshagbemi (in Crossman & Harris, 2006) posits that mature teachers cope well with job associated matters since they have established more approaches in the teaching field. Nonetheless, the 16.7% of young teachers depicted in Table 5.1 is possibly associated with the damaging sentiments due to the changing educational system in South Africa, curriculum changes, poor service delivery, lack of learner discipline and bullying linked with lack of safety and security in schools, as supported by interviews and observation in the qualitative phase.

The study reveals that there are more Setswana-speaking teachers in the selected schools as one would expect in a predominantly Setswana-speaking area. The aspect of language barriers is closely associated with different cultures. However, it is inconclusive that poor performance of some learners is due to language barriers in the selected schools yet Sepeng (2015) found that language has an impact on mathematics performance. It further suggests that race and culture is not an issue as there were no incidents of racial discrimination reported so far in these schools.

This study reported that 100% of the respondents in the selected secondary schools have higher education qualifications. These teachers’ qualifications are a good picture of what a qualified cohort of mathematics teachers must be made up of. This is not in contrast with DBE (2016) which reported that the number of employees with a tertiary education is increasing and made up 20% of the labour force in 2014, while graduates consist of 7% of the employed labour force.
6.3. RESULTS RESPONDING TO RESEARCH QUESTIONS

The results of the data analyses presented in chapter 5 are used to address the research sub-questions in this study. The questions are addressed in the following section. The researcher’s main research question was: What are the possible (learner, teacher and school) determinants of effective mathematics teaching in the selected secondary schools? (Chapter 1, page 5).

The focus of this research problem investigated was best expressed by upholding the following sub-problems:

- What factors/conditions of teachers facilitate effective mathematics teaching?
- What learner conditions enable effective mathematics teaching?
- What school conditions enable effective mathematics teaching?
- How do the views of teachers, learners and heads of the department support their perspectives on school effectiveness and effective mathematics teaching towards the development of an improved learner outcomes model?

6.3.1. Question 1: What factors/conditions of teachers facilitate effective mathematics teaching?

To address this question, data were collected through the use of both quantitative and qualitative methods. In response to this question, findings suggested that several factors influence effective mathematics teaching. The findings from both quantitative and qualitative sets of data indicated that the identified teacher attributes are strong contenders which may facilitate quality mathematics teaching. Teacher attributes investigated are technically aligned to the constructivist ideology. The technicality is that the teacher creates a mathematical environment to allow learners to build mathematical knowledge (Prawat & Floden, 1994; Larochelle & Bednarz, 1998; Jonassen et al., 1996; Morrison & Collins, 1996; Jonassen, 1991, Doolittle, 2014; Krahenbuhl, 2016). This environment is most likely to afford learners with chances to link their understanding, use resources, make assumptions and test their thinking in the direction to construct mathematical knowledge.
The findings in Table 5.4 with a high mean score of 3, 25 showed that mathematics teachers in the selected schools have a high regard for classroom management. The findings suggest that these teachers command good classroom management. These mean scores being higher than the mid-point range advocate that these mathematics teachers carry out rigorous but flexible measures as far as classroom management skills are concerned. A majority of interviewed teachers indicated that timeous arrival in the classroom for a lesson greatly assists in the management and organisation of the classroom. The view such as “I always emphasise the arrival on time, time management towards the learners as well as me as a teacher……and also encourage them to respect each other, not to disrespect the teacher…” supports the quantitative responses in Table 5.4. Nonetheless, the above issue is in contrast to what I observed. Most teachers and learners arrive five minutes late in their classrooms which seem to suggest that the school management must attend to this challenge, as advocated by Burns (1978), who describes leadership as the inspiration of individuals in their respective institutions.

Findings depict that all participants in Table 5.5 are in agreement about the creation of a conducive environment in mathematics classrooms and it is also supported by respondents in the qualitative phase like “I start the lesson …. I motivate these kids that to perform to try to do their best. I always tell them our background doesn’t make us….” I further observed that teachers were able to introduce lessons by effectively motivating learners hence creating a conducive environment for learning. In actual fact, various studies seemed to suggest that in a compassionate learning atmosphere, emphasis on learning holds academic significance as well as full usage of existing learning time which have been revealed to be the basic components of effective schools (see for example Molnar et al., 1999; Nye, Hedges & Konstantopoulos, 2001; Fidler, 2000; Reynolds & Teddlie, 2000; de Frondeville, 2009). The most widely accepted explanation is that classroom climate is one among the significant predictors of learner success (Brophy & Good, 1986; Mortimore, 1993; Muijs & Reynolds, 1999). Lastly, an effective mathematics teacher views quality teaching and learning develops a good classroom atmosphere as well as school climate that value civic-mindedness and diversity (Goe et al., 2008).

Table 5.6 depicts that all mathematics teachers in the selected schools use the assessment data to improve their teaching. The interview supports the issue as “We do item analysis, we check in which topic or subtopic do the learners have some difficulties and then … make a diagnostic
report and then our educators use it in their lesson preparation just to reinforce their lesson preparation ...”. The respondents agreed that the diagnostic report will assist in identifying challenges or threats to teaching and learning of mathematics in their schools. In actual fact, the researcher observed that teaching and learning tasks made good sense with learning activities with the help of the CAPS document, which concurs with DBE (2013); Lieberman and Miller (2000) and Krahenbuhl (2016).

Table 5.20 is not in contrast with the findings above that teachers do motivate learners frequently and their achievements are celebrated. Aiken, Hawley and Vanjani (2007) report that there are regularly high correlations between learners’ assessments of the amount learned in subject matter and their overall assessments of the teacher. The researcher observed that the impact of the teacher in a classroom defines the learners’ outcomes (Aiken et al., 2007).

Table 5.7 depicts that teachers are in agreement with the issues related to realistic attributes. Respondents argued that they face various challenges on daily basis. Learner discipline is generally a great challenge in most schools but they manage to continue with their work in this trying condition, though a few disagree that some learners do not submit their tasks timeously. This is corroborated by this statement “we have got a serious challenge of learner-behaviour in our schools nowadays. They are rude, they are rowdy, and they are disrespectful towards the teachers”. Although there are excellent intervention policies like the South African Schools Act, 1996 Section 12(b) which deals with discipline, interviewees indicate that it is very difficult to implement and handle these learners because they are so wayward. Respondents are concerned about the frequency and level of disruptive behaviour, cheekiness, disorderly conduct, laziness, uncooperativeness, and verbal abuse of learners by other learners which are the most important aspects of learner control. These participants believe that lack of learner control detrimentally affects school effectiveness and effective mathematics teaching as it affects their teaching and assessment processes. The researcher concludes that lack of learner discipline as well as commitment is a direct connection to poor learner performance (Van der Westhuizen et al., 2002).

Respondents indicate that they have high workloads which impede them from doing their best. High workloads are the obstacles to productive roles of teachers since they create a threat to efficient and effective assessment and immediate feedback (Masitsa et al., 2004). The results
further indicate that most mathematics teachers undergo various, difficult and continuously changing demands in teaching learning conditions due to the intolerable rise of workload, but still move on.

Table 5.8 depicts that the average scores on ‘knowledge of error analysis’ are greater than their corresponding standard deviations. This confirms that the teachers are informed and practised about knowledge of error analysis in schools (Black & William, 2006). By contrast, interviewees commonly agreed that “most of the time, item analysis is done at those formal tasks, therefore that’s only when we analyse. I only analyse learner errors during examinations at a point whereby we are doing the item analysis. Honestly we only check that error analysis at item analysis”. This view suggests that respondents do not do error analysis throughout the teaching and learning process to the fullest unless they did not understand the concept ‘knowledge of error analyses’ during the interviews.

The researcher’s observations support Table 5.8 in which mathematics teachers were hard at work during their lessons correcting methodical mistakes when learners come across new problems and inaccurately generalise solutions, particularly when they were unclear of what to do. Black and William (2006) maintain that teachers are dynamically carrying out formative assessment the minute they respond to learners’ errors in their classrooms, throughout the lesson. Schunk (2004) also agrees that learners and adults often construct procedures to solve mathematical problems; nonetheless, the errors are unplanned but rather methodical mistakes. Methodical mistakes reveal the constructivist beliefs that learners form methods based on analysis of their experiences. In some cases, learners were able to question the pros and cons in using specific approaches to solve problems. Some learners were able to identify incorrect ways and used alternative approaches to solve problems based on the concept taught which characterised the constructivist philosophy (Krahenbuhl, 2016). Most teachers were able to unpack mathematical procedures while probing learners’ errors, as the results lend strong support to quality teaching as maintained by Ball et al. (2005); Black and William (2006); Hugo (2015). Some teachers were able to go beyond stating the actual error by using probing questions to try and follow the way learners reasons about the error, which agrees with the views of Sapire et al., (2016).
Table 5.9 reveals that all the average scores for instructional capacity and content knowledge are greater than the standard deviations confirming that teachers are in agreement as far as this attribute is concerned. Mathematics teachers in these selected schools have in-depth understanding of instructional capacity and content knowledge on how to address mathematics topics in their classes.

The understanding of instructional capacity and content knowledge are corroborated by Bransford, Brown and Cocking (2004) who maintain that for teaching to be effective, teachers need “to have a deep understanding of the subject matter and its structure, as well as an equally thorough understanding of the kinds of teaching activities that help students understand the subject matter in order to be capable of asking probing questions.’’ Smith (cited in Rohaan et al., 2010) maintained that subject content knowledge is of utmost importance for effective teaching for the reason that “strong and useful pedagogical content knowledge cannot be built on a shaky content foundation”. The researcher reports that teachers from both selected schools demonstrated excellent content knowledge (CK) and the skills to teach specific content (PCK) by using mathematical explanations, models, technologies, activities, etc. whilst managing their classrooms effectively, which is emphasised by Doolittle (2014).

The teacher quality is one of the main determinants of the learners’ performance; having powerful content knowledge is critical for effective mathematics teaching. Ball, Thames and Phelps (2008) posit that there is enough evidence of the correlation between mathematical content knowledge of the teachers and their capability for sound teaching in classrooms. Schmidt (1999) advocates that “what teachers teach and how they teach it are affected by their subject matter, belief and preferred pedagogical approaches, things that are consequences of their training and experiences”

The respondents have shown in Table 5.10 that they are all in accord as far as professional development is concerned. All the mean scores are in excess of 2, suggesting that respondents have the strong belief that professional development is one of the determinants of effective mathematics teaching which could mark schools and teachers effective in improving learner performance in mathematics. The assessment of the effectiveness of professional growth stands in relation to its influence on pupil learning and development of teaching enactment. It does not purely reveal the attitudes of member fulfilment (Hawley & Valli, 1999).
About 70% of the interviewees share the same sentiment as in Table 5.10. They argue that professional development deepens their subject matter knowledge and are able to solve problems with sound decisions. Views like this “I’m not sure about other schools but in our schools there is that internal professional development for each and every department” supports table 5.10. The same sentiments are shared by Analoudi (2007) who acknowledged that empowerment prepares employees (teachers) to make quality decisions, set their individual goals (good mathematics outcomes) as well as solve problems. It is therefore imperative for schools to ensure that there is PD available for empowerment and capacity building of mathematics teachers which will possibly have a positive effect on teaching of mathematics. PD is vital as it contains learning activities capable of improving individual performance through changes in knowledge, attitudes and skills (Rothwell & Kazanas, 2003).

Many scholars and development institutions like SACE’s CPTD policy (see Boyle et al., 2005; Gravani, 2007; William, 2007; SACE, 2012; UN (1963) in Aworti, 2013) advise that professional development is the route which employers and professions must follow if they want to keep professionals up to date, deepen their knowledge and sharpen their skills. The view of the respondents is aligned with the findings by Van der Heijden, Boon, Van der Klink and Meijs (2009) that it is through continuous professional teacher development that teachers can keep themselves up to date with what is happening in their field. Observation shows that teachers are able to reflect on how to improve their work in both schools. PD encourage teachers to employ integrated communal work.

Table 5.11 depicts that all respondents embrace the attribute of teacher collaboration, as the mean scores are above the value of 2, which suggests that teachers draw their strength and effectiveness from teamwork in the selected schools. Pugach and Johnson (1995:178) argue that collaboration is intended to “promote the most effective teaching possible for the greatest number of students” which is supported by the Table 5.11. Interviewees share this common view that “all the stakeholders must play their role of effective communications; if we become effective communicators amongst ourselves then effective teaching can be achieved”. It suggests that communication and collaborations between learners and teachers play a critical role to achieve effective teaching of mathematics. Participants also say that they have a jovial relationship with most of their colleagues and learners grounded on shared respect.
Shannon and Bylsma (2007) maintain that consistent and flawless communication is essential in an effective school in corroboration with Table 5.11. This suggests that parents and society should play a part in school activities at various levels like governance, implementation and development including teaching and learning. Research has revealed that family participation is a crucial element in a learner’s improved school performance (Henderson and Mapp, 2002). Learners are most likely to benefit from improved achievement, improved attendance in classrooms, conduct and social expertise (Griffith, 1996; Henderson & Mapp, 2002). Constantino (2005) further affirms that the learners perform better when their parents inspire and support them, which forms the basis of a parent-learner-teacher relationship for effective mathematics teaching.

Lastly but not least, the correlation matrix was studied with a view of optimising the target values or attributes according to their relevance in Table 5.13. The results are clear according to the coefficient matrix associated with the nine variables. There is evidence of a mixture of negative and positive significant and insignificant relationships between the nine attributes. It is evident that there are high correlations (in excess of 0.3) looking at the correlation matrix. It is also evident that the correlation matrix is not unitary, providing a strong relevance between the nine mathematics teachings attributes. This further endorses the viability of the multiple relationships between these attributes.

Attributes like classroom management, creation of conducive classroom environment, use assessment data to improve teaching; instructional capacity and content knowledge are among the nine attributes which are found to be highly and strongly related with other attributes in Table 5.13. The correlation coefficients of these factors are reported to be highly significant in some cases, i.e. in excess of 0.3, than in others. Other coefficients are lower but still are an indication of a type of relationship to some extent (Norusis, 1994). This could imply that the nine factors cannot be treated as individuals. The performance of one attribute in one way or the other has a certain influence on the other attributes. The p-value of most of the attributes is less than 0.01 and 0.05 levels of significance, confirming the interrelations between the attributes. Consequently, none of the factors is achievable without considering the others. The general perception of teachers about attributes impacting on effective mathematics teaching is positive as
presented in Table 5.12. This therefore implies that the nine (9) attributes may be used as teacher conditions to build a model towards improved learner outcomes in mathematics.

6.3.2. Question 2: What learner conditions enable effective mathematics teaching?

The investigation was concerned to understand learner outcomes or learner attitudes regarding mathematics to the extent that it informed the study about the learners’ classroom teacher. The learner conditions identified to enable effective mathematics teaching are rooted in the constructivist philosophy. Constructivist theories are grounded on the certainty that learners construct their theoretical understanding and specific knowledge from their own action (Fleury, 1998). The philosophy puts prominence on learner abilities and interests as indicated in the conditions below. This theory of learning is viewed as an interior process where the learner builds meaning by processing fresh information as well as knowledge to integrate and grow previously assimilated knowledge and skills (Krahenbuhl, 2016). It is generally the approach that learners construct their own knowledge from interpreting their experiences which are immersed in the conditions investigated in this study.

The data reflected in Table 5.16 suggests that the respondents concur with all the ten learner conditions investigated. The correlation coefficients of the rotated factors are generally significant as a confirmation that respondents agreed with these factors. Self-perception on mathematics is one condition that schools should pay attention to. This can be best described by this view “…mathematics can be quite frustrating….mmm….some kind of anxiety, especially when you see that you can do something but you don’t know how to do it, because in various circumstances …”.

Mathematics anxiety has certain consequences revealed in different forms, which may affect learners suffering from it. It manifests in different ways, such as memory loss during mathematics examinations, panic or total avoidance to execute mathematics related tasks. Thus, mathematics anxiety reduces mathematics performance regardless of the learner’s level of mathematics competence (Ashcraft & Moore, 2009). Many learners fear and get frustrated when doing mathematics. Avoidance of doing mathematics can be observed in terms of learners’ involvement and participation in mathematics lessons, time spent studying mathematics and regular submission of homework. Hemree (1990) maintains that mathematics anxiety really
relates with poor performance in mathematics assessment, but also with participation in the subject, which indirectly influences attitudes towards mathematics and avoidance of it.

Each learner is unique; nonetheless, certain elements impede the performance of a learner regardless of his/her uniqueness. Some of the respondents have negative perceptions about mathematics and are frustrated when trying to solve mathematics problems. Most of the learners felt strongly driven to learn mathematics, and believe that the mathematics they learn at school is useful to them and will also be useful to assist them to get work in the future. Learners’ self-perceptions/beliefs, like teachers’ beliefs, can make learners to be either obstacles to or conveyers of curriculum requirements (Prawat, 1990).

Learner attitude regarding mathematics is the second worrisome condition which schools try to address. A general view of respondents is the hard attitude like “I am lazy and there is this thing in mathematics, so my problem is that, whenever I am lazy, I cannot practise and I am unable to obtain average marks ... .. the only reason I can give as to why I do not think that I am good right now in maths, is maybe attitude...”. Their negative attitude towards mathematics and lack of self-confidence possibly result in learner underperformance in mathematics. “I think that what contributes to my success in mathematics is having a positive mind”. Some respondents believe that a positive mind-set made them to enjoy mathematics and hence achieve better performance in mathematics.

Learners were able to access resources and skills necessary to achieve learning autonomy. In both schools, teachers systematically collect and use classroom data to assess their classroom practice and to find ways to withstand improvement efforts about recognised priorities (see Beresford & Payne, 1997; Hopkins, West & Ainscow, 1996; West & Beresford, 1998). It is also documented that learners do reflect on what they have done at school and that homework is often a catalyst for such reflection. Teachers elucidate and substantiate to learners the purpose of different teaching strategies they use like homework, collaborative groupwork, etc. which enhance learning. They give guidance to learners on the meaning of hard work. A culture in which learners are not scared to ask for assistance is driven by continuous dialogue among teachers and learners using a shared language of teaching and learning in both schools. It appeared that self-assessment is the activity in which learners normally take part including those who happen to dodge classes. Learners in both schools especially that in Grade 12, appeared to
be able to use various learning strategies which enabled them to learn independently. This capability of independent learning is equally a vital condition in their growth as learners. Learners in the selected schools were able to organise themselves with this complementary skill (independent learning) for effective mathematics teaching. Constructivist philosophy encourages independent learning and learner autonomy. Learner autonomy is the notion that learners are dynamic participants in the learning procedure and in due course are in charge of their own learning (Doolittle, 2014). This all-inclusive viewpoint is a non-reductionist method that gives emphasis to learning in the environment. One of the key roles of a teacher is to develop learner autonomy so as to build in learners a craft knowledge which makes sense of the learning situation, and actively construct as well as develop learning techniques.

“…I’m not a bright student; I have to work hard to get to understand properly in class, I have to do extra work on top of the time that I’m given with the teacher ….. when I practise at home and when I put extra work on top of the work that I do with the teacher, I find maths very interesting and I enjoy it”. The statement shows that learners have developed a battery of independent learning as well as problem-solving techniques so as to practise and learn from a wide range of information at their disposal. Respondents reveal that they benefit from different learning situations which emanate from this refined skill of independent learning.

Learners in the selected schools were able to maintain a positive relationship during my observation period. The issue enabled them to seek and receive support and help needed in mathematics. The findings concur with the research by Wang, Haertel and Walberg (1993), who advocate that the quality of learner-teacher relationship contributes to effective teaching.

Constructivist theorists hold the view that teachers should deal not only with learners having high abilities or high motivation for mathematics, but they should also look at the learners’ cognitive and affective dimensions. “Our teacher is a good man he explains nicely in a good way as compared to other educators that I’ve been with in the past I understand him in a good level than I did with the other teachers …. sends question papers that we cannot print here at school … gives us rewards which makes us feel more comfortable”. Findings reveal that respondents felt strongly that their mathematics teachers knew how to explain mathematics to them, were able to demonstrate to them how mathematics may benefit them in real life, and made them feel that they could learn mathematics.
Research by Hopkins et al. (1997, 1998) and West et al. (1995) indicate that an all-embracing teaching repertoire is one of the classroom conditions essential for school effectiveness. Based on my observation premise, the findings reveal that teachers provided learning experiences that made learners feel good about themselves by presenting the range of learning prospects. This interpretation is supported by earlier work of Joyce et al. (1997), who maintain that “increasing the range of learning experiences provided in our schools increases the likelihood of more students becoming more adept learners”.

The findings revealed that teachers tend to integrate more divergent and accommodative activities in their lessons and assessment tasks. One respondent said that “my teacher presents a new topic she does not only give it to us in one method but she gives us alternative methods of doing it and that the other thing is that for instance is the linking is the linking of concepts or linking of topics in she tells us or she makes us aware that this topic may be in algebra it links in functions it links in calculus...”. The results show that learners learn best when their mathematics teachers use different methods during the lessons. Moreover, they alleged that when their teachers use various methods for teaching mathematics, it assisted them to think for themselves in mathematics, and tried to support everyone to do well in mathematics.

Documentary findings reveal that teachers do much to change attitudes of learners towards mathematics. Teachers use a range of teaching approaches to make the lessons more interesting. The observation evidence shows that learners feel that their hard work is acknowledged by their mathematics teachers. Target-setting is further reported to have a motivational and improvement effect in mathematics performance.

Documentary findings in Table 5.19 reveal that schools do buy some learning teaching support materials (LTSM) using DBE section 21 allocations. LTSM sometimes take time to reach schools due the snail’s pace of delivery caused by some fraudulent suppliers. This issue compromises school effectiveness and effective teaching a lot because these materials enhance better teaching and learning of mathematics. Johnson and Thomas (1992) agree that LTSM have value for better teaching and learning as they maintain that constructivism is a process of adding new information to what is already known. The learners will not be able to add new information on their own if they do not have textbooks to refer to. Results concur with Weber (2008), who opines that the provision of LTSM make lessons more fun for learners as well as teachers and
assists them in comprehending certain mathematical concepts. It is therefore of the utmost importance for teachers to emphasise the use of LTSM in the endeavour to make the lessons better and to enhance teaching and learning.

Daily learner conduct in a school also influences school effectiveness and effective mathematics teaching. Evidence revealed from the observation report suggested that some learners are wayward and ill-disciplined. Tactics to maintain discipline are frequently constructed on an individual’s views of what incorporates “good behaviour” and a number of expectations relating to people as well as how they are supposed to behave (Van Wyk, 2001). Almost all participants in both quantitative and qualitative phases indicate that there is a serious challenge of learner discipline in their schools. Learners should adapt to the rules and regulations of the school community. Results show that teachers are prepared to motivate and negotiate with learners about matters within which they function.

A respondent argued that “I remember when I had failed my Grade 11 she couldn’t help me with maths because she hadn’t done maths”. Studies by Masitsa et al. (2004) aver that some parents are too committed to grace children with their presence in school activities, and some who are illiterate think they cannot contribute to the education of their children. This is reflected in this study that parents do not monitor their children’s work, which suggests that there is lack of parental support for their children’s education. Another respondent indicated that “…they cannot really help me because they do not understand what’s going on so I can say that no, they are not”. Similar results have been found in Steyn (2003) who maintains that parents are potentially powerful educational partners in society, suggesting that learners who are supported by their parents usually perform better in their school work which is associated with effective mathematics teaching by that partnership. Teachers need support if they encounter challenges with parents who do not assist their children with school work and wayward behaviour.

6.3.3. Question 3: What school conditions enable effective mathematics learning?

The researcher framed this question from the complexity viewpoint. The philosophy is based on the concepts which include, amongst others, fitness, development, hierarchy, internal models, schemas, regularity and randomness, self-organisation, selection and selection pressures, systems as well as system dynamics (Battram, 1999; Cilliers, 1998; Scheerens, 2015). The fitness,
development, organisation of the teachers in a school is challenged by changing complex issues like human behaviour (learners and school leadership) and political matters (curriculum and various policies) for their effectiveness in mathematics teaching.

Teachers react and sense their surroundings, in this manner varying their environment, which modifies the teachers again, thus proactively varying the environment; the procedure, in redoing the situation, yields continuous and dynamic change regularly (Cilliers, 1998). Teachers have to adapt to the situations to yield required results. Schools admit learners with different behavioural patterns and policies are reviewed annually; therefore they need to acclimatise so as to improve the learner outcomes. The elements of school conditions, from the teachers and learner questionnaires data, are highly supported by documents analysis, observation sessions, semi-structured interviews and focus-group discussions.

Table 5.12 and 5.13 reveal the correlation of the teacher attributes which are aligned to the school condition. These tables concur with reflected revised school characteristics viz. effective monitoring and leadership, effective teaching and improvement, supportive learning environment, communication and group effort, clear and shared focus as well as high expectations and standards in Table 3.1, utilised as a context for an effective school in this project. The worrisome elements from the findings are poor school leadership, teaching and learning as well as proper support.

Participants said that they are dissatisfied with the dictatorial kind of leadership in their schools. Another respondent said “autocratic principals, as an example, management which is found in the different schools, if principals of different school impose instruction or come up with their own ideology on how things must be done then teachers will not be satisfied. They impose things that teachers cannot perform on their daily basis obviously teachers will never be effective”. This suggests that management imposes binding instructions on them irrespective of the circumstances, which is in contrast with Burns (1978), who maintains that leadership is the inspiration of individuals to accomplish goals. This suggests that the SMT’s sometimes exploited their authority or demonstrated inappropriate diplomacy when they address concerns. The respondents indicate that sometimes SMT lack clear decision-making and vision for mathematics. Respondents argue that poor teacher relationships in schools lead to gross insubordination and teachers neglecting their work.
Nonetheless, the document analysis revealed another dimension in Table 5.18, that the selected secondary schools have a hidden treasure to improve learner outcome in mathematics. Mathematics teachers’ work is well planned in advance and committees within their department are representative of teachers and parents. The table reveals that there are good communication strategies and record-keeping in these institutions. Teachers are developed, mentored and motivated in these schools. Good school improvement plans are in place. Inclusive decision-making, quality strategic management and interpersonal relationships are some of the elements which could mark these schools as effective.

Another observation finding is that the teachers were essential symbols in the mathematics classroom, and not learners which is reflected in Tsang et al.’s (2014) findings. The teacher designed learning activities, which are the key to effective teaching. This issue is maintained by Christie et al. (2007), who show that impact in the classroom defines the learners’ outcomes. Consequently, one condition of effective school teaching might be both teacher-led, yet learner-orientated. Researchers adopting this position include Rowe, Holmes-Smith and Hill (1993); Wright et al. (1997); Muijs and Reynolds (2001) and Rowe (2004) who maintain that schools are effective if there are effective teachers in a school.

“There are issues of overcrowding in schools due to population explosion. A teacher has more than enough learners in his class or in his or her classroom”. Findings indicate that there is overcrowding in the classes in Table 5.2. Teachers argue that overcrowding leads to more workload as they are unable to handle many learners at the same time in classes. They indicate that teaching and learning is not that effective due to high numbers in classes. An average classroom is entitled to have 35 learners, which means that a classroom with more than 40 learners is actually overcrowded (Masitsa et al., 2004). The position of Masitsa et al. (2004) is DBE policy driven which specifies that teacher-learner ratio in class must be 1:35.

Some participants in both quantitative and qualitative phases indicate that there is no proper support in these selected secondary schools. They argue that their schools are not well resourced, though the school documents show something different. This suggested that maybe the resources were bought and stolen, or inventory is not properly kept, but at the end of the day teachers allege that there is inadequate LTSM. This issue compromises school effectiveness and effective teaching a lot, because these materials enhance better teaching and learning of mathematics.
Observation and school documents indicate that both schools provided PD to teachers to improve their work in both selected schools, though some teachers generally say “my school doesn’t show any professional development of mathematics teachers because we are aware of events like AMESA but in our school it’s a challenge for us to even attend that conference of mathematics teacher development”.

It is worrisome that almost all respondents maintain that the support from all the stakeholders is insufficient. They indicate that guardians/parents do not assist them with their learners’ work at home and these are very serious problems which affect the effectiveness of teaching. This is in contrast with Meighan (1994), who maintains that there is a strong significant association between parental interest and encouragement of their learners at school, and educational performance in academic tasks.

Teacher effectiveness and learner outcomes are not linear, closed processes but open, recursive processes developing from and intertwined in diverse, often commonly constitutive factors in the connection between:

- The institutional character, (in)efficiencies, disciplinary domains as well as departmental and institutional resource allocation and planning;

- Teachers and learners’ habits, characters and life-worlds;

- Macro-collective factors influencing teachers, learners and parents including the school and its networks;

and

- The unfolding of an individual, compromised, challenging as well as changing teaching and learning journey as contextualised in the two selected secondary schools.
6.3.4. Question 4: How do the views of teachers, learners and heads of the department support their perspectives on school effectiveness and effective mathematics teaching towards the development of an improved learner outcomes model?

The proposed model is a complementary diagnostic tool regarding how learner outcomes could be improved. This model is goal-orientated. The proposed model can yield the intended results if the conditions are addressed on a complementary basis. The conditions are interrelated and thus impact on one another. The model as it appears is multidimensional and grounded on the constructivist paradigm and complexity viewpoint. It is based on the latter in the sense that schools and their all-inclusive stakeholders, even those who are under no circumstances consulted for the purpose of keeping up effectiveness, get the opportunity to share their concerns, questions and statements devoid of any constraints in relation to the learner performance in mathematics. The creation and use of mathematical models can help schools and teachers develop new improvement perspectives and make the difference to the envisaged learner outcomes in mathematics.

Outcomes and better achievement in mathematics are facilitated by effective teaching and learning materials and suitable opportunities. Effective academic development and support promote student success. The development and support can provide adequate orientation to quality learner outcomes in mathematics. Effectively coordinated teaching and learning facilitate the attainment of its intended learner outcomes in mathematics.

Well-functioning schools have adequate mechanisms ensuring accountability for the implementation of policies and regulations. Furthermore, effective policies and systems should be in place for monitoring student performance in mathematics and opportunities existing for student input and participation in learner outcomes in mathematics.
Figure 6.1. Model towards improved Learner Outcomes in Mathematics (tiLOM)

**School effectiveness and effective mathematics teaching**

**Teacher attributes**
- setting correct mind-set
- classroom management skills
- creation of conducive classroom environment
- use of assessment data to improve teaching
- realistic attributes
- knowledge of error analysis
- instructional capacity & CK
- professional development
- teacher collaboration

**School conditions**
- effective school management
- teaching and learning
- proper stakeholders’ support

**Learner conditions**
- self-perception in mathematics
- attitudes concerning mathematics
- self-assessment
- independent learning
- affinity to teachers
- learning repertoire
- orientation to learning
- integration of project and problem based learning
- LTSM
- adjustment to school
- parental support

**Intervention**
- constant monitoring
- constant motivation & mentoring
- responsive & cyclical PD
- time management
- provide support
- parent-learner-teacher partnership
- good record keeping
- integration of technology in teaching & learning
6.3.4.1. Teacher attributes

The aspects on teacher attributes share multiple ways on school effectiveness and effective mathematics teaching to improve learner outcomes in mathematics.

Attribute 1: Setting a correct mindset

The most crucial thing is that a teacher should be in a position to believe that all learners can learn by affording them space to be responsible and finding ways to make school work interesting and relevant (Fennema & Franke, 1992). In actual fact, the teacher should provide effective support to the learners by employing diverse teaching strategies (Nickson, 1992). Teachers should know students as thinkers and learners.

Attribute 2: Classroom management skills

It is key that teachers should be in authority in their classroom. They should make sure that they are always on time for their lessons; that would make discipline of learners more spontaneous (Gallagher, 2004). The teacher should have in mind the diversity inside the classroom with no element of favouritism. As corporal punishment is outlawed by SASA of 1996, teachers should find alternative ways to make learners self-disciplined. Without compromising respect there should be a harmonious relationship between teachers and the learners. As teachers are in loco parentis, learners should feel free to confide in the teacher with regard to their personal experience without fear (Yang, 2010). The teacher should be the source of holistic knowledge as he/she is the custodian of such.

Attribute 3: Creation of a conducive learning classroom environment

Teachers' attitudes, beliefs as well as expectations regarding mathematics, teaching and learning, structure their planning, their decisions, their overall practice and, therefore, their effects in the classroom (Pajares, 1992). If the teacher sees mathematics as interesting, pleasant and useful, learners are likely to develop more positive attitudes towards mathematics (Mailloux, 2012). The teacher should create a classroom where learners are free to give their views and be at liberty to explore without limitations. It should be viewed that self-respect will automatically lead to mutual respect, so creating a safe learning environment by the teacher is essential. Teachers should craft and manage learning environments. They should be in a position to develop
classroom norms which support classroom discourse as part of “teaching for understanding.” Teachers should build the relationship that supports learning. Learners should be taught that there is nothing like a stupid answer but trying all the time might lead to getting the sum right one way or the other. So learners should be encouraged to try until they get it right.

Attribute 4: Use assessment data to improve teaching

The good remedy for good outcomes is, that the teacher must teach, mark learners work, do remedial work and must give feedback timeously. The teacher must come up with the approach that will cater for learners of different cognitive levels (Faulkner & Cain, 2013). Intervention should be done spontaneously to all learners.

Attribute 5: Realistic attributes

The programme of Assessment and the Annual Teaching Plan often limit the teacher’s creativity and learners who have a slow pace of study. It is the responsibility of the teacher to curb the behavioural attitude of learners towards the subject and make the lesson very interesting so that it caters for all learners. No learner should feel that he/she is left behind. Learners should be encouraged to do and submit their work timeously (Mortimore, 1993; Moore, 1998). They should be praised if they are showing any positive effort. If learners receive continuous praise, they tend to develop a positive attitude towards the specific subject.

Attributes 6: Knowledge of error analysis

Learners are not empty vessels, they come with one or two things from their background, and so the recognition of prior learning is essential in dealing with learners. Teachers must acknowledge learner errors. Learners should know that it is not a crime to make a mistake and mistakes are there to be rectified. Therefore, it is then the responsibility of the teacher to make the environment acceptable for errors and come up with ways to deal with errors without offending or making a learner offended by his/her peers (Martin et al., 2000). Teachers should be in a position to clarify and demonstrate mathematical content hence giving right procedures and solutions. Teachers should show responsiveness to errors and investigative activities. The teacher should encourage critical thinking in his/her class and allow learners to think out of the box. Learners should be made aware that positive criticism is not meant to destroy but to correct
common faults. A mathematics teacher should be in a position to interpret learner activities and to explain the steps needed to get to the correct answer with appropriate conceptual links.

**Attribute 7: Instructional capacity and content knowledge**

The teacher must plan the lesson in advance to avoid embarrassment in front of learners. It should be noted that learners can use technological sources and other sources of information; so the teacher should always be equal to the task. Different teaching methods should be explored so that the lesson becomes interesting. Learner involvement and acknowledgement of learners who try their best should not be compromised (Townsend, 2001). Learners should know that being assessed is their daily bread and this will elicit hard work and thorough preparation from their side. The teacher should not fail to be a lifelong learner; a variety of pedagogical strategies and concepts should be employed (Steyn, 2008). The teacher should know his learners’ strengths and weaknesses that will help him/her in planning the lesson. The use of technological sources of information should not be neglected. The involvement of parents, social partners and the community at large in the education of their children should be encouraged continually. After getting the results of learners, it is essential to do individual and item analysis for further remedial activity.

**Attribute 8: Professional development**

The teacher should assist the SMT in ensuring that there is continuous teacher professional development. The planning of professional development should be discussed by both the teachers and SMT (Rosenholtz, 1987). SMT should lead in the monitoring of learner performance and feedback. A School Improvement Plan should be drawn based on the feasibility study done by SMT. Integrated quality management system should be used to help teacher development and to come up with ways that would speak to their contextual factors that deter the culture of teaching and learning. Technology should be used in teaching and learning since it includes formal and informal means of helping teachers not only to learn new skills but also develop new insight into PCK as well as their own practice, and explore new or advanced understanding of content and resources (Faulkner & Cain, 2013). In this technological era teachers’ professional development includes using various kinds of technology to foster teachers’
growth. It should further incorporate a collaborative, communal approach and create a more dynamic and engaged learning community (Johnson, 2012).

**Attribute 9: Teacher collaboration**

True teacher collaboration is best accomplished through a designed process for exchanging perceptions and content; even casual teacher-to-teacher collaboration can have a positive influence on learners (Henderson & Mapp, 2002). The teacher should learn to team-up with peers on lesson planning. A teacher should help other teachers and allow other teachers to assist him/her in imparting knowledge to learners. A teacher should further encourage learners to work as teams. Integration of learner performance will not only help the teacher in question but will also help the entire school result, so team teaching and results analysis is essential (Wagner, 2002).

**6.3.4.2. Learner conditions**

**Attribute 1: Self-perception on mathematics**

Learners’ mathematics self-perception, or belief in their specific abilities, is an essential outcome of education as well as being strongly associated with effective learning (Marsh, 1986; Marsh & O’Mara, 2008). Parallel studies of self-perception and success show that they are equally associated over time (Marsh, Xu & Martin, 2012; Marsh & Martin, 2011). The teacher should command his/her subject so as to instill a positive mind-set to learners in mathematics. Learners should enjoy mathematics’ complexity in its simplicity with the perception that they are talented in mathematics. Learners should have the winning mentality and erase the ‘fail’ word from their vocabulary.

**Attribute 2: Attitudes concerning mathematics**

The teacher should use all available resources to make learners enjoy mathematics and to have a positive mind set in learning mathematics. Positive learners’ attitudes enhance their reflection on their school work. Some degree of reflection enables learners to exercise an element of control over, as well as responsibility for their own learning (West & Beresford, 1998).
Attribute 3: Self-assessment

Learners should be encouraged and taught to learn from their mistakes. Learners should monitor their own learning, use assessment feedback from their teacher, themselves, as well as classmates to determine next steps, and set individual learning goals (Martin et al., 2000). Learners should have learning objectives and success criteria of each lesson written in user friendly language. They should be able to explore learning through their own thinking and problem solving (Marsh & Martin, 2011). Learners should be made aware that they should use their teachers as a source of reference.

Attribute 4: Independent learning

Group and peer discussions should be encouraged in problem solving. It should again be enforced that working alone after group and peer discussion is essential to check one’s individual ability. Direction is needed to help learners identify areas of difficulty but too much direction detracts from their sense of ownership of the learning activities (Marsh, Xu & Martin, 2012). Learners need to develop a sense of independent learning as well as problem-solving skills and techniques in order to learn and process from a wide scope of information available. Independent learning builds in learners the skills with which they sense their learning circumstances and actively construct and develop their specific learning strategies (Hubbard, 1997).

Attribute 5: Affinity to teachers

Learners should forever engage with mathematics teachers and accept assistance. Furthermore, they should negotiate challenges in mathematics and areas they are not sure of. The closeness of the learner-teacher association appears to have a strong bearing on the effectiveness of teacher assessment (Black & William, 1998) and even on the improvement of literacy skills (Campbell, 1999).

Attribute 6: Learning repertoire

The learner should embrace being taught new ways of mathematics and find mathematics more interesting (see Gipps, 1992; Harris, 1995). Learners should forever acknowledge and experience various teaching strategies used by the mathematics teacher. Joyce et al. (1997) maintain that
“increasing the range of learning experiences provided in our schools increases the likelihood of more students becoming more adept learners”.

Attribute 7: Orientation to learning

Acknowledgement and reward or praise from the mathematics teacher should make learners look forward to a mathematics class. Learners should work very hard and not depend on the teacher. Learners’ learning is enhanced by the shared love of mathematics within themselves (Barth, 1996). Learners should always look forward to lessons and new topics.

Attribute 8: Learner-teacher support materials

Stockard (1980) maintains that the provision of adequate resources helps in promoting learners’ outcomes. The researcher believes that if schools function with inadequate or no resources, there is the likelihood that ineffective teaching and learning will take place, leading to poor academic outcomes of learners. This can be attributed to the fact that learners will not be able to perform certain functions that could in turn improve their academic performance. Learners should, therefore, be accommodated in classrooms with equality of access to resources which can create a conducive atmosphere for teaching and learning.

Attribute 9: Adjustment to school

Learners should accept the sense of having school rules and hence conform to rules that regulate the school community (Hopkins et al., 1997). Another important issue is that learners must attend school regularly and never be late to their classes, including mathematics sessions. Learners should feel secure in their schools, which is supported by Hopkins et al.(1997)

Attribute 10: Parental support

It is key that learners allow and remind their parents/guardians to monitor their mathematics work/books regularly. An understanding of ways in which mathematics is used out-of-school can help improve learners’ attitudes towards mathematics and to assist learners understand the value as well as relevance of mathematics in several contexts (Meighan, 1994). Learners should encourage their parents to have a parent-teacher relationship, especially in mathematics.
6.3.4.3. School conditions

Effective school mathematics teaching can be affected by the conditions within the school (Ingvarson et al., 2004). The school conditions outlined are consistent with Scheerens’ (2015) correlation of a system: school effectiveness points out that the input influences the process and the subsequent output of the education system. Furthermore, Taylor (2011) concurs with DBE that systems which boost performance in South African schools are curriculum planning, systematic use of assessment to focus on teaching and learning, time management, procurement and retrieval of textbooks, school management, involvement of teachers as well as parents.

Attribute 1: Effective school management

School leadership should provide quality strategic management with the tools to gain a competitive advantage in effective mathematics teaching. Schools as a complex system should use strategic planning to predict and anticipate changes in the teaching and learning environment as well as other situations for the schools to respond (Stacey, 1996; Cairny, 2012). The researcher believes that there is no single correct management leadership style; the best leadership style is the one that meets the challenges one is facing and the needs of the people one is leading, as supported by Cairny (2012). Effective leaders are often flexible and are able to change their style of leadership to suit changing circumstances, as emphasised in the complexity theory (Cilliers, 1998; Scheerens, 2015). The school leadership should have a good management of interpersonal relations, and performance reviews should be conducted regularly. All learners should be included in the learning process. There should be a profitable use of time allocated and all work well-planned in advance. Effective school leadership should ensure that period registers and time-tables are in place for control and monitoring of teachers and learners (Burns, 1978; Stacey et al, 2002). Teachers should be developed and mentored to improve mathematics performance with good communication strategies as one of the guiding principles.

Attribute 2: Teaching and learning

Schools should provide practical resources and activities that foster the enhancement of teaching and learning of mathematics as well as to develop greater commitment across the school, and programmes to support mathematics teaching in diverse ways as supported by Krahéenhul
The school management ensures and monitors that all mathematics teachers make daily lesson plans as well as keeping good records of both learners and teachers. In the teaching and learning of mathematics, various assessments should be done continuously (Lieberman & Miller, 2000).

Schools should create teaching and learning environments where learners are active participants in mathematics, both as individuals and as members of collaborative groups (Elmore, 1996). Effective project based learning integrated well with problem based learning is ideal for better learner outcomes in mathematics. Schools should motivate learners and nurture their desire to learn in a safe, healthy and supportive environment which develops compassion as well as mutual respect (Shannon & Bylsma, 2007). Schools should cultivate cross-cultural understandings and the value of diversity (Naude et al., 2002). Schools should encourage learners to accept responsibility for their own learning and accommodate the diverse learning needs of all learners. Schools display effective and efficient classroom management that includes classroom routines that promote comfort, order and appropriate learner behaviours (Van der Westhuizen et al., 2002). Schools should provide learners with equitable access to technology, space, tools as well as time. Schools should effectively allocate time for learners to engage in hands-on experiences, discuss and process content as well as make meaningful connections.

**Attribute 3: Proper stakeholders' support**

Schools should be supportive to all teachers and learners. Schools should apply appropriate models and principles of organisational development and management which include inclusive decision-making. Attention should be given to indicators of effectiveness, equity, development of a school improvement plan (SIP) focused on management of finances, teachers and LTSM with emphases on curriculum, learning, safety and teaching, as supported by Cilliers (1998). Schools should collaborate with stakeholders like teachers, learners and community members to gather information needed to plan for improvement of the educational environment and share decision-making (Shannon & Bylsma, 2007). School management should promote and monitor a culture of communication and collaboration with learners, teachers, parents/guardians/caregivers/social partners and the community at large.
6.3.4.4. Intervention strategies

The research findings showed that there are challenges which may impede the effectiveness of the school and effective mathematics teaching. Challenges were experienced in relation to elements outlined below. To achieve goals of tiLOM, appropriate interventions strategies are required.

It is paramount that schools should have a responsive management which can articulate a vibrant strategic planning process and implement policies fairly. The school management should be in a position to involve teachers in decision-making processes. The SMT should be capable of addressing the challenges raised by mathematics teachers timeously to exclude cabals and dissatisfaction in the workplace.

In an effective school, there should be constant monitoring for effective mathematics teaching. It should develop a roster of the programme of activities. Furthermore, the school should develop a monitoring tool and use it regularly. The school should ensure that feedback is provided on time. An effective school should develop a motivation and mentoring tool for effective mathematics teaching.

An effective school should have responsive and cyclical professional development of mathematics teachers. There should be consistent training and a clearly articulated pledge towards professional development of mathematics teachers. The school policy should make provision of time for professional development (PD). Schools should align the key performance areas of mathematics teachers with its PD policy. An effective school should align the PD plans with mathematics teaching plans (Villeneuve-Smith, West & Bhinder, 2009).

An effective school should have a good time management plan i.e. effective use of period registers, timeous class attendance, constant monitoring of time as well as movement registers. It is key that all adhere to the lost-time policy. Resources should be allocated rationally for maximal use and need. The SMT, together with teachers should establish a functional peer review mechanism to benefit teaching and learning of mathematics as supported by Westoby & Van Blerk (2012).

An effective school should have a good parent-learner-teacher partnership i.e. it should provide consistent update to parents on planning processes (Katz & Earl, 2010). Parents should be
regularly invited to school, to collect reports and even on open days in advance. Schools must enforce manual and electronic record-keeping. Lastly, an effective school should use assessment data to evaluate effective mathematics teaching and learner outcomes.

Well functioning schools have adequate mechanisms ensuring accountability for the implementation of policies and regulations. Furthermore, effective policies and systems should be in place for monitoring student performance in mathematics and opportunities existing for student input and participation in learner outcomes in mathematics.

In a nutshell, the study will possibly assist to implement and support the execution of excellent strategies within the teaching and learning of mathematics in the selected schools. The findings will possibly lead and co-ordinate the effective and impactful implementation of academic programmes for the teaching and learning needs of teachers and learners enhancing better performance in mathematics in the selected schools. The findings will also assist in monitoring and tracking the effectiveness of teaching and learning of mathematics programmes’ implementation including the integration of digital technology into teaching and learning of mathematics.

6.4. SUMMARY

The discussion of the findings of the study were presented in this chapter. It involved the discussions leading to answering of the cardinal investigation of this study. It covered the critical analysis of findings and intervention strategies resulting in tiLOM. The following chapter will focus on conclusions and recommendations to the study.
CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1. INTRODUCTION

In this chapter, the researcher presents the conclusions and recommendations of critical discussion findings presented in Chapter 6. This chapter restates the main argument of the study. This chapter also covers the critical suggestion regarding the best course of action to improve learner outcomes in mathematics. The intent and outcomes of the investigation were clarified in this chapter 7. The author further outlined the limitations and contributions of the study.

7.2. CONCLUSIONS

The results of the study within its framework successfully responded to the basic unit of investigation. In the mission for a school to become more effective through bringing into being more effective mathematics teachers, it should provide teachers with clear indications of the quality of their teaching and the path to improvement. Effective teachers need to retool their approaches in order to teach learners effectively. Effective teaching necessitates teachers choosing “what aspects of a task to highlight, how to organize and orchestrate the work of students, what questions to ask students having varied levels of expertise, and how to support students without taking over the process of (mathematical) thinking for them” (Khalid, 2009). Effective mathematics teaching must be complemented by teachers’ in-depth knowledge of mathematics content, their understanding of what improves learners’ learning, and their best instructional capacity in the classroom.

Classroom teaching needs the undivided support of the school management. What school leadership can do to support learner learning is to support the teachers. Specifically, they must be more supportive by providing LTSM that teachers require in order to plan their lessons effectively, allow access to essentials like the internet, LCD projectors, visualisers, copiers and
so on. They must not only inspire teachers to explore new teaching concepts and improvements, but also assist the teachers in getting mathematics resources for the school.

These findings are consistent with scholarly consensus that teacher effectiveness is the foremost factor impacting learner academic development (Sanders & Rivers, 1996). This conclusion also agrees with previous research by Stronge (2010), who has averred that the quality of teachers is so strategic to the lives of learners. Teachers are of significant value when it comes to school improvement, as well as learner learning, surrounded by multiple elements within schools. The results lend strong support to Stronge (2010) who posits that “there is no more powerful influence on student success than the teacher”.

Teachers and learners require an engagement in critical self-evaluation leading to identification of areas of best practices, areas needing improvement, and other interventions required in order to enhance effective mathematics teaching with quality learner outcomes in mathematics. This process should culminate in the preparation of a self-evaluation report that evaluates the extent to which the programme performs against the set conditions i.e quality learner outcomes in mathematics. Self-evaluation is generally accepted as one means of conducting an authentic review of academic practice. Most effective validation of a practice (effective mathematics teaching) and critical self-evaluation have the potential to yield effective mathematics teaching, outcomes and services to underpin improvement in mathematics performance.

7.3. RECOMMENDATIONS

7.3.1. Recommendations for policy makers and school management team

The way in which some policies are effected has been noted to influence negatively school effectiveness and effective mathematics teaching in this study. This section outlines the recommendations to policy planners as well as SMTs and can positively impact on school effectiveness and effective mathematics teaching through proper implementation of the policy changes. Through realistic channels, the outcomes of this study recommend actions to embark on in schools to address the teachers, learners and school conditions which influence policies.

Policies should be developed to assist the teaching and learning culture of mathematics which can also assist in co-ordinating year-level outcomes to ensure both vertical and horizontal articulation in secondary schools. There should be effective monitoring and tracking of the
Academic Action Plans per school, ensuring that reporting formats reflect performance in mathematics. There is a need to gather both formal and informal feedback from teachers and parents at information sessions which may be relayed to relevant stakeholders. There should be a creation of platforms or forums where global trends are shared and disseminated across the school with mathematics teachers and the DBE.

It is critical for DBE to supply schools with resources and support they require for effective teaching and learning of mathematics in secondary schools. The preservation of infrastructure in schools is one of the functions of the SGB (DoE, 1996). Conversely, due to financial restrictions, ignorance and lack of awareness, the functionality of management/SGBs deters schools to have adequate resources; as a result, school effectiveness and effective mathematics teaching is adversely affected. The DBE needs to necessitate systematic capacity-building workshops to enhance the functionality of management/SGBs. The main focus area should be on financial and school buildings management, selection process of the SMT members as well as other related matters for better functionality of the schools. Although schools are given Section 21 funds, the state should further assist underprivileged schools to pay for some services like internet, copiers, security services, electricity, sanitation as well as water bills.

The national treasury and the DBE should look into market-related remuneration packages for good performance, especially in subjects like mathematics. All teachers who are prepared to go the extra mile during weekends/holidays or even giving morning classes and afternoon classes must be given some incentives by either the SGB or DBE to elevate their job satisfaction.

Learners are the sole purpose of the existence of the learning institutions. Older learners must not be allowed general admission with the normal age cohort as is currently being done, and that the policy should be amended to cater for these learners. Learners who have some learning challenges or special needs, and multiple repeaters with behavioural challenges should be placed in relevant learning institutions like adult basic education and training (ABET) centres, special schools or even TVET colleges.

Teachers believe that their workloads are heavy and the anticipations of the DBE are unrealistic. This basically pulls down the levels of teacher effectiveness in mathematics classes. This study
suggests that the DBE should take action on the approved 1:35 teacher-learner ratios in secondary schools. The DBE needs to address the matter of teachers’ various roles.

Teaching is being interested in the transfer of SKAV i.e. skills, knowledge, attitudes as well as values from teachers to learners. Teachers need support from all stakeholders to succeed in that endeavour. A harmonious atmosphere grounded on mutual collaboration need to exist between the SMT and teachers to expedite a good culture of learning and teaching in mathematics. Staff improvement programmes (PD), teachers’ summits, etc. should be strengthened. Related democratic principles, consensual decision-making as well as transparency should be emphasised so that teachers can perform their teaching tasks without any impediments or prejudice. Nonetheless, support from community, guardians and parents are of vital importance as the school is the pivotal centre of the society. These stakeholders need to be taken on board or encouraged as partners in the education of their children, irrespective of their socio-economic status or literacy levels. Mindful endeavours to stimulate interpersonal relations will assist to positively elevate learner outcomes in mathematics.

Principals and their SMTs must have democratic leadership styles as well as reducing excessive bureaucracy, prompt participation from teachers and involve them in decision-making as well as policy-making. The above-mentioned are likely to enhance teacher autonomy as well as control in schools, hence leading to effective mathematics teaching. An efficient management style should become accustomed to changing desires of teachers and learners in an effort to realise achievement for all involved. A democratic leadership nurtures and sustains a school climate and culture in which teachers are inspired and dedicated, hence producing improved learner outcomes in mathematics.

Schools should provide conducive infrastructure and environment for its mathematics students. Adequate facilities and specialised equipment must be provided such as IT facilities, library facilities, study and laboratory spaces for mathematics. Each school’s strategic priorities, output and impact criteria should be well described for mathematics performance outcomes as well as throughput rates.
7.3.2. Recommendations for future research

The following areas are recommended for supplementary research:

An investigation could be done into the effects of school effectiveness attributes like leadership support for teachers on learners’ performance in mathematics. This is imperative because changing and challenging workplace conditions can have a powerful effect on the quality of teaching and learning of mathematics.

Further study should be done to investigate the influence of mathematical pedagogical content knowledge (PCK) and subject content knowledge (SCK) on effective mathematics teaching. The study could explore using various topics such as differential calculus, trigonometry, financial mathematics, etc.

Other suggestions refer to the use of language in mathematics, such as how multilingual learners can be effectively assisted to learn mathematical English while they learn mathematics. What does research in multilingual education suggest would be useful to assist learners in mathematics? What approaches might assist such learners? An investigation is needed on the effects of mother tongue as a language of teaching and learning mathematics in a classroom in South Africa.

School effectiveness and effective mathematics teaching are constructs dealing with the views and challenges experienced by human beings at their workplace. Views are subject to change, the study may be replicated in which results can be the same or different. These indicate that a longitudinal research is required to understand further school effectiveness and effective mathematics teaching towards the development of improved learner outcomes in mathematics in a broader area like a region or province.

7.4. LIMITATIONS OF THE STUDY

The study is limited to two secondary schools, one of which is in a rural and the other in an urban area of Mahikeng. However, the perceptions and conditions of primary school teachers and learners in Mahikeng will remain unknown. There are some constraints in these schools especially in terms of the infrastructure and delivery services. Consequently, the character and
culture of schools in Mahikeng could vary from other areas. Hence, the outcomes of this research may not be automatically relevant to other schools within the province or country.

The survey instrument was only applied to mathematics teachers and learners in the selected schools. The research was originally intended to include more schools such as private and farm schools but there were serious challenge of lack of cooperation and sponsors. The researcher used his associates in some of these schools in the one-on-one interviews but at least achieved the goal of intended sampling. Furthermore, the research echoed the opinions of teachers and learners from predominantly one race group, as Mahikeng is mainly composed of African Setswana-speaking communities. Hence, the outcomes may not be generalised to other schools hence the limitations of this study.

7.5. CONTRIBUTIONS OF THE STUDY

Human beings are intricate, and comprehending their demeanours demands an extensive insight and proficiency. There is a review of literature on school effectiveness and effective mathematics teaching, which hopefully contributed an explicit understanding and clarifications of school conditions and mathematics teacher attributes with changing and challenging learner conditions in public schools. From the findings, the researcher was able to identify the determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in secondary schools.

Even though there is no static rule for good practices, some of the attributes/conditions can be proposed in the direction of creating improved practices. The study provided the model towards improved Learner Outcomes in Mathematics (tiLOM) which could be an intervention tool for effective mathematics teaching and hence lead to improved learner performance in mathematics. The study further offered SMTs objective findings to identify areas where teachers and learners may need specific support in terms of various methods of facilitating teaching and learning. An opinion was obtained from the findings about the school effectiveness and effective mathematics teaching which will improve the school conditions.

It appears from the results of the study that effectiveness of classroom teaching starts from the teacher. Mathematics teachers have to be well-prepared, well-versed and systematically
supported for deviations in the curriculum as well as instructional approaches to follow. A particular pedagogy observed in the exemplar teacher that stood out from the rest was the teacher’s questioning abilities.

The research added factual aspects to some of the issues that need specific scrutiny in the teaching fraternity. The study also attempted at elucidating issues related to school effectiveness and effective mathematics teaching which affects the outputs, confidence, organisational dedication, human relations, PD as well as intervention strategies (Spector, 2008). The study can therefore serve as a diagnostic tool which may be utilised to inform relevant decision-making in developing and implementing appropriate interventions to improve teaching and learning in basic education

7.6. SUMMARY OF THE STUDY

The purpose of the research was to investigate the possible determinants of quality mathematics teaching which could mark schools and teachers effective in mathematics teaching in selected Mahikeng secondary schools of the North-West province. The literature study was carried out on relevant theories, outcomes of previous studies involving similar issues and empirical inquiry. The researcher used a sequential explanatory strategy i.e. mixed methods, starting with a quantitative method followed by a qualitative method. The structured questionnaire i.e. survey instrument, was used in the quantitative phase in which 12 and 360 survey instruments for teachers and learners respectively were issued, and 321 learners and 12 teachers responded. The second part, the qualitative phase used semi-structured interviews for teachers and focus group interviews for learners. Furthermore, the classroom observations and documents analysis were done.

It was found that learner and school conditions complement teacher attributes for effective mathematics teaching which enhance improved learner outcomes in mathematics. However, there were some worrisome elements impacting negatively on mathematics teachers’ effectiveness which include lack of proper support, learner indiscipline, challenges in learner assessments, school location, congested work schedules, overcrowding and work overload as well as poor leadership styles. The study drew a model towards improved learner outcomes in mathematics (tiLOM). The model is open for criticism to strengthen it.
The study recommended approaches on how to elevate school effectiveness and effective mathematics teaching to policy makers/planners and SMTs for improvement of learner outcomes in mathematics. Most educational management approaches and policies need to be re-visited. In addition, educators need prospects to exhibit their potential to succeed in various facets of their jobs. The limitations and contributions of this study were also presented in this chapter. Since change in the real world is unavoidable, it is important for mathematics teachers to continually learn and bring up-to-date their instructional skills with the aim to promote and furnish learners with the necessary mathematical understanding to face the challenges of the 21st century.
REFERENCES


Collias, K., Pajak, E., & Rigden, D. 2000. One cannot teach what one does not know: training teachers in the United States who know their subjects and know how to teach their subjects.
http://www.c-be.org/PDF/OneCannotTeach. pdf Date of access: 30 Jul. 2016.


Department of Basic Education. 2013. Investigation into the implementation of maths, science and technology. Pretoria: Government Printers.


Huber, S.G. 1999. Dovetailing school effectiveness and school improvement: towards a model of effectiveness and improvement features; a panorama over the educational effectiveness and
improvement landscape. Paper prepared for the International Congress for School Effectiveness and Improvement, Manchester, 4-7 January 1998.


Kline, S. J. 1999. The low-down on entropy and interpretive thermodynamics: for the students and teachers of the subject. La Canada: DCW Industries.


Onwu, G. O. M. 1999. An Investigation of the availability and use of learning materials in grade 12 science classes in some selected schools in the Northern Province. Thohoyandou: University of Venda.


Preedy, M. 2016. Distributed leadership: where are we now? Management in education, 30(4): 139-140.


Appendix A

LEARNER SURVEY INSTRUMENT

Research: School effectiveness and effective mathematics teaching: Towards a model of improved learner outcomes.

Dear Participant

You are invited to participate in an academic research study conducted by K. Gilbert Pule, a doctoral degree student of mathematics education at the North West University, Mafikeng Campus. The survey may take you approximately 15-20 minutes to complete.

The intent of this study is to investigate effectiveness in mathematics teaching and learning in secondary schools.

Please note the following:

- This survey is anonymous and you do not have to indicate your name on the questionnaire.
- The answers you provide will be treated as strictly confidential as a result you cannot be identified in person based on the answers you give.
- Your participation in this study is very important to us.
- Please answer the questions in the attached questionnaire as completely and as honestly as possible.
- The results of the study will be used for academic purposes only and may be published in an academic journal. Summary of the findings will be availed on request.
- For further enquiry please contact my supervisor, Professor Percy Sepeng, online at Percy.Sepeng@nwu.ac.za if you have any questions or comments regarding the study.

K.G.Pule

Signature

Research conducted by:

Mr. K.G. Pule
**SECTION A**  
**BIOGRAPHICAL DATA**

Tick the appropriate box

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>17 yrs</th>
<th>18 yrs</th>
<th>19 yrs &amp; above</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>15 yrs</td>
<td>16 yrs</td>
<td>17 yrs</td>
<td>18 yrs</td>
<td>19 yrs &amp; above</td>
</tr>
</tbody>
</table>

**Home Language**

| Home Language | Setswana | English | Afrikaans | Sesotho | Other |

**Race**

| Race       | African | White | Coloured | Indian | Other |

**Number of years in a grade**

| Number of years in a grade | First attempt | Second attempt | Third attempt | Many years | Other |

**Number of learners in your class school**

| Number of learners in your class school | Below 30 | 31-40 | 41-45 | 46-50 | Above 50 |

**How would you rank yourself**

| How would you rank yourself | Top 5 | Top 10 | Middle | Lower 10 | Etc |
SECTION B

LEARNER CONDITIONS ELEMENTS/ASPECTS

The purpose of this section is to identify possible learner conditions influencing school effectiveness and effective mathematics teaching. Please tick the appropriate answer in the tick boxes provided. Please complete all statements.

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SELF-PERCEPTION ON MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I would like mathematics much more if it were not so challenging.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>I am talented in mathematics.</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Mathematics is an easy subject.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Mathematics is my strength area.</td>
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</tr>
<tr>
<td>I passed mathematics in my previous examination.</td>
<td></td>
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</tr>
<tr>
<td>ATTITUDES CONCERNING MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I enjoy learning mathematics.</td>
<td></td>
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<tr>
<td>Mathematics is taking a lot of my time.</td>
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<tr>
<td>I like mathematics</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>I would like a work which involves the usage of mathematics</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>SELF-ASSESSMENT</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>I am responsible for my own learning.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I reflect and analyse errors on my learning of mathematics on daily basis.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I explore my learning through my thinking and problem solving strategies.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I developed a culture of asking for help from teachers and learners</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**INDEPENDENT LEARNING**

| I am ready to access mathematics resources needed in lessons. | 1 | 2 | 3 | 4 |
| I can apply the knowledge acquired in problem-solving situation. | 1 | 2 | 3 | 4 |
| I regularly take part in mathematics group-work. | 1 | 2 | 3 | 4 |
| I undertake independent mathematics research activities. | 1 | 2 | 3 | 4 |

**AFFINITY TO TEACHERS**

| I get on well with my mathematics teacher. | 1 | 2 | 3 | 4 |
| My mathematics teacher always motivates me. | 1 | 2 | 3 | 4 |
| I learn mathematics best when assisted by my teacher. | 1 | 2 | 3 | 4 |
| I negotiate with my teacher over the work to be done. | 1 | 2 | 3 | 4 |

**LEARNING REPERTOIRE**

| I learn best when my teacher use various teaching methods/strategies in mathematics lessons. | 1 | 2 | 3 | 4 |
| I am able to cope with the range of teaching methods/strategies. | 1 | 2 | 3 | 4 |
| I find mathematics lessons interesting. | 1 | 2 | 3 | 4 |
| Learners are taught new ways of learning and working. | 1 | 2 | 3 | 4 |
### ORIENTATION TO LEARNING

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I always look forward to mathematics lessons and new topics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I work hard on my studies e.g. class-works, assignments and tests.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I work hard on daily mathematics home-works and submit them timeously.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>My mathematics teacher acknowledges my hard work.</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### LEARNER TEACHER SUPPORT MATERIALS

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our school is well resourced with mathematics materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>There is vandalism and theft of resources in our school.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Technological resources are always available for learners.</td>
<td></td>
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</table>

### ADJUSTMENT TO SCHOOL

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I accepted the sense of having school rules.</td>
<td></td>
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<tr>
<td>There are good support structures for learners.</td>
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<td></td>
</tr>
<tr>
<td>I attend school regularly.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I attend my mathematics classes regularly.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I conform to rules that regulate the school community.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### PARENTAL SUPPORT

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>My parent(s)/ guardian monitor my mathematics work regularly.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>My parents(s), siblings and friends assist me with my mathematics work.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>My parent(s) usually visit(s) my school to check on my progress.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>There is a good parent-teacher relation especially in mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

THANK YOU VERY MUCH
Appendix B

MATHEMATICS TEACHER SURVEY INSTRUMENT

Research: School effectiveness and effective mathematics teaching: Towards a model of improved learner outcomes.

Dear Participant

You are invited to participate in an academic research study conducted by K.Gilbert Pule, a doctoral degree student mathematics education at the North West University, Mafikeng Campus. The survey may take you approximately 25-30 minutes to complete.

The intent of this study is to investigate effectiveness in mathematics teaching and learning in secondary schools.

Please note the following:

- This survey is anonymous and you do not have to indicate your name on the questionnaire.
- The answers you provide will be treated as strictly confidential as a result you cannot be identified in person based on the answers you give.
- Your participation in this study is very important to us.
- Please answer the questions in the attached questionnaire as completely and as honestly as possible.
- The results of the study will be used for academic purposes only and may be published in an academic journal. Summary of the findings will be availed on request.
- For further enquiry please contact my supervisor, Professor Percy Sepeng, online at Percy.Sepeng@nwu.ac.za if you have any questions or comments regarding the study.

K.G.Pule

Signature

Research conducted by:
Mr. K.G Pule
Cell: +27 84 720 5192
# SECTION A

## BIOGRAPHICAL DATA

Tick the appropriate box

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>41-50yrs</th>
<th>51-60</th>
<th>60 &amp; above</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Age</strong></td>
<td>20-30yrs</td>
<td>31-40yrs</td>
<td>41-50yrs</td>
<td>51-60</td>
<td>60 &amp; above</td>
</tr>
<tr>
<td><strong>Home Language</strong></td>
<td>Setswana</td>
<td>English</td>
<td>Afrikaans</td>
<td>Sesotho</td>
<td>Other</td>
</tr>
<tr>
<td><strong>Race</strong></td>
<td>African</td>
<td>White</td>
<td>Coloured</td>
<td>Indian</td>
<td>Other</td>
</tr>
<tr>
<td><strong>Nature of Employment</strong></td>
<td>Contract</td>
<td>Substitute</td>
<td>SGB Post</td>
<td>Permanent</td>
<td>Other</td>
</tr>
<tr>
<td><strong>Highest teaching qualification</strong></td>
<td>Diploma</td>
<td>Bachelor’s degree</td>
<td>ACE/PGCE</td>
<td>Honours degree</td>
<td>Masters Degree</td>
</tr>
<tr>
<td><strong>Current Position</strong></td>
<td>Educator</td>
<td>Senior Teacher</td>
<td>HOD</td>
<td>Deputy Principal</td>
<td>Principal</td>
</tr>
<tr>
<td><strong>Working Experience (years)</strong></td>
<td>Below 3</td>
<td>3-10</td>
<td>11-15</td>
<td>16-20</td>
<td>Above 20</td>
</tr>
<tr>
<td><strong>Average class size</strong></td>
<td>Below 31</td>
<td>31-40</td>
<td>41-50</td>
<td>51-60</td>
<td>above 60</td>
</tr>
<tr>
<td><strong>School location</strong></td>
<td>Urban</td>
<td>Township</td>
<td>Semi-rural</td>
<td>Rural</td>
<td>Farm</td>
</tr>
</tbody>
</table>
SECTION B
MATHEMATICS TEACHER’S ATTRIBUTES/PRACTICES

The purpose of this section is to identify possible attributes/practices of teachers which impact on effective mathematics teaching. Please tick the appropriate answer in the tick boxes provided. Please complete all statements.

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SETTING A CORRECT MINDSET</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I believe my learners can learn.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I have high expectations from my learners.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I give extra help to my learners.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I find ways to make mathematics interesting, relevant and easy.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I use diverse instructional strategies.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>CLASSROOM MANAGEMENT SKILLS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I enforce rules fairly without favouritism.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I react to minor situations in class.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I allow learners to prevent others from learning.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I prevent instructional time from interventions.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>CREATION OF CONDUSIVE CLASSROOM ENVIRONMENT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I encourage a mutual respect within a classroom.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I make a classroom a safe learning community.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I provide an inspirational learning environment.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
All learners are at liberty to ask me questions concerning their subject mathematics

<table>
<thead>
<tr>
<th>USE ASSESSMENT DATA TO IMPROVE TEACHING</th>
</tr>
</thead>
<tbody>
<tr>
<td>I constantly assess my learners informally and formally.</td>
</tr>
<tr>
<td>I give feedback timeously to learners.</td>
</tr>
<tr>
<td>I usually reflect on my work and do interventions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REALISTIC ATTRIBUTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>I understand that some learners will misbehave.</td>
</tr>
<tr>
<td>Learners reject my efforts to provide relations.</td>
</tr>
<tr>
<td>I focus on what I can do (teaching) best in an uncontrollable class.</td>
</tr>
<tr>
<td>All learners do their homework.</td>
</tr>
<tr>
<td>All learners submit their tasks timeously.</td>
</tr>
<tr>
<td>Pressure to cover large amounts of work forces me to move on quickly through the syllabus.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KNOWLEDGE OF ERROR ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I am able to interpret learner activities and performance.</td>
</tr>
<tr>
<td>I am able to explain the steps needed to get to correct answer and the appropriate conceptual links amongst the steps.</td>
</tr>
<tr>
<td>I am able to unpack a mathematical procedure while probing learner’s errors.</td>
</tr>
<tr>
<td>I gloss over errors in the course of the lesson.</td>
</tr>
<tr>
<td>I am able to look on patterns of learners’ errors.</td>
</tr>
<tr>
<td>I am able to explain and provide a rationale for the way learners were reasoning when producing errors.</td>
</tr>
<tr>
<td>I have developed a repertoire of explanations with a view to address differences in a classroom.</td>
</tr>
<tr>
<td>I am able to identify the error around which the conversation is focused.</td>
</tr>
<tr>
<td>I am able to go beyond stating the actual error by using probing</td>
</tr>
</tbody>
</table>
questions to try and follow the way a learner is reasoning about the error.

I enable learners to clarify their own thinking hence developing a deeper understanding of the mathematical concepts underpinning the errors.

| INSTRUCTIONAL CAPACITY AND CONTENT KNOWLEDGE |
|---------------------------------------------|---|---|---|---|
| I frequently evaluate learners to check their progress | 1 | 2 | 3 | 4 |
| I use a variety of instructional strategies appropriate to the topic. | 1 | 2 | 3 | 4 |
| I make adjustments in my teaching based on learner capabilities. | 1 | 2 | 3 | 4 |
| I seek information about my learners’ strengths and weaknesses. | 1 | 2 | 3 | 4 |
| I team up with parents to motivate my learners | 1 | 2 | 3 | 4 |
| I frequently use various assessment data to adjust. | 1 | 2 | 3 | 4 |
| I give learners individual feedback on their progress. | 1 | 2 | 3 | 4 |
| I have access to my learners’ standardised marks. | 1 | 2 | 3 | 4 |
| I demonstrate sound knowledge of the curriculum. | 1 | 2 | 3 | 4 |
| I use mathematical language in my lessons. | 1 | 2 | 3 | 4 |
| I clearly explain procedures for solving problems. | 1 | 2 | 3 | 4 |
| I plan my lessons in advance. | 1 | 2 | 3 | 4 |
| I create opportunity for learner participation in classroom problem solving. | 1 | 2 | 3 | 4 |
| I support lessons with useful classwork and homework. | 1 | 2 | 3 | 4 |
| I usually give more than enough activities. | 1 | 2 | 3 | 4 |
| I use different resource materials. | 1 | 2 | 3 | 4 |
### PROFESSIONAL DEVELOPMENT (PD)

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>My school has continuous professional development in mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>My knowledge of mathematics content is improved by PD</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PD deepens my subject matter understanding.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I am able to identify how learners learn the content.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PD provides an adequate time to reflect on how to improve my work.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I am actively involved in meaningful discussions, planning and practice.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I have the opportunity to observe and be observed in using new concepts in a classroom.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I am able to plan how to use new curriculum and teaching methods in the classroom.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I am able to review learners’ work.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PD relates to content standards and assessment.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PD builds on my previous knowledge.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PD encourages an integrated communal approach</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Mathematics teachers in my school work together to create a more dynamic and engaged learning community.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

### TEACHER COLLABORATION

<table>
<thead>
<tr>
<th>Activity</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departmental teachers meet on lesson planning.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>We discuss with each other on how to assist specific learners.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>We meet to analyse and address learner results.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>There is peer classroom teaching observations.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>We engage in mentoring each other to improve.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>We work collaboratively with subjects specialists</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
Appendix C

OBSERVATION SCHEDULE

1. How does teaching and learning of Mathematics occur? (Please list e.g. whole class)
   (i) ____________________________________  (ii) ________________________________
   (iii) ___________________________________  (iv) ________________________________

2. How is the classroom arranged?
   (Furniture) ________________________________________________________________

3. What methodology/ approach is being used?
   __________________________________________________________________________

4. Which resources are used?
   __________________________________________________________________________

5. How does the teacher deal with correct or incorrect responses?
   __________________________________________________________________________
   __________________________________________________________________________
   __________________________________________________________________________
   __________________________________________________________________________

The PEER system underlies the lessons in a classroom situation. It might not be possible to incorporate all of them in a particular lesson but each lesson will contain some aspects of this system. Please tick (√) your rating.
<table>
<thead>
<tr>
<th></th>
<th>Learners</th>
<th>PRODUCTIVE SKILLS</th>
<th>Excellent</th>
<th>Good</th>
<th>Average</th>
<th>Needs more attention</th>
<th>Not applicable to the lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Learners are able to do reading on the concept being taught</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Learners write notes on the concept taught</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Learners are able to solve problems given as exercises</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4.</td>
<td>Learners are able to relate and apply the concept in real life problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>5.</td>
<td>Learners are able to use their knowledge of and experience in the concept in formulating their own responses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Learners are able to accomplish work given on the concept independently</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Learners are able to define and describe learned terms encountered when dealing with the concept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Learners are able to follow steps in solving exercises based on the concept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Learners competently use technology (calculators) in areas where it is required in the concept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Learners are able to deal with problems in real and abstract context using the concept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Learners' ways of making decisions in problem solving is enhanced</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**B**

<table>
<thead>
<tr>
<th></th>
<th><strong>EQUIVOCATIVE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Learners ask questions for clarification</td>
</tr>
<tr>
<td>2.</td>
<td>Learners ask questions to consolidate their understanding of the concept</td>
</tr>
<tr>
<td>3.</td>
<td>Learners are puzzled by certain areas of the concept and hence very inquisitive.</td>
</tr>
<tr>
<td>4.</td>
<td>Learners are able to interpret new information on the concept</td>
</tr>
<tr>
<td>5.</td>
<td>Learners ask critical questions to ensure that methods used are appropriate</td>
</tr>
<tr>
<td>6.</td>
<td>Learners use their referencing skills to acquire better understanding of the concept</td>
</tr>
</tbody>
</table>

**C**

<table>
<thead>
<tr>
<th></th>
<th><strong>EVALUATIVE SKILLS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Learners are able to do self-assessment tasks in the concept learned.</td>
</tr>
<tr>
<td>2.</td>
<td>Learners are capable of evaluating their own work on the concept.</td>
</tr>
<tr>
<td>3.</td>
<td>Learners are able to evaluate procedures followed in problem solving in the concept.</td>
</tr>
<tr>
<td>4.</td>
<td>Learners are able to identify errors committed when dealing with the concept.</td>
</tr>
<tr>
<td>5.</td>
<td>Learners are able to discuss pros and cons in using specific methods to solve problems.</td>
</tr>
<tr>
<td>6.</td>
<td>Learners are able to identify incorrect ways of solving problems.</td>
</tr>
<tr>
<td>7.</td>
<td>Learners have alternative ways to solve problems based on the concept.</td>
</tr>
</tbody>
</table>

**D**

<table>
<thead>
<tr>
<th></th>
<th><strong>REFLECTIVE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Learners are constantly engulfed in the world of &quot;exploration in errors.&quot;</td>
</tr>
<tr>
<td>2.</td>
<td>Learners reflect on errors committed in solving problems and work towards eliminating those errors.</td>
</tr>
<tr>
<td>3.</td>
<td>Learners are able to respond to questions testing their comprehension of the learned concept.</td>
</tr>
<tr>
<td>4.</td>
<td>Learners are able to select and use appropriate methods in solving problems.</td>
</tr>
<tr>
<td>5.</td>
<td>Learners are able of hypothesising in problem solving.</td>
</tr>
<tr>
<td>6.</td>
<td>Learners can reflect on the decision they made in solving a particular problem.</td>
</tr>
</tbody>
</table>
GENERAL CONDUCT AROUND THE SCHOOL

Please provide the general conduct around the school.
Appendix D

TEACHER SEMI-STRUCTURED INTERVIEW SCHEDULE

1. What constitute effective mathematics teaching in your classroom? Please give examples.
2. What do you think are the factors that influence the effectiveness of mathematics teaching and learning in this school?
3. How many years of experience do you have in teaching mathematics?
4. What was the highlight of teaching mathematics over that period?
5. What do you associate such highlight with?
6. How do you put a correct mind-set for your class/learners?
7. How do you manage your class?
8. What are the elements of good planning as part of your work?
9. How do you adhere to the assessment processes and procedures as outlined in the CAPS document?
10. How do you draw the learners’ attention to important aspects of mathematics activity?
12. What intervention(s) are in place for learners at risk?
13. What measures do you have in place to help learners with special educational needs?
14. What measures do you take in developing and capacitating yourself on issues relating to knowledge for the teaching of mathematics as well as mathematics content knowledge?
15. How does the school ensure professional development of mathematics teachers?
16. What role(s) do you have in supporting your HOD on issues of teaching and learning processes?
17. Do you always use mathematical language during your lessons? Please explain.
18. How do you involve your learners during the lesson?
19. Do you analyse learner errors in their various activities? Please explain.
20. Do you usually reflect on your lessons? Please explain.
21. Does the school supply adequate mathematics LTSM?
22. Are parents involved in any way towards the teaching and learning of mathematics? What are they roles?
23. What kind of support do you get from SMT including HOD?
24. Do you have anything to add?
Appendix E

SEMI-STRUCTURED INTERVIEW SCHEDULE FOR HOD

1. What is your experience (in years) in teaching mathematics?
2. What was your highest pass percentage in the school you taught mathematics?
3. What do you associate that particular performance with?
4. What do you think are the school factors that influence effective teaching and learning of mathematics in your school?
5. How does the school ensure professional development of mathematics teachers? Can you name few off such initiatives/PD activities?
6. What is the role of the mathematics subject advisors in relation to professional teacher development?
7. How often do you conduct quality assurance and monitoring of teaching mathematics in the classrooms through class visits in your department?
8. Who else administer class visits in your school? Is it perhaps part of your monitoring and evaluation processes/school policy?
9. How do you motivate teachers and learners in your department?
10. Does your school participate in mathematics Olympiads or any other mathematics benchmark assessments? Please explain.
11. How often do you analyse /review the performance of your department? How do you use these data to inform teaching practice and for what purpose(s)?
12. What interventions are in place for learners at risk, e.g. academically struggling and/or progressed learners?
13. How does the school ensure that all learners have mathematics learning and teaching resources such as textbooks?
14. What strategies does the school use to keep good performance in mathematics? /What went wrong for this school to have poor performance in mathematics and how is it going to be addressed?
15. Are parents involved in any way towards the teaching and learning of mathematics? Your opinion please?
16. Are there any general comments about mathematics teaching and learning?
Appendix F

LEARNERS’ FOCUS GROUP SEMI-STRUCTURED INTERVIEW GUIDE

The purpose of the interview is to collect information about experiences, beliefs and attitudes of learners in the mathematics class and to get some specific factors which impede their learning in mathematics. The identities of learners will remain confidential. Learners will be required to give their honest opinions without any fear of victimisation.

1. How would you describe your experience as mathematics learner?
2. Can you describe your feelings about mathematics? Do you like mathematics or can you please say something about it?
3. Do you think that you are good in mathematics? Can you please explain?.
4. What do you think contribute to your success or failure in mathematics?
5. What do you think will improve learning and results of mathematics?
6. Do you feel comfortable to ask and answer questions during mathematics lesson? Explain.
7. Does your teacher assist learners with errors made during various mathematics activities?
8. How is your relationship with your fellow learners?
9. How is your relationship with your mathematics teacher?
10. Do you believe that your teacher is doing his/her best during the lessons? Explain.
11. Are your parents actively involved in your mathematics work? Explain.
12. Is your school doing its best to assist you to be a better mathematics learner? Explain.
13. Is there anything to add concerning teaching and learning of mathematics?
## Appendix G

### DOCUMENTARY ANALYSIS SHEET

The following documents will be analysed in the two selected secondary schools

<table>
<thead>
<tr>
<th>AREA</th>
<th>DOCUMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>School management</td>
<td>Year plan, school development plan, school improvement plan, departmental improvement plan, HOD records, control of work, files, committee’s composition, SMT minutes, teacher development, staff minutes, departmental minutes, staff attendance and leave register, movement register, filling system, circulars and communication, quality management, stock control and register, strategic management, teacher discipline, motivation of staff and learner, interpersonal relations, decision making, conflict resolution, control of work, performance review, delegation, visitors register, induction, inclusion and extra-curriculum.</td>
</tr>
<tr>
<td>Functionality</td>
<td>Organogram, organizational structure, duty lists, timetables.</td>
</tr>
<tr>
<td>Governance</td>
<td>Vision and mission, admissions, policies, SGB year plan, SGB minutes, parent’s minutes, financial management, projects, budgeting, control, procurement, reporting, auditing, fundraising, duties, teacher recruitment, disciplinary records, cleanliness.</td>
</tr>
<tr>
<td>Teaching and learning</td>
<td>Personal teacher file, learner file, LTSM, teacher development</td>
</tr>
<tr>
<td>Learner attainment</td>
<td>Assessment records, intervention records, quarterly examination schedule, end year examination schedule, rewards and awards</td>
</tr>
<tr>
<td>Resources</td>
<td>Procurement procedure, retrieval procedure</td>
</tr>
<tr>
<td>School safety</td>
<td>Safety policy, safety procedures</td>
</tr>
<tr>
<td>Teacher discipline</td>
<td>Records of both formal and informal hearings</td>
</tr>
</tbody>
</table>
Appendix H

ETHICS CLEARANCE CERTIFICATE
ETHICS APPROVAL CERTIFICATE OF PROJECT

Based on approval by the Human Resource Research Ethics Committee (HRREC) on 02/11/2017 after being reviewed at the meeting held on 12/09/2017, the North-West University Research Ethics Regulatory Committee (NWU-RERC) hereby approves your project as indicated below. This implies that the NWU-RERC grants its permission that, provided the special conditions specified below are met and pending any other authorisation that may be necessary, the project may be initiated, using the ethics number below.

**Project Title:** School effectiveness and effective mathematics teaching: towards a model of improved learner outcomes.

**Project Leader/Supervisor:** Prof Pontso Christine Moorosi

**Student:** K G Pule

**Ethics number:** NWU-HS-2017-0187

**Application Type:**

- Commencement date: 2017-06-12
- Expiry date: 2020-06-11
- Risk: Minimal

**Special conditions of the approval (if applicable):**

- Translation of the informed consent document to the languages applicable to the study participants should be submitted to the HRREC if applicable.
- Any research at governmental or private institutions, permission must still be obtained from relevant authorities and provided to the NWU-RERC. Ethics approval is required BEFORE approval can be obtained from these authorities.

**General conditions:**

While this ethics approval is subject to all decrees, undertakings and agreements incorporated and signed in the application form, please note the following:

- The project leader (principal investigator) must report in the prescribed format to the NWU-RERC via HRREC:
  - Annually for any other monthly reports on the progress of the project, and upon completion of the project.
  - Without any delay in case of any adverse event (or any matter that interrupts sound ethical principles) during the course of the project.
- The HREC retains the right to:
  - Request access to any information or data at any time during the course or after completion of the project.
  - To ask further questions, seek additional information, require further modification or monitor the conduct of your research or the informed consent process.
  - Withdraw or postpone approval.
- For ethical principles or practices of the project are revealed or suspected, it becomes apparent that any relevant information was withheld from the NWU-RERC or that information has been false or misrepresented, the request for renewal and reporting of adverse events was not done timely and accurately, new institutional rules, national legislation or international conventions deem it necessary.

The RERC would like to remain at your service as scientist and researcher, and wishes you well with your project. Please do not hesitate to contact the RERC or NWU-RERC for any further enquiries or requests for assistance.

Yours sincerely,

Prof Reilwile Phaswana-Mafuya

Chair NWU Research Ethics Regulatory Committee (RERC)
Appendix I

LANGUAGE EDITOR CERTIFICATE

9 Sunbird Place
Arendskloof
Safari Gardens
Rustenburg
30/08/2019

This is to certify that the thesis entitled

SCHOOL EFFECTIVENESS AND EFFECTIVE MATHEMATICS TEACHING:
TOWARDS A MODEL OF IMPROVED LEARNER OUTCOMES

Submitted by KERENG GILBERT PULE
ORCID Number: orci.org/0000-002-9438-6826

For the degree of DOCTOR OF PHILOSOPHY
(MATHEMATICS EDUCATION)

In the FACULTY OF EDUCATION
NORTH WEST UNIVERSITY

Has been edited for language by

Mary Helen Thomas (B.Sc. Hons. PGCE)
Appendix J

STATISTICAL CONSULTANT DECLARATION

Date: 12 August 2019

Cell: 0825912655

TO WHOM IT MAY CONCERN

CERTIFICATE OF STATISTICAL SERVICES RENDERED

I, Ntebogang Dinah Moroke, confirm and declare that I have provided statistical services for the student Mr KG Pule, student number 125438353, thesis “School effectiveness and effective mathematics teaching: Towards a model of improved learner outcomes”, in fulfilment of the requirement for the degree Doctor of Philosophy in Education with Mathematics Education, at the Mafikeng campus of the North West University. The service was rendered in June 2017.

I am a qualified statistician who holds a PhD in Statistics and provided the services in response to the data type collected and the study objectives.

Yours sincerely,

Ntebo Moroke

Prof ND Moroke

BComm, BComm Hons, MComm, PhD (Statistics)
Appendix K

BIBLIOGRAPHIC EDITOR DECLARATION

I Sabelo Chizwina confirm and declare that I have provided bibliographic editing for Mr KG Pule Student Number 125438353 a registered student at the North West University Potchefstroom Campus for the thesis titled:

School effectiveness and effective mathematics teaching: Towards a model of improved learner outcomes.

for

Doctor of Philosophy in Education with Mathematics Education

I am a qualified librarian with a Masters of Information Science Degree and have extensive experience in referencing.

Yours Sincerely

Sabelo Chizwina

(+27 72 753 9065)