

Incremental FTTH deployment planning

Dissertation submitted in fulfilment of the requirements for the degree
Magister in Computer and Electronic Engineering at the Potchefstroom campus of
the
North-West University

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Supervisor: Dr. M.J. Grobler

Co-supervisor: Prof. S.E. Terblanche

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Declaration

I, Jonabelle Laureles hereby declare that the thesis entitled “Incremental FTTH deployment planning” is my own original work and has not already been submitted to any other university or institution for examination.

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For with God nothing shall be impossible - Luke 1:37

Abstract

The use of optical fibres is favoured due to its desirable physical properties and especially for its high bandwidth transmission capability. The challenges faced in the design of fibre-based networks, specifically FTTH deployment, prompts the application of advanced mathematical models and computing technology.

Single-period planning is the current design approach used by service providers. The shortcomings presented with this method have led to the creation of incremental planning. Declared benefits with this approach include eliminating post network deployment modifications, whereby, the unnecessary use of resources can be avoided.

The primary objective of this research is to mathematically model the incremental FTTH planning problem and to evaluate the feasibility of the model by means of a number of case studies.

A Mixed Integer Linear Programming formulation is proposed to model the incremental FTTH planning problem. Three case studies are conducted. The first, a 1-5-20 tree-network (1-central office, 5-splitters and 20-optical network units), used for error detection throughout the formulation of the model. The second, a 1-5-20 street-network, used to determine whether the model can be implemented on a small-scale real-world street scenario, and the third, a 1-8-40 network, to determine the model's scalability.

It was seen from the results that fewer splitters needed to be installed when using the incremental planning approach. This means that fewer trenches and fibres would need to be dug and placed respectively. The results obtained from Case Study 3 proved the model's scalability, indicating that model can solve large-scale incremental networks.

From the results gained, it can be concluded that a Mixed Integer Linear Programming formulation can be used to optimally design an incremental FTTH network. This then proving the feasibility of the proposed mathematical model.

Keywords: *FTTH, MILP, Incremental Time-Periods, Optical Fibres, Optimisation, PON*

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List of Acronyms

AI Artificial Intelligence

AON Active Optical Network

AWG Arrayed Waveguide Grating

BB Branch and Bound

CAPEX Capital Expenditures

CD Cable Distribution

CO Central Office

CSV Comma Separated Values

DBA Dynamic Bandwidth Allocation

EP2P Ethernet Point-to-Point

EPON Ethernet-Based PON

FCP Fibre Concentration Point

FSAN Full-Service Access Network

FTTB Fibre-to-the-Business

FTTC Fibre-to-the-Curb

FTTH Fibre-to-the-Home

FTTN Fibre-to-the-Node

FTTP Fibre-to-the-Premises

FTTx Fibre-to-the-x

GPON Gigabit-Capable PON

HCO Head Central Office

HDTV High Definition Television

ID Identification

ILP Integer Linear Programming

ITU-T International Telecommunication Union - Telecommunication Standardisation
Sector

JC Joint Cabinet

LED Light Emitting Diode

LP Linear Programming

MAC Medium Access Control

MILP Mixed Integer Linear Programming

NP Non-Deterministic Polynomial-Time

NP-complete Non-Deterministic Polynomial-Time Complete

NP-hard Non-Deterministic Polynomial-Time Hard

NP-problem Non-Deterministic Polynomial-Time Problem

NPV Net Present Value

NSP Network Service Provider

OAM Operation and Maintenance

ODF Optical Distribution Frame

OLT Optical Line Terminal

ONT Optical Network Terminal

ONU Optical Network Unit

ONU ID ONU Identification Tag

OPEX Operational Expenditures

OSP Outside Plant

OTB Optical Terminal Box

OVALO Optical Network NPV Optimisation

P2P Point-to-Point

P2MP Point-to-Multiple-Point

PM Percentage Margin

PON Passive Optical Network

QoS Quality of Service

SA South Africa

SATNAC Southern African Telecommunications and Networks Access Conference

SC Splitter Cabinet

SoS System-of-Systems

SP Splitter

Tbps Terabits per Second

TDM Time Division Multiplexing

TIR Total Internal Reflection

VoD Video-on-Demand

WDM Wavelength Division Multiplexing

WIP Work in Progress

Chapter 1

Introduction

The following chapter serves as an introduction to this dissertation. The study is introduced by providing motivation for initially conducting the research, identifying the problem at hand, stating the plan set out to solve the said problem and listing any research restrictions identified. The chapter concludes by providing an overview of the document.

1.1 Background

Higher bandwidth is a growing demand in today's telecommunications industry. There is a constant increase in subscribers making use of services such as Video-on-Demand (VoD), video conferencing, High Definition Television (HDTV) and unlimited content downloading. Network Service Provider (NSP)s are thus under pressure to meet these increased bandwidth demands. According to the authors in [1] ^a there has been an exponential increase in the worldwide international bandwidth used between 2009 and 2013 alone. This demand was measured at 30 Terabits per Second (Tbps) in 2009, whereas, just four years later in 2013, it was measured at a much larger 138 Tbps.

^awith permission from *TeleGeography*

For NSPs to meet these demands, but still keep the upgrade and development costs at a minimum, a fibre based solution known as Fibre-to-the-x (FTTx) was developed. Of all the FTTx structures, Fibre-to-the-Home (FTTH) is the most commonly found and preferred. This is due to forming the ultimate broadband architecture for fixed access networks, by making use of fibre connections all the way to subscriber's homes. In 2005, there were approximately 11 170 FTTx subscribers, which grew to 108 262 by 2012 [2]. This growth of fibre based networks in the telecommunications industry forms the source for the gradual elimination of the older, slower copper-based networks.

From an NSP's point of view, an optimal network topology results when cost and throughput form the basis of the design criteria. The unreliability and impracticality of the conventional point-to-point method made a pathway for the Passive Optical Network (PON) solutions to be considered as good candidates [3].

The approach currently in use when designing a network, such as an FTTH PON, is to base the final topology on the current demands. In other words, the design of the network is chosen specifically to only meet the demands that are currently available. Therefore, when future demands arise, the network will have to be modified accordingly to meet these demands. This approach is known as single-period planning, also referred to as just-in-time planning. The shortcomings to this approach become evident when future demands become available, and network modification is required. The modifications often result in the unnecessary allocation of expenses, labour and time [4].

In recent years, an alternative approach to single-period planning has been found, whereby a network is designed based on both future and current demands. This approach is known as the incremental network planning approach, also referred to as multi-period planning. It has been found that by taking future demands into consideration during the design phase of a network [4,5], the modifications normally required when future demands become available, are omitted, thus avoiding the unnecessary use of resources.

Mathematical programming has become a popular technique used in network design, especially incremental network planning. Various studies have been conducted, whereby, the use of mathematical formulations to design a network based on specified constraints, are investigated. Related work using mathematical programming for PON designs include the work by [6], [7] and [8]. Although these studies include PON network designs, none of them specifically address the incremental network planning problem. The anticipation is the realisation of a greater benefit with an increment network planning approach due to the evolving nature of bandwidth requirements over time and the fact that every planning project is subjected to budget limitations.

1.2 Motivation

Single-period planning is the current design approach used by NSPs. The shortcomings presented with this method has led to the creation of incremental planning. Declared benefits with this approach include eliminating post network deployment modifications, whereby, the unnecessary use of labour and resources can be avoided [4].

Financial concerns, specifically profitability, forms a vital aspect of network design. A network is considered ideal when network expenditures are done as late as possible and revenue is achieved as early as possible. In addition, the worth of deploying a network increases when profit can be expected, this can be calculated with the aid of a Net Present Value (NPV) calculation [9].

The following subsections discuss the above in more detail.

1.2.1 Network planning: Single-period vs. Incremental

Figure 1.1 depicts an example of a result after implementing the single-period planning approach that is currently used by many service providers. The plan operates on a "rise-of-demands" basis, whereby the network is laid out according to the current

demand. As the demand increases, so too, does the network expand accordingly. $T1$ and $T2$ represent time-period 1 and 2 respectively.

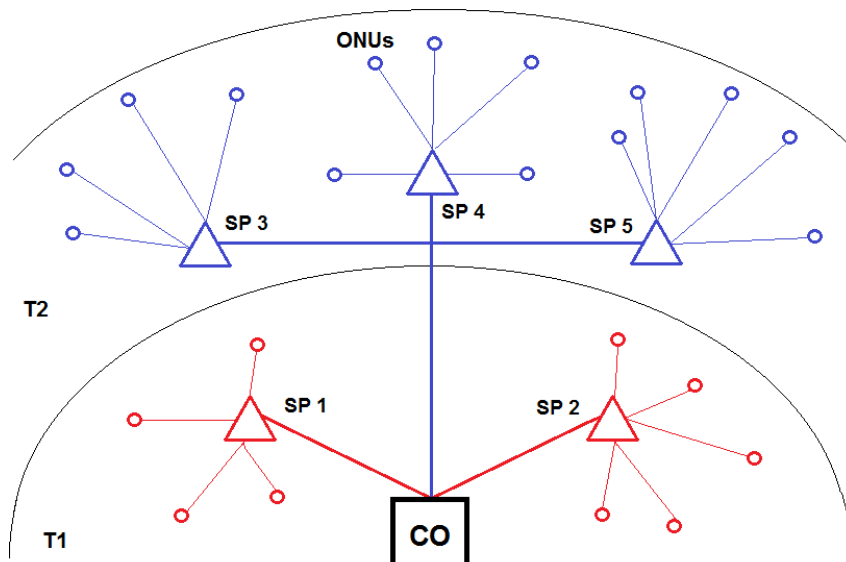


Figure 1.1: An example of a Single-period Plan

Referring to the first time-period, whereby installation is depicted by the colour red, 9 demands become available, presented as Optical Network Unit (ONU)s that are represented by circles. The two lower Splitter (SP)s are selected to distribute to these 9-ONUs and are thus installed in the first time period when the demands become available. As can be seen, the Central Office (CO) directly feeds both of these SPs, this too occurring in the first time-period.

Installation in the second time-period is represented by the colour blue. As can be seen, 3-SPs are chosen to distribute to the additional 14-ONUs that become available, and each SP is directly fed by the CO. Overall, within the time span of two time-periods, a total of 5-SPs are selected, and thus installed, to distribute to the network's total of 23-ONUs.

An incremental planning approach, on the other hand, takes future demands into account when designing the layout of a network. For example, consider the same network scenario as previously mentioned, whereby a network, comprising 23-ONUs, realises over two time-periods. If the network was designed using an incremental ap-

proach, the SP(s) would be selected based on their optimal position(s) for catering to all 23 ONUs, regardless of the time-period in which they become available. Figure 1.2 depicts an example of an optimal network after implementing the incremental planning approach.

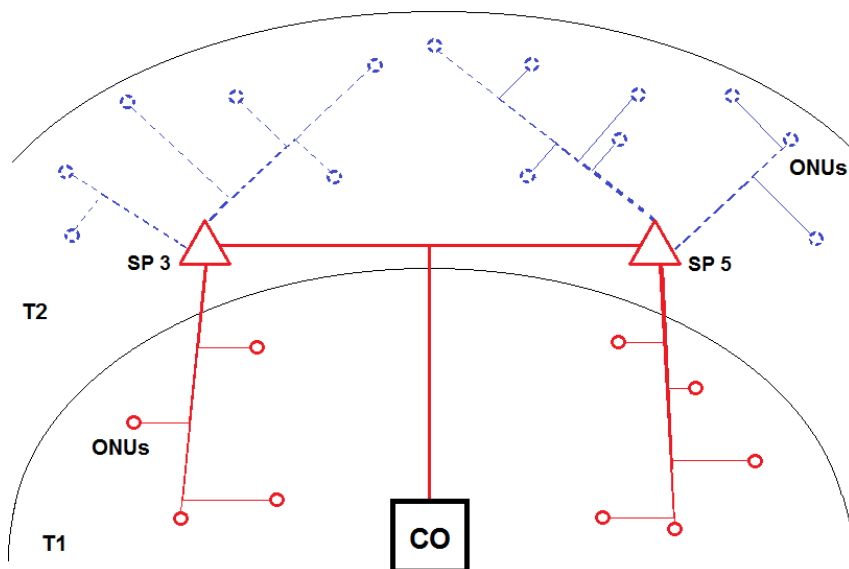


Figure 1.2: An example of an Incremental plan

As can be seen, SP-3 and SP-5 have been selected to distribute to the 23-ONUs over the two time-periods. Each SP is selected based on its optimal position to cater for all demands comprising the network. Both SPs are installed in *T1* as each cater to demands that become available in the first time-period. Thereafter, in the second time-period, the two SPs remain installed to cater to the new demands that become available, as indicated by the colour blue.

It is important to note that, by taking the future demands into consideration when designing the network, the optimal SPs and their locations are selected. This then eliminating the need for network modifications when new demands become available. Therefore, instead of installing an SP to cater for just the current demands, the SP chosen is in a position to cater for both current and future demands.

A decrease in the number of splitters required is immediately noticeable when comparing figure 1.1 to figure 1.2. Fewer SPs indicates fewer trenches that have to be dug. The

fewer trenches and SPs needed to build a network over multiple time periods lowers the overall cost of implementing and expanding a network.

1.2.2 A potential network's net present value

An NPV calculation determines the profitability of an investment. In other words, the calculation aids in determining the future value of a current project. The worth of a network is defined by its profitability, i.e. once deployed, will the network produce a profit? One way to determine this before deployment begins is by performing an NPV calculation. Financially, NSPs will be able to determine the discount rate required for a network to produce a profit, and based on the rate obtained, whether it is worth deploying the network.

This research is focused on investigating the feasibility of a mathematical formulation that models the incremental FTTH planning problem. The process of using a mathematical formulation automates the design phase, thus eliminating the dated method of designing a network by hand. In addition, the implementation of FTTH, i.e. optical fibres, address the current increased bandwidth demand problem.

1.3 Research goal

The overall goal of this research is to mathematically model the incremental FTTH planning problem and to evaluate the feasibility of the model by means of a number of case studies. To accomplish this goal, the focus of this dissertation is simplified into the following:

- Is the aforementioned model feasible?
- Can the integration of an NPV calculation with the model indicate the worth of the deploying the network?

- Can the model be used for large-scale incremental FTTH PONs?

1.4 Research objectives

The following lists the proposed research objectives:

- Construct a small-scale incremental FTTH PON problem instance which captures certain obvious characteristics that allow for verification of the model.
- Formulate a mathematical model for the incremental FTTH planning problem.
- Use CPLEX [10] to solve the model and determine the feasibility of the solution.
- Improve the model with the addition of an NPV calculation.
- Construct an incremental FTTH problem instance to simulate real-world scenarios.
- Determine the feasibility of the model by evaluating the results obtained.

1.5 Methodology

The methodology comprises 3 phases, namely define and design; prepare, collect and analyse; and findings and conclusion. Figure 1.3 depicts a diagram of these 3 phases along with *key steps* describing what each phase entails.

Phase 1: Define and design

The first step is to thoroughly understand the research problem by performing a literature review. This assists in defining the overall goal of the research (section 1.3), listing the objectives to be achieved (section 1.4), and stating the motivation for conducting the study (section 1.2). The information obtained is briefly discussed under the background (section 1.1) of this chapter.

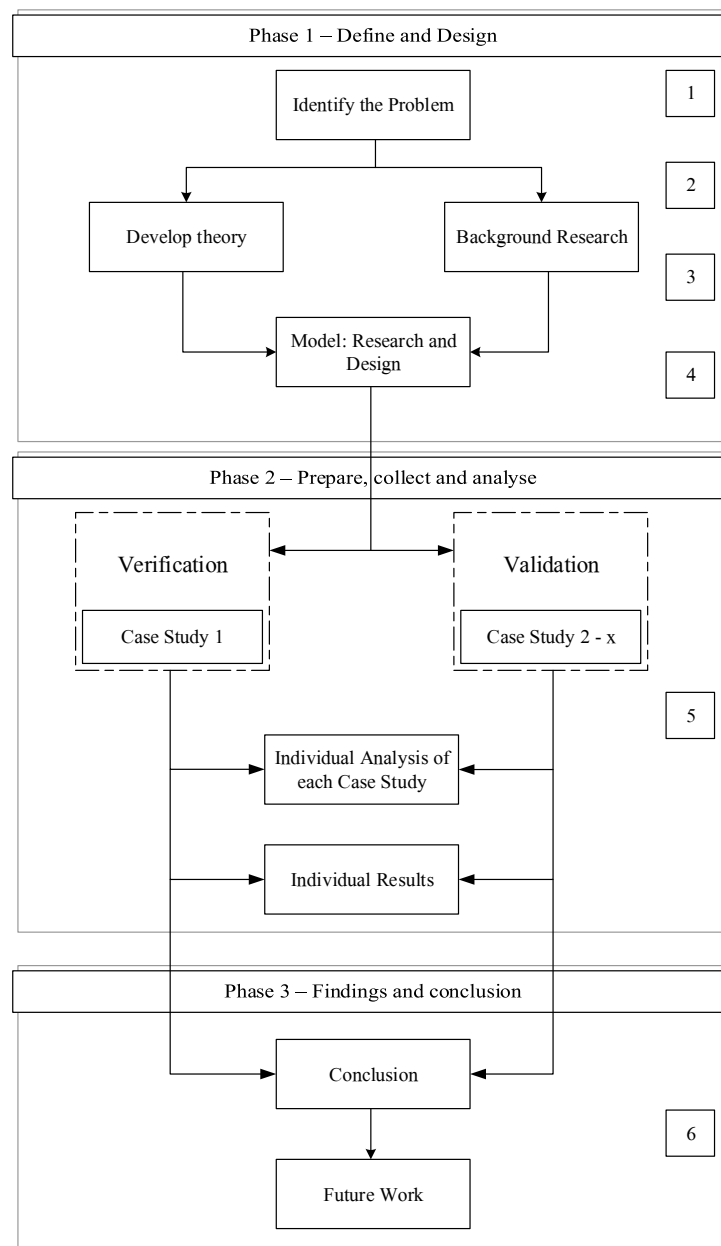


Figure 1.3: Diagram of research layout

The second step in this phase is to do the necessary research to fully understand the relevant concepts brought forth in this study. This will be divided into two chapters, namely Chapter 2 where the technical background will be discussed, and Chapter 3 where a mathematical background will be presented.

The last step in this phase will be to formulate the proposed mathematical model based on the background research conducted and presented in Chapter 4.

Phase 2: Prepare, collect and analyse

In this phase, the evaluation of the model will take place. This will involve the construction of case studies that will be used for verification and validation purposes. A total of three case studies will be investigated, each comprising a different problem instance. The first problem instance will represent a small-scale tree-like network structure that will be used to verify the model. The second problem instance, with the same number of elements as the first, will represent a real-world scenario that will be used to validate the model. The last problem instance will also represent a real-world scenario, however, on a larger scale than the previous two, and will also be used to validate the model, as well as to determine the model's scalability.

Phase 3: Findings and conclusion

The last phase will be to conclude the research by summarising the findings and discussing any future work that can be done.

1.5.1 Verification and validation

As previously mentioned, this research will be verified and validated by the means of case studies.

The first case study will comprise a small-scale network and will be used to verify the model. The idea behind this case study is to determine whether the model can solve an incremental FTTH planning problem by means of a simulation. The simplicity of the network, due to its size, makes it possible to easily perform manual calculations. These calculations will be compared to the simulated results determining the accuracy of the output gained and thus verifying the model.

The second and third case studies will be used to validate the model. This will be

done by constructing real-world potential networks to determine whether the model can solve an incremental FTTH planning problem based on real-world scenarios.

1.6 Research restrictions

The research presented in this dissertation focuses on designing networks for real-world scenarios. It would, therefore, be appropriate to validate the model based on these scenarios. However, due to public restrictions, it is not possible to obtain the required data from NSPs, which results in having to construct problem instances based on real-world information. Therefore, although the problem instances used do not comprise real-world data, they are formulated as close to real-world as possible by selecting actual areas on a map and building a network that realises in the form of suburbs. Any and all information that is required but could not be obtained is modelled as realistically as possible so as to maintain model accuracy.

1.7 Dissertation overview

This dissertation is divided up into a total of 6 chapters, as indicated along the right-hand side of figure 1.3. This chapter, Chapter 1, introduces the research. This includes identifying the chosen problem to be solved, the motivation for performing the study, and stating the objectives and methodology that will be followed so as to achieve the desired results.

The following chapter, Chapter 2, provides all the necessary literature required for a thorough understanding of this research. A description of the FTTx structures can be found, along with descriptions of optical access network technologies, practical considerations to consider when designing a network, and incremental network planning. Chapter 3 discusses all the mathematically-related literature; this includes all algorithms, terminologies and calculations associated with this research.

Chapter 4 presents the proposed mathematical model for the incremental FTTH planning problem. Here the objective function, together with all the model constraints are described. The NPV calculation, as mentioned under the objectives in section 1.4, can also be found in this chapter. Chapter 5 presents the simulations performed and the results obtained. This includes an investigation of three case studies. *Case study 1*, a small-scale incremental FTTH PON problem instance, used to verify the model, and *case study 2*, a modification of *case study 1* to simulate a real-world scenario, used for validation purposes. The last case study, *case study 3*, comprises a larger problem instance when compared to the first two case studies, is also used for validation purposes, as well as to determine whether the model can be used on large-scale FTTH PONs. This case study, as with *case study 2*, is also constructed to simulate a real-world FTTH scenario.

The final chapter, Chapter 6, concludes the research by briefly summarising the problem, concluding the findings, and discussing any recommendations for possible future work. Here is where the last objective listed in section 1.4 is performed whereby the results obtained are evaluated and the feasibility of the model determined.

Chapter 2

Literature Study

The following chapter discusses the literature concepts and methods required throughout the duration and for the completion of this research. This includes the key concepts optical fibres, FTTH, PON, and incremental networks.

2.1 Introduction

The complexity of network design sources from the various methods, techniques, and designs currently available in the telecommunications industry. Before a network can be designed, NSPs are required to know specific details about the desired specifications, regardless of the network's magnitude. Such details are: what type of network will satisfy the demand; what materials would best accomplish the desired task; and how to design an optimal network with a minimum cost objective.

Four key concepts form the basis of this research, namely multi-period, FTTH, PON and NPV. All of which are discussed in this chapter except for NPV, which is fully defined in Chapter 3. Together, these concepts aid in obtaining one goal: Designing an

optimal incremental network, with a minimum cost objective and a profitable future income thereof.

2.2 Fibre optic networks: Fibre-To-The-X

Optical fibres have been found as the optimal solution for the increased bandwidth demand problem, one that metallic cables, specifically the old copper connections, have been unable to meet. The availability of fibre optics was first made possible by Corning Glass Works in the 1970s [11]. They were able to produce a single strand of fibre that had a loss of 20dB/km . Used today, however, and depending on the optical fibre used, a loss that ranges from 0.5dB/km to 1000dB/km can be expected [11]. The year 1997 saw the first commercial installation of the fibre optic system which has since increased quite rapidly throughout the years [11]. There are multiple advantages of fibre optics over copper cables, including those mentioned in table 2.1.

Optical fibres operate in the following way [11]: Various signals, from text, to images, to voice and video, are modulated by pulses of light that serve as electromagnetic wave carriers. These modulated signals are transmitted through a glass tube i.e. a strand of fibre, capable of expanding over far distances. A big advantage when using optical fibres over other alternatives is that signals travelling through the fibre, experience very little loss and attenuation, especially over far distances. This is due to the principle of Total Internal Reflection (TIR). Figure 2.1 depicts a single fibre strand with two light signals entering simultaneously. Through reflection, the TIR principle, light signals can reach each destination point regardless of the number of curves experienced or the angle at which each curve is experienced.

Communication between two points is possible through what is known as a *fibre optic communication link* or simply, a *fibre channel*. The basic design of this communication technique consists of a data transmitter, transmission fibre, and a receiver. As depicted in figure 2.2, the data transmitter comprises of a Light Emitting Diode (LED) which is capable of converting electric signals into light. The light then navigates through

Table 2.1: Comparison between Copper and Fibre cables [11, 12]

Copper Cables	Optical Fibre Cables
The longer the distance, the slower the data transfer	Transmits data much faster over long distances
More signal degradation in comparison	Less signal degradation
Requires a lot of maintenance	Requires a lot less maintenance
Requires a lot of repair actions	Does not require a lot of repairs
Costly to operate	Cheaper to operate
Not immune to RFI and EMI	Immune to RFI and EMI
Uses more power in comparison	Uses less power in comparison
Heavier weight in comparison	Lighter weight in comparison
Vulnerable to outside conditions	Far less vulnerable to outside conditions
Cannot keep up with bandwidth increase	Can keep up with bandwidth increase
Lower security, easy to infiltrate	Higher security, capable of preventing infiltration
Shorter distances, cheaper alternative	Shorter distances, expensive to implement

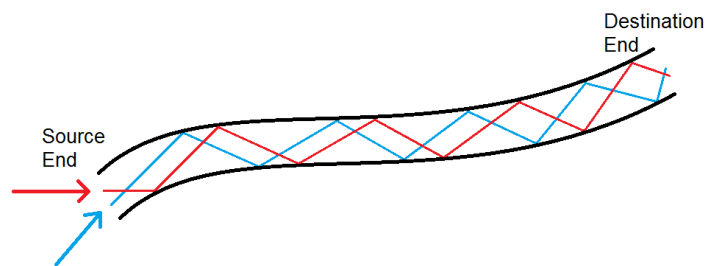


Figure 2.1: Property of a single strand of fibre

the transmission fibre where it is received by the receiver. The receiver consists of a photodetector that is capable of converting the light back into an electric signal [11]. Hereafter, based on the format of the originally transmitted data, is the signal processed accordingly.

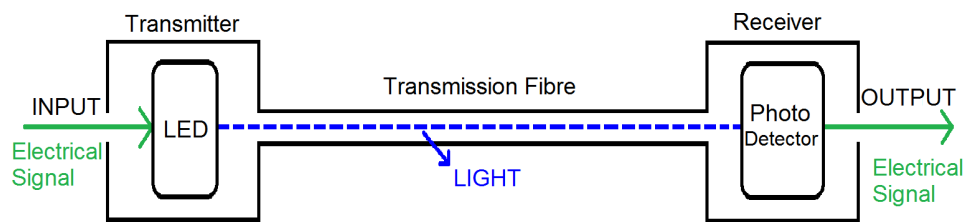


Figure 2.2: Basic design of a fibre optic communication link

With properties such as those found in optical fibres, networks are either being upgraded or developed to include them in their designs. This brought on the creation of fibre optic networks, specifically FTTx infrastructures. These infrastructures have been found as the optimal solution to the world's telecommunication supply and demand problem [2].

In China, construction of FTTx networks first begun in 2007. By the end of 2009, one of China's leading mobile network, China Mobile Zhejiang, had a successful deployment of over 250 000 FTTx lines and had also attracted nearly 40 000 subscribers [13].

An exponential growth of FTTx has been observed since its first implementation over a decade ago. The authors in [2] graphically represent this growth between the years 2005 to 2012. According to this graph, the year 2005 only saw 11 170 FTTx subscribers, while just seven years later, in 2012, it had expanded to 108 983 FTTx subscribers. Figure 2.3 graphically presents this exponential growth experienced throughout the years.

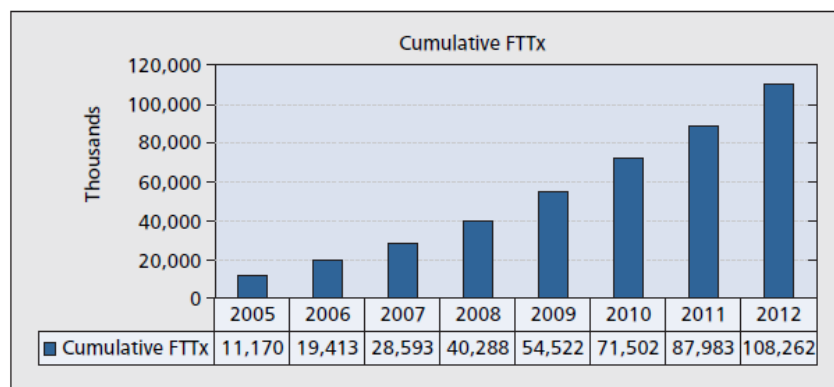


Figure 2.3: Global growth accumulation of FTTx between 2005 - 2012 [2]

There are four well known FTTx structures, namely FTTH, Fibre-to-the-Node (FTTN), Fibre-to-the-Curb (FTTC) and Fibre-to-the-Business (FTTB). Figure 2.4 depicts these four FTTx infrastructures on an *optical fibre versus metallic cabling* basis. As the figure depicts, FTTN is made up of a brief section of fibre, exiting the CO. After a short distance switches over to the dominating metallic cabling. The cable then leads to the end user's premises, where switches/routers/hubs are implemented as necessary. With a similar layout, FTTC also has optical fibres exciting the Head Central Office (HCO). However, they extend for just over half the networking distance before switching over to the metallic cabling, a smaller ratio to that of FTTN. The second last structure, FTTB, consists majority of optical fibres, leading straight to the end user's premises and only upon reaching it, does it switch over to metallic cabling. On an almost identical basis, FTTH only switches over to metallic cabling at the very end, within the premises, where the actual connections are made.

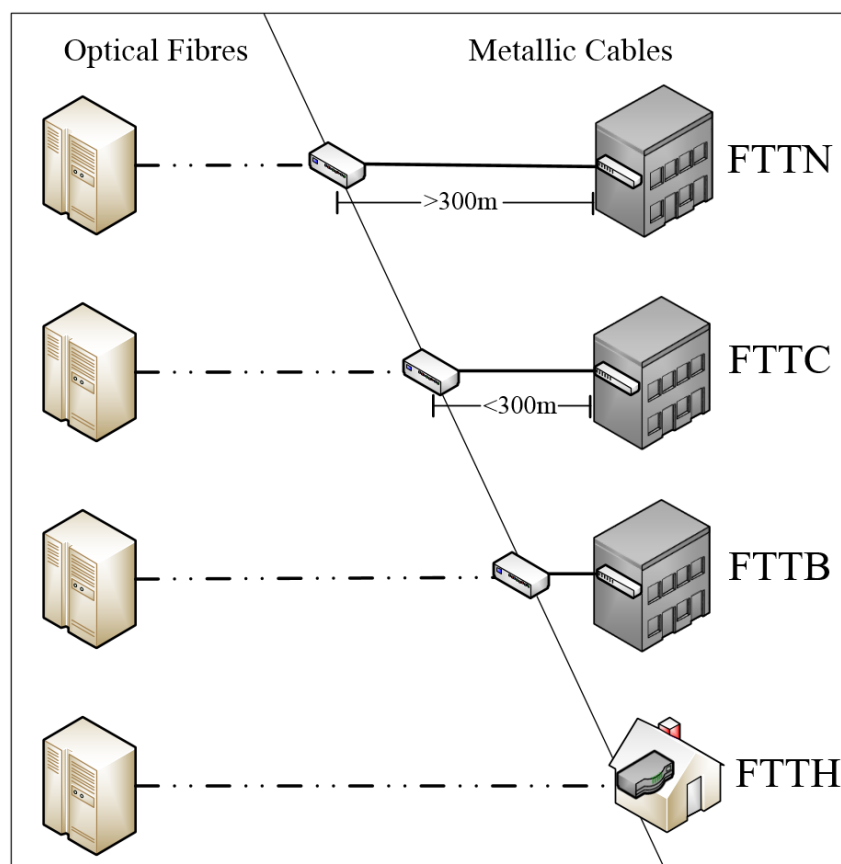


Figure 2.4: FTTx Infrastructures: Optical fibres vs. Metallic cabling

As mentioned earlier in the chapter, fibre dominates over other telecommunication transport materials, which is why FTTH has been found to be the dominating structure followed closely by FTTB. Together, the two terms are commonly referred to as Fibre-to-the-Premises (FTTP) [2], however, this research is based, and will focus only on implementing FTTH networks.

Deployed massively together with PONs, FTTH currently provides the desired broadband services. This combined technique is the most cost-effective solution that minimises the investment needed for deployment. By avoiding the use of repeaters and active electrical components located at the Outside Plant (OSP), operation and maintenance costs are decreased. This takes place between the Optical Line Terminal (OLT) at the CO, and the ONUs found at the user's premises [14].

The FTTH network structure can be divided up into five sections, namely the HCO, feeder, distribution, dispersion, and user networks, as depicted in figure 2.5. The HCO consists of the OLT and Optical Distribution Frame (ODF), which links to the feeder network through the OSP. The OSP connects to the Splitter Cabinet (SC) which then connects to a splitter. The splitter, with the specifications of an appropriate split ratio, then links to the distribution network through the Optical Terminal Box (OTB). If need be, another splitter can be used before entering the dispersion network. The dispersion network is typically the wiring within the building located between the OTB and ONU. Then finally, at the user, the ONU is located. The use of another splitter(s) may be used, based on the requirements of the network to be served [14].

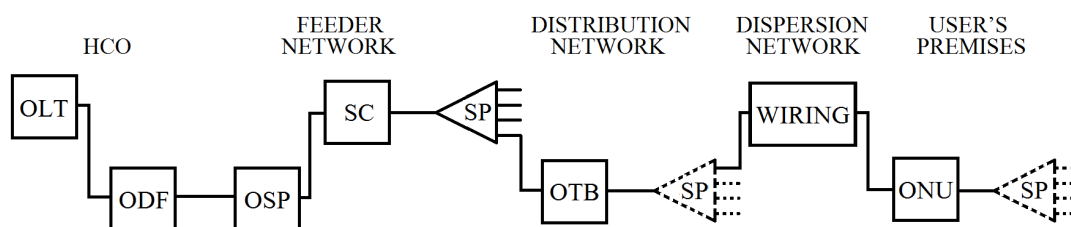


Figure 2.5: The five sections forming the basic structure of a FTTH network

When designing an FTTH network, a choice has to be made with regards to what networking configuration will be used. Two configurations exist, namely Point-to-Point (P2P) and Point-to-Multiple-Point (P2MP). The two configurations are discussed under the following [14]:

Point-to-Point

The utilisation of the traditional Ethernet Point-to-Point (EP2P) topology is used with the P2P technological category. Here exists a dedicated optical fibre between the Ethernet switch, situated at the CO and the ONU. Thus, high Capital Expenditures (CAPEX) and Operational Expenditures (OPEX) are required for each user due to exclusive ports used at the CO and OSP. *The paragraph below discusses CAPEX and OPEX in more detail.* It is, therefore, important to note that EP2P is suitable in a business environment, such as FTTB access networks. This is due to its capabilities of using Ethernet rates up to Gb/s in an easy custom configuration.

When referring to the financial side of telecommunications, CAPEX, as the name suggests, involves capital expenditures. These include investments made towards the network's infrastructure, and systems used for charging and billing clients. OPEX, as too suggested by its name, includes all the operational costs involved in the business section of telecommunications. These include labour costs required for network and customer relationship management, software support, and any marketing expenses [9, 15].

Point-to-Multi-Point

In a P2MP access network, there exists a fibre that exits the OLT, which spreads out to all ONUs, in the network, in a tree topology fashion. Users making use of the same set of components and infrastructures will, therefore, have to share the costs. Hence why it is the most cost-effective option for users located in a residential area. However, collision avoidance and Dynamic Bandwidth Allocation (DBA) protocols will be needed to efficiently distribute the available optical bandwidth among the users.

DBA is a term used in telecommunications to describe the method for allocating bandwidth on an *as-needed* basis in a network. What this means is that the allocation of bandwidth is based on the number and type of activities taking place, rather than reserving a certain amount of users for each task.

A comparison between DBA characteristics was done by the authors in [16]. These characteristics include bandwidth utilisation, delay, and jitter at different traffic loads. This comparison was completed within two major PON standards, namely Gigabit-Capable PON (GPON) and Ethernet-Based PON (EPON). The significant differences between these two standards bring forth many implications for DBA approaches but also shows how to design an efficient bandwidth allocation scheme, specifically for the aforementioned standards. Further information regarding DBA can be found in [16].

2.3 Optical access network technologies

Currently, two FTTH structures exist, namely Active Optical Network (AON) and PON. The difference between these two structures, leading to the popularity and preference for the one over the other, results from the type of equipment used [17].

As stated in its name, AONs make use of *active* components which is why it is also known as active or P2P Ethernet. Europe found the use of AONs desirable, which is why at the end of 2009, AONs represented 84% of their total FTTH/FTTB roll outs [18].

In addition, as stated in its name, a PON makes use of *passive* components in its designs. These components include attenuators, dispersion compensators, splitters, taps, and directional couplers, just to name a few. This structure forms around the concept of connecting a CO, via none-powered passive optical components, to as many users as possible [3,19].

Although both of these structures have advantages and disadvantages in their performance and operational requirements, both are very much capable of making FTTH

connections possible. The final choice between the two comes down to what the NSPs would like to achieve with the network. Whether it is to reach far distances or have low building and maintenance costs, or maybe be able to serve as many users as possible. It is important for NSPs to know their status on these factors as they play an important role in deciding which structure is best suited to achieve the desired results.

Time Division - and Wavelength Division - Multiplexing

An FTTH PON can transmit data in either of two techniques, namely Time Division Multiplexing (TDM) or Wavelength Division Multiplexing (WDM). The commercial TDM PON deployments currently in existence are normally of the GPON or EPON type [18]; these two standards are discussed in more detail in the following subsection. The authors in [18] provide a model whereby a comparison between the different technologies, including TDM PON and WDM PON can be found.

The authors in [20] performed a System-of-Systems (SoS) cost analysis between WDM and TDM FTTH Networks. SoS is an emerging and multidisciplinary research area that comes from the need brought on by consumers for systems that provide advanced features. Although not a new concept, SoS has received an increase in attention over the last decade.

The basic PON architecture makes use of a shared fibre to connect the CO with an intermediate node, and short dedicated fibres to connect said intermediate node to the end users. There are two architectures to choose from, namely TDM-PON and WDM-PON. Figure 2.6, simplified from [20], provides a visual idea of how each of these function.

The TDM-PON architecture, as seen in figure 2.6-a connects all end users to the CO via the aid of a power splitter and a single wavelength. Here, time slots are made available whereby each user is allocated a different slot. Referring to the downlink direction, the OLT's broadcast data over a single wavelength channel, which can be shared via the aid of the Ethernet protocol, on a frame basis. A different wavelength channel is implemented in the uplink direction. This channel transports the data being

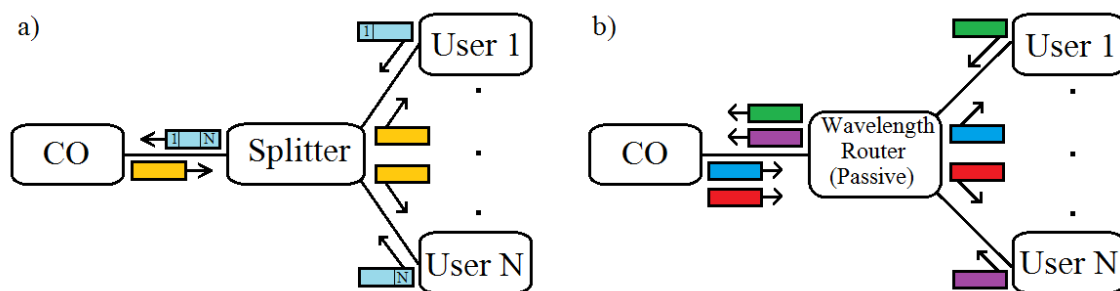


Figure 2.6: The two PON architectures: a) TDM-PON and b) WDM-PON

transmitted upstream by the Optical Network Terminal (ONT)s. All ONTs, part of the PON, make use of the same wavelength. With time slots available with TDM, every user is then able to transmit their own data to the CO. The implementation of TDM is an easy one; however, the use of a splitter does bring forth splitter losses. These losses are linearly in scale with $1/N$, where N indicates the number of users [20].

Taking a look at the WDM-PON architecture, in figure 2.6-b, the use of a multi-wavelength source, specifically a WDM bank, can be found at the CO. The reason for the use of this source is based on the architecture's assignation of a different wavelength for each user in the network. The distribution of the wavelengths to the various users is done so with the aid of a static wavelength router, such as an Arrayed Waveguide Grating (AWG). The use and placement of this device are located in the cabinets of the network. Its cyclic spectral properties make it possible for a single AWG to simultaneously route all the up- and down- stream wavelengths [20]. The equipment used, however, has been found to be more expensive due to the use of more advanced optical components.

Other than the TDM architecture containing time slots and the WDM containing multiple wavelengths, the big difference between the two, narrows down to the *optoelectronic* equipment used at the three main locations. These locations are the CO, intermediate node and user's premises.

2.3.1 PON standards

As mentioned earlier, two standards of PON exist, GPON and EPON. The difference between these two lies in the philosophy chosen. GPON is a versatile concept, based on a rather complex standard, with tight hardware requirements. It also focuses a great deal on Quality of Service (QoS). EPON, on the other hand, is a more economical alternative, based on a much simpler standard with hardware requirements that are much looser, such as accuracy of timing and physical signal levels [3, 16].

The GPON standard is defined by the International Telecommunication Union - Telecommunication Standardisation Sector (ITU-T) G.984.x series of Recommendations, sponsored by the Full-Service Access Network (FSAN) [16]. Several upstream and downstream rates up to 2.48832 Gb/s are specified.

Although lower, at 1.25 Gb/s, EPON too provides both upstream and downstream line rates. However, because 8B/10B line encoding is used, the transmission of data has a resulting bit rate of 1Gb/s [16].

Before choosing a standard, one should first determine which of the two is more economical, and in which circumstances that benefit would apply [3]. Issues to consider include compatibility with other transport systems; transport link utilisation; the cost to build a PON network; network segmentation required when the number of connected users increases; and granularity of the offered transport service.

With regards to compatibility, GPON has been found to be the better choice due to its ability to adapt to the other transport concepts. EPON, on the other hand, supports only Ethernet. Both concepts are considered equally good choices concerning granularity. This is because they both provide connection rates that vary in fragments of the transport link's capacity, from very small to very large [3].

It would, therefore, be inaccurate to say which of the two standards outweighs the other but rather that, each problem dictates which standard is best suitable to accomplish the desired task.

2.3.2 Designing an active PON

The following paragraphs, obtained from [3], describe the practical decisions and steps followed during the development of an active PON.

PONs were designed with the basic idea of connecting an X amount of users to a CO with the aid of non-powered PON components. The CO connects to the various users through the aid of a single fibre that splits into separate strings upon arrival at each destination. The OLT, located at the CO, and the ONUs, located at each end-user, perform the same task, which is to receive and transmit optical signals.

The OLT functions by transmitting data to all the ONUs in the network through the method of broadcasting. A common light wave channel is used to broadcast data frames, and only the frames addressed to a specific ONU are received by said ONU. ONU Identification Tag (ONU ID)s are used to help each ONU identify which frames to accept. The size and form of each ONU ID are dependent on the transport concept used. Transmission in the opposite direction i.e. from the ONUs to the OLT is a more complicated task due to the commonly shared upstream light wave channel.

The use of a mechanism is required to allocate time slots for the transportation needed to be done by each ONU. Due to the unequal distances found between the OLT and all the ONUs in the network, the use of additional mechanisms becomes a requirement to adjust signal levels and propagation delays. With the use of a specific DBA mechanism, an OLT is capable of reserving time slots as required, this occurs in both EPON and GPON scenarios. When an ONU requires transport capacity, it will send a queue length report to the OLT and then based on the information obtained in that report, will a time slot be assigned to the ONU. Another method that can be used to assign time slots is the more common fashion known as *taking it in turns*. What this means is that a cyclic manner is used to allocate transmission turns to each ONU. The transport delay, and, therefore, the efficiency of the network, are significantly affected by the DBA scheme selected and the length of the cycle time.

Upon getting powered up, the first step an ONU has to take is to register as an active

ONU. There are several initialising procedures involved in a registration phase; these procedures should at least include the performance of ranging and registration. A ranging procedure involves measuring the round-trip delay that occurs individually between the OLT and each ONU, while the registration procedure simply selects an ONU ID for each active ONU. As each ONU registers itself, the remaining ONUs remain silent. Programmable intervals are used to present the registration period. The length of the period is dependent on the longest distance between an ONU and OLT, therefore making use of the maximum distance. For example, if the maximum distance is 20km , the period should last for at least $200\text{ }\mu\text{s}$. These procedures are followed in both EPON and GPON. The difference comes in where, with EPON, Ethernet frames are used while GPON makes use of synchronised frame-based transport. Additional differences come in with Medium Access Control (MAC) protocols and the line coding methods used.

2.4 Practical considerations of network design

In practice, there are many factors that influence the overall network layout and choice of equipment. It is, however, important to note that it is not always possible to take all practical factors into consideration when simulating a real life model.

There are many factors to consider, on a practical level, when designing a network. The decisions made should take CAPEX and OPEX into account. By performing trade-offs between the possible choices, from the equipment to the possible layouts and future expansions, CAPEX is affected. OPEX, on the other hand, is affected when operational costs, such as reducing Operation and Maintenance (OAM), are worked upon [14].

However, apart from taking CAPEX and OPEX into account, there are still several other factors that need to be considered. These can be divided up into *three* categories, namely Served sites and positions, Materials and Techniques. The subsequent sections provide a discussion of these three categories.

2.4.1 Served sites and positions

Obtaining details about the site to be served forms a vital step in network design. Firstly, NSPs need to know the nature of the served site i.e. whether it is of a greenfield, brownfield or overbuilt type. Depending on the nature of the site, accurate information on the geography; population distribution; location of present infrastructures must be acquired. If the served site is of a Greenfield deployment, the best course of action would be to put the access technology in place when building the infrastructure. If multiple zones are to merge, i.e. become a multi-area zone, of a Brownfield or Overbuilt type, a single HCO must be chosen from all the legacy COs, to serve the area. The position of the HCO is based on the population distribution and infrastructure facility [8, 14].

As mentioned in Chapter 2, a typical FTTH network structure can be divided into the feeder network, the distribution network and the dispersion network. The feeder network consists of the HCO, OSP and SCs while the distribution network consists of the OTBs. The dispersion network exists between the OTBs and ONUs, located at the user's premises. Under each network, NSPs have to decide what components will be used and what the optimal position of placement will be.

Utilising the link capacity affects the segmentation need of an optical network, which in return, affects the overall access network cost. Therefore, when building a new AON, if the area to be covered has a known population total, NSPs have to decide on the number of segments needed to supply acceptable transport capacity to each end user. As the demand rate increases, NSPs are required to know when to divide the access network into a larger number of segments [3].

2.4.2 Materials

By acquiring information on the geography, infrastructures and population distribution, NSPs can make informed decisions on what materials to use and where. For

instance, at an HCO, the indoor cables should be of a TKT type whereas, by the OSP they should be of a PKP type [14]. The reason for this is due to the different properties of the two cables. TKT cables are manufactured to suppress/delay the production of flames, have a low smoke property should the cable catch fire, and comprises a halogen free thermoplastic cover. These properties are all ideal for indoor use at the HCO. PKP cables on the other hand, are coated with polyethylene, ideal for outside use due to properties enabling it to withstand outdoor extremities, such as what would be experienced at the OSP [14]. Furthermore, a decision should be made on the split ratios of the various splitter types, based on the split ratio required, section 2.4.3 discusses this in further detail.

A typical ODF, situated at the HCO, consists of 256 connections and enables fibre control, thus making it possible for the OSP to connect to any of the OLT ports. For distances of several kilometres to be reached, high-capacity optical cables, consisting of a maximum of 512 fibres, connect the ODF to a primary Fibre Concentration Point (FCP). With Joint Cabinet (JC)s located in the FCP, lower capacity cables can be used to reach out and connect to other FCPs [14].

OTBs located within a building should possess the correct capacity required to serve all of its apartments. In addition, due to its low bending radius ability, KT fireproof, mono-tube type cables are ideal for the dispersion part of the network [14].

The choice of materials is vital for a successful network but varies from network to network based on the specifications and requirements.

2.4.3 Techniques

In addition to selecting the most appropriate FTTx structure, that is either FTTC, FTTN, FTTB or FTTH, the next vital decision to make is whether a P2P or a P2MP configuration will be used. As discussed in the previous section, a P2P solution is where a direct connection between the CO and the subscriber exists, and with a P2MP, multiple subscribers share a single fibre [7]. Using dedicated fibres will, of course, provide the best

bandwidth option, but at a prohibitive price. Therefore, most networks are based on a P2MP configuration.

Another consideration is whether PON or an AON implementation will be done. The main advantage that PONs have over AONs is that it avoids the use of repeaters and any active electrical components. Not only are AONs costly to implement but they require considerably more maintenance.

There are currently two PON based solutions that exist, namely EPON and GPON. EPON is commonly assumed to be an economical alternative whereas GPON is considered to be more versatile. Deciding between the two solutions requires evaluating certain criteria. Examples of this include evaluating the compatibility with other transport systems, transport link utilisation, costs, the network segmentation needed as the connected users increase, and granularity of the offered transport service [3].

The link capacity in a GPON is often dimensioned based on the upstream link utilisation. It is, however, important to note, that with residential subscribers, generated traffic mainly occurs in the downstream direction, flowing from the OLT to the ONU. Dimensioning the downstream link capacity of a GPON needs to be done with care so as to accommodate VoD, HDTV, and the other services available. An accurate estimation of the capacity required by subscribers is an important step in selecting the correct optical split ratio [21].

Neighbourhoods can be made up from more than one *type* of set of subscribers, i.e. not only residence or only businesses but maybe a combination of both. Services deployed in residential areas are done so in *unprotected mode* i.e. not protected against a possible connection failure, leading to minimum expenses. Whereas, in areas comprising businesses, *protected mode* is required i.e. a backup path is defined in order to protect the connection from a single/multiple failure(s), which can thus lead to high expenses [21].

The bandwidth allocation per subscriber, sharing a single PON link, is solely determined by the split ratio. High costs are expected when a small split ratio is selected. However, a higher split ratio has the tendency for reducing the bandwidth allocation

per subscriber and as a result, subscribers may experience a quality decrease in the services received [21].

According to [21], there are 8 *Rules* to follow when choosing the optimal split ratio.

- **Rule 1:** A survey should be conducted with the aim of obtaining information such as the nature of the population to be served. This will then aid in allocating and configuring the resources needed. The idea is to have the survey conducted alongside questionnaires with the same focus. These questionnaires should be completed by customers who request an internet connection.
- **Rule 2:** Multiple dimensioning parameters should be considered to obtain an optimal dimensioning GPON capacity. These parameters include the number of subscribers and the required bandwidth for the generated traffic. Knowing the number of channels, such as HDTV and VoD, aids in determining the capacity required for all the subscribers. Subsequently, the split ratio selection process can proceed with a guaranteed supply of the required bandwidth.
- **Rule 3:** For optimal GPON capacity dimensioning, valuable inputs are required, these include understanding subscribers' profiles and determining the busiest hour(s) that would be experienced on a daily basis. It is, however, important to note that, although it is possible to dimension GPON's capacity based on the requested resources during peak hour, the volume of generated traffic should also be considered. Doing so will allow trade-offs to be performed between the cost of requested resources, QoS and potential network revenue.
- **Rule 4:** A conservative approach for dimensioning GPON link capacity is one where the sum of services peak rates is calculated.
- **Rule 5:** The network's performance should be monitored carefully by constantly measuring the flow of traffic and then reporting the value of the quality observed. Doing so makes it possible to monitor the evolution of traffic as it unfolds, and network planners can then anticipate any upgrades that will occur in the access network.

- **Rule 6:** With the knowledge of the GPON link utilisation and service penetration values, network providers can, more accurately, select an optical split ratio that can support the required services, such as video.
- **Rule 7:** When selecting a split ratio, what should always be kept in mind is that that ratio should guarantee a link utilisation of around 75%. Thus leaving some resources to manage, if any, unexpected growing traffic rates.
- **Rule 8:** It is important to remember that a specific split ratio might be required for building locations and network applications.

A design technique that should be considered, especially when cost minimisation is the desired design criterion, is *fibre duct sharing* or *path sharing*. This technique deems useful when common routes are found among fibres i.e. when fibres share a part of or the entire route. Trenching is considered to be one of the largest cost contributors when deploying a PON. However, this expense can be kept at a minimum through the implementation of fibre duct sharing. The basic principle, as depicted in figure 2.7 is that, instead of having a dedicated trench for each fibre, fibres following the same route can share a trench for as long as possible. Path sharing thus saves on having to dig up new trenches along the same routes [7].

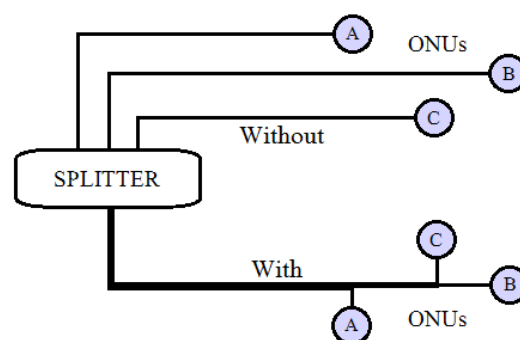


Figure 2.7: The basic concept of fibre duct sharing

A popular network design approach is to make use of splitting stages also known as tree splitting as seen in figure 2.8. A single splitting stage, for example, is when the CO connects to a splitter and that splitter connects directly to an ONU located at each

client. A two stage splitter, on the other hand, would also have the CO connected to a splitter but then instead, have that splitter connect to multiple splitters. Those splitters then connect to the individual client terminals and so grows the splitting stages [8].

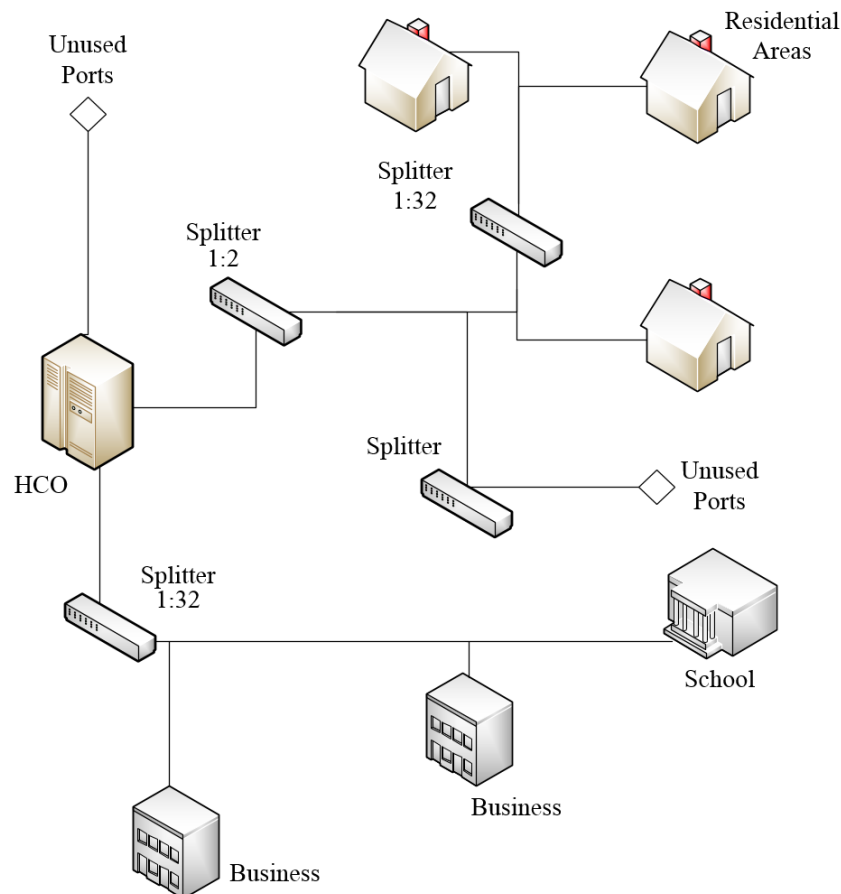


Figure 2.8: A tree splitting network

Positioning the HCO is highly dependent on the housing density of the served zone. With a uniform density, the HCO would most likely be placed at the centre, however with more variability, the HCO would be placed closer to where the most demands are located [14].

The feeder network connects the HCO to the distribution network. The design of this network must include a long-term evolution, as it is very costly to install new cables at the dawn of each new network. It is, therefore, more practical to have fibres unused and stagnant than to spend, unnecessarily, as the need arises [14].

The deployment process completes within 3 phases, namely, strategic, high-level and detailed. The strategic phase focuses on the area and population to be served, and what techniques should be applied to fulfil said demands. The high-level phase focuses mainly on the geography of the served zone, i.e. what components to use and where to implement them ensuring a minimum cost yet effective solution. The last phase, the detailed phase, involves sending the final network design, the one to be constructed, to the relevant companies for installation [14].

2.5 Incremental network planning

Incremental network planning also referred to as multi-period planning, has been studied extensively within the context of transmission networks, see *e.g.* [22], [23] and [24]. However, the application of incremental network design methodologies for access networks, specifically PON, are still very limited.

A comparison between the single-period- and the multi-period- planning problem can be found in [4]. The focus of the paper was to study the long-term planning of survivable WDM networks. The aim of their study was to assess the cost difference, both for installation and maintenance, when comparing long-term planning (with a planning horizon of 2 to 5 years) v.s an ad hoc scenario, to deploy connections between cities. A multi-period model was formulated based on the combination of network topology and capacity expansion. The single-period approach involved separately designing the network for every time-period, whereas, the multi-period approach considered all time periods simultaneously. Exactly 30 different problems were generated and then solved using 6 different methods, 3 algorithmic approaches and 3 network design cost models, which resulted in a total of 180 experiments.

The integrated multi-period approach was found to save on expenses up to an average of 4.4%, calculated by doing a comparison on a global level. The authors concluded that the result gained was based purely on better scheduling of investment over time.

A similar, real life scenario can be found in [5], where an entire city's network, Santa Monica, was converted to run on fibre optics. In the year 1998, the city announced their "*concrete yet visionary Telecommunications Master Plan*" which involved adopting an incremental approach to constructing networks based on optical fibres. To date, the result is the most successful *dig once* policy implemented in the United States.

The plan was initiated with a \$530,000.00 investment, spent on connecting municipal facilities, the school district, and the Santa Monica College with the city's owned fibre. It was also decided to lay down extra fibre, so that when large firms like *Google* requested access to their fibre, they could assist with no hassle.

When the project started, the City Information Systems Department carefully chose locations whereby fibre would be a great necessity. By combining fibre and conduit installation with other capital projects and performing joint trenching together with other entities, the cost of laying fibre was reduced by up to 90%. During the first year, after the initial migration, the City was able to save roughly \$400,000.00, which increased to \$700,000.00 per year thereafter.

2.6 Chapter conclusion

This chapter is the first of the two literature-based chapters in this dissertation. Presented here are all the theoretical terms vital to this research.

Firstly, a large section is dedicated to discussing optical fibres and FTTx structures. The advantageous properties of fibre over copper clearly prove why fibre is the ideal solution to the problems and limitations experienced with network design and performance. This brought about the birth of FTTx structures, specifically FTTH, the most popular FTTx structure.

Many techniques currently exist, some are improvements on others, such as P2P and P2MP, while others differ in technologies, such as GPON and EPON or TDM and

WDM. The choice of techniques and technologies play a vital role in the success of a network. However, if a certain technique works well for one network, it does not necessarily mean it will for every network it gets implemented in, this is due to specifications differing from network to network depending on the requirements.

The last two sections discuss incremental network planning and practical considerations of a network design. With this research focused on incremental networks, it is important to note any previous studies conducted and the results gained thereof, thus providing a frame of reference for comparison purposes. Lastly, practical factors play a vital role in network design. Some may greatly affect the design of the network, while others may not. However, to have a successful network, the design should consider as many practical factors as possible.

Chapter 3

Optimisation and Mathematical Modelling Techniques

The Oxford Dictionary defines the word optimisation as follows: "Make the best or most effective use of (a situation or source)". In mathematics, optimisation is a technique used to find the optimal or best solution to a mathematical problem, by either minimising or maximising the objective function. The following chapter discusses how to achieve optimisation with the aid of mathematical modelling.

3.1 Linear Programming

As defined in [25], a *Linear Programme* is made up of continuous variables and linear constraints, which can be either inequalities or equalities. The objective of the formulation is to optimise the linear cost function that aims at obtaining either a minimum or maximum cost. Linear Programming (LP) can be written in many forms; the following is an example of one of these standard forms:

$$\begin{array}{llllll}
\min & c_1x_1 & + & c_2x_2 & + & \cdots & + & c_nx_n \\
\text{subject to} & a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n = b_1 \\
& a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n = b_2 \\
& \vdots & & & & & & \vdots \\
& a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n = b_m \\
& x_1, \cdots, x_n & \geq & 0
\end{array}$$

which can be simplified in matrix notation as:

$$\min\{c^T x | Ax = b, x \geq 0\} \quad (3.1)$$

where $c, x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m \times n}$. Here the cost vector is represented by c while x represents the vector of variables.

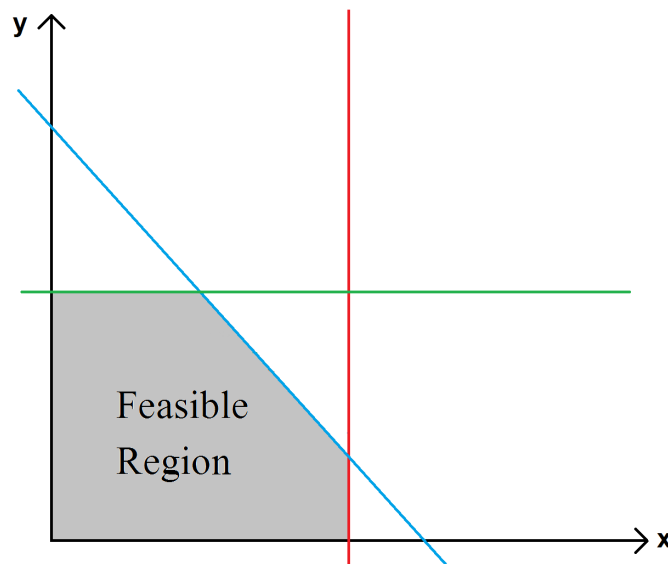


Figure 3.1: Linear Programming - Identifying the feasible region

So to simplify, a typical format for an LP formulation can be broken up into the *Objective Function* subject to *Constraints*. By knowing what would like to be achieved, stated so in the objective function, and based on the various constraints forming the bound-

aries of the model, it is possible to obtain the feasible region. Figure 3.1 depicts this region whereby the optimal solution(s) can be found.

For simplicity, assume that the rank (the number of linearly independent rows or columns of the matrix) of A is m , in other words, no redundant equations exist in (3.1).

Let $A = (a_1, a_2, \dots, a_n)$ where a_j is the j -th column of A . An "index set" $I \subseteq \{1, \dots, n\}$ is denoted by A_I where A_I is the sub-matrix of A and consists of columns a_i where $i \in I$.

As mentioned earlier, the rank of A is m , therefore, there exists an index set $B = \{B_1, \dots, B_m\}$ such that the $m \times m$ submatrix $A_B = (a_{B1}, \dots, a_{Bm})$ is non-singular and therefore invertible. The sub-matrix A_B is called the *basis* of A . Let $N = \{1, \dots, n\} \setminus B$. By rearranging the columns of A in such a way that $A = (A_B, A_N)$, $Ax = b$ from (3.1) can be written as:

$$A_B x_B + A_N x_N = b \quad (3.2)$$

where $x = (x_B, x_N)$.

A solution to $Ax = b$, known as a *basic solution*, is provided by $x_B = A_B^{-1}b$ and $x_N = \overleftarrow{0}$. A basic solution is considered feasible if $A_B^{-1}b \geq \overleftarrow{0}$. The *basic variables* can be found in vector x_B while the *non-basic variables* can be found in vector x_N . By rearranging c in such a manner that $c = (c_B, c_N)$, the objective value that corresponds to it is then $c^T x = c_B^T A_B^T b + c_N^T \overleftarrow{0} = c_B^T A_B^{-1} b$

Given a basis A_B and from (3.1), an improvement in the objective function is reflected in $(c_N^T - c_B^T A_B^{-1} A_N) x_N$. The cost vector for x_N is now renamed the *reduced-cost* vector. From both a basic and non-basic variable point of view, the reduced-costs vector is defined as $\bar{c}^T = c^T - c_B^T A_B^{-1} A$.

Based on the information above, the following theorem is defined:

Theorem 1 [25] (x_B, x_N) is an optimal solution if and only if $\bar{c} \geq \overleftarrow{0}$

Apart from improving the objective function, the reduced-costs vector also provides information on the marginal rate. That is, by inserting a non-basic variable into the solution, or simply by giving the solution a non-zero value.

The *Simplex Method*, invented by Dantzig, was first published in the 1940s [25], and is often used to solve LP formulations. The application of *Theorem 1* is found in this method. In very basic terms, this method functions by replacing a column in A_B by a column in A_N . This process continues until a very basic, feasible solution is found, whereby all reduced-costs are non-negative. In practice, this method's performance has been found to be relatively fast, especially because it entails a polynomial worst-case time complexity. More information on LP programming can be found in [25].

As mentioned earlier, an LP formulation consists of an objective function and constraints, both comprising unknown variables. If the model requirement is to have all the variables be of an integer type, the problem is then labelled as an Integer Linear Programming (ILP) problem. If, however, only a few of the variables need to be of an integer type, the problem is known as a Mixed Integer Linear Programming (MILP). Both ILP and MILP are considered as Non-Deterministic Polynomial-Time Hard (NP-hard) in many practical situations, a great contrast to LP, of which worst case problems can be efficiently solved.

3.2 Computational complexity

Within *theoretical computer science and mathematics* resides a section called *theory of computation*, of which a section entitled *computational complexity theory* exists. The focus of this section is to classify computational problems according to their difficulty and then to relate those classes to one another. Computational problems have been defined to be

solved by a mechanical application of mathematical steps, for example, an algorithm. Keeping this in mind, it is now possible to define a Non-Deterministic Polynomial-Time (NP) problem.

NP was originally defined in terms of non-deterministic machines, hence its name. However, with regards to computational complexity theory, NP has been classified as an essential complexity class. It can be seen as a set of all decision problems, whereby each instance that has the answer "yes" has efficient proofs, that can be verified, fully confirming the answer. From a more mathematical point of view, a deterministic Turing machine should be able to verify these proofs in polynomial time. Two classes have been mentioned in this chapter, namely NP-hard and Non-Deterministic Polynomial-Time Complete (NP-complete), and, as a result, will be briefly discussed in the following subsections. Further information regarding NP and the two aforementioned classes can be found at [26].

3.2.1 NP-hard

In computational complexity theory NP-hard is defined as follows:

"Assume there exists a problem H . H is considered NP-hard if and only if every problem L in NP can be transformed in polynomial time to H ."

In other words, if there exists an algorithm to solve a problem A , and that algorithm can be translated into one that can be used to solve any Non-Deterministic Polynomial-Time Problem (NP-problem), problem A is then classified as an NP-hard problem. NP-hard can therefore be defined to be *"at least as hard as any NP-problem"*.

3.2.2 NP-complete

As with NP-hard, NP-complete is also defined regarding computational complexity theory, which states:

"If a decision problem is found in both NP and NP-hard, it is classified as NP-complete."

The most highly known characteristic of the NP-complete problem is that "no fast solution" to them exists. It is, therefore, important to note that even though it is possible to quickly verify a given solution in polynomial time, no efficient method to obtain the solution in the first place exists. It is also noteworthy to know that the growth of a problem is directly proportional to the time required to solve the problem. Implying that, as the problem grows, so does the time required to solve the problem, increase.

3.3 Solution methodologies: Exact vs. Heuristic

3.3.1 Exact approaches

Exact approaches are solution methodologies which either solve problem instances up to optimality, or if terminated prematurely, provide a quantification of the solution quality [7]. Examples of such algorithms are *simplex* and *interior point* that can be used to solve LPs, and *Branch and Bound (BB)* that can be used to solve MILPs. Examples of the commercial solvers that can be used are *CPLEX Optimisation Studio*, a product by IBM [10], and *Gurobi Optimiser* [27].

An exact approach aims to solve problems up to a *proven* optimality i.e. to produce a globally optimal solution.

Although this approach is the ideal methodology when an optimal solution is desired, there exist a disadvantage to using it. The algorithms used thoroughly assess a problem in order to determine the optimal solution, therefore, the entire process can become computationally expensive when medium to large sized problems have to be solved [7]. Therefore, a decision should be made whether an optimal solution is vital or whether a heuristic approach (discussed later in this section) would be the ideal method.

The following discusses one of the aforementioned algorithms in more detail, namely BB.

Branch and bound

The BB algorithm is a solution paradigm specifically for optimisation problems such as those that are of the discrete and combinatorial type. The algorithm functions by making use of a *state space search* in order to obtain the possible solutions to a problem. The search space is then systematically enumerated or split into smaller regions. The combination of the enumerated solution sets then begins to form what looks like a *rooted tree*, whereby the root is formed by the full set of solutions. The subsets of the solutions are represented by the *branches* of the tree. The branches are then investigated further by the algorithm, whereby a comparison between them and the upper and lower bounds, estimated on the optimal solution, is performed before enumerating the possible solutions. If, during each comparison process, a branch does not produce a better solution than the current best solution, it gets discarded. This process is repeated until eventually, a final optimal solution is found [28].

Combinatorial optimisation problems, such as the *shortest path* and *minimum spanning tree* problems, make use of polynomial algorithms, however, there is no known polynomial method for their solutions. A few examples are crew scheduling, vehicle routing and production planning, which are all labelled as *NP-Hard* problems. When an *NP-Hard* combinatorial optimisation problem needs to be solved, the BB algorithm has been found to be the widely preferred tool for tackling said problem. It is, however, important to note that, BB is a solution paradigm, which, for each specific problem type, has to be filled out. In addition, there exists multiple choices for each of the components [28].

Just as described in subsection 3.1, there exists an objective function whereby the optimal solution is desired based on either minimising or maximising said solution, subject to a set of constraints. Figure 3.2 depicts an example of BB. The set S is the *search space*

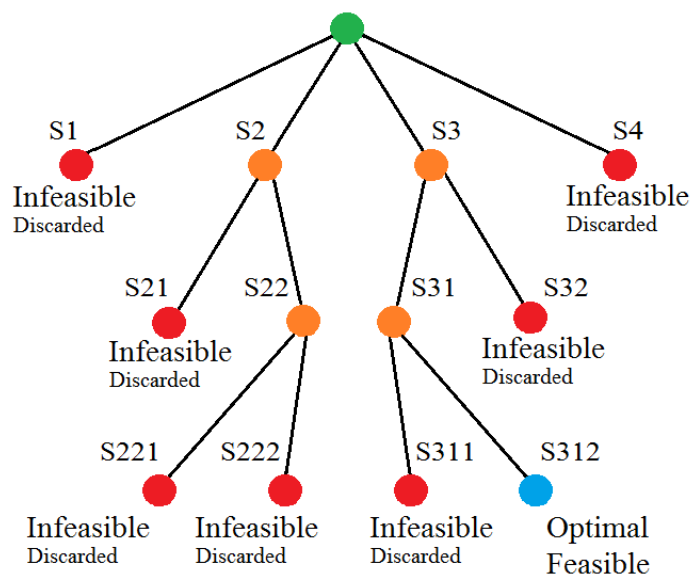


Figure 3.2: An example of branch and bound

that contains all the possible solutions. If a subset is found to be infeasible, indicating that one or more of the constraints has not been met, it is immediately discarded as seen by the red nodes in the figure. If a subset meets all the constraints, it is then "branched" further, as depicted by the orange nodes. As the "tree" expands it gets narrower and narrower, eventually revealing the optimal solution whereby the objective function is met, finding an optimal solution within all the declared constraints.

3.3.2 Heuristic approaches

Heuristic approaches are solution methodologies that provide feasible solutions to a problem at only a fraction of the computational cost. In other words, this approach is unable to prove the optimality of a solution however, the time required to solve a problem, with a close-to-optimal solution, is mostly faster than other alternatives [7,29]. Heuristics are preferred when other methodologies, such as the exact approach, are unable to solve a problem due to their high time-complexity which is unacceptable for NP-complete tasks.

The implementation of this approach can be found in the various research fields, such

as those listed below [30]:

- limiting large search spaces,
- behavioural studies of animals,
- Artificial Intelligence (AI), and even
- biology, specifically human error.

The reason behind the initial purpose of researching and implementing the heuristic process in the field of AI was due to *NP-complete* problems i.e. problems of which logic and probability were unable to solve.

The use of heuristics spread to many more fields, eventually providing a vital contribution to mathematical modelling, specifically where optimisation is concerned. It has even been defined as "a step in an optimisation algorithm [7]". Heuristics is implemented by analysing the data collected thus far, and determining which of the solutions from the search space to iterate next. In other words, which solution shows possible promise and should thus be investigated next. By following this process, the algorithm is guided into the direction of a possible optimal solution, thus avoiding exhaustive enumeration.

The following discusses the greedy algorithm, a popular heuristic technique.

Greedy algorithms

Greedy algorithms are best known for the manner of which solutions are obtained. This specific manner entails *building up* the solution piece by piece, by selecting solutions that prove to be the most obvious and have an immediate benefit based on the objective. The following lists greedy-based algorithms each of which are described in full detail in [31].

- Minimum spanning trees,
- Huffman encoding,
- horn formulas, and
- set cover.

This brief discussion on greedy algorithms is based on the focus of this research. The purpose of mentioning the algorithm at all is to provide the reader with proof that multiple algorithms exist whereby the optimal or, as most often experienced, a close-to-optimal solution is obtained, known as heuristics. Although the algorithm has been found to be very unsuccessful for some computational tasks, there exists many, for which it has been found to be very successful.

3.4 Formulate the FTTH planning problem as a MILP

As discussed earlier in this chapter, a problem is declared a MILP if only some variables are required to be of an integer type. The following points list the benefits of using a MILP formulation as an automating tool [32]:

- Deployment costs are minimised; these include installation and material costs
- The design time is shortened as a result of implementing the automation process
- Any changes made in the planning requirements can be met by comparing various scenarios together
- For easier contract control and installation, any sudden changes in costs can be implemented without any hassle
- Network planners will be able to produce automatically a cable design and bill-of-materials

- Prior to performing network optimisation, network planners can identify pre-determined locations for the elements

As listed above, it is clear why an ILP formulation, specifically a MILP, would be the ideal tool for formulating the incremental FTTH planning problem. An additional factor to consider when formulating the problem is the financial aspect, i.e. all the expenses that have to be made, the income expected, and whether the network will make a profit or loss. One such way to formulate this as part of the planning problem is by determining the NPV of the network, a popular calculation used in financial mathematics. The following subsection discusses NPV in more detail.

3.4.1 Net Present Value

Financial success involves taking financial risks; this includes making investments, i.e. placing money in one or another financial scheme with the hopes of achieving a profit. Specifically, from a networking point of view, NSPs have to decide on whether the investment (the design, deployment and expansion) of a network will indeed make a profit. This is where the maximisation of NPV becomes useful. The basic aim of this calculation is to determine whether or not, in the future, a financial gain or a financial loss will be experienced, by investing now, in the present. The calculation, therefore, takes the future income of a specific investment, subtracts the initial investment and any additional expenses, and determines whether or not a profit will be achieved [9]. This calculation provides the *worth* of the investment.

A single formula for the NPV calculation exists, this is captured in equation (3.3). This formula can be used for any scenario but has been found to be preferable for scenarios with an even cash inflow. By rearranging equation (3.3), equation (3.4) is formed. This equation has been found to be very useful and easier to use for scenarios with an uneven cash inflow.

$$NPV = R * [\frac{1 - (1 + i)^{-n}}{i}] - x \quad (3.3)$$

$$NPV = [\frac{R_1}{(1 + i)^1} + \frac{R_2}{(1 + i)^2} + \dots + \frac{R_n}{(1 + i)^n}] - x \quad (3.4)$$

Where, for both equations, R is the cash inflow; x is the initial investment; i is the rate of return in decimals and n the number of periods.

Equation 3.5 captures NPV in terms of revenue, CAPEX, and OPEX:

$$NPV = \sum_{t=1}^Y [\frac{Revenue_t - OPEX_t}{(1 + r)^t}] - CAPEX \quad (3.5)$$

where: Y is the total time periods (in years) considered, $t \in Y$, and r represents the annual discount rate chosen. Other than the layout used, this equation is no different to equation 3.3. The format of equation 3.5 is ideal for the Optical Network NPV Optimisation (OVALO) problem whereby NPV is maximised. Further information regarding this problem can be found in [9].

As was mentioned earlier, the result of the NPV calculation aids in determining the worth of an investment. Therefore, a positive result indicates a profit while a negative result indicates a loss. The greater the NPV, the greater the profit and, therefore, a greater the cash inflow can be expected.

The advantage of using the NPV technique is that while other techniques do not take payback periods and the rate of return into account, NPV takes the time value of money into account. The disadvantage, however, is that the values used in the calculation are based on estimated cash inflows, and as with any estimation, has the possibility of being far from the actual value.

3.5 Related work

A study was done in [32], whereby a MILP formulation was used to demonstrate how the process of designing an FTTH network could be automated. The formulation was solved using *IBM ILOG CPLEX* [10] and applied to 3 FTTH GPONs, specifically 2 medium networks and 1 big network. FTTH GPON was chosen based on the ability of being able to eliminate any bottleneck that occur in the last kilometre or two of the network. This bottleneck is experienced in speed and services between the core and customer's premises. The focus of this article was to deploy a network, at a minimum cost expenditure, and optimally choose and place the required elements.

However, before solving the model, certain assumptions have to be made about what information regarding the planning area is provided. The following lists all the information pre-provided about each network:

- The location of the CO
- The location of the customer's premises, whereby an ONU is situated
- All possible locations for each SP node and Cable Distribution (CD) points
- How many customers there are per premises, thus identifying how many PON connections are required
- The civils layer network specifying all the connections between the various elements
- How much additional capacity is required for future expansion

The two medium networks comprise 2 SPs, 94 possible CD points and 214 plots. The first, consisting of 314 customers, gained an optimal result within *12 seconds*, making use of 25 CD points. The second, containing no specific customer total but rather that there are two customers per plot, also gained an optimal result, obtained within *5 seconds*, making use of 34 CD points. The big network comprises 5 SPs, 487 possible

CD points and 1033 plots. After an execution time of fewer than 20 minutes, a near to optimal cost was obtained. From what was understood from CPLEX, the result gained was just 2% higher than the actual optimal cost.

The authors were able to conclude that it is indeed feasible to design an FTTH network based on using a MILP formulation. The output of the last execution proved that even large networks can be investigated and still output satisfactory results. For all three networks, the locations generated for the SP nodes and CD points were gained with a satisfactory degree of symmetry. The authors lastly and most importantly concluded that the formulation truly did aid in automating the FTTH design process, capable of laying out a medium sized network in a matter of seconds.

3.6 Chapter conclusion

This is the last of the three literature-based chapters in this dissertation. Presented here are all the mathematical algorithms, terminologies and calculations both directly and indirectly related to this research. The aim of these algorithms is to satisfy the objective function subject to constraints based on the requirements of the problem. Although there are many algorithms ideal for this research, the ones discussed in this chapter are best found to determine the optimal solution to a problem similar to that which is under investigation. Each algorithm follows a different method for determining the optimal solution to a problem; some may take longer than others. Although these algorithms share the same goal, not all of them are capable of solving every problem. Choosing an algorithm is, therefore, based on the problem to be solved, this includes its complexity, size and of course, the main objective. The uncommon terms mentioned while describing the various algorithms are individually defined for a more mathematical understanding of this research.

Chapter 4

Model Formulation

The following chapter introduces the mathematical model, specifically an MILP formulation, formulated for the study of this research. An in-depth description of the objective function and for each constraint is provided, explaining the purpose of each equation. Keywords in this chapter are: MILP, FTTH, NPV and PON.

4.1 Model sets, parameters and variables

The proposed mathematical formulation captures essential features of the incremental FTTH PON planning problem. It entails the maximisation of NPV, subject to constraints that model the technological requirements of an FTTH network. The most important aspect considered is the evolving bandwidth requirements over time. To formulate the problem as an MILP problem, several sets and parameter definitions are required. Listed in table 4.1 are the different index sets used while table 4.2 summarises all the input parameters.

Table 4.1: Index sets

Notation	Description
V	Set of vertices corresponding to either ONU's or splitters
E	Set of edges corresponding to potential duct installations
T	Set of time periods
CO	The central office
S	Set of splitters
ONU	Set of ONUs
$\sigma^{+1}(i)$	Set of arcs exiting node $i \in V$
$\sigma^{-1}(i)$	Set of arcs entering node $i \in V$

It should be noted that the demand parameter d_{ti} is defined in such a way, that when demand becomes available for ONU i in a time-period t , it remains available for the remainder of the planning horizon. For example, consider the time-periods $T = \{1, 2, 3\}$. If demand for ONU i becomes available in time-period $t = 2$, the corresponding demand vector is $\{d_{1i}, d_{2i}, d_{3i}\} = \{0, 1, 1\}$.

Table 4.2: Input parameters

Notation	Description
d_{ti}	Bandwidth demand for ONU i in time-period $t \in T$
Δ	Total network demand
K	The maximum number of down-fibres per splitter
l_e	Length (in meters) of edge $e \in E$
δ	The discount rate for the NPV calculation
PM	The percentage margin used to calculate revenue
C_{pi}	The yearly average inflation rate calculated over the years 2012 -2014
C_{co}	Cost for the use and maintenance of the CO
C_{sp}	Cost per splitter installation
C_{onu}	Cost per ONU installation
C_{fpm}	Cost of fibre per meter
C_{tpm}	Cost per trench per meter

In order to express logical decision making related to the topological design of the network and the timing of infrastructure deployment, the variables, listed in table 4.3, are required.

Table 4.3: Decision variables

Variable	Description
$Z_{et} \in \{0, 1\}$	Trenching of edge $e \in E$ in time-period $t \in T$
$x_{ijt}^D \in \mathbb{Z}_+$	Fibre installation for the distribution network from node i to j in time-period $t \in T$
$x_{ijt}^F \in \mathbb{Z}_+$	Fibre installation for the feeder network from node i to j in time-period $t \in T$
$\psi_{it} \in \{0, 1\}$	Splitter installation at node $i \in V$ in time-period $t \in T$
$F_t \in \mathbb{Z}_+$	The number of down-fibres at the CO in time-period $t \in T$
$D_{it} \in \mathbb{Z}_+$	The number of down-fibres for a splitter at node $i \in V$ in time-period $t \in T$
$H_{ct} \in \{0, 1\}$	Head central office installation at node $c \in CO$ in time-period $t \in T$

The objective function comprises an NPV calculation. This calculation comprises unknown variables that are listed in table 4.4. These variables each represent financial terms as described.

Table 4.4: Unknown variables

Notation	Description
m_t	Network maintenance cost per time-period $t \in T$
p_t	Network expenses per time-period $t \in T$
R_t	Revenue per time-period $t \in T$

After introducing the model's *sets, parameters and variables*, it is now possible to introduce the MILP formulation formulated for this research.

4.2 Mathematical model

The mathematical formulation of the incremental FTTH PON design problem is described in the subsequent subsections.

4.2.1 Objective function

Maximise

$$\sum_{t \in T} \frac{R_t - m_t - p_t}{(1 + \delta)^t} \quad (4.1)$$

The objective function (4.1), maximises NPV over all time periods. As described in Chapter 3, NPV is calculated by subtracting expenses from income. Defined more specifically for this research, by subtracting maintenance costs m_t and initial expenses p_t from revenue R_t , calculated for each time-period. That value is then divided by $(1 + \delta)^t$ where δ is the discount rate over the time-period $t \in T$.

4.2.2 Constraints

It is now possible to define the constraints relative to the design problem. It can therefore be said that the objective function is *subjected to* the following constraints.

Node balance constraints

$$\sum_{j \in \sigma^+(i)} x_{ijt}^D - \sum_{j \in \sigma^-(i)} x_{jit}^D = \begin{cases} -d_{ti}, & : i \in ONU, \\ D_{it}, & : i \in S, \\ 0, & : otherwise. \end{cases} \quad \forall i \in V, \forall t \in T, t > 0 \quad (4.2)$$

$$x_{ijt}^D + x_{jit}^D \leq \Delta Z_{et}, \quad \forall e \in E, \forall t \in T, t > 0 \quad (4.3)$$

The demand requirements for the distribution part of the network are captured in (4.2) and constraint set (4.3) allows fibre to be installed at an edge, provided that trenching was done for that specific edge.

$$\sum_{j \in \sigma^+(i)} x_{ijt}^F - \sum_{j \in \sigma^-(i)} x_{jit}^F = \begin{cases} -\psi_{it}, & : i \in S, \\ F_t, & : i \in CO, \quad \forall i \in V, \forall t \in T, t > 0 \\ 0, & : otherwise. \end{cases} \quad (4.4)$$

$$x_{ijt}^F + x_{jit}^F \leq \Delta Z_{et}, \quad \forall e \in E, \forall t \in T, t > 0 \quad (4.5)$$

Similarly, the demand requirements for the feeder part of the network are formulated by constraints (4.4). While constraint set (4.5), just as constraint set (4.3), allows fibre to be installed at an edge, but yet again, only if trenching has been done for that specific edge.

Logical decision-making constraints

The following constraints are required to model logical decision making with regards to the installation of the edges, splitters and fibres.

$$D_{it} \leq K\psi_{it}, \quad \forall i \in S, \forall t \in T, t > 0 \quad (4.6)$$

Constraint set (4.6) ensures that the ratio of ONUs to a splitter will not be exceeded, where K represents the maximum down fibres per splitter.

$$F_t = \sum_{i \in S} \psi_{it}, \quad \forall t \in T, t > 0 \quad (4.7)$$

The total number of splitters used also indicates the number of down fibres from the CO, which is depicted in (4.7).

$$Z_{et} \geq Z_{e,t-1}, \quad \forall e \in E, \forall t \in T, t > 0 \quad (4.8)$$

Constraint set (4.8) ensures that once trenching has been done for an edge, it remains available for duct sharing for the remainder of the planning horizon.

$$\psi_{it} \geq \psi_{i,t-1}, \forall i \in S, \forall t \in T, t > 0 \quad (4.9)$$

Once a splitter has been installed, constraint set (4.9) is then used to ensure that it remains installed and thus available for further use in the upcoming time periods.

$$H_{ct} \geq H_{c,t-1}, \forall c \in CO, \forall t \in T, t > 0 \quad (4.10)$$

The HCO is automatically and immediately put into use during the first time-period, as it is the main source for the feeder and distribution system of the network. Constraint set (4.10) is thus used to ensure that the HCO always remains installed.

$$Z_{et} = 0, \forall e \in E, \forall t \in T, t = 0 \quad (4.11)$$

$$\psi_{it} = 0, \forall i \in S, \forall t \in T, t = 0 \quad (4.12)$$

$$H_{ct} = 0, \forall c \in CO, \forall t \in T, t = 0 \quad (4.13)$$

If, for example, $T = \{1, 2, 3\}$ indicating that the network realises over 3 time-periods, an additional time-period $t = 0$ where $t \in T$ is added for simulation purposes. This then resulting in $T = \{0, 1, 2, 3\}$ ensuring that if need be, the installation of an element remains installed as stated in constraint sets (4.8) - (4.10). Constraint sets (4.11) - (4.13) are thus used to ensure that the binary variables for an edge, splitter and HCO installation at $t = 0$ is 0 indicating that it has not yet been installed.

The NPV constraint sets

The following constraints focus on the unknown variables that are used in the NPV calculation in the objective function.

$$m_t = 0, \forall t \in T, t > 0 \quad (4.14)$$

The constraint set (4.14) determines the maintenance costs for each time-period $t \in T$. The implementation of this variable is purely for formulation purposes, adding to the expenses that is subtracted from the revenue. With there being no way of determining what maintenance costs could occur, m_t has been set to equal 0.

$$\begin{aligned} p_t = & (1 + Cpi)^t * (\sum_{ee \in E} (x_{ijt}^D + x_{jit}^D + x_{ijt}^F + x_{jit}^F) * (l * C_{fet}) + \sum_{ee \in E} (Z_{et} - Z_{e,t-1}) * (l * C_{tet}) \\ & + \sum_{ie \in S} (\psi_{it} - \psi_{i,t-1}) * C_{it} + \sum_{j \in ONU} (d_{tj} - d_{t-1,j}) * C_{jt} + \sum_{ce \in CO} (H_{ct} - H_{c,t-1})) * C_{co}, \forall t \in T, t > 0 \end{aligned} \quad (4.15)$$

The expenses encountered while the network design process is calculated in constraint set (4.15) for all time periods. These include the following:

- the total cost of fibre per edge, calculated for both the feeder and distribution network,
- the total trenching costs,
- the total cost for all splitter installations,
- the total cost for each ONU, and
- the HCO's start-up and maintenance costs.

$$TC = \sum_{t>0} p_t, \forall t \in T \quad (4.16)$$

$$d_{tot_t} = \sum_{i \in ONU} d_{ti}, \forall t \in T, t > 0 \quad (4.17)$$

$$R_t = \frac{TC * d_{tot_t}}{\Delta} * PM, \forall t \in T, t > 0 \quad (4.18)$$

Constraint set (4.18), which calculates the revenue over each time-period, involves two calculations, namely the total cost over all time-periods, from equation (4.16), and the total demands per time period from equation (4.17). The desire to determine the exact amount of revenue is impossible which means that an alternative calculation is required. Therefore, for simulation purposes revenue, equation (4.18), is calculated by multiplying TC by d_{tot_t} , divided by d_{tot_t} and then multiplying that answer by Percentage Margin (PM), making R_t a function of demand d_{ti} .

4.3 Employed model techniques

The problem investigated in this research is defined to be NP-complete therefore requiring heuristics, refer to section 3.2 and section 3.3 located in the previous chapter for a detail description thereof.

As the aim of this research is to mathematically model the incremental FTTH deployment problem, the problem instances investigated have dataset-sizes small enough to be solved, optimally, within a reasonable amount of time. Heuristics will however be required if time becomes a limiting factor when larger datasets are implemented.

4.4 Model verification and validation

The following subsections describe how the mathematical model will be verified and validated in terms of a simulation and feasibility study, and with the aid of the commercial solver *CPLEX Optimisation Studio*.

4.4.1 Simulation and feasibility study

After defining the mathematical model that will be implemented for the purpose of this research, the use of a simulation and feasibility study can now be described.

The chosen method for determining the feasibility of the model is to run various simulations whereby the mathematical model is repeatedly used to determine the optimal FTTH layout for three different case studies, each with their own problem instance. In other words, the model's feasibility will be determined by performing multiple simulations and evaluating the results obtained thereof. Each case study will differ from the other in one or another way whereby each will provide a better insight into the mathematical model used. A more detailed explanation can be found in subsection 4.4.3.

4.4.2 CPLEX Optimisation Studio

CPLEX Optimisation Studio, a product by IBM, is a software package used for optimisation purposes [10]. The optimiser can be used to solve programming problems such as large linear problems with the aid of either primal or dual variants of the simplex method, integer problems, convex and non-convex quadratic problems, and convex quadratically constrained problems. In mathematics, CPLEX has become a popular software package to use when optimisation is desired.

The simplex method

The simplex algorithm, or more commonly known as the simplex method, is a very popular algorithm used for linear programming. The final solution obtained from this method is often a very basic, feasible solution [25]. A more detailed discussion can be found in the previous chapter under section 3.1, Theorem 1.

4.4.3 Verification and validation

A total of 3 case studies will be conducted for the purpose of this research, the following chapter discusses these in full detail. As mentioned in Chapter 1, public restrictions have deemed it impossible to base the case studies on actual real-world data, resulting in the construction of problem instances based on real-world information. Based on this information, the 3 case studies have all been formulated in the following way in order to test the feasibility of the mathematical model.

Case study 1 comprises a small problem instance of just 20 ONUs and 5 SPs. The chosen structure for the layout of the entire network is a tree-network whereby the branches and expansion of the network can be clearly observed. This is specifically chosen so as to determine whether the simulated optimal network layout gained from the mathematical model is indeed correct. The tree-like structure of the network enhances the ease of observing the incremental demand growth making it easier to determine whether the optimal network, according to the model, is indeed optimal. The selected size of the network is also specifically chosen so as to determine whether the model can be used on a non-complex network layout comprising a few ONUs and SPs, thus minimising the execution time. Case study 1 is therefore used to verify the model by means of a simulation whereby a small theoretically-based problem instance is investigated.

Once satisfactory results are gained from the first case study, the next step is to determine whether the model can be used on a network with a more real-world layout. Case

study 2 is specifically constructed on a real-world basis so as to determine and observe how the model selects the optimal FTTH network. Once again, a small network comprising 20 ONUs and 5 SPs is used, however in this problem instance the network is designed on an actual section of a map obtained from a Google Maps [33]. This case study is therefore used to verify the model, because of its real-world design, and the network size, as with case study 1, is used to minimise the execution time and simplify the evaluation process of the results gained.

Once satisfactory results are gained from the previous two case studies, the last case study can be simulated and evaluated. As with case study 2, case study 3 is also used to verify the model due to having a problem instance based on real-world information. Once again, Google Maps is used to design a layout of a network based on a randomly selected area, however, the dataset now comprises more ONUs and SPs. This is to determine whether the model can indeed perform on a larger dataset with real-world properties. Therefore, in addition to verifying the model, this case study is also used to test its scalability, i.e whether the model is capable of determining the optimal network layout of large datasets.

4.5 Chapter conclusion

Presented in this chapter is the mathematical model formulated for the study of this research. The chosen algorithm is an MILP formulation proposed to solve the *incremental FTTH deployment planning* problem. The objective of this formulation is to maximise NPV, subject to constraints that model the technological requirements of an incremental FTTH network. The model is designed with the *evolving bandwidth requirements over time* as the most important design aspect.

The NPV calculation, as stated in Chapter 3 equation 3.3, involves subtracting all the expenses from the income, calculated for each time-period. While the constraints constrain the network in an optimal fashion, specifically for an FTTH network, they too are based on an incremental time-period design.

The overall aim of model is to maximise NPV and minimise expenses. The desired output, when solved with CPLEX, is to obtain an optimal network, designed on an incremental basis.

Chapter 5

Simulations and Results

This chapter presents the three case studies conducted whereby the proposed mathematical model is evaluated and thereafter verified and validated accordingly. The first case study investigates a 1-5-20 network with a tree-like structure. The second case study also investigates a 1-5-20 network but with more realistic street layout and scenario. The third case study investigates a 1-8-40 network whereby results on two scenarios are presented, one where the splitters have a split ratio of 1:16 and the other, a split ratio of 1:32. Each case study presents a potential network topology, which expands incrementally in time-periods, along with the optimal solution to that network, as obtained by the simulated results of the model.

5.1 Case study 1 - Model verification

The following case study was used to verify the model. This was done by solving a small-scale tree-like structured network with the use of the model. The simulated results obtained were compared to manually-calculated results to determine the accuracy of the model.

In order to illustrate the benefits of employing the proposed mathematical model, pre-

sented in Chapter 4, to solve the incremental FTTH planning problem, a problem instance was constructed. This problem instance comprises 1-CO, 5-SPs and 20-ONUs, from here onwards defined as a 1-5-20 network. Throughout the formulation of the model, the 1-5-20 network was repeatedly used to test the model, thus aiding in identifying any inconsistencies and errors. This made it possible to rectify the identified problems until the simulation gave an output with satisfactory results. The following subsections describe the simulation process followed, and the results obtained thereof.

5.1.1 Simulation

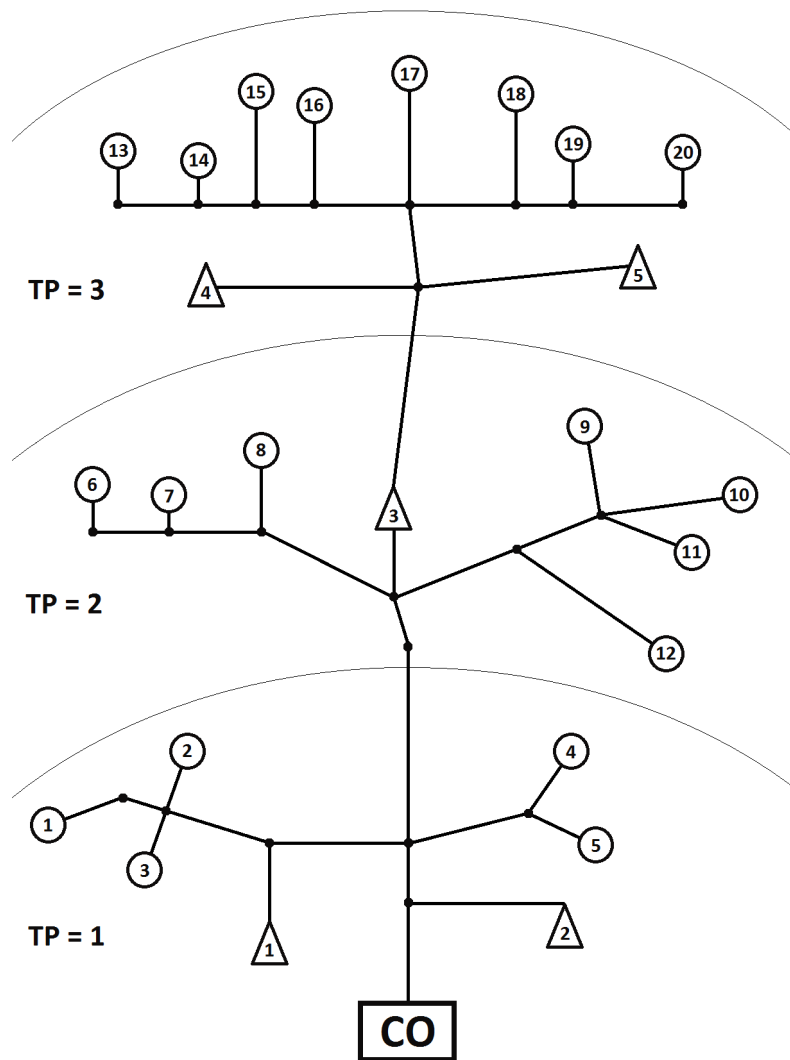


Figure 5.1: The potential network topology for case study 1: 1-5-20 tree-network

Figure 5.1 depicts the potential network topology of the tree-like structured 1-5-20 network, from here onwards referred to as the 1-5-20 tree-network. As can be seen in the figure, there is 1-CO, represented by the square labelled CO, 5-SPs, represented by the 5 triangles numbered from 1-5 and 20-ONUs, represented by the 20 circles numbered from 1-20. The 3 semi-circles represent the 3 time-periods, starting with time-period 1 at the bottom increasing upwards, depicting how the network expands incrementally.

This specifically chosen structure increases the ease of visually observing the design and growth of the network. This exact reason is what makes it possible to immediately identify errors in the proposed mathematical model. Throughout the formulation of the model, this case study was repeatedly used to monitor the simulated outputs until an error-free output was gained. An output is considered error-free if the simulated design for the 1-5-20 tree-network remained within the modelling constraints whereby all demands are met, installed SPs and edges are used, and the CO and SPs feed and distribute to the installed SPs and ONUs respectively.

The starting costs are listed in table 5.1. Each cost is subjected to an inflation rate of 5.442% per year, the calculated average rate of inflation for South Africa (SA) for the years 2012, 2013 and 2015. It should be noted that the costs expressed here are not actual costs but rather educated guesses based on research done.

Table 5.1: Starting Costs

Element	Cost Value
CO	10 000 R/unit
SP	3000 R/unit
ONU	200 R/unit
Fibre	100 R/m
Trench	300 R/m

The discount rate, δ , implemented in the NPV calculation (the objective function - equation 4.1) has a value of 10%, while the percentage margin, PM , implemented in equation 4.18 the revenue calculation, has a value of 60%. 10% is a commonly used value in discount rate calculations and is thus found ideal for the implementation of this simulation. The 60% value chosen for the percentage margin was allocated after

a process of trial and error was completed until a positive NPV value was obtained. In other words, the value chosen as the percentage margin is the first multiple of 10 whereby the NPV calculation has determined it favourable, from the point of view of an NSP, to design the network.

The following lists the input files used, each in the format of a dot Comma Separated Values (CSV) file. As stated earlier, the network comprises 1-CO, 5-SPs, and 20-ONUs, as well as 22-intermediate nodes, and thus a total of 48-Vertices. In addition, the network also comprises a total of 47-Edges, expanding over 3-time-periods. As seen from this information and figure 5.1, this case study presents a relatively *small* problem instance, ideal for testing the model.

- CO
- SP
- ONU
- Intermediate nodes
- Vertices
- Edges
- Time-Periods
- Demands
- Starting Costs

With such a small problem instance comprising 20-ONUs, a decision was made to give each SP a split ratio of 1:16 instead of a ratio of 1:32 for example. This is because a split ratio of 1:32 is large enough for one SP to feed all twenty ONUs. However, the idea behind this case study is to determine whether an optimal network, comprising multiple SPs, can be obtained.

5.1.2 Results

The commercial software *CPLEX Optimisation Studio*, a product by IBM [10], is used on an *HP Z1 All-in-one Workstation* to run the simulation. The computer's system specifications are listed in table 5.2. In order to gain an average execution time for this problem instance, the simulation was executed a total of five times, resulting in an average time of *44 hundredths of a second (sec/100)*. These execution times are listed in table 5.3.

Table 5.2: Specifications of the computer used to run the simulations

System Specifications	
Processor	Intel® Xenon® CPU E3-1245 V2 @ 3.40 GHz
RAM	16 GB
Operating System	Windows 8.1 64-bit

Table 5.3: Case study 1: 1-5-20 tree-network execution time

1-5-20 Tree-Network	
Execution no.	Time (sec/100)
1	53
2	41
3	39
4	48
5	40
Average	
Execution time	44

Calculations were manually done to verify the accuracy of the simulated results, these calculations did not consume too much time due to the size of the network. After rectifying all errors and inconsistencies, it was possible to obtain an optimal output of the network based on the proposed mathematical model. The optimal layout for the incremental FTTH 1-5-20 tree-network is depicted in figure 5.2. As can be seen, two SPs are installed, SP-1 located in the first time-period and SP-3 located in the second time-period.

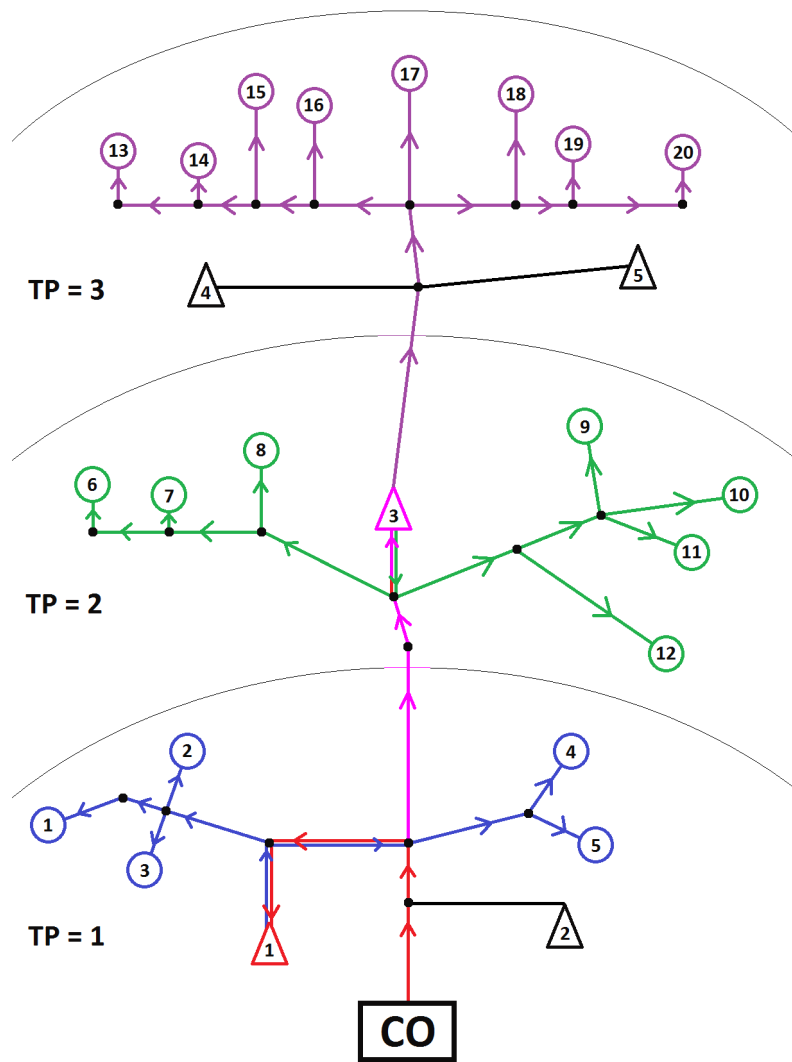


Figure 5.2: The optimal network layout for case study 1: 1-5-20 tree-network

Figure 5.3 depicts a very basic layout of the feeder and distribution part of any network. The feeder part of network starts at the CO and ends at an SP while the distribution part of the network starts at an SP and ends at an ONU. Based on this, and referring back to figure 5.2, the feeder part of the network is seen to start at the CO and expand outwards to the two installed SPs, as indicated by the colours red (time-period 1) and pink (time-period 2). The first section of the distribution part of the network starts at SP-1 and expands out to the first five ONUs located in the first time-period, as indicated by the colour blue. The second section of the distribution part of the network starts at SP-3 and expands out to ONU-6 - ONU-12, located in the second time-period,

as indicated by the colour green. The last section of the distribution part of the network also starts at SP-3 and expands out to ONU-13 - ONU-20, located in the last time period, as indicated by the colour purple. Table 5.4 provides a summary of this, indicating which colour belongs to which network and in what time period it occurs.

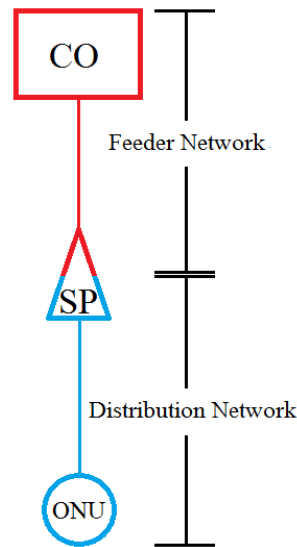


Figure 5.3: A simple design of the feeder and distribution network

Table 5.4: Case study 1: 1-5-20 tree-network colour key

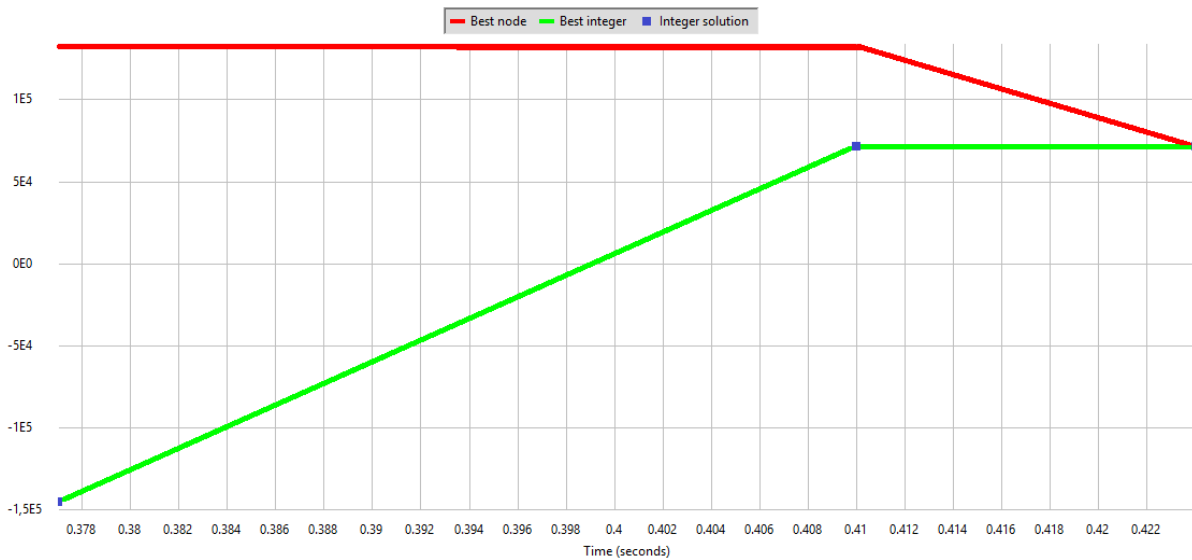
Key	Network Type	
	Feeder	Dist.
Red	TP 1	-
Pink	TP 2	-
Blue	-	TP 1
Green	-	TP 2
Purple	-	TP 3

The additional outputs obtained from the simulation are captured in table 5.5. This includes information on the objective function, total trenches required, optical fibres used, expenses, and revenue. Figure 5.4 depicts the graph obtained from CPLEX upon gaining the optimal solution to the network. From the graph, it can be seen how the best node, indicated in red, and the best integer, indicated in green, converge towards the optimal integer solution, indicated in blue. This then graphically presenting the gain of the optimal layout for the 1-5-20 tree-network.

Table 5.5: Case study 1: 1-5-20 tree-network additional simulated outputs

Additional Outputs	
Objective Function (R)	71447.07
Total no. of trenches	44
Total trench length (m)	1072.08
Total fibre length (m)	2495.54
Total costs (R)	877580
Initial expense per TP (R)	[144300 303750 429530]
Cash inflow per TP (R)	[307150 737170 1228600]
d_tot per TP	[5 12 20]

Figure 5.4: Case study 1: 1-5-20 tree-network CPLEX statistics graph



According to the simulated output of the optimal layout for the 1-5-20 tree-network, the following summary can be made. SP-1 meets all the demands in the first time period. In the following time-period SP-3 is installed to meet the next seven demands while SP-1 still distributes to the first five ONUs. In the last time period, when the last set of demands become available, SP-1 is again used to distribute to the first five ONUs while SP-3 now distributes to the remaining 15 ONUs. All demands that realise over the three time-periods are met, and all edges and SPs that are installed are used accordingly.

5.2 Case study 2 - Model validation

The following case study was used to validate the model by determining whether it can solve a real-world network of the same size used in case study 1, but with a more realistic design.

After completing case study 1 and having a confirmed working mathematical model, a decision was made to investigate a problem instance of the same size based a real-world set-up. The idea behind this case study is to determine whether the model, by means of a simulation, can determine the optimal layout for a real-world non-tree-like structured network. Due to public restrictions preventing the gain of actual data, the problem instances used in this research had to be constructed, and are done so accordingly and as realistically as possible.

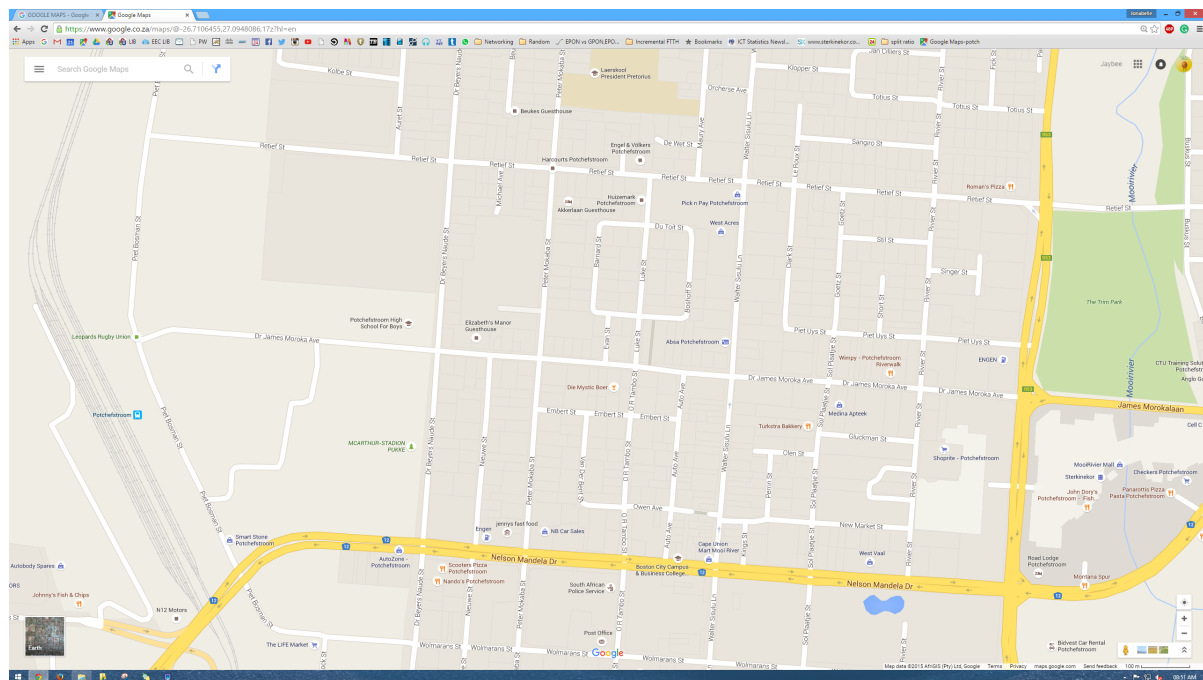


Figure 5.5: The potential street layout for case study 2: 1-5-20 street-network [33]

This will be done by taking the same sized 1-5-20 network and changing its network topology by re-routing it across an actual area on a map and allowing the time-periods to realise over suburbs. The first step in converting this network into a real-world scenario is to choose a location on a map upon which to design the potential network.

Figure 5.5 depicts a portion of a map, obtained from *Google Maps* [33], for which the design of the new 1-5-20 network is considered, from here onwards referred to as the 1-5-20 street-network. The attraction to this specific section of the map is attributed to the overall layout of the streets. This includes curves on the roads, the multiple routes to get to a destination and how, instead of adjoining roads everywhere, there exist multiple roads that come to a dead end. This section of the map consists of multiple, real life scenarios ideal for network mapping.

5.2.1 Simulation

As mentioned earlier, the 1-5-20 network is a relatively small network and because of this, the portion of the map considered for the street-network is narrowed down to a specific area as depicted in figure 5.6. The reason for choosing this area from the figure is due to having a majority of the curved and dead-end roads found here, ideal for the design of the potential 1-5-20 street-network.

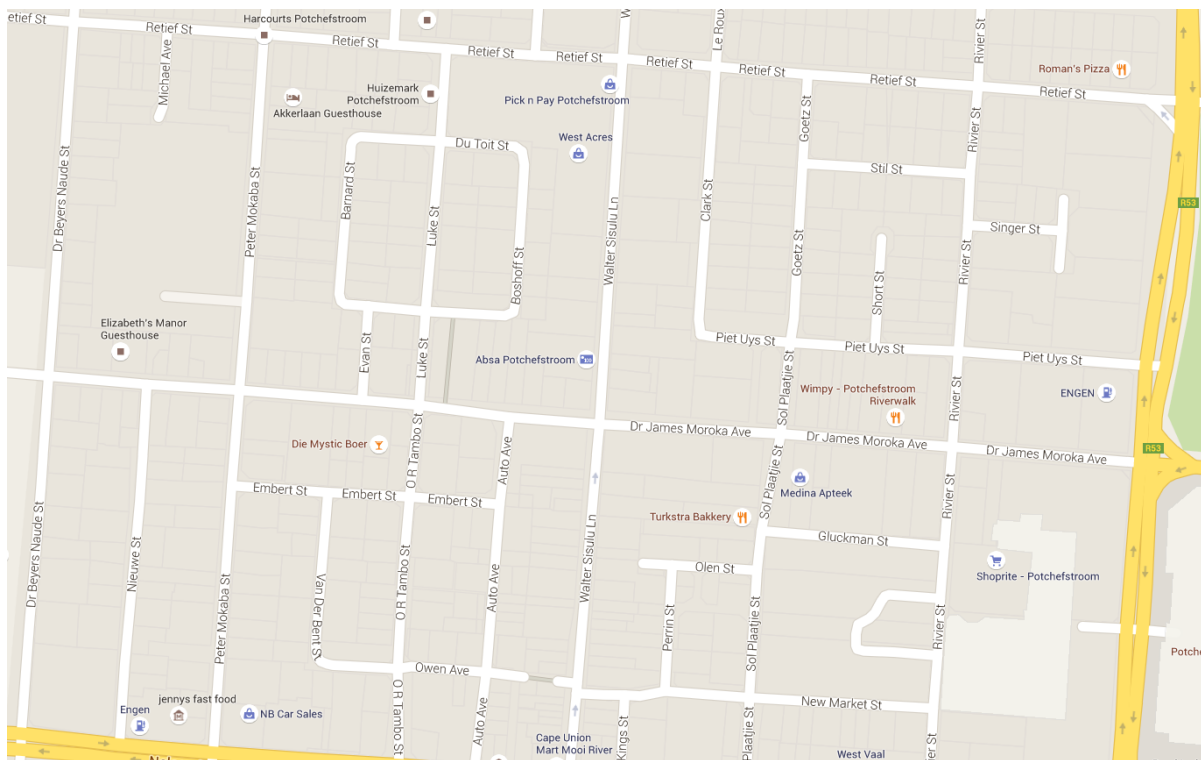


Figure 5.6: The selected area for case study 2: 1-5-20 street-network [33]

Figure 5.7 depicts the potential topology for the 1-5-20 street-network; unlike the 1-5-20 tree-network the incremental expansion of the network is not easily seen. The SPs are randomly placed on the network, and the CO is now located within the network and no longer on the outskirts. Table 5.6 provides a key to the symbols and colours used. The lengths of the edges are drawn to scale, where $23mm = 100m$, and is used so accordingly to maintain the realism of the network.

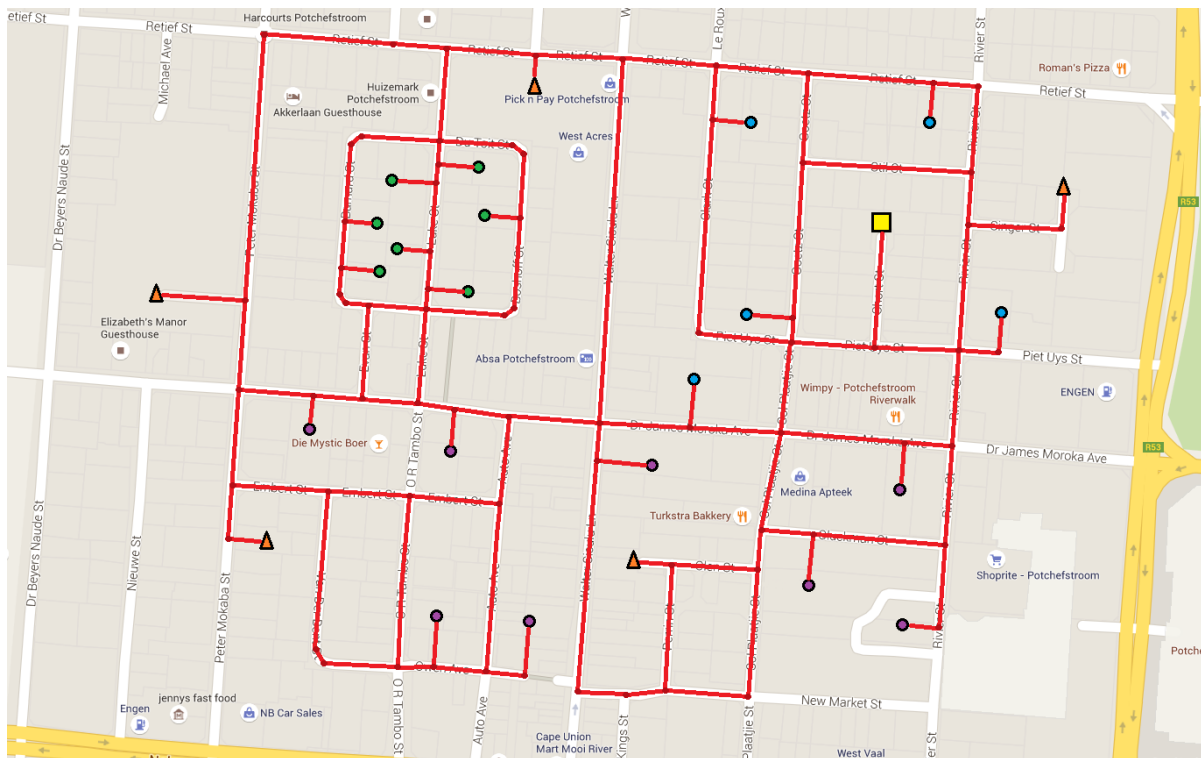


Figure 5.7: The potential network topology for case study 2: 1-5-20 street-network [33]

Table 5.6: Case study 2: 1-5-20 street-network general network key

Unit ID	Shape	Colour	TP-available
CO	Square	Yellow	1,2,3
SP	Triangle	Orange	1,2,3
ONU	Circle	Blue	1
		Green	2
		Purple	3

Just as the 1-5-20 tree-network, the 1-5-20 street-network realises over three time-periods although it is not easily seen. The chosen colours provide an easy way for observing the incremental growth of the network; this is also tabulated in table 5.6. As can be

seen in the figure, the suburb surrounding the CO is considered the first time-period. The second time-period can be identified by the green ONUs forming a suburb while the third time-period makes use of the colour purple to represent the last suburb.

Each ONU has its own unique Identification (ID) in the format *ONU_70x* where *x* is a placeholder for the number allocated to each of the twenty ONUs. Table 5.7 provides information on the ONUs, specifically specifying which ONU, and thus demand, become available in each of the time-periods. The first time-period and thus first suburb, comprises a total of 5-ONUs, the second time-period, and thus the second suburb, comprises a total of 7-ONUs and the last time-period representing the last suburb, comprises a total of 8-ONUs. There are, therefore, a total of 20-ONUs found in this network.

Table 5.7: Case study 2: 1-5-20 street-network colour coded ONU identification

ONU ID (<i>ONU_70x</i>)									
TP 1	-	3	6	12	15	20			
TP 2	-	1	2	9	11	13	17	18	
TP 3	-	4	5	7	8	10	14	16	19

The SPs are placed randomly on the network. They too have their own unique ID in the format *SP_140x* where *x* is a placeholder for the unique number allocated to each of the five SPs; these numbers can be found in figure 5.8 at each of the five orange triangles.

Based on the new network layout, the data of some of the files change. The network still comprises 1-CO, 5-SPs and 20-ONUs, however, due to the expansion of the network, there are now 72-intermediate nodes, and thus a total of 98-Vertices can be found. In comparison to the 47-Edges in case study 1, there are now 113-Edges that expand over the three time-periods.

Once again, for simulation purposes, the starting costs listed in table 5.1 are used. Once again, it should be noted that the costs expressed here are not actual costs but rather educated guesses based on research done. The same inflation rate of 5.442% per year is used to increase each cost accordingly in the following years. The discount rate and

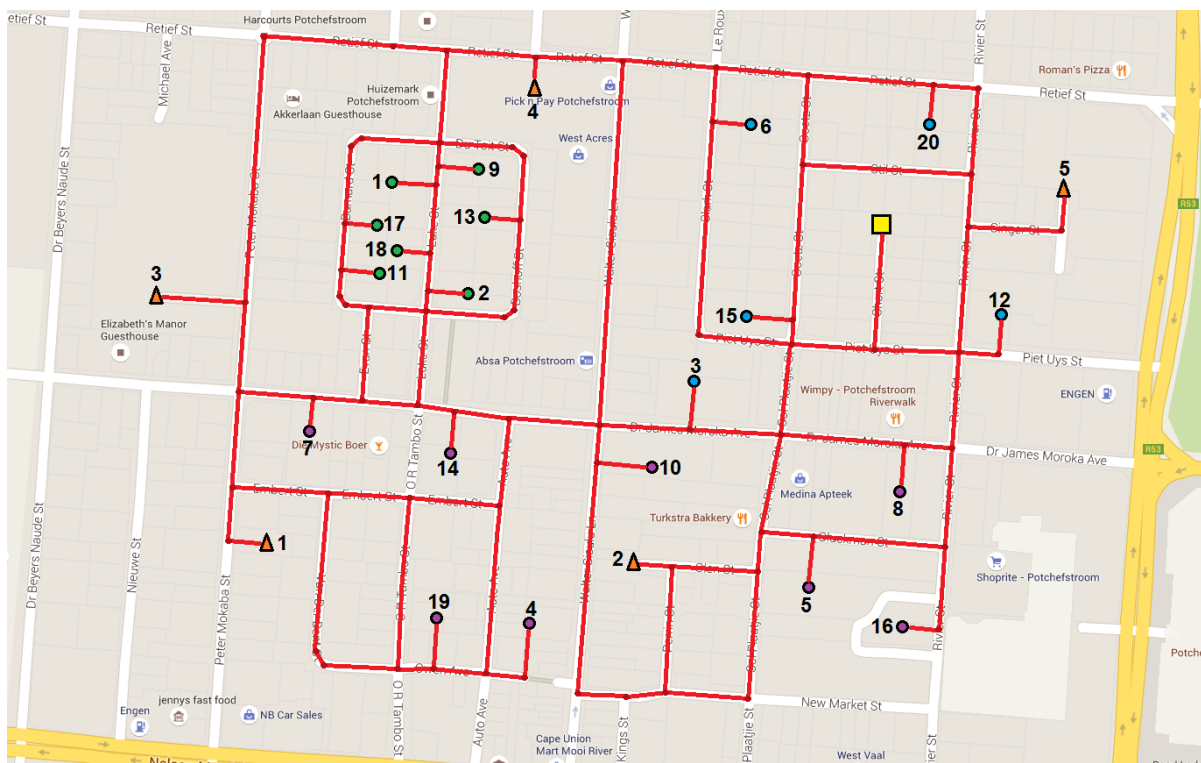


Figure 5.8: Element identification for case study 2: 1-5-20 street-network [33]

percentage margin remains at 10% and 60% respectively. The same data files with the same format are used, just as in case study 1. However, the information listed in them differ, specifically for the edges and demand realisation, intermediate nodes, and thus vertices. The split ratio for all five SPs is kept at 1:16 i.e. 1 SP can accommodate a maximum of 16 ONUs.

5.2.2 Results

The optimal network layout found for the 1-5-20 street-network is depicted in figure 5.9. Two SPs have been chosen for this network, SP-2 and SP-4. The optimal routes from the CO to each of the SPs and from each of the SPs to the twenty ONUs are indicated in the colour red. Unused SPs and edges are greyed out accordingly. Table 5.8 provides a key to the colours and symbols used to depict the optimal network.

As can be seen from figure 5.9, the colour of the SPs is no longer orange but blue.

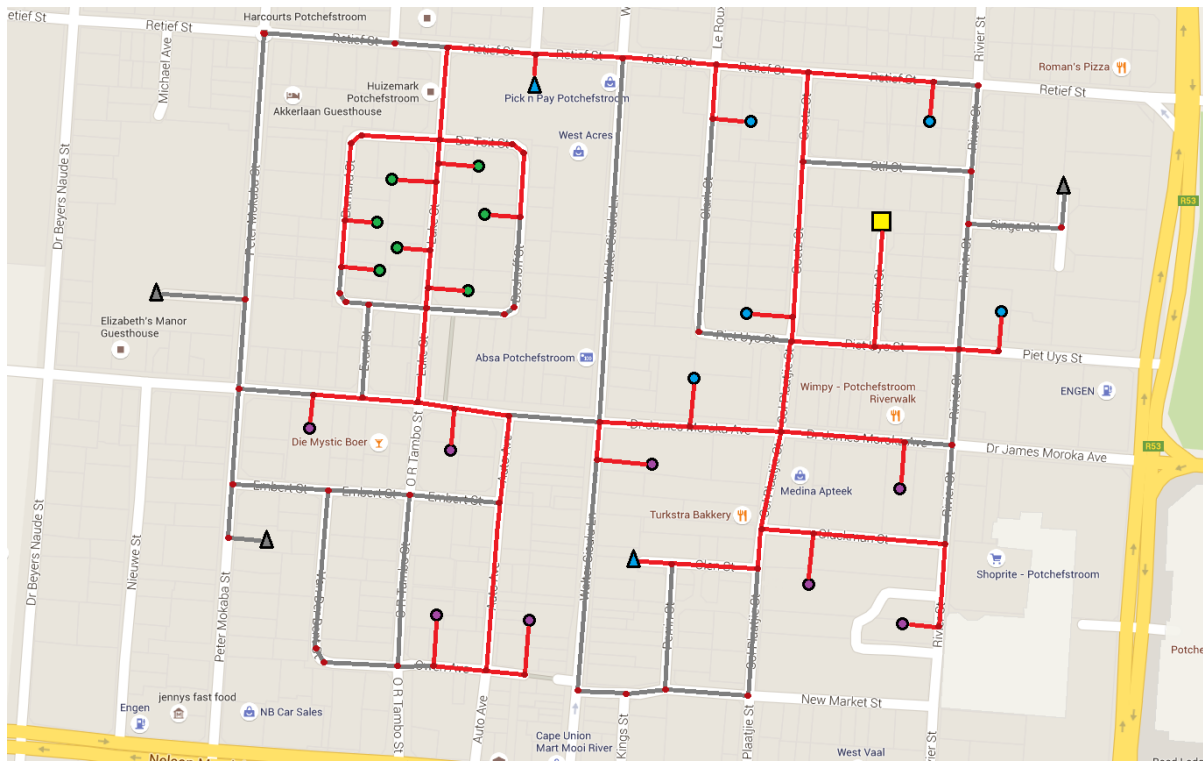


Figure 5.9: The optimal network layout for case study 2: 1-5-20 street-network [33]

Table 5.8: Case study 2: 1-5-20 street-network optimal network key

Object	Shape	Colour	TP-installed
CO	Square	Yellow	1
SP	Triangle	Blue	1
ONU	Circle	Blue	1
		Green	2
		Purple	3
-	-	*Grey	Not installed

This indicates that both SPs are installed in the first time-period, and therefore, both distribute to the first suburb where 5-ONUs realise. Thereafter, each SP distributes to the second and third suburbs that realise in the second and third time-periods respectively. The edges and SPs not considered as part of the optimal network are presented in grey and are thus disregarded when referring to the optimal result. Refer to Appendix C.1 for additional information regarding the IDs of the installed edges and the time-periods in which they are installed and distribute accordingly. The IDs allocated to each edge are in the format *ED_150x* where *x* represents the edge number.

Table 5.9 and table 5.10 tabulate the results for SP-2 and SP-4 respectively. The SPs have all been given an SP ID in the format SP_140x where x represents the SP number.

Table 5.9 provides information on the installed SP-2 whereby it distributes to a total of 3 ONUs in both the first and second time-periods, and then to a total of 7 ONUs in the third time-period.

Table 5.9: Case study 2: 1-5-20 street-network SP_ 1402-to-ONU distribution

SP_1402									
TP	ONU Total			ONU ID (ONU_70x)					
1	3	-		3	12	15			
2	3	-		3	12	15			
3	7	-		3	5	8	10	12	15 16

Table 5.10 provides information on the installed SP-4. As can be seen, SP-4 meets most of the demands, especially in the second and third time-period. In the first time-period, 2 ONUs are distributed to, which increases to 9 in the second time-period, and once more to 13 in the third time-period. Altogether a total of 20 demands are met, 5 in the first time-period, 12 in the second and 20 in the third.

Table 5.10: Case study 2: 1-5-20 street-network SP_ 1404-to-ONU distribution

SP_1404													
TP	ONU Total			ONU ID (ONU_70x)									
1	2	-		6	20								
2	9	-		1	2	6	9	11	13	17	18	20	
3	13	-		1	2	4	6	7	9	11	13	14	17 18 19 20

Once again, *CPLEX* was used on the same *HP Z1 All-in-one workstation* to achieve these results. The system specifications for the computer are listed in table 5.2.

In order to determine the average execution time for this problem instance, the simulation is executed a total of five times resulting in an average time of 16.47 sec. Table 5.11 captures the final time gained after each execution.

Table 5.11: Case study 2: 1-5-20 street-network execution time

1-5-20 Street-Network	
Execution no.	Time (sec)
1	16.97
2	16.34
3	16.27
4	16.26
5	16.54
Average	
Execution time	16.47

The additional outputs obtained from the simulation are captured in table 5.12. This includes information on the objective function, total trenches required, optical fibre used, expenses, and revenue. Figure 5.10 depicts the graph obtained from CPLEX upon gaining the optimal solution to the network. From the graph, it can be seen how the best node, indicated in red, and the best integer, indicated in green, converge towards the optimal integer solution, indicated in blue. This then graphically presenting the gain of the optimal layout for the 1-5-20 street-network.

Table 5.12: Case study 2: 1-5-20 street-network additional simulated outputs

Additional Outputs	
Objective Function (R)	297253.42
Total no. of trenches	68
Total trench length (m)	4573.91
Total fibre length (m)	11539.1
Total costs (R)	4022500
Initial expense per TP (R)	[996700 1090200 1935700]
Cash inflow per TP (R)	[1407900 3378900 5631500]
d_tot per TP	[5 12 20]

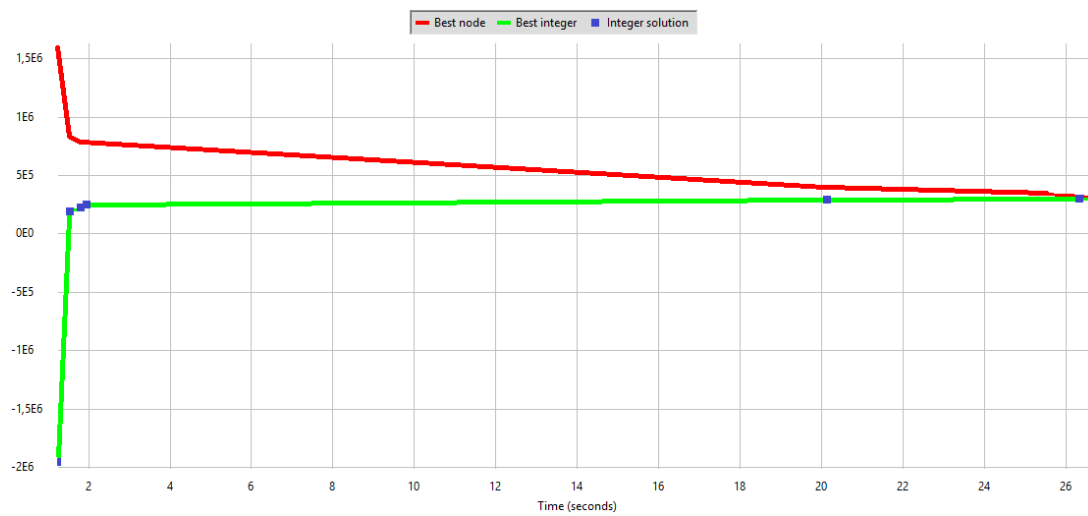


Figure 5.10: Case study 2: 1-5-20 street-network CPLEX statistics graph

5.3 Case study 3 - Model validation

This last case study was also used to validate the model. The idea behind this case study was to determine whether the model can solve a larger scaled real-world network with accurate results.

5.3.1 Simulation

The network presented in this case study is a 1-8-40 network. This means that the network comprises 1-CO, 8-SPs and 40-ONUs that realise over 4 time-periods. As can be seen, the network is relatively larger than the 1-5-20 street-network presented in case study 2. The increase in the number of SPs, ONUs and time-periods all contribute to the size of the network, ideal for testing the scalability of the model.

For this case study, it was decided to select a different area on the map on which to design the network. The first reason for this is to determine whether a network, based on a different problem instance as opposed to those investigated thus far, can be simulated and an optimal network gained thereof. This will then determine whether the model can indeed perform on different sized networks, where no two edges or placements of elements are the same as those in the other two case studies. The second reason is to

have a larger area with multiple paths, of different lengths, leading to each destination. This will then investigate how the model simplifies the design by selecting the optimal paths and thus an optimal layout of the network.

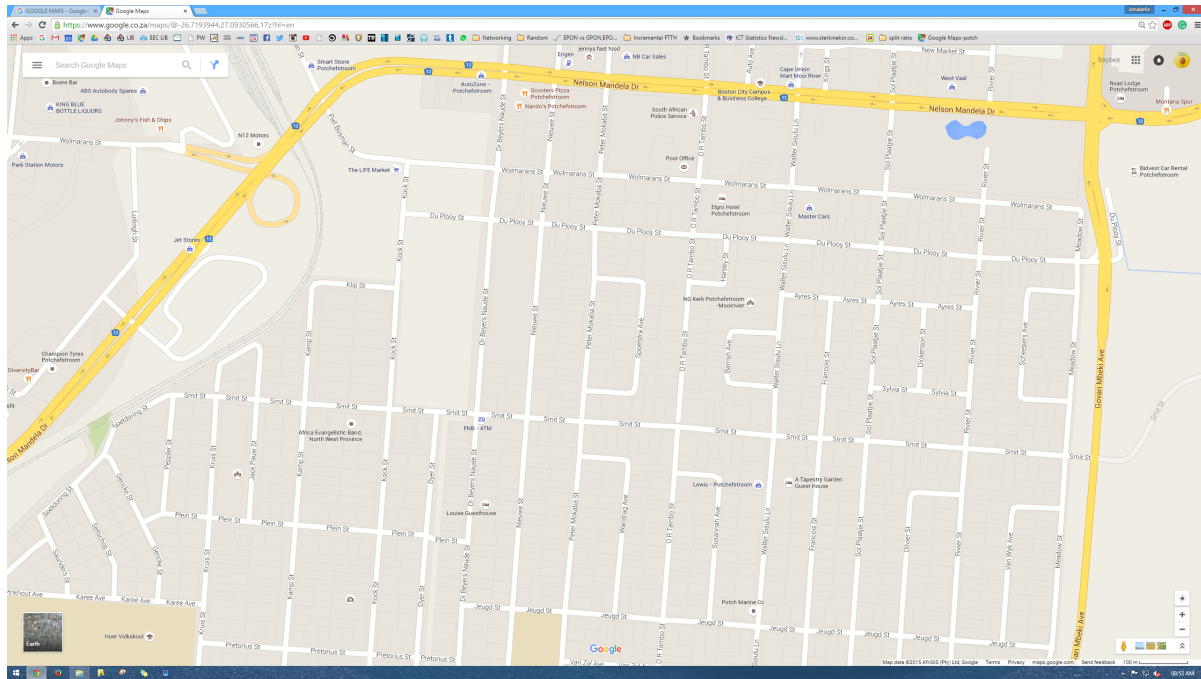


Figure 5.11: The potential street layout for case study 3: 1-8-40 [33]

The area considered for the 1-8-20 network is depicted in figure 5.11. This section of the map, obtained from *Google Maps* [33], portrays the desired street layout ideal for this case study.

Figure 5.12 is obtained by narrowing down a specific area, from figure 5.11, upon which to design the potential network. As can be seen from the figure, there are multiple routes to choose from to get to any destination. This then creates multiple paths for the model to work through before determining the optimal layout of the 1-8-40 network.

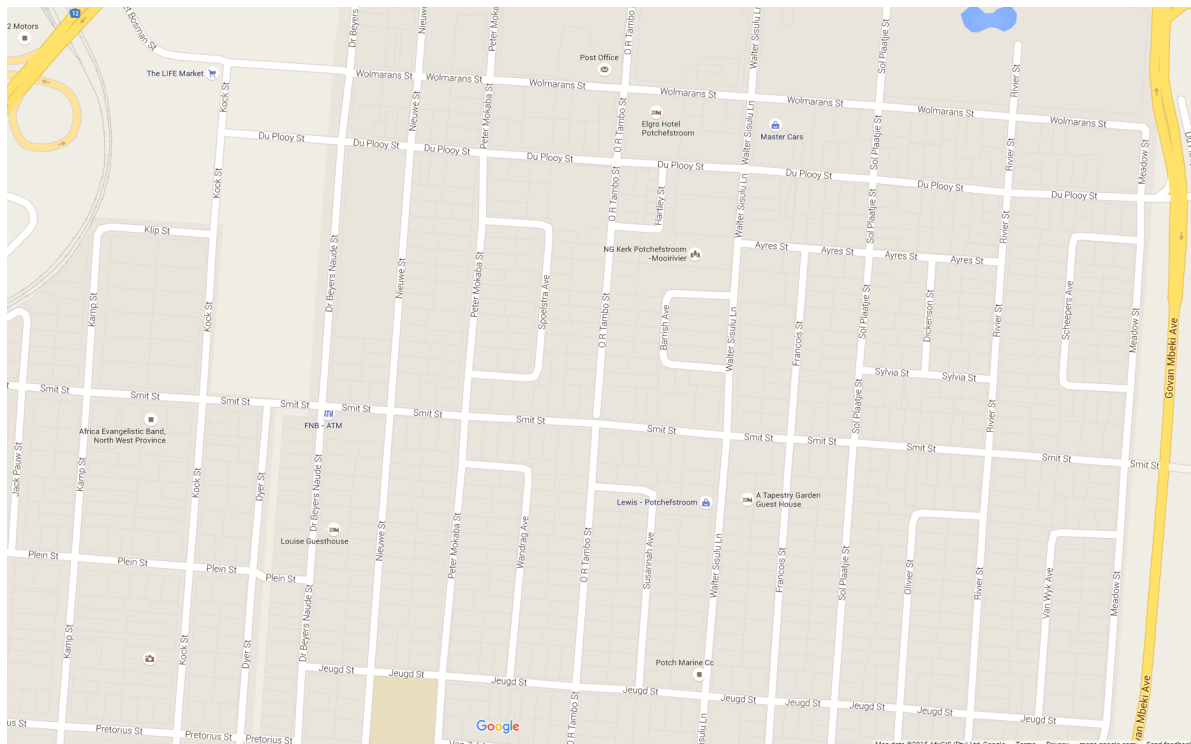


Figure 5.12: The selected area for case study 3: 1-8-40 [33]

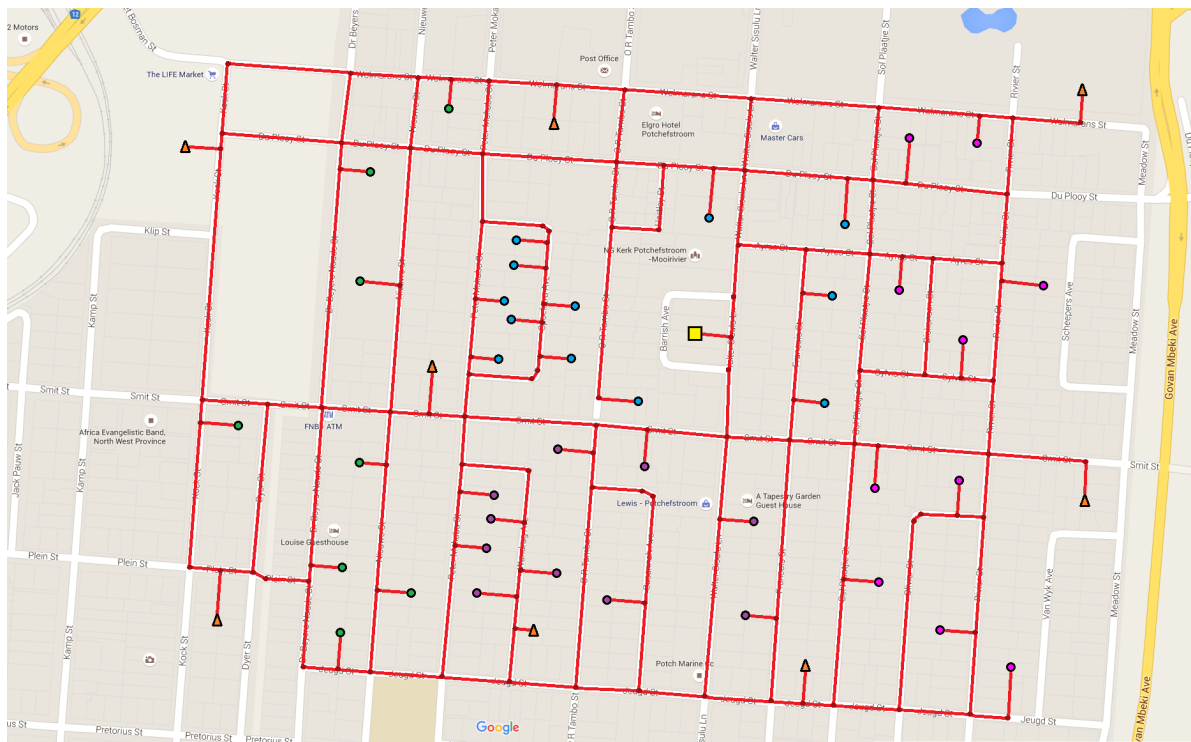


Figure 5.13: The potential network topology for case study 3: 1-8-40 [33]

The potential network topology for the 1-8-40 network is depicted in figure 5.13. Table 5.13 can be used as a key to the symbols used to represent the network. The CO is located in the centre of the network identified as the yellow square. The 8 SPs can be identified by the orange triangles located in various positions in the network. Lastly, the ONUs are represented by the circles that have been placed in such a manner forming the suburbs of the network. The various colours indicate the time-periods in which the ONUs realise, thus also helping identify the location of each suburb, *remember: each suburb represents a time-period*. The lengths of the edges are drawn to scale, where $16mm = 100m$, and is used so accordingly to maintain the realism of the network.

Table 5.13: Case study 3: 1-8-40 general network key

Object	Shape	Colour	TP-available
CO	Square	Yellow	1,2,3,4
SP	Triangle	Orange	1,2,3,4
ONU	Circle	Blue	1
		Green	2
		Purple	3
		Pink	4

Each element has been allocated a unique ID, as depicted in figure 5.14. This is used firstly to identify the element, i.e. whether it is the CO, an SP or an ONU, and secondly to identify the number of the element, i.e. whether it is the 3rd SP, or the 20th ONU.

The realisation of the demands are represented by the ONUs, and as previously mentioned, the ONUs of the same colour are installed in the same time-period. Table 5.14 provides information on the installation of each ONU in each time-period. It is important to note that once a demand becomes available in time-period t , where $t \in T$, it remains available in the remaining time-periods. As can be seen from the table, the first time-period, or suburb, has a total of 12 demands that become available, the second, a total of 8, the third, a total of 10 and the fourth, an additional total of 10. So to summarise, the first time period sees the network distributing to a total of 12 ONUs, the second to a total of 20 ONUs, the third to a total of 30 ONUs, and the fourth and final time period, to a total of 40 ONUs.

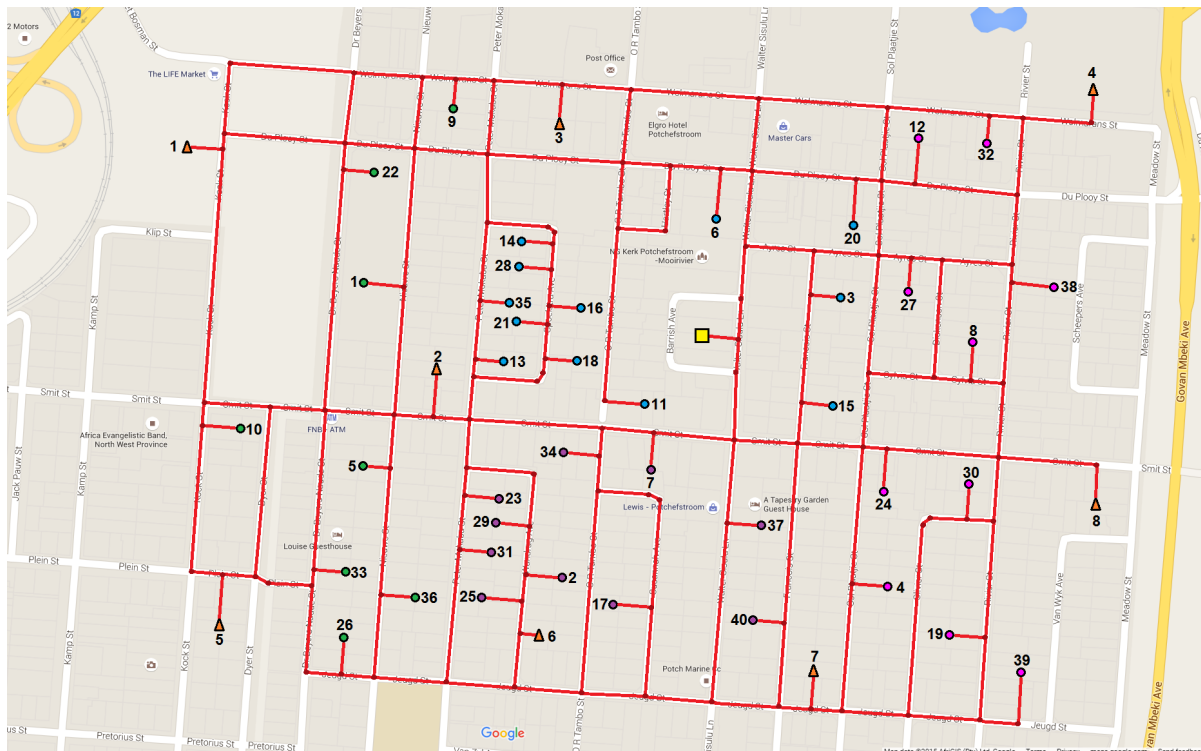


Figure 5.14: Element identification for case study 3: 1-8-40 [33]

Table 5.14: Case study 3: 1-5-20 colour coded ONU identification

	ONU ID (ONU_70x)												
TP 1	-	3	6	11	13	14	15	16	18	20	21	28	35
TP 2	-	1	5	9	10	22	26	33	36				
TP 3	-	2	7	17	23	25	29	31	34	37	40		
TP 4	-	4	8	12	19	24	27	30	32	38	39		

Table 5.1 can once again be referred to for the starting costs used in the simulations. As a reminder from the previous case studies, the costs reflected in this table are not based on actual costs but are rather educated guesses based on research done. The inflation rate of 5.442% is used once again to increase the amounts after each time-period.

The discount rate, δ , implemented in the NPV calculation (the objective function - equation 4.1) remains at 10%, while the percentage margin, PM , implemented in equation 4.18 the revenue calculation, is now given a value of 40%. 10% is a commonly used value in discount rate calculations and is thus found ideal for the implementation of this simulation. The 40% value chosen for the percentage margin was allocated after

a process of trial and error was completed until a positive NPV value was obtained. In other words, the value chosen as the percentage margin is the first multiple of 10 whereby the NPV calculation has determined it favourable, from the point of view of the NSP, to design the network.

The input files used have the same format as the previous case studies, a dot CSV format. This was specifically chosen so as to have the input files remain consistent throughout the research. As stated earlier, the network comprises 1-CO, 8-SPs, and 40-ONUs, as well as 117-intermediate nodes, and thus a total of 166-Vertices. In addition, the network also comprises a total of 197-Edges, expanding over 4-time-periods.

This case study is aimed at investigating the scalability of the proposed mathematical model, hence the increase in the network's size. However, a decision was made to also investigate the difference in the optimal network outputs when the split ratios of the splitters are changed. This will then not only determine the scalability of the model but also show the effectiveness that split ratios have on a network. The two chosen split ratios, ideal for a network comprising 40-ONUs, is 1:16 and 1:32, both below the total value of demands, and both preventing the instalment of a single SP for distributing over all 4 time-periods.

Once again the commercial software *CPLEX Optimisation Studio*, a product by IBM [10], is used on the same *HP Z1 All-in-one Workstation* to run the simulations.

5.3.2 Results for the 1-8-40 network-16

The optimal network gained from the potential network depicted in figure 5.13 is depicted in figure 5.15. Each of the eight SPs are subjected to a split ratio of 1:16, hence the name 1-8-40 network-16.

Table 5.15 can be used as a key to help identify the elements and the time-periods in which they become available or are installed. From the eight available SPs, the optimal network comprises five of them, namely SP-2, SP-3, SP-4, SP-6 and SP-7. As seen from

Table 5.15: Case study 3: 1-8-40 optimal network key

Object	Shape	Colour	TP-installed
CO	Square	Yellow	1
SP	Triangle	Blue	1
		Purple	3
		Pink	4
		Blue	1
ONU	Circle	Green	2
		Purple	3
		Pink	4
		*Grey	Not installed
-	-		

remaining 4, which total up to the 12 demands that become available in the first time-period. In the second suburb, SP-2 distributes to an additional 6 ONUs while SP-3 distributes to an additional 2. This adds up to a total of 20 demands being met in the second time-period.

Table 5.16: Case study 3: 1-8-40 network-16 SP_ 1402-to-ONU distribution

SP_1402																		
TP	ONU Total	ONU ID (<i>ONU_70x</i>)																
1	8	-	13	14	15	16	18	21	28	35								
2	14	-	1	5	10	13	14	15	16	18	21	26	28	33	35	36		
3	16	-	1	5	10	13	14	15	16	18	21	23	26	28	31	33	35	36
4	16	-	1	5	10	13	14	15	16	18	21	23	26	28	31	33	35	36

Table 5.17: Case study 3: 1-8-40 network-16 SP_ 1403-to-ONU distribution

		SP_1403							
TP	ONU Total	ONU ID (ONU_70x)							
1	4	-	3	6	11	20			
2	6	-	3	6	9	11	20	22	
3	6	-	3	6	9	11	20	22	
4	5	-	3	6	9	11	22		

SP-6 is installed in the third time-period, and from then onwards, aids in the distribution part of the network, as tabulated in table 5.18. When the demands in the third time period become available, an additional 2 ONUs are added to SP-2's distribution part of the network, while SP-3 remains the same. SP-6 distributes to 8-ONUs, which gives a total of 30 demands being met in the third time-period.

Table 5.18: Case study 3: 1-8-40 network-16 SP_ 1406-to-ONU distribution

		SP_1406									
TP	ONU Total	ONU ID(ONU_70x)									
3	8	-	2	7	17	25	29	34	37	40	
4	6	-	2	7	17	25	29	34			

In the last time-period, time-period 4, SP-4 and SP-7 are installed. The ONU distribution per splitter is captured in table 5.19 and table 5.20. The distribution network is now as follows, SP-2 remains at 16 ONUs while SP-3 decreases by an ONU, which is now fed by SP-4 while SP-6 also decreases but by 2 ONUs that are now fed by SP-7. SP-4 and SP-7 thus distribute to 6 and 7 ONUs respectively. The total demands met in the last time-period add up to 40 ONUs, the total number of ONUs that comprise the network.

Table 5.19: Case study 3: 1-8-40 network-16 SP_ 1404-to-ONU distribution

		SP_1404									
TP	ONU Total	ONU ID(ONU_70x)									
4	6	-	8	12	20	27	32	38			

Table 5.20: Case study 3: 1-8-40 network-16 SP_ 1407-to-ONU distribution

		SP_1407									
TP	ONU Total	ONU ID(ONU_70x)									
4	7	-	4	19	24	30	37	39	40		

Table 5.21 provides a list of additional information of the optimal network obtained from the simulation. This includes the objective function, information on the trenches, the total number of optical fibres required, and the total costs. The expenses and revenue are presented in time-periods, along with the total demands per time-period.

Figure 5.16 depicts the graph obtained from CPLEX upon gaining the optimal solution to the network. From the graph, it can be seen how the best node, indicated in red, and the best integer, indicated in green, converge towards the optimal integer solution, indicated in blue. This then graphically presenting the gain of the optimal layout for the 1-5-20 street-network.

Table 5.21: Case study 3: 1-8-40 network-16 additional simulated outputs

Additional Outputs	
Objective Function (R)	29856.66
Total no. of trenches	135
Total trench length (m)	9923.8
Total fibre length (m)	20953.2
Total costs (R)	9737400
Initial expense per TP (R)	[1794500 1850100 2563600 3529200]
Cash inflow per TP (R)	[4089700 6816200 10224000 13632000]
d_tot per TP	[12 20 30 40]

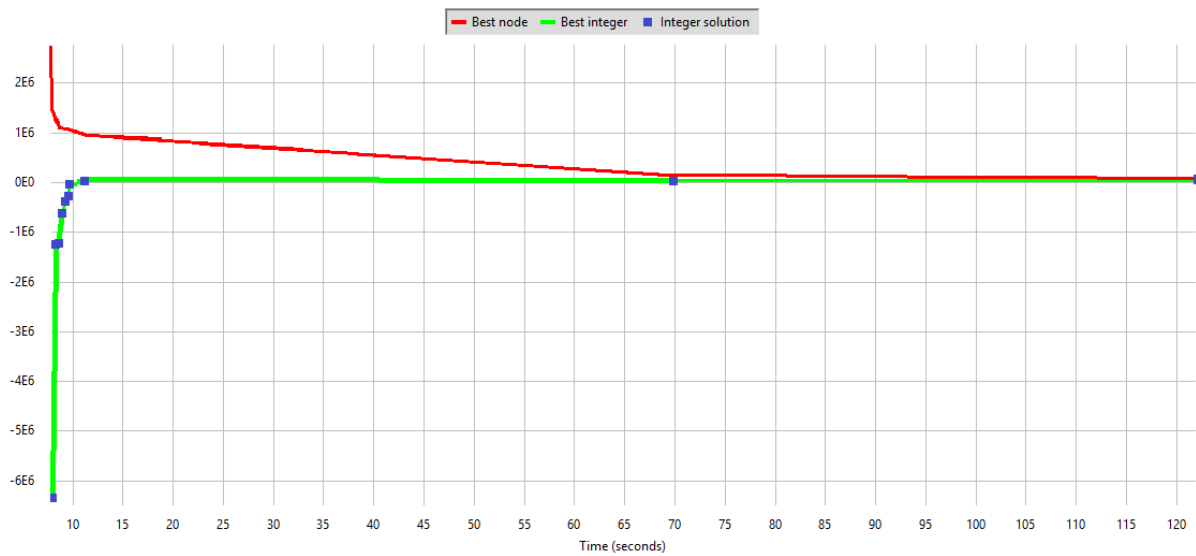


Figure 5.16: Case study 3: 1-8-40 network-16 CPLEX statistics graph

The last result for the 1-8-40 network-16 is tabulated in table 5.22. This table captures the execution times of the simulation. In order to gain an average execution time for this problem instance, the simulation was executed a total of five times, resulting in an average time of *1.18.88 min.*

Although not a micro network such as the 1-5-20 street-network, the 1-8-40 network-16 is of an average size and has a perfectly acceptable execution time of just over a minute. The results obtained are ideal for this research because vital information regarding network size and execution time is provided.

Table 5.22: Case study 3: 1-8-40 network-16 execution time

1-8-40 Network - 1:16	
Execution no.	Time (min)
1	1.19.20
2	1.19.15
3	1.18.71
4	1.18.56
5	1.18.78
Average	
Execution time	1.18.88

5.3.3 Results for the 1-8-40 network-32

The commercial software *CPLEX* [10], is used once again on the same *HP computer* to run the 1-8-40 network with a split ratio of 1:32, from here onwards referred to as the 1-8-40 network-32. The data and information provided remain unchanged from the 1-8-40 network-16 with the split ratio being the only change. The costs used, also remain the same, as well as the discount rate at 10% and the profit margin at 40%.

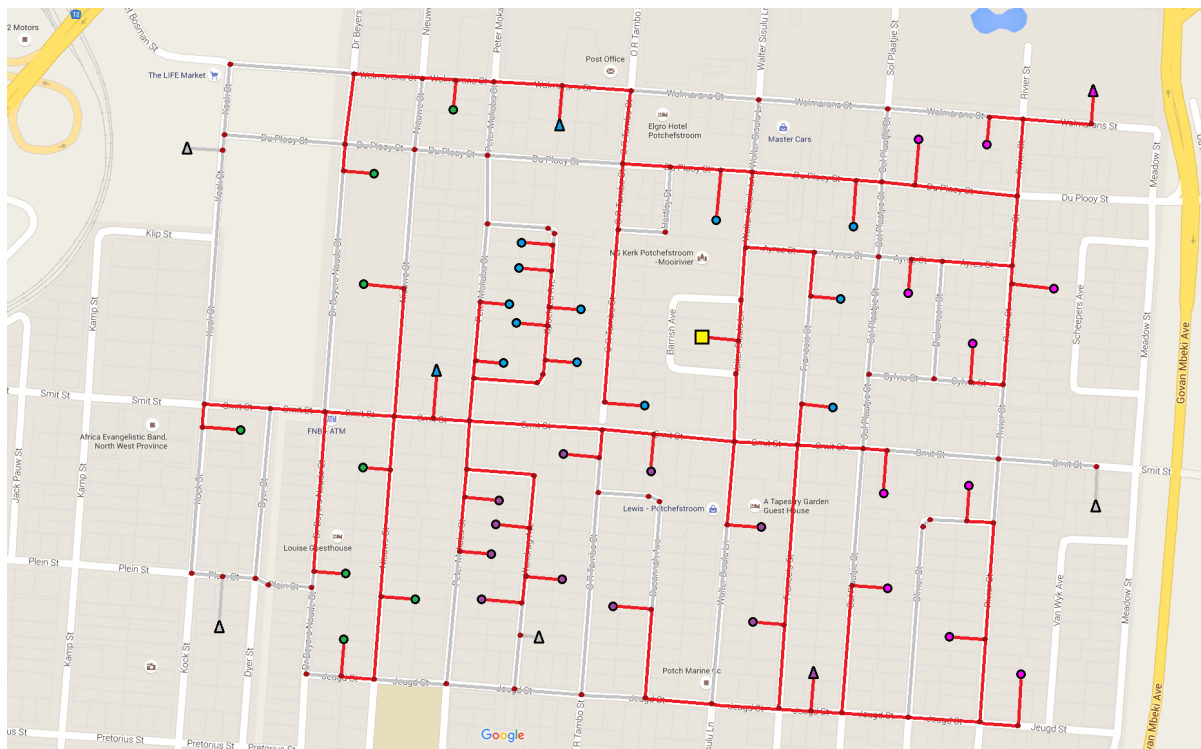


Figure 5.17: The optimal network layout for case study 3: 1-8-40 network-32 [33]

Figure 5.17 depicts the optimal network obtained from the simulation. Table 5.15 can again be referred to for a tabulated list of information regarding the elements, colours and time-periods involved in the network. The edges and SPs not considered as part of the optimal network are presented in grey and are thus disregarded when referring to the optimal result. Appendix C.3 can be referred to for further information regarding the IDs of the installed edges and the time periods in which they are installed and distribute accordingly. The IDs allocated to each edge are in the format *ED_150x* where *x* represents the edge number.

The optimal network involves the installation of 4 SPs, namely SP-2, SP-3, SP-4 and SP-7. The new colours allocated to each of these SPs, as opposed to the orange colour depicted in the potential network in figure 5.13, indicate that both SP-2 and SP-3 are installed in the first time-period. SP-7 is installed in the third time-period while SP-4 in the last time-period.

Table 5.23: Case study 3: 1-8-40 network-32 SP_ 1402-to-ONU distribution

SP_1402																									
TP ONU Total			ONU ID (<i>ONU_70x</i>)																						
1	8	-	13	14	15	16	18	21	28	35															
2	14	-	1	5	10	13	14	15	16	18	21	26	28	33	35	36									
3	21	-	1	2	5	7	10	13	14	15	16	18	21	23	25	26	28	29	31	33	34	35	36		
4	21	-	1	2	5	7	10	13	14	15	16	18	21	23	25	26	28	29	31	33	34	35	36		

Table 5.24: Case study 3: 1-8-40 network-32 SP_ 1403-to-ONU distribution

SP_1403										
TP	ONU Total	ONU ID (<i>ONU_70x</i>)								
1	4	-	3	6	11	20				
2	6	-	3	6	9	11	20	22		
3	6	-	3	6	9	11	20	22		
4	5	-	3	6	9	11	22			

Table 5.23 and table 5.24 depict the SP-to-ONU distribution per time-period for SP-2 and SP-3 respectively. In the first time-period SP-2 distributes to a total of 8 ONUs while SP-3 distributes to a total of 4. This adds up to a total of 12 demands in the first time-period.

In the second time-period SP-2 distributes to an additional 6 ONUs while SP-3 dis-

tributes to an additional 2. This then raises the demands to a total of 20 in the second time-period.

SP-7 is installed in the third time-period. This then aids SP-2 and SP-3 with the distribution part of the network. In the third time-period, SP-2 now distributes to an additional 7 ONUs while SP-3 remains at 6 ONUs. SP-7 distributes to a total of 3 ONUs that then bring the total demands up to 30 ONUs over the three time-periods. The SP-to-ONU distribution per time-period for SP-7 is tabulated in table 5.25.

Table 5.25: Case study 3: 1-8-40 network-32 SP_1407-to-ONU distribution

		SP_1407								
TP	ONU Total	ONU ID(ONU_70x)								
3	3	-	17	37	40					
4	8	-	4	17	19	24	30	37	39	40

In the last time period, time-period 4, SP-4 is installed. The final network distribution is as follows: SP-2 remains at 21 ONUs, SP-3 decreases from 6 to 5 ONUs, an additional 5 ONUs are fed by SP-7, and SP-4 distributes to 6 ONUs. This amounts up to a total of 40 demands, the total amount of ONUs that comprise the network. The SP-to-ONU distribution per time-period for SP-4 is tabulated in table 5.26.

Table 5.26: Case study 3: 1-8-40 network-32 SP_1404-to-ONU distribution

		SP_1404								
TP	ONU Total	ONU ID(ONU_70x)								
4	6	-	8	12	20	27	32	38		

Table 5.27 provides additional information on the optimal network obtained from the simulation. This includes the final objective function value, information regarding the trenches dug, the lengths of fibre required, and the total deployment cost. In the form of time-periods, information regarding the initial expenses, the cash inflow, and total demands per period are provided.

Figure 5.18 is captured from CPLEX after obtaining the optimal solution for the 1-8-40 network-32. As seen from the graph, the best node, indicated in red, and the best integer, indicated in green, converge towards the final integer solution, indicated by the last blue square. This thus graphically showing how optimality is achieved.

Table 5.27: Case study 3: 1-8-40 network-32 additional simulated outputs

Additional Outputs	
Objective Function (R)	23589.72
Total no. of trenches	130
Total trench length (m)	9559.09
Total fibre length (m)	20817.9
Total costs (R)	9615300
Initial expense per TP (R)	[1794500 1850100 2494600 3476000]
Cash inflow per TP (R)	[4038400 6730700 10096000 13461000]
d_tot per TP	[12 20 30 40]

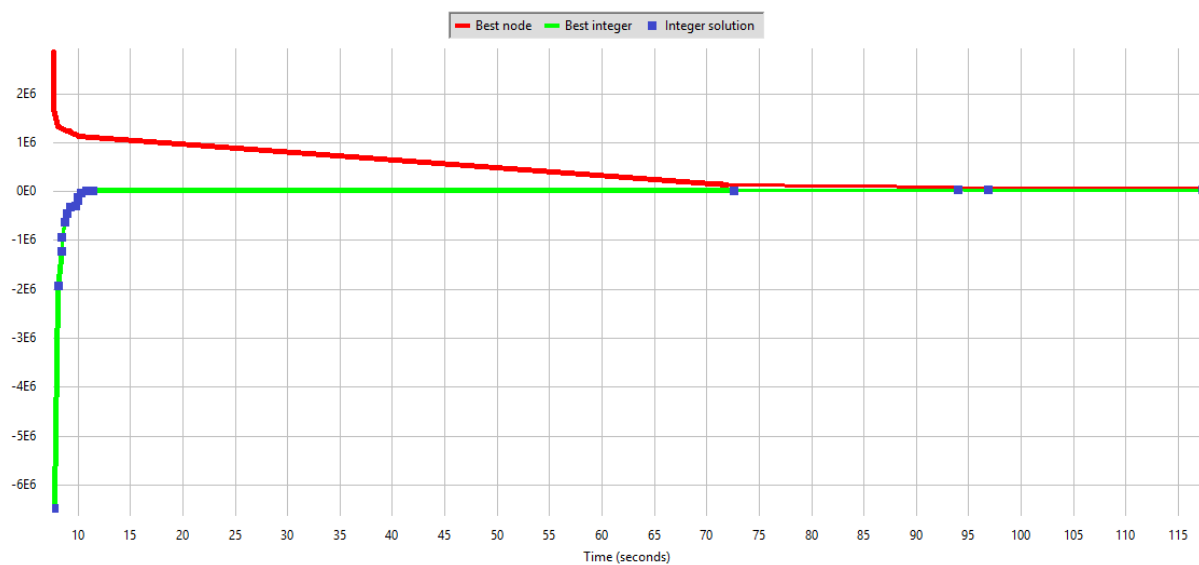


Figure 5.18: Case study 3: 1-8-40 network-32 CPLEX statistics graph

Table 5.28: Case study 3: 1-8-40 network-32 execution time

1-8-40 Network - 1:32	
Execution no.	Time (min)
1	1.19.00
2	1.19.88
3	1.19.29
4	1.19.45
5	1.18.78
Average	
Execution time	1.19.28

The last result for the 1-8-40 network-32 is tabulated in table 5.28. This table captures execution times of the simulation. In order to gain an average execution time for this

problem instance, the simulation was executed a total of five times, resulting in an average time of *1.19.28 min*. The change in the split ratio, from 1:16 to 1:32, does not have a great effect on the execution times of the simulation, differing by just a second during the last three executions.

5.4 Chapter conclusion

This chapter presents the simulations performed and the results obtained from three case studies, namely case study 1, case study 2 and case study 3. Case study 1 involved a problem instance comprising 1-CO, 5-SPs and 20-ONUs that realise over 3 time-periods. The chosen structure for this network was a basic tree-like structure whereby network expansion and distribution could be easily seen. This case study aided in verifying the proposed mathematical model as it was used to simulate and confirm the results gained. The small and simple design of this network made it easy to visually observe the output of the simulation and determine whether any errors or inconsistencies exist. The simulated results were compared to manually-calculated results to determine the model's accuracy. The manually-calculated results did not take long to conduct due to the simplicity and size of the network. Once satisfactory results were gained, modification on the model ceased. The simulated results outputted an optimal network layout by installing SP-1 and SP-3 to feed the network accordingly.

A decision was made to use the same data, i.e. 1-CO, 5-SPs and 20-ONUs, but with a different network design. After having confirmed that the mathematical model could indeed simulate and gain an optimal layout for a 1-5-20 tree-network, the next step was to test it on a network based on a real-world scenario. This network was investigated in case study 2 which was used to validate the model. The case study investigated the performance of the mathematical model on a network with a more complex, real-world design. Although the number of SPs and ONUs remained the same, the actual data grew in terms of edges, intermediate nodes and distances. In addition, the demands were realised in the form of suburbs, whereby each suburb is directly linked to a time-

period. The results gained thereof showed the design of an optimally gained network whereby SP-2 and SP-4 were selected. Both splitters were installed in the first time-period and distributed to the ONUs accordingly.

Case study 3, also used for model validation, involved analysing a problem instance comprising 1-CO, 8-SPs and 40-ONUs over 4 time-periods. Two scenarios involving the split ratios of the SPs were analysed, one where all 8 SPs had a split ratio of 1:16 and the other, a split ratio of 1:32. The potential network layout, initial costs, the discount and inflation rates, and the profit margin remained the same in order for a comparison to be done on the effect split ratios have on the final design. Although the final outputs both resulted in optimal networks that looked identical, there were significant differences. Firstly, the optimal 1-8-40 network-16 layout made use of 5 SPs while the optimal 1-8-40 network-32 layout made use of 4 SPs. With one less SP the number of trenches decreased, thus decreasing the amount of fibres required and, of course, the expenses required.

Together the three case studies verified and validated the proposed mathematical model, and in addition, case study three proved its scalability. The optimally gained network layouts proved the success of mathematically modelling the incremental FTTH planning problem, thus confirming the feasibility of model.

Chapter 6

Conclusions and Recommendations

The following chapter is the final chapter of this dissertation and concludes this research. The research problem, proposed model and case studies are summarised, followed by an overall conclusion based on the findings obtained. Lastly, the recommendations for possible future work is discussed after which the dissertation concludes with a few final words.

6.1 Summaries

6.1.1 Research problem

The many shortcomings of single-period planning have brought about the creation of incremental network planning. Previous studies have been conducted whereby the two methods were compared and the results obtained proved the utility of incremental network planning. It was found that the network modifications normally required for future demands were omitted, thus preventing the unnecessary use of resources.

Financial aspects, specifically network profitability, is one of the main concerns in network design, i.e. will the network produce a profit? In finance, a popular method used to determine the future value of a current project is the implementation of the NPV calculation. Therefore, before an investment is made, the calculation can be used to determine whether it is worth making the investment. Regarding network design, the NPV calculation can be used to determine the worth of a simulated network before actual deployment commences. This calculation can thus aid NSPs in determining whether to deploy a network or not.

Therefore, the goal of this research is to mathematically model the incremental FTTH planning problem, whereby the worth of the network is also determined, and to evaluate the feasibility of the model by means of a number of case studies.

6.1.2 Proposed model

Mathematical programming has become a popular technique used in network design, favoured due to its speed and accuracy when determining the optimal topology of a network. The mathematical model, an MILP formulation, proposed for this study was presented in Chapter 4. The objective function involved maximising an NPV calculation, thus determining the worth of deploying a network. The set of constraints captured essential features of the incremental FTTH planning problem. The model addressed the incremental factor in the form of time-periods. By taking future demands into consideration during the initial design of a network, unnecessary modifications can be avoided.

The overall aim of the model was to maximise NPV and minimise expenses while determining the optimal network topology for any given potential layout of an incrementally designed network.

6.1.3 Case studies

Case study 1 presented a problem instance for a 1-5-20 tree-network that realises over 3 time-periods. This problem instance was repeatedly used to aid in testing the proposed mathematical model as it made it easier to identify any errors that had to be rectified. This was made possible due to the chosen structure and size of the network. The 5-SPs and 20-ONUs made it a relatively small network, and the tree-like structure used, made it easier to understand the selection of the SPs and observe the growth of the simulated optimal incremental network. The outputs gained from the simulation were compared to and confirmed by manually-calculated results in order to ensure their accuracy. Once satisfactory results were gained, the model was considered accurate and ready to be tested on more real-world problem instance. Therefore, due to the results gained from the simulation, this case study was used to **verify** the proposed model.

The next step was to test whether the model could be used on a potential network based on real-world scenarios. Case study 2 was conducted for this purpose. It was decided to once again use the 1-5-20 network over 3 time-periods but with a different structure. Instead of using the tree-like structure, a real-world street layout was selected. This was to determine whether the model could be used on an actual network, with a more complex structure than before. The gain of an optimal network with satisfactory results confirmed that the model could be used on a small street-network. This case study was therefore used to validate the model on a real-world scale.

Case study 3 also involved a network with on a real-world topology. However, the main focus was placed on the size of the network. The network, presented in this case study, comprised 1-CO, 8-SPs and 20-ONUs that realised over 4 time-periods, a relatively larger network than what has already been used. A set of two simulations were conducted; both investigated the effect split ratios have on the final design. After running the two simulations, an optimal network was gained for each of them, confirming that the model can be used on large-scaled networks and that the split ratios chosen for the splitters greatly affect the final network topology, and thus expenses. This case

study was therefore used for **validation** purposes and to test the model's scalability.

6.2 Overall conclusion

The goal of this research was to mathematically model the incremental FTTH planning problem and to evaluate the model by means a number of case studies. This was simplified into the following three questions and are thus answered accordingly:

Is the aforementioned model feasible?

Based on the success of optimally designing three incremental FTTH networks, as obtained from the three case studies conducted and used for verification and validation purposes, it was found that the model is indeed feasible. The three networks, differing in size, layout and scenarios, were all solved within an acceptable time period, with satisfactory results, whereby optimal network topologies were gained, thus proving the feasibility of the model.

Can the integration of an NPV calculation indicate the network's worth?

The addition of an NPV calculation enables NSPs to determine whether the simulated optimal network topology, based on the discount rate used, will produce a profit. This can, therefore, aid NSPs in determining whether it will be worth deploying the network as obtained through simulation. By varying the discount rate chosen, it was possible to, through trial and error, determine the smallest rate required in order for the network to produce a profit. Thus proving that in network design, NPV can aid in determining the network's worth.

Can the model be used for large-scale incremental FTTH PONs?

Case study 3 was used to test the model's scalability. This was investigated by constructing a larger network comprising more SPs, ONUs, and edges over an additional time-period. The results obtained from this case study proved that the model was capable of optimally designing a larger network. Although the average execution time was found to be greater than what was obtained in the

other two case studies, the execution time itself was still feasible. Therefore, it can be concluded that the model can be used for large-scale FTTH PONs.

After successfully conducting the research and answering the three main questions set out at the beginning of this study, it can be concluded that the model is indeed feasible.

6.3 Recommendations

As with any research conducted, there is always room for improvement. One of the suggestions came as feedback during an article presentation at a conference. It was suggested to possibly look into disabling the "once installed, it remains installed" rule for the SPs. By doing so, the optimal network would consist of an SP being installed for a certain time-period and uninstalled when a different SP would benefit the network instead. What would then happen is, the split ratio for that SP would be big enough to handle just the demands in the time-periods in which it is installed, instead of having a split ratio that is too large with connections that will never be used.

In addition to that, further research can be done into the split ratios of each SP. This can be done by investigating the results that would be obtained if each SP had a different split ratio during the design process.

Another alternative would be to investigate how expenses get affected if SP positions are kept, but the SPs and their split ratios are changed. This would entail using SPs with a split ratio large enough to just meet the demands currently available, and then change the SP to an SP with a larger ratio to meet the added demands in the next time-period. This process would then be repeated for each time-period as the demands arise.

6.4 Final Words

Together the three case studies **verified** and **validated** the proposed mathematical model, and in addition, case study 3 proved its scalability. The optimally gained network layouts and outputs proved the success of mathematically modelling the incremental FTTH planning problem, thus confirming the feasibility of model.

A final note concluding this research: Although the proposed mathematical model has been proven feasible, this research can be expanded further to improve the results and accuracy of the model.

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Appendix A

Mini Glossary

1-5-20 Network

The *1-5-20 network* is an incremental FTTH network with an expansion that occurs over 3 time-periods. The network comprises 1 central office, 5 splitters and 20 optical network units. This is the general term used when describing both the *1-5-20 Tree-Network* and *1-5-20 Street-Network*.

1-5-20 Tree-network

The *1-5-20 tree-network* is an incremental FTTH network with an expansion that occurs over 3 time-periods. The network comprises 1 central office, 5 splitters and 20 optical network units. It has a tree-like structure whereby the central office is considered as the root from which the splitters and optical network units branch out accordingly. This basic network structure simplifies visual observation, thus making it easier to detect errors and observe network expansion during the design phase.

1-5-20 Street-network

The *1-5-20 street-network* is an incremental FTTH network with an expansion that occurs over 3 time-periods. The network comprises 1 central office, 5 splitters and 20 optical network units. The design of the network is based on the street layout of an area located on a section of a map obtained from *Google Maps*. Network expansion is not easily seen due to the complex layout of the network. However, although small, the network has a realistic layout realising over suburbs.

1-8-40 Network

The *1-8-40 network* is an incremental FTTH network with an expansion that occurs over 4 time-periods. The network comprises 1 central office, 8 splitters and 40 optical network units. This is the general term used when describing both the *1-8-40 Network-16* and *1-8-40 Network-32*.

1-8-40 Network-16

The *1-8-40 network-16* is an incremental FTTH network with an expansion that occurs over 4 time-periods. The network comprises 1 central office, 8 splitters and 40 optical network units. The design of the network is based on the street layout of an area located on a section of a map obtained from *Google Maps*. The optimal network is designed with each splitter having been allocated a split ratio of 1:16, i.e. 1 splitter can distribute to a *maximum* of 16 optical network units.

1-8-40 Network-32

The *1-8-40 network-32* is an incremental FTTH network with an expansion that occurs over 4 time-periods. The network comprises 1 central office, 8 splitters and 40 optical network units. The design of the network is based on the street layout of an area located on a section of a map obtained from *Google Maps*. The optimal network is designed with each splitter having been allocated a split ratio of 1:32, i.e. 1 splitter can distribute to a *maximum* of 32 optical network units.

Appendix B

Proposed Mathematical Model: Reference of Equations

Table B.1: Equation References

Equations	Description
Objective Function	
5.1	Where NPV is maximised on a time-period basis.
Node Balance	
5.2	Demand requirements for the distribution part of the network.
5.3	Distribution network: Total flow may not exceed maximum demands.
5.4	Demand requirements for the feeder part of the network.
5.5	Feeder network: Total flow may not exceed maximum demands.
Logical Decision Making Constraints	
5.6	Total ONUs to a splitter may not exceed the split ratio of that splitter.
5.7	Maximum down fibres from the central office.
5.8	Once trenching has been done, it remains available for fibre duct sharing.
5.9	Once a splitter has been installed, it remains installed.
5.10	Ensures that the central office remains installed from time-period 1.
5.11	Binary value for an edge is set to 0 at time-period 0.
5.12	Binary value for a splitter is set to 0 at time-period 0.
5.13	Binary value for the central office is set to 0 at time-period 0.
NPV Constraint Sets	
5.14	Maintenance cost calculation per time-period.
5.15	Calculation of initial costs per time-period.
5.16	Calculation of the total cost of the network.
5.17	Total demands per time-period.
5.18	Calculation of the revenue on a time-period basis.

Network Edges - Additional Information

C.1 Case study 2: 1-5-20 street-network

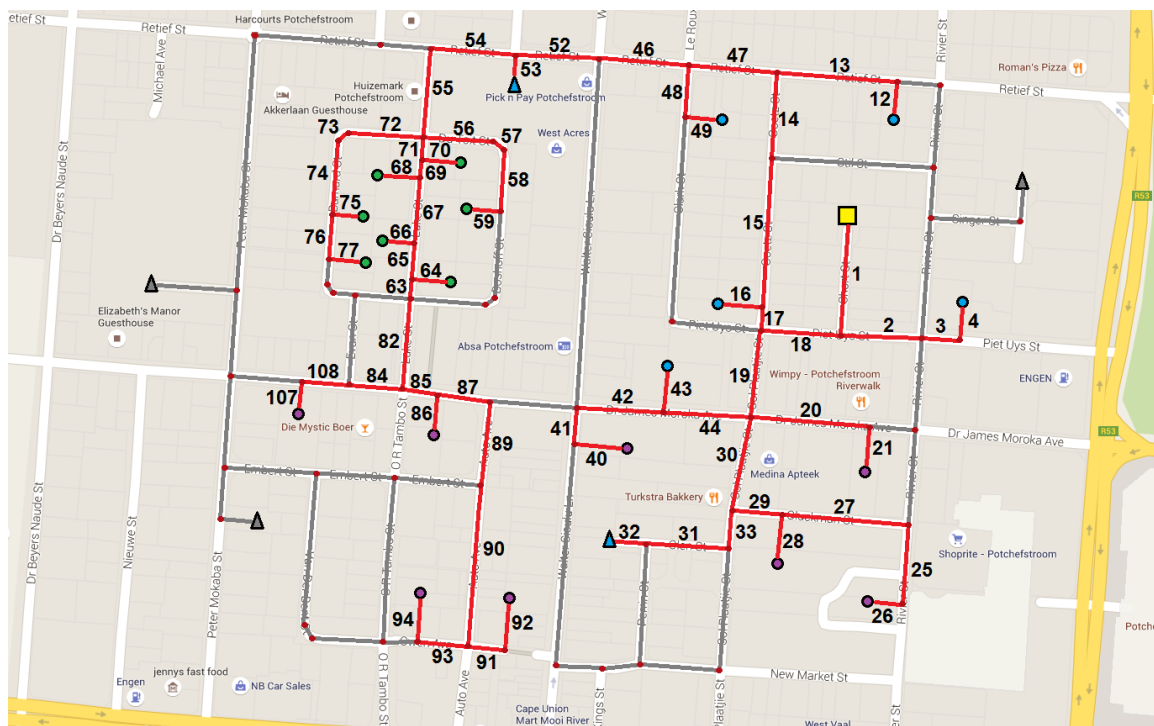


Figure C.1: Case study 2: 1-5-20 street-network optimal network's edge identification

Table C.1: Case study 2: 1-5-20 street-network edge instalment per time-period

ED_150x	Time Period		
	1	2	3
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
12	x	x	x
13	x	x	x
14	x	x	x
15	x	x	x
16	x	x	x
17	x	x	x
18	x	x	x
19	x	x	x
20			x
21			x
25			x
26			x
27			x
28			x
29			x
30	x	x	x
31	x	x	x
32	x	x	x
33	x	x	x
40			x
41			x
42			x
43	x	x	x
44	x	x	x
46	x	x	x
47	x	x	x
48	x	x	x
49	x	x	x
52	x	x	x
53	x	x	x

ED_150x	Time Period Cont'		
	1	2	3
54		x	x
55		x	x
56		x	x
57		x	x
58		x	x
59		x	x
63			x
64		x	x
65		x	x
66		x	x
67		x	x
68		x	x
69		x	x
70		x	x
71		x	x
72		x	x
73		x	x
74		x	x
75		x	x
76		x	x
77		x	x
82			x
84			x
85			x
86			x
87			x
89			x
90			x
91			x
92			x
93			x
94			x
107			x
108			x

C.2 Case study 3: 1-8-40 network-16

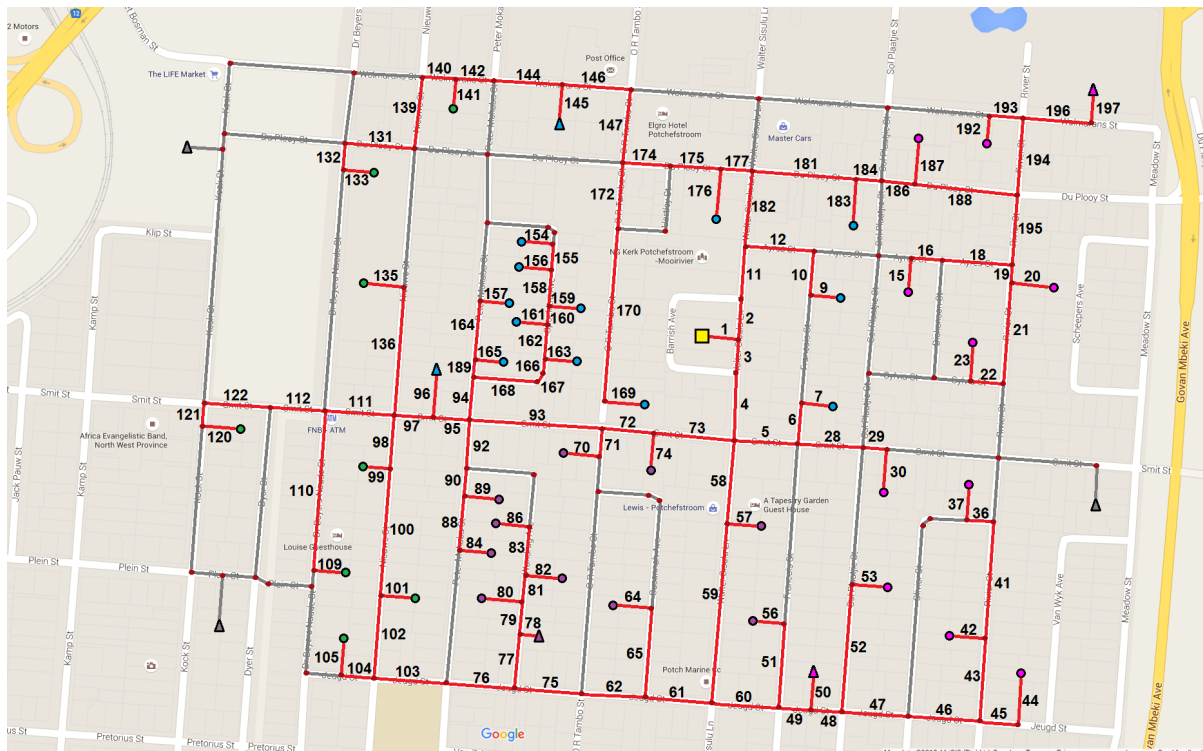


Figure C.2: Case study 3: 1-8-40 network-16 optimal network's edge identification

Table C.2: Case study 3: 1-8-40 network-16 edge instalment per time-period

ED_150x	Time Period			
	1	2	3	4
1	x	x	x	x
2	x	x	x	x
3	x	x	x	x
4	x	x	x	x
5	x	x	x	x
6	x	x	x	x
7	x	x	x	x
9	x	x	x	x
10	x	x	x	x
11	x	x	x	x
12	x	x	x	x
15				x
16				x
18				x
19				x
20				x
21				x
22				x
23				x
28				x
29				x
30				x
36				x
37				x
41				x
42				x
43				x
44				x
45				x
46				x
47				x
48				x
49				x
50				x
51			x	x
52				x
53				x
56			x	x
57			x	x
58			x	x
59			x	x
60			x	x
61			x	x
62			x	x
64			x	x
65			x	x
70			x	x
71			x	x
72	x	x	x	x
73	x	x	x	x
74			x	x
75			x	x
76			x	x
77			x	x
78			x	x
79			x	x
80			x	x
81			x	x
82			x	x
83			x	x
84			x	x
86			x	x
88			x	x
89			x	x
90			x	x
92			x	x
93	x	x	x	x
94	x	x	x	x

ED_150x	Time Period Cont'			
	1	2	3	4
95	x	x	x	x
96	x	x	x	x
97		x	x	x
98		x	x	x
99		x	x	x
100		x	x	x
101		x	x	x
102		x	x	x
103			x	x
104		x	x	x
105		x	x	x
109		x	x	x
110		x	x	x
111		x	x	x
112		x	x	x
120		x	x	x
121		x	x	x
122		x	x	x
131		x	x	x
132		x	x	x
133		x	x	x
135		x	x	x
136		x	x	x
139		x	x	x
140		x	x	x
141		x	x	x
142		x	x	x
144		x	x	x
145	x	x	x	x
146	x	x	x	x
147	x	x	x	x
154	x	x	x	x
155	x	x	x	x
156	x	x	x	x
157	x	x	x	x
158	x	x	x	x
159	x	x	x	x
160	x	x	x	x
161	x	x	x	x
162	x	x	x	x
163	x	x	x	x
164	x	x	x	x
165	x	x	x	x
166	x	x	x	x
167	x	x	x	x
168	x	x	x	x
169	x	x	x	x
170	x	x	x	x
172	x	x	x	x
174	x	x	x	x
175	x	x	x	x
176	x	x	x	x
177	x	x	x	x
181	x	x	x	x
182	x	x	x	x
183	x	x	x	x
184				x
186				x
187				x
188				x
189	x	x	x	x
192				x
193				x
194				x
195				x
196				x
197				x

C.3 Case study 3: 1-8-40 network-32

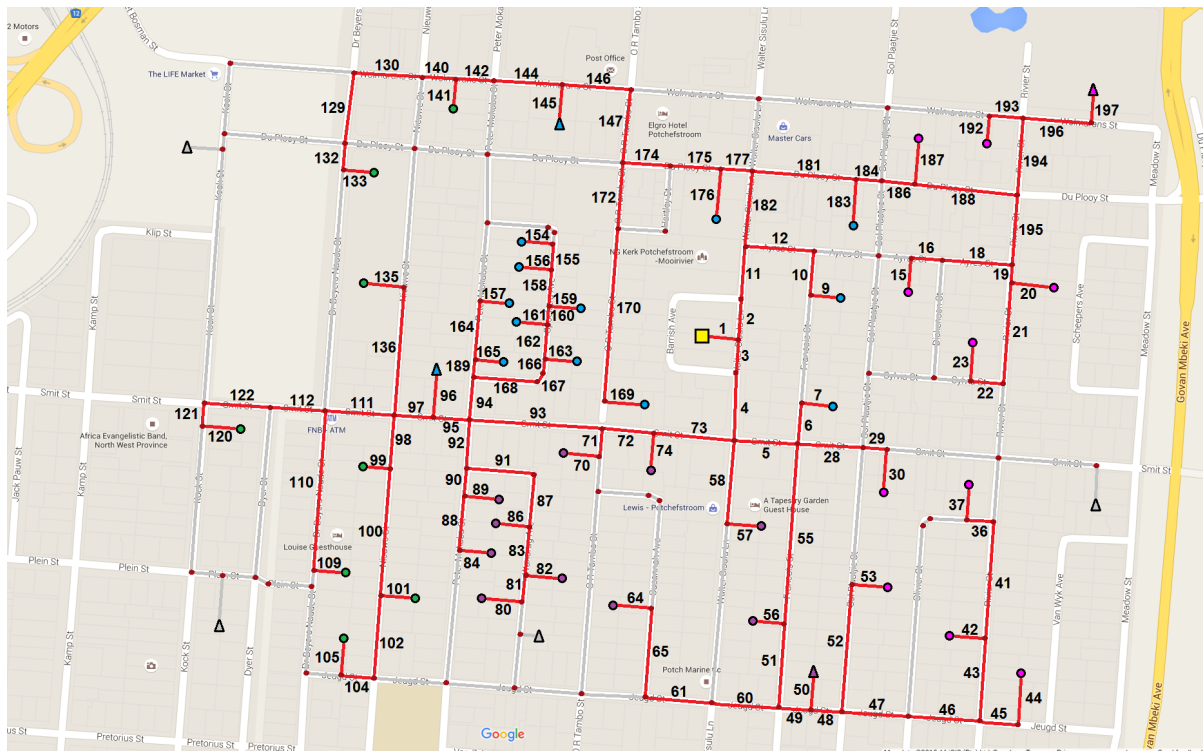


Figure C.3: Case study 3: 1-8-40 network-32 optimal network's edge identification

Table C.3: Case study 3: 1-8-40 network-32 edge instalment per time-period

ED_150x	Time Period			
	1	2	3	4
1	x	x	x	x
2	x	x	x	x
3	x	x	x	x
4	x	x	x	x
5	x	x	x	x
6	x	x	x	x
7	x	x	x	x
9	x	x	x	x
10	x	x	x	x
11	x	x	x	x
12	x	x	x	x
15				x
16				x
18				x
19				x
20				x
21				x
22				x
23				x
28				x
29				x
30				x
36				x
37				x
41				x
42				x
43				x
44				x
45				x
46				x
47				x
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83			x	x
84			x	x
86			x	x
87			x	x
88			x	x
89			x	x
90			x	x
91			x	x
92			x	x
93	x	x	x	x
94	x	x	x	x
95	x	x	x	x

ED_150x	Time Period Cont'			
	1	2	3	4
96	x	x	x	x
97		x	x	x
98		x	x	x
99		x	x	x
100		x	x	x
101		x	x	x
102		x	x	x
104		x	x	x
105		x	x	x
109		x	x	x
110		x	x	x
111		x	x	x
112		x	x	x
120		x	x	x
121		x	x	x
122		x	x	x
129		x	x	x
130		x	x	x
132		x	x	x
133		x	x	x
135		x	x	x
136		x	x	x
140		x	x	x
141		x	x	x
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155	x	x	x	x
156	x	x	x	x
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158	x	x	x	x
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161	x	x	x	x
162	x	x	x	x
163	x	x	x	x
164	x	x	x	x
165	x	x	x	x
166	x	x	x	x
167	x	x	x	x
168	x	x	x	x
169	x	x	x	x
170	x	x	x	x
172	x	x	x	x
174	x	x	x	x
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177	x	x	x	x
181	x	x	x	x
182	x	x	x	x
183	x	x	x	x
184				x
186				x
187				x
188				x
189	x	x	x	x
192				x
193				x
194				x
195				x
196				x
197				x

Appendix D

Conference Contributions

Work in Progress (WIP)

- J Laureles, M.J. Grobler, S.E. Terblanche, "*Incremental FTTH deployment planning*", in Southern African Telecommunications and Networks Access Conference (SATNAC), Port Elizabeth, South Africa, August/September 2014

Article

- J Laureles, M.J. Grobler, S.E. Terblanche, "*Incremental FTTH deployment planning with budget constraints*", in Southern African Telecommunications and Networks Access Conference (SATNAC), Hermanus, South Africa, September 2015

Incremental FTTH Deployment Planning with Budget Constraints

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Abstract—The use of optical fibres is favoured due to its desirable physical properties and especially for its high bandwidth transmission capability. The challenges faced in the design of fibre based networks, specifically Fibre-to-the-home (FTTH) deployment, prompts the application of advanced mathematical models and computing technology. In this paper, an Integer Linear Programming (ILP) formulation is proposed to solve the incremental FTTH deployment problem, by considering a planning budget constraint. Computational results for a synthetic problem instance are presented that illustrate the advantage of using an incremental planning approach over a once-off model, when considering evolving bandwidth demand over a short to medium planning horizon.

Index Terms—FTTH, Incremental Network Deployment, ILP, Optical Fibres, Optimisation, PON

I. INTRODUCTION

HIGHER bandwidth is a growing demand in today's telecommunications industry. With a constant increase in subscribers making use of services such as video-on-demand (VoD), video conferencing, high definition television (HDTV) and unlimited content downloading, Network Service Provider (NSP)s are under pressure to meet these increased bandwidth demands. According to the authors in [1] (with permission from *TeleGeography*) there has been an exponential increase in the worldwide international bandwidth used, between 2009 and 2013 alone. This demand was measured at a mere 30 Tera-bits per second (Tbps) in 2009, whereas, just 4 years later in 2013, the demand was measured at a whopping 138 Tbps.

In order for NSPs to meet these demands, but still keep the upgrade and development costs at a minimum, a fibre based solution known as Fibre-to-the-x (FTTx) was developed. Due to forming the ultimate broadband architecture for fixed access networks, by making use of fibre connections all the way to subscriber's homes, FTTH is the most commonly found and preferred of all the FTTx structures. In 2005 there were approximately 11 170 FTTx subscribers, which grew to 108 262 by 2012 [2]. This growth of fibre based networks in the telecommunications industry forms the source for the gradual elimination of the older, slower copper based networks.

From a NSP's point of view, an optimal network topology results from when cost and throughput form the basis of the design criteria. The unreliability and impracticality of the conventional point-to-point method made a pathway for the Passive Optical Network (PON) solutions to be considered as good candidates [3]. The two most commonly known

PON solutions being Ethernet based PON (EPON) and Gigabit-capable PON (GPON).

Related work using mathematical programming for PON designs include the work by [4], [5] and [6]. Although these studies include PON network design, none of them specifically address the incremental network planning problem. It is anticipated that a greater benefit can be realised with an incremental network planning approach due to the evolving nature of bandwidth requirements over time and the fact that every planning project is subjected to budget limitations.

The remainder of this paper is structured as follows: Section II discusses the differences between single- and multi-period network planning; real life scenarios where multiple periods have been considered and what benefits have been gained by planning ahead. The following section, Section III, brings forth all the considerations required when designing an incremental network. This includes, the type of site to be served, what materials are preferable over others and the various techniques to be considered for achieving an optimal result. Section IV describes the research problem including the mathematical model used. Section V describes the network scenario, how the empirical test was carried out and what the results were. Finally, the article is concluded in Section VI with a small discussion on what future work will be done.

II. INCREMENTAL NETWORK PLANNING

Incremental network planning, also referred to as multi-period planning, has been studied extensively within the context of transmission networks, see *e.g.* [7], [8] and [9]. The application of incremental network design methodologies for access networks, specifically PON, are still very limited.

A comparison between the single-period- and the multi-period- planning problem can be found in [10]. The aim in their study was to assess the cost difference, both for installation and maintenance, when comparing long term planning v.s an ad hoc scenario, in order to deploy connections between cities. Exactly 30 different problems were generated and then solved using 6 different methods, 3 algorithmic approaches and 3 network design cost models, which resulted in a total of 180 experiments. The integrated multi-period approach was found to save on expenses up to an average of 4.4%, calculated by doing a comparison on a global level. This result was gained based purely on better scheduling of investment over time.

A similar, real life scenario can be found in [11], where an entire city's network, Santa Monica, was converted to run on fibre optics. To date, the result is the most successful *dig once* policy implemented in the United States. The plan was initiated with a \$500,000.00 investment, spent on connecting municipal facilities, the School district, and the Santa Monica College with the city's owned fibre. By combining fibre and conduit installation with other capital projects and/or performing joint trenching together with other entities, the cost of laying fibre was reduced by up to 90%. During the first year, after the initial migration, the City was able to save roughly \$400,000.00, which increased to \$700,000.00 per year thereafter.

III. PRACTICAL CONSIDERATIONS

In practice, there are many factors that influence the overall network layout and choice of equipment. Apart from taking capital and operational expenditure into account, there are several practical factors that need to be considered. These can be divided up into 3 categories, namely Served Sites and Positions, Materials and Techniques. The subsequent sections give a summary of the 3 categories as described by [12].

A. Served Sites and Positions

Obtaining details about the site to be served, forms a vital step in network design. First and foremost, NSPs need to know the nature of the served site *i.e.* whether it is Greenfield, Brownfield or Overbuilt. Depending on the nature of the site, accurate information on the geography, population distribution and location of existing infrastructure, must be acquired. If the served site is a Greenfield deployment, the best course of action would be to put the access technology in place when building the infrastructure. If multiple zones are to merge, *i.e.* become a multi-area zone, of a Brownfield or Overbuilt type, a single Head Central Office (HCO) must be chosen from all the legacy Central Office (CO)s, to serve the area. The position of the HCO is based on the population distribution and infrastructure facility.

A typical FTTH network structure can be divided into the feeder network, the distribution network and the dispersion network. The feeder network consists of the HCO, Outside Plant (OSP) and Splitter Cabinet (SC)s, while the distribution network consists of the Optical Terminal Box (OTB)s. The dispersion network exists between the OTBs and Optical Network Unit (ONU)s, located at the user's premises. Under each network, NSPs have to decide what components will be used and what the optimal position of placement will be.

B. Materials

By acquiring information on the geography, infrastructure and population distribution, NSPs can make informed decisions on what components to use and where. For instance, at an HCO, the indoor cables should be different to the ones installed at an OSP. Furthermore, a decision should be made on the various splitter types, based on the split ratios required. The subsection below discusses this in further detail.

A typical Optical Distribution Frame (ODF), situated at the HCO, consists of 256 connections and enables fibre control, thus making it possible for the OSP to connect

to any of the Optical Line Terminal (OLT) ports. In order for distances of several kilometres to be reached, high-capacity optical cables, consisting of a maximum of 512 fibres, connect the ODF to a primary Fibre Concentration Point (FCP). With Joint Cabinet (JC)s located in the FCP, lower capacity cables can be used to reach out and connect to other FCPs.

OTBs located within a building, should possess the correct capacity required to serve all of its apartments. In addition, due to its low bending radius ability, KT fireproof, mono-tube type cables are ideal for the dispersion part of the network.

C. Techniques

In addition to selecting the most appropriate FTTx structure, that is either Fibre-to-the-curb (FTTC), Fibre-to-the-node (FTTN), Fibre-to-the-business (FTTB) or FTTH, the next vital decision to make is whether a point to point (P2P) or a point to multiple point (P2MP) configuration will be used. A P2P solution is where a direct connection between the CO and the subscriber exist, and with a P2MP, multiple subscribers share a single fibre. Using dedicated fibres will of course provide the best bandwidth option, but at a prohibitive price. Therefore, most networks are based on a P2MP configuration.

Another consideration is whether PON or an Active Optical Network (AON) implementation will be done. The main advantage that PONs have over AONs is that it avoids the use of repeaters and any active electrical components. Not only are AONs costly to implement but they require considerably more maintenance.

There are currently two PON based solutions that exist, namely EPON and GPON. EPON is commonly assumed to be an economical alternative whereas GPON is more versatile. Deciding between the two solutions requires evaluating certain criteria such as the compatibility with other transport systems, transport link utilisation, costs, the network segmentation needed when the number of connected users increases, and granularity of the offered transport service [3].

The link capacity in a GPON is often dimensioned based on the upstream link utilisation. It is however important to note, that with residential subscribers, generated traffic mainly occurs in the downstream direction, flowing from the OLT to the ONU. Dimensioning the downstream link capacity of a GPON needs to be done with care in order to accommodate VoD, HDTV, and the other services available. An accurate estimation on the capacity required by subscribers is an important step in selecting the correct optical split ratio.

Neighbourhoods can be made up from more than one *type* of set of subscribers, *i.e.* not only residence or only businesses but maybe a combination of both. Services deployed in residential areas are done so in *unprotected mode*, leading to minimum expenses, whereas, in areas consisting of businesses, *protected mode* is required and can thus lead to high expenses.

The bandwidth allocation per subscriber, sharing a single PON link, is solely determined on the split ratio. High costs are expected when a small split ratio is selected. However, a higher split ratio has the tendency for reducing the bandwidth allocation per subscriber and as a result,

subscribers may experience a quality decrease in the services received. According to [13], there are 8 Rules that should be followed when choosing the optimal split ratio.

- **Rule 1:** A survey should be conducted with the aim at obtaining information such as the nature of the population to be served. This will then aid in allocating and configuring the resources needed.
- **Rule 2:** Multiple dimensioning parameters should be considered in order to obtain an optimal dimensioning GPON capacity. These parameters include, the number of subscribers and the required bandwidth for the generated traffic.
- **Rule 3:** Valuable inputs required, in order to obtain an optimal GPON capacity, include understanding subscribers' profiles and determining the busiest hour(s) that would be experienced on a daily basis.
- **Rule 4:** A conservative approach for dimensioning GPON link capacity, is one where the sum of service peak rates is calculated.
- **Rule 5:** The network performance should be monitored carefully by constantly measuring the flow of traffic and reporting the quality of the services.
- **Rule 6:** By knowing the GPON link utilisation and service penetration values, network providers can better select a split ratio that can support the required services.
- **Rule 7:** A link utilisation of around 75% should be guaranteed by the split ratio chosen.
- **Rule 8:** A specific split ratio might be required at building locations and network applications.

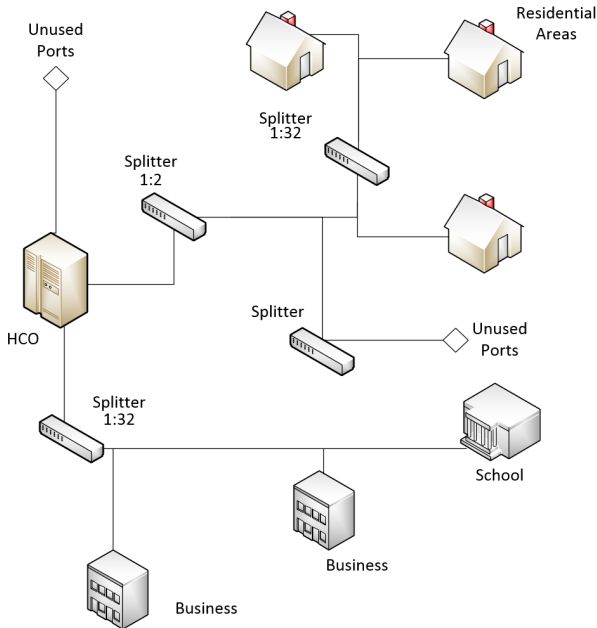


Figure 1: A tree splitting network

A popular network design approach is to make use of splitting stages, also known as tree splitting, as seen in Figure 1. A single splitting stage, for example, is when the CO is connected to a splitter and that splitter connects directly to an ONU located at each individual client. A two stage splitter on the other hand, would also have the CO connected to a splitter but then instead, have that splitter connected to other multiple splitters, which then connects to the individual client terminals, and so grows the splitting

stages.

A design principle that should be considered, especially when cost minimisation is a desired design criterion, is *fibre duct sharing* or *path sharing*. This technique deems useful when common routes are found among fibres, *i.e.* when fibres share a part of or the entire route. Trenching is considered to be one of the largest cost contributors when deploying a PON, however, this expense can be kept at a minimum through the implementation of fibre duct sharing. The basic principle, as depicted in Figure 2 is that, instead of having a dedicated trench for each fibre, fibres following the same route can share a trench for as long as possible, thus saving on having to dig up new trenches along the same routes.

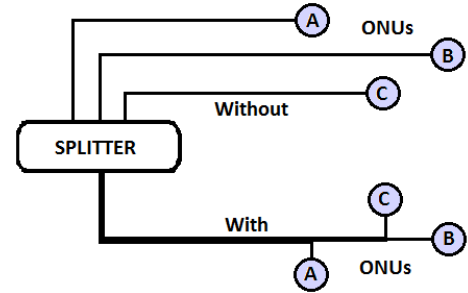


Figure 2: The basic concept of fibre duct sharing

Positioning the HCO is highly dependent on the housing density of the served zone. With a uniform density, the HCO would most likely be placed at the centre, however with more variability, the HCO would typically be placed closer to where the most demands are located.

The feeder network connects the HCO to the distribution network. The design of this network must include a view on long term evolution, as it is very costly to install new cables at the dawn of each new network. It may therefore be more practical to have fibres unused and stagnant than to spend, unnecessarily, as the need arises.

IV. PROBLEM FORMULATION

The proposed mathematical formulation captures the most essential features of the incremental FTTH planning problem. It entails the minimisation of deployment costs, subject to constraints that model the technological requirements of an FTTH network. The most important aspect considered is evolving bandwidth requirements over time. In order to formulate the problem as an ILP problem, several sets and parameter definitions are required. Listed in Table I are the different index sets used, while Table II summarises all the input parameters.

TABLE I: Index sets

Notation	Description
V	Set of vertices corresponding to either ONU's or splitters
E	Set of edges corresponding to potential duct installations
T	Set of time periods
S	Set of splitters
ONU	Set of ONUs

It should be noted that the demand parameter d_{it} is defined in such a way, that when demand becomes available for ONU i in a time period t , it remains available for the

remainder of the planning horizon. For example, consider the time periods $T = \{1, 2, 3\}$. If demand for ONU i becomes available in time period $t = 2$, the corresponding demand vector is $\{d_{i1}, d_{i2}, d_{i3}\} = \{0, 1, 1\}$.

TABLE II: Input parameters

Notation	Description
d_{it}	Bandwidth demand for ONU i in time period $t \in T$
Δ	Total network demand
K	The maximum number of down-fibres per splitter
l_e	Length (in meters) of edge $e \in E$
b_t	Budget constraint for time period $t \in T$
C_{co}	Cost for the use and maintenance of the CO
C_{sp}	Cost per splitter installation
C_{onu}	Cost per ONU installation
C_{fpm}	Cost of fibre per meter
C_{tpm}	Cost per trench per meter

In order to express logical decision making related to the topological design of the network and the timing of infrastructure deployment, the variables, listed in Table III, are required.

TABLE III: Decision variables

Variable	Description
$Z_{et} \in \{0, 1\}$	Trenching of edge $e \in E$ in time period $t \in T$.
$x_{ijt}^D \in \mathbb{Z}_+$	Fibre installation for the distribution network from node i to j in time period $t \in T$
$x_{ijt}^F \in \mathbb{Z}_+$	Fibre installation for the feeder network from node i to j in time period $t \in T$
$\psi_{it} \in \{0, 1\}$	Splitter installation at node $i \in V$ in time period $t \in T$
$F_t \in \mathbb{Z}_+$	The number of down-fibres at the CO in time period $t \in T$
$f_{it} \in \mathbb{Z}_+$	The number of up-fibres for a splitter at node $i \in V$ in time period $t \in T$
$D_{it} \in \mathbb{Z}_+$	The number of down-fibres for a splitter at node $i \in V$ in time period $t \in T$

The mathematical formulation of the incremental FTTH network design problem is the following:

Minimise

$$\sum_{t \in T} \sum_{e \in E} (Z_{et} - Z_{e,t-1}) * [(l_e * C_{fpm}) + (l_e * C_{tpm})] + \sum_{t \in T} \sum_{i \in S} (\psi_{it} - \psi_{i,t-1}) * C_{sp} + (\Delta * C_{onu}) + C_{co} \quad (1)$$

Subject to

$$\sum_{j \in \sigma^+(i)} x_{ijt}^D - \sum_{j \in \sigma^-(i)} x_{jit}^D = \begin{cases} -d_{it}, & : i \in ONU, \\ D_{it}, & : i \in S, \quad \forall i \in V, \forall t \in T \\ 0, & : otherwise. \end{cases} \quad (2)$$

$$x_{ijt}^D + x_{jit}^D \leq \Delta Z_{et}, \quad \forall e \in E, \forall t \in T \quad (3)$$

$$\sum_{j \in \sigma^+(i)} x_{ijt}^F - \sum_{j \in \sigma^-(i)} x_{jit}^F = \begin{cases} -\psi_{it}, & : i \in S, \\ F_t, & : i \in CO, \quad \forall i \in V, \forall t \in T \\ 0, & : otherwise. \end{cases} \quad (4)$$

$$x_{ijt}^F + x_{jit}^F \leq \Delta Z_{et}, \quad \forall e \in E, \forall t \in T \quad (5)$$

$$\sum_{e \in E} (Z_{et} - Z_{e,t-1}) * [(l_e * C_{fpm}) + (l_e * C_{tpm})] + \sum_{i \in S} (\psi_{it} - \psi_{i,t-1}) * C_{sp} - b_t = 0, \quad \forall t \in T \quad (6)$$

$$D_{it} \leq K \psi_{it}, \quad \forall i \in S, \forall t \in T \quad (7)$$

$$F_t = \sum_{i \in S} \psi_{it}, \quad \forall t \in T \quad (8)$$

$$Z_{et} \geq Z_{e,t-1}, \quad \forall t \in T, t > 0 \quad (9)$$

The objective function (1), minimises the total cost over all time periods, specifically the costs for trenching, splitter installation, fibre installation, the cost for each ONU in the network and, finally, the CO's start up and/or maintenance costs.

The demand requirements for the distribution part of the network are captured in (2) and constraint set (3) allows fibre to be installed at an edge, provided that trenching was done for that specific edge. Similarly, demand requirements for the feeder part of the network are formulated by constraints (4) and, once again, constraint set (5) allows fibre to be installed at an edge, only if trenching was done for that specific edge.

The implementation of the budget constraints are reflected under constraint set (6). These constraints affect the final network design according to the available budget for any chosen time period.

The remaining constraints are required to model logical decision making with regards to the installation of splitters and fibres. Constraint set (7) ensures that the ratio of ONUs to a splitter will not be exceeded, where K represents the maximum down fibres per splitter. The total number of splitters used, also indicates the number of down fibres from the CO, which is depicted in (8).

The last constraint set (9) ensures that once trenching has been done for an edge, it remains available for duct sharing for the remainder of the planning horizon.

V. EMPIRICAL RESULTS

In order to illustrate the benefits of employing the proposed model to solve the incremental FTTH planning problem, a fictitious data set comprising 20 ONUs and 5 splitters was created. The resulting model for this problem instance was implemented by using the commercial software *CPLEX Optimisation Studio*, a product by IBM.

Figure 3 is a depiction of the potential topology used as input. The evolving nature of the bandwidth demand is captured by the availability of ONU's during each of the time periods overlaying the potential topology.

For comparison purposes, the problem instance was solved initially for only a single time period. That is, all demands were assumed to be immediately available with no notion of an incremental plan. The total deployment cost for this problem instance amounted to a total of $R\ 437\ 606$. In practice such an amount may exceed an annual allowable

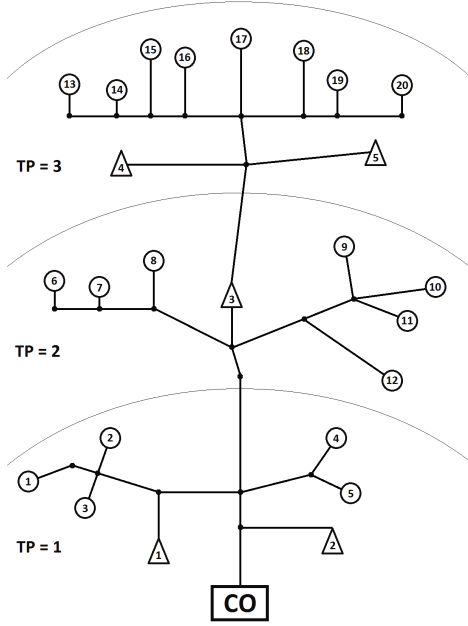


Figure 3: The potential network

budget, and therefore, a deployment plan, over several periods, with a cost budget per period becomes a necessity.

The problem instance, labelled *Budget Plan A*, was subsequently solved by considering a fictitious cost budget of $R\ 150\ 000$ for the 1st time period, the 2nd and 3rd time periods remained without any budget constraints. Figure 4 depicts the final topology for *Budget Plan A* and it clearly shows how the deployment plan realises over the 3 period planning horizon. Table IV tabulates the colour-coded key indicating when demands become available, in either the feeder or distribution network. It also shows how the budget constraint influences the final network design i.e. in which Time Period (TP)s should deployment commence in order to avoid exceeding the allowable budget.

TABLE IV: Budgeted Multi-Period Network Key

Key	Network Type	
	Feeder	Dist
Pink	TP 1	-
Blue	-	TP 1
Red	-	TP 2
Green	-	TP 3

Three separate sets of demands become available during the 3-time-period network topology, and from what can be seen in Figure 4, the entire can be designed within the first 2 time periods, even though the last set of demands only become available in the 3rd time period.

Figure 5 depicts the final topology obtained when an additional fictitious cost budget of $R\ 200\ 000$ is added for the 2nd time period. Inflation was considered when deciding on the value of the second budget, hence why it is relatively more than the first one. The 3rd time period remains without any budget constraint. This problem instance was labelled *Budget Plan B*. Table IV can again be referred to for a colour-coded indication for this plan's final topology.

The budget constraints for the 1st and 2nd time period create a limit as to what can be done, on a financial basis, during the first two time periods. It is clear that the entire

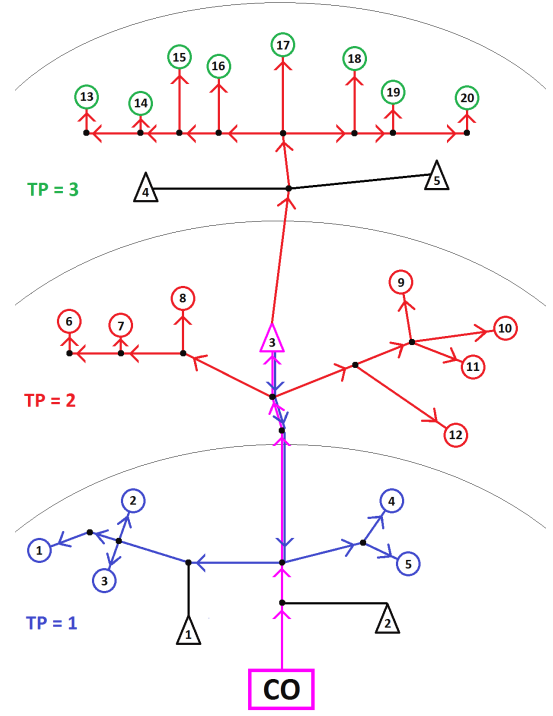


Figure 4: Optimal multi-period Budget Plan A

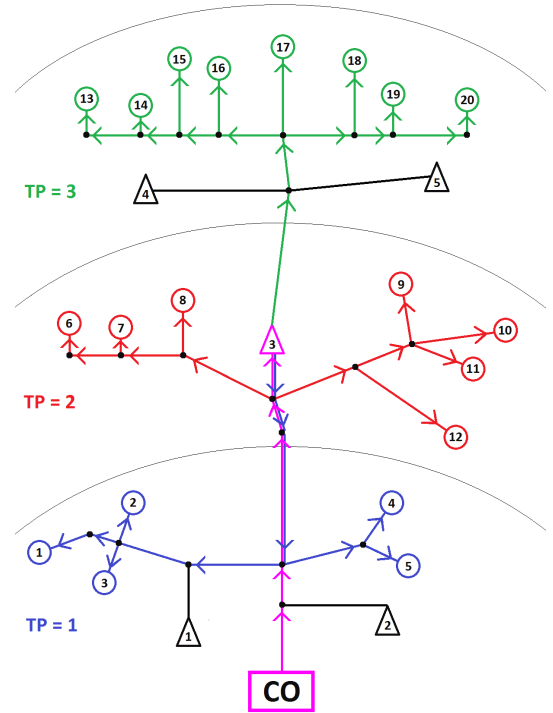


Figure 5: Optimal multi-period Budget Plan B

network can only be designed within the 3 time periods, not any time sooner, as a result of these constraints.

VI. CONCLUSION AND FUTURE WORK

In this paper, an ILP formulation was proposed to solve the incremental FTTH deployment problem, by considering a planning budget constraint. A short to medium planning horizon, over 3 time periods, was fictitiously formulated consisting of 20 ONUs and 5 splitters. A total of three different deployment plans were obtained, namely a single-period plan, where all demands were assumed to be immediately

available; a multi-period plan with a budget constraint for the first time period; and another multi-period plan with budget constraints for the first and second time periods.

The single-period plan had a total deployment cost of *R 437 606*, an amount possibly exceeding the budget originally set out by the NSP. Thus, the need for a cost budget per period was realised. Two multi-period scenarios were formulated. The first, by setting a cost budget of *R 150 000* for the first time period and leaving the other two time periods without any budget constraints. The second, by adding a cost budget of *R 200 000* for the second time period and leaving the last time period without a budget constraint.

It is evident that, even though certain demands only became available at later stages of the network, it was still possible and financially preferable, to build and extend the network as the budget allowed. This approach made it possible to obtain an optimal deployment plan for an FTTH PON, that is mapped over several time periods, within a set budget. It can therefore be concluded, from a NSP's point of view, that implementing an incremental planning approach over a once-off model, perfectly ties financial planning together with network planning. An optimal network, satisfying all demands, can be obtained and designed well within the allocated budget, making it feasible for NSPs to save on resources, expenses and time.

Future work could involve the extension of the objective function to cater for *net present value*, thus enabling the use of a revenue model and taking future cash flows into account.

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